The influence of the Gauss-Bonnet interaction on the properties of boson stars and hairy black holes

Betti Hartmann

Instituto de Física de São Carlos (IFSC)
Universidade de São Paulo (USP), Brazil

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Outline

1. The model and (some) analytical solutions
2. Numerical solutions
   - Asymptotically flat space-time
     - Non-rotating boson stars
   - Asymptotically Anti-de Sitter (aAdS) space-time
     - Black holes with hyperbolic horizon ($k = -1$)
     - Black holes with planar horizon ($k = 0$)
     - Black holes with spherical horizon ($k = 1$)
     - Boson stars in AdS
3. Conclusions & Outlook

The model

The model in $d = 4 + 1$:
Einstein-Gauss-Bonnet + scalar field Ψ + U(1) gauge field A_M

$$S = \int d^5x\sqrt{-g}\left[\frac{R-2\Lambda}{16\pi G_5} - (D_M\Psi)^* D^M\Psi - \frac{1}{4}F_{MN}F^{MN} - V(|\Psi|) + \frac{\alpha}{64\pi G_5}\left(R^{MNKL}R_{MNKL} - 4R^{MN}R_{MN} + R^2\right)\right]$$

with

$$D_M = \partial_M - ieA_M \ , \ F_{MN} = \partial_M A_N - \partial_N A_M$$

$\Lambda = -6/L^2$: cosmological constant
G_5: Newton's constant α: Gauss–Bonnet coupling
e: gauge coupling $V(|\Psi|)$: scalar field potential

Ansatz for non-rotating solutions

- Metric

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}d\Sigma^2_{k,3}$$

with

$$d\Sigma^2_{k,3} = \begin{cases} d\Xi^2_3 & \text{for } k = -1 \text{ hyperbolic} \\ dx^2 + dy^2 + dz^2 & \text{for } k = 0 \text{ flat} \\ d\Omega^2_3 & \text{for } k = 1 \text{ spherical} \end{cases}$$

- Matter fields

$$A_M dx^M = \phi(r)dt \quad , \quad \Psi = \exp(i\omega t)\psi(r)$$

Solutions without scalar fields

(Boulware & Deser, 1982; Cai, 2003)

$$\psi(r) \equiv 0 \ , \ \phi(r) = \frac{Q}{r_h^2} - \frac{Q}{r^2}$$

$$f(r) = k + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 - \frac{4\alpha}{L^2} + \frac{4\alpha M}{r^4} - \frac{\alpha \tilde{Q}^2}{r^6}}\right) \ , \ a(r) \equiv 1$$

M: mass parameter , Q: charge (density) , $\tilde{Q} \propto Q$

Uncharged solutions without backreaction

$G_5 = 0$, $Q = 0$, $V(|\Psi|) = m^2|\Psi|^2$, $k = 1$

$$\partial_x \left(x^3 f \partial_x \psi\right) + L^2 x^3 \psi \left(\frac{\omega^2}{f} - m^2\right) = 0 \ , \ \ f(x) = 1 + x^2 \ , \ \ x = \frac{r}{L}$$

has general solution

$$\Psi = \sum_{k=0}^{\infty} \exp(i\omega_k t)\psi_k(x)$$

with oscillon basis

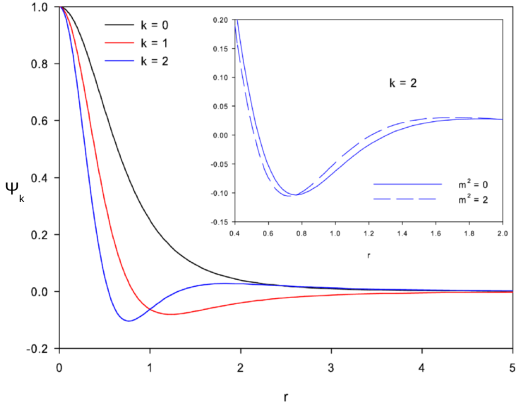
$$\psi_k(x) = c_k(1+x^2)^{-2-\kappa} \ {}_2F_1\left(\frac{4-L\omega}{2} + \kappa, \frac{4+L\omega}{2} + \kappa; 3+2\kappa, \frac{1}{1+x^2}\right)$$

$$\kappa = -1 + \sqrt{1 + L^2 m^2} \ , \ \ \omega_k = \frac{4 + 2\kappa + 2k}{L} \ , \ \ k = 0, 1, 2, ...$$

