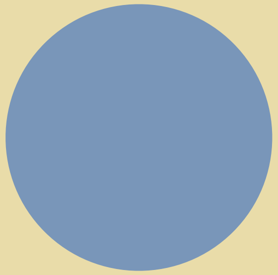
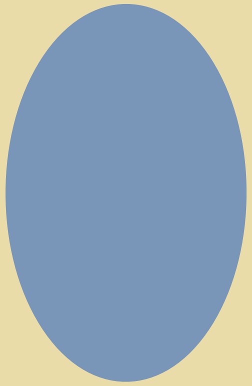
# Polarized Black Holes in AdS



Lauren Greenspan



Work in Progress with M. Costa, J. Penedones, and J. Santos

# Motivation

Find interesting new geometries in AdS.

What can we learn about the dual field theories?

# Motivation

Find interesting new geometries in AdS.
What can we learn about the dual field theories?

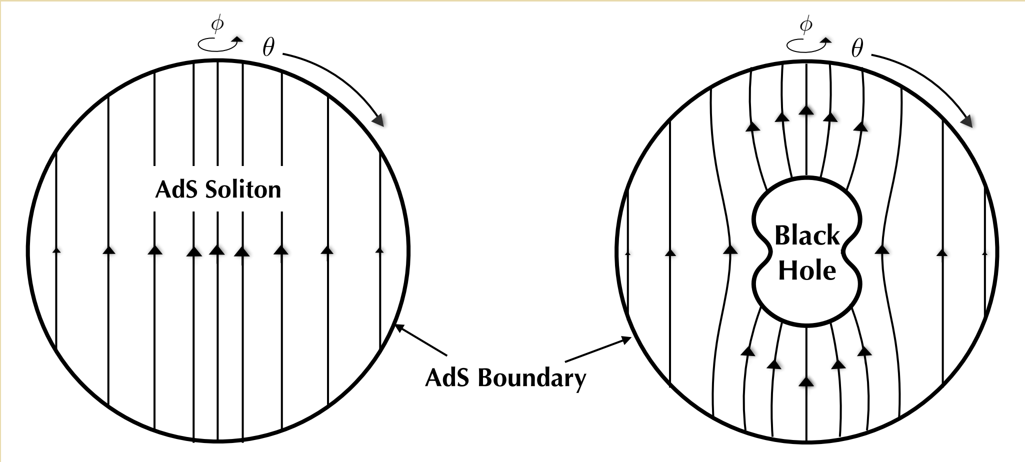
# Idea

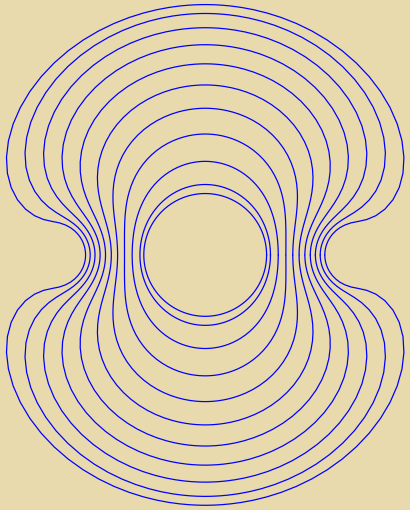
Study the thermodynamics of deformed 4-dimensional black holes in dual to 3-dimensional ABJM theory.