Uncharged solutions without backreaction

$G_5 = 0$, $Q = 0$, $V(|\Psi|) = m^2|\Psi|^2$, $k = 1$

(Y.Brihaye, B.H. & J. Riedel, Phys. Rev. D 92, 044049 (2015))



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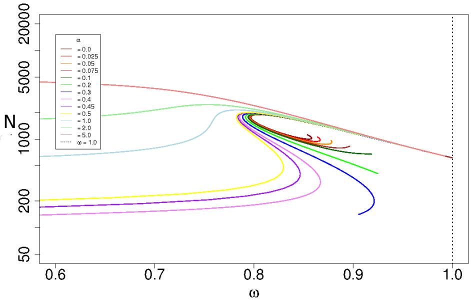
Gauss-Bonnet boson stars

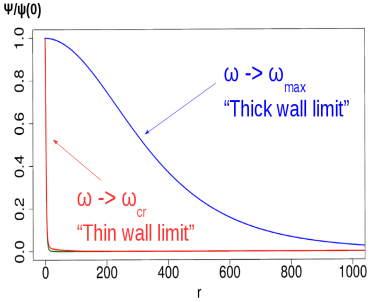
(B.H., J.Riedel, R. Suciu, Phys.Lett. B726 (2013) 906)



- exist for $\omega \in [\omega_{\text{min}} : \omega_{\text{max}}]$
- for α small: spiraling behaviour ending at $\omega_{\text{cr}} > \omega_{\text{min}}$

Gauss-Bonnet boson stars

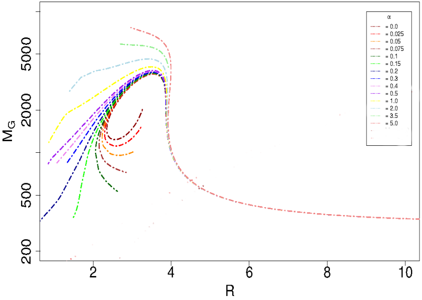
(B.H., J.Riedel, R. Suciu, Phys.Lett. B726 (2013) 906)



- practically no change for thick wall limit
- strong influence for thin wall limit

Gauss-Bonnet boson stars

(B.H., J.Riedel, R. Suciu, Phys.Lett. B726 (2013) 906)



- for sufficiently large α: unique mass-radius relation
- density increases with increasing α

Black holes with scalar hair in Anti-de Sitter

- (Gubser, 2008) **scalar field charged** under $U(1)$, charge e

$$m_{\rm eff}^2 = m^2 - e^2|g^{tt}|A_t^2$$

  if $m^2 \geq m^2_{\rm BF,d}$: asymptotic AdS_d stable
  $e^2|g^{tt}|$ large close to horizon of black hole $\Rightarrow m^2_{eff} < m^2_{\rm BF,d}$ close to horizon $\Rightarrow$ black hole forms scalar hair

- **uncharged** scalar field
  near-horizon geometry of **extremal** black holes given by $AdS_2 \times M_{d-2}$ (Robinson, 1959; Bertotti, 1959; Bardeen & Horowitz, 1999)
  if $m^2_{\rm BF,2} > m^2 > m^2_{\rm BF,d} \Rightarrow$ asymptotic AdS_d stable, but black hole forms scalar hair

Outline

Uncharged black holes for $k = -1$

- Uncharged black holes $Q = 0$; uncharged scalar field $e = 0$
- for $k = -1$ extremal solution with $T_{\rm H} = 0$, $r_h^{(\rm ex)} = L/\sqrt{2}$ exists
- close to **extremality** horizon topology is $AdS_2 \times H^3$
  (Astefanesei, Banerjee & Dutta, 2008)
- hyperbolic Gauss-Bonnet black holes in $d = 5$ have AdS_2 radius

$$R = \sqrt{L^2/4 - \alpha}$$

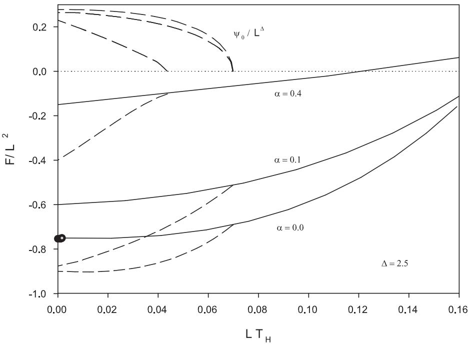
  (Y. Brihaye & B.H., Phys.Rev. D84 (2011) 084008)
- asymptotic AdS_5 stable, near-horizon AdS_2 unstable for

$$m^2_{\rm BF,5} \le m^2 \le m^2_{\rm BF,2}$$

Black holes with scalar hair, $G_5 \neq 0$, $\alpha \neq 0$

(Y. Brihaye & B.H., Phys.Rev. D84 (2011) 084008)

- black holes with scalar hair **thermodynamically preferred**



Black holes with scalar hair, $G_5 \neq 0$, $\alpha \neq 0$

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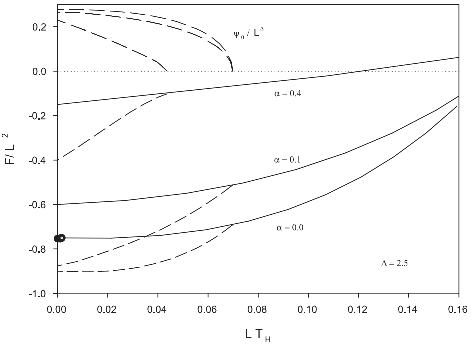
- the larger α the lower $T_{\rm H}$ at which instability appears



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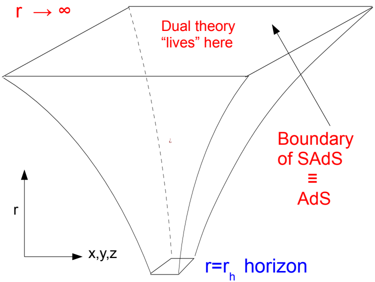
Black holes with planar horizon in AdS

- $k = 0$: planar horizon
- charged scalar field $e \neq 0$
- $r \to \infty$: planar AdS boundary

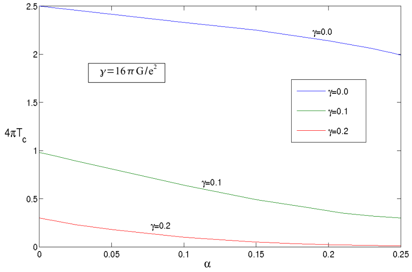


Taken from arxiv: 0808.1115

Including Gauss-Bonnet corrections

(Brihaye & B.H., Phys. Rev. D81 (2010) 126008)

- Gauss-Bonnet coupling $0 \leq \alpha \leq L^2/4$



$\Longrightarrow$ **condensation gets harder for $\alpha > 0$, but not suppressed**

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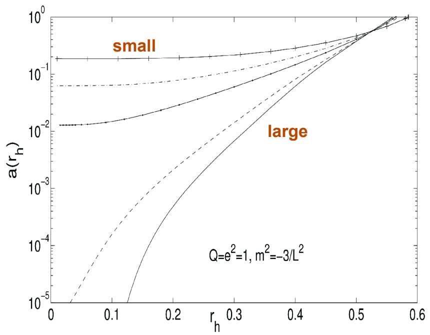
Charged black hole with scalar hair, $k = 1$

(Y. Brihaye & B.H., Phys.Rev. D85 (2012) 124024)

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}d\Omega_3^2$$

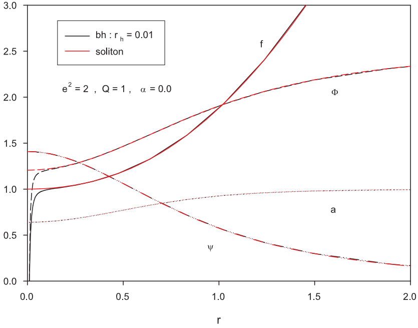


- For small α: solution exists down to $r_h = 0$
  $\rightarrow$ **soliton?**
- For large α: solution has $a(r_h) \rightarrow 0$ for $r_h \rightarrow r_h^{(cr)} > 0$
  $\rightarrow$ **extremal black hole?**

Charged black hole with scalar hair, $k = 1$

(Y. Brihaye & B.H., Phys.Rev. D85 (2012) 124024)

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}d\Omega_3^2$$



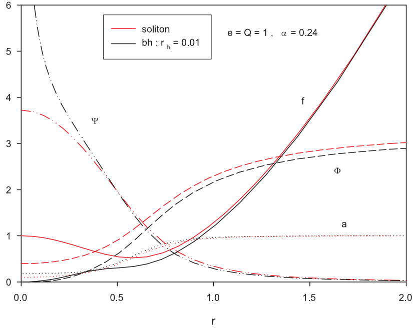
For $\alpha = 0$:

- Black hole tends to soliton solution in the limit $r_h \to 0$

Charged black hole with scalar hair, $k = 1$

(Y. Brihaye & B.H., Phys.Rev. D85 (2012) 124024)

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}d\Omega_3^2$$



$\alpha \neq 0$:

- Gauss-Bonnet solitons with scalar hair exist
- black holes with scalar hair do **not** tend to corresponding solitons for $r_h \to 0$

Charged black hole with scalar hair, $k = 1$

(Y. Brihaye & B.H., Phys.Rev. D85 (2012) 124024)

There exist no extremal Gauss-Bonnet black holes with scalar hair.