# Background - Polarized Black Holes in $AdS_4$

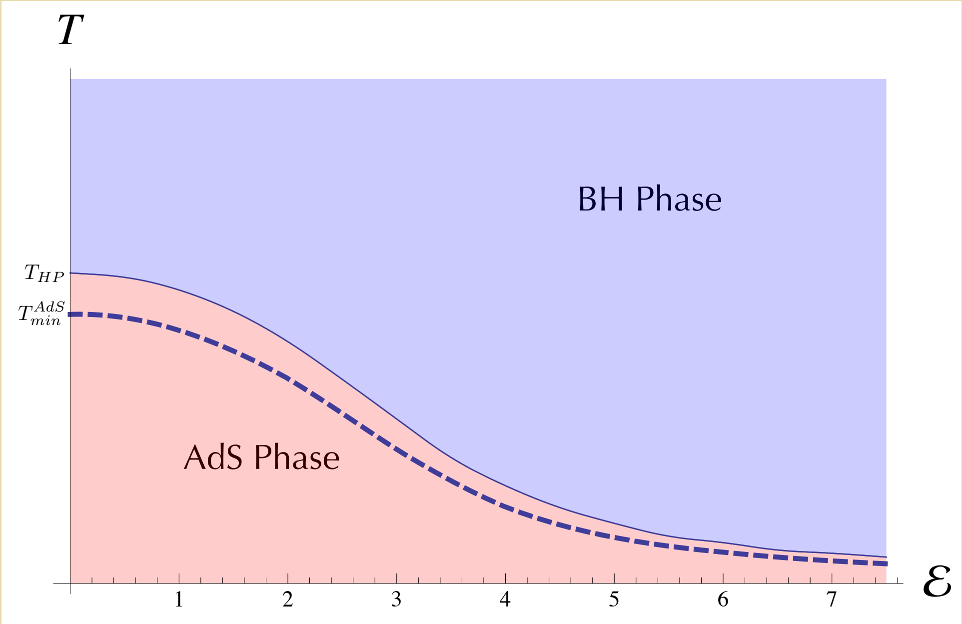
[hep-th/1511.08505 Costa, LG, Oliveira, Penedones, Santos]

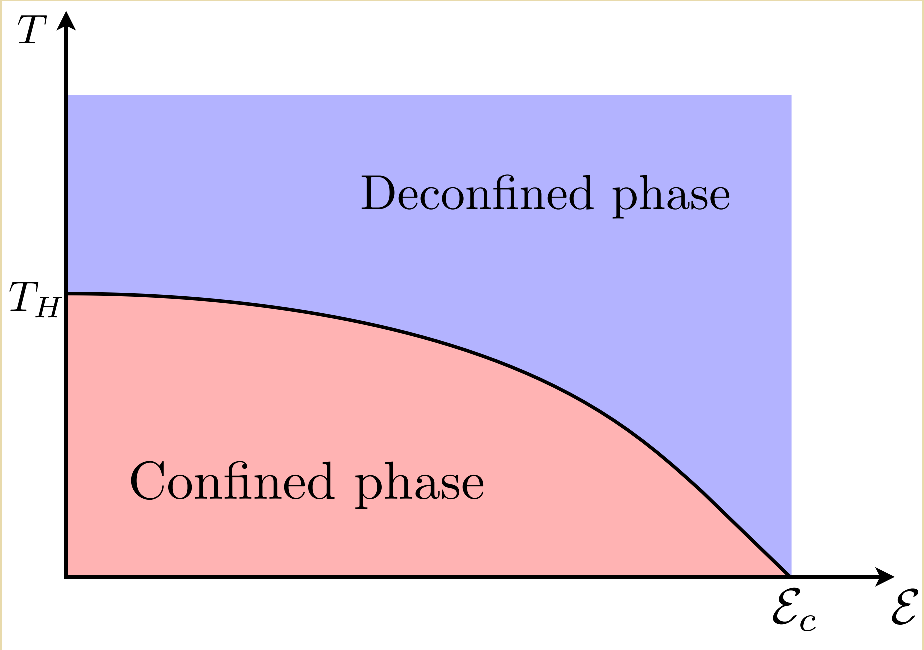
Studied numerical solutions Einstein-Maxwell gravity with a dipolar potential source. $A_\tau = i\mathcal{E}\cos\theta$



- STRONG COUPLING: phase Diagram from BH thermo- can increase electric field without bound.
- WEAK COUPLING: Partition function of free Bosons on a sphere- maximum electric field.



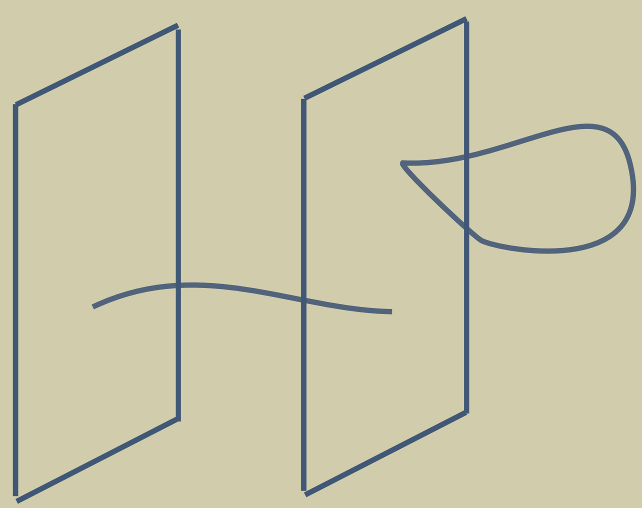




What is the correct gauge/gravity description for polarized black holes?