Charged black hole with scalar hair, $k = 1$

Proof:

- assume near-horizon geometry to be $AdS_2 \times S^3$:

$$ds^2 = v_1 \left(-\rho^2 d\tau^2 + \frac{1}{\rho^2} d\rho^2\right) + v_2 \left(d\psi^2 + \sin^2\psi \left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right)$$

  v_1, v_2: positive constants

- Combination of equations of motion yields

$$0 = 16\pi G \left(\frac{\rho^2}{v_1}\psi'^2 + \frac{e^2\phi^2\psi^2}{\rho^2 v_1}\right)$$

  This leads to: $\psi' = 0$ and $\phi^2\psi^2 = 0$ in near horizon geometry

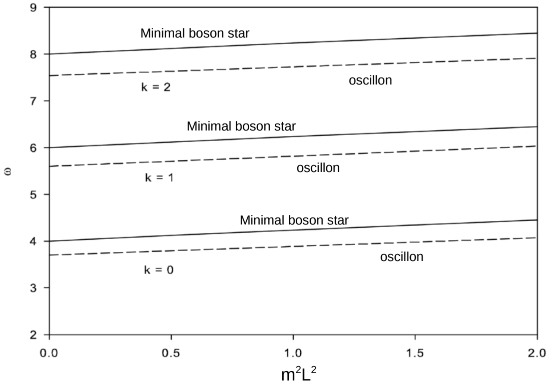
- $\phi^2 = 0$ ruled out $\rightarrow \psi \equiv 0$ **in hear horizon geometry** q.e.d.

Outline

From massive oscillons to minimal boson stars

(Y.Brihaye, B.H. & J. Riedel, Phys. Rev. D 92, 044049 (2015))

$G_5 \neq 0$, $Q = 0$, $V(|\Psi|) = m^2|\Psi|^2$

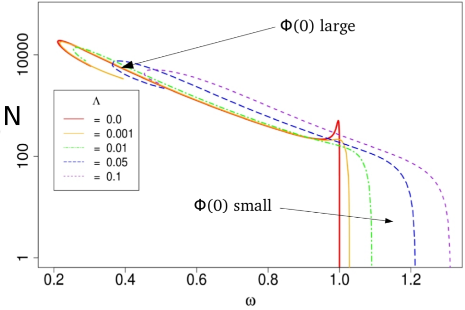


Self-interacting boson stars

(B.H.& J. Riedel, PRD87 (2013), 044003; PRD86 (2012) 104008)

$Q = 0$, $V(|\Psi|) = m^2|\Psi|^2 - \lambda|\Psi|^4 + |\Psi|^6$

$\phi(0) = 0 \Rightarrow \phi(r) \equiv 0 \Rightarrow M = N = 0 \Rightarrow$ AdS vacuum

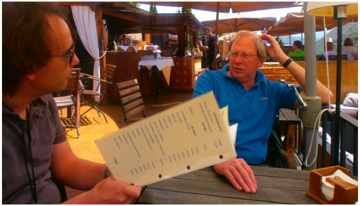


Conclusions

- Gauss-Bonnet (GB) interaction influences (mainly) the spacetime close to $r = 0$ (for solitons)
  - see also *rotating* GB boson stars
  (Y. Brihaye & B.H., Class. Quant. Grav. (2016))
- With increasing α, GB black holes become unstable to form scalar hair at decreasing temperatures
- GB black holes with scalar hair *thermodynamically preferred*
- *extremal* GB black holes do not support scalar hair
- Very compact rotating boson stars possess *ergoregion*: Gauss-Bonnet interaction has only small influence on the ergoregion
  (Y. Brihaye & B.H., Class. Quant. Grav. (2016))

Jürgen Riedel
Universität Oldenburg, Germany

Yves Brihaye
Université de Mons, Belgium



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Thanks for your attention