# ABJM [Aharony, Berenstein, Jafferis, Maldacena 2008]

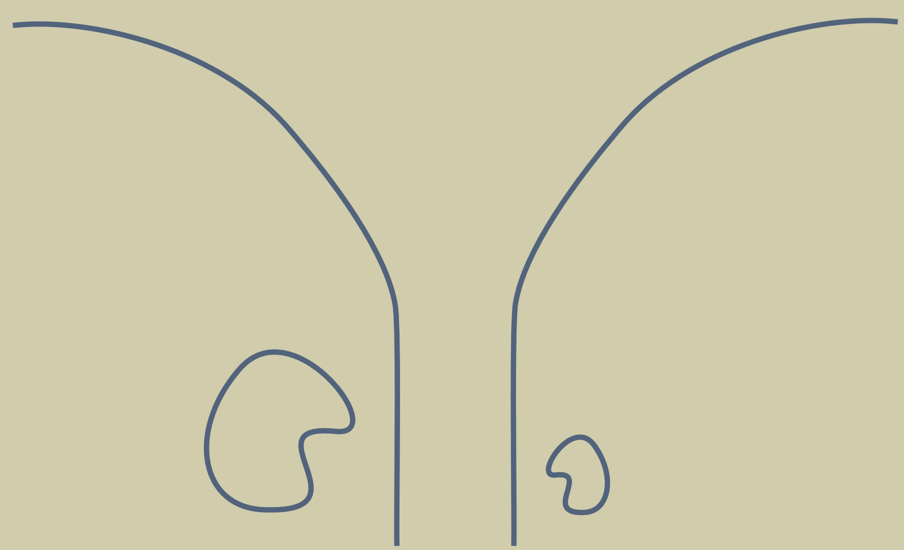
A precise dual of BH polarization is deformed 3 d ABJM theory

- $\mathcal{N}=6$ Superconformal CS matter theory
- 12 Real Supercharges
- gauge group $U(N)_k \times U(N)_{-k}$
- Conformal fixed points of CM systems



ABJM

M2 branes ⟷



M-theory on $AdS_4 \times S^7$

Concrete Realization of the gauge/gravity duality!

[Itzhaki, Maldacena, Sonnenschein, Yankielowicz '98]

# Gravity Dual

[Cvetic, Duff, Hoxha et. al. 1999]

$AdS_4 \times S^7$ → 4 d gauged SUGRA that can be truncated to include

- gauge field $\tilde{A}_\mu$
- scalar field $\Phi$
- metric $g_{\mu\nu}$

With gravitational action

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left( R - \frac{1}{2} \nabla_\mu \Phi \nabla^\mu \Phi + \frac{2}{l^2} (\cosh \Phi + 2) - \frac{1}{2} e^{\Phi} F^2 \right) + S_{bdy}$$

$$S_{bdy} = \frac{1}{8\pi G_N} \int d^3x \sqrt{h} \left( K - \frac{1}{2} n_\mu \Phi \nabla^\mu \Phi \right)$$

Some boundary observables:

$$T_{\mu\nu} = \frac{2}{\sqrt{h}} \frac{\delta S}{\delta h^{\mu\nu}} = \frac{1}{8\pi G_N} \left( K_{\mu\nu} - K\, h_{\mu\nu} + G_{\mu\nu} - 2h_{\mu\nu} + \frac{1}{8} \Phi n^\alpha \nabla_\alpha \Phi h_{\mu\nu} \right)$$

$$Q = \frac{1}{4\pi G_N} \int d\Omega_2 \left( e^{\Phi(r,\theta)} \star F \right) \Big|_{r=1}$$

# Ansatze

$$ds^2_{sol} = \frac{1}{(1-r^2)^2}\left(f(r)A(r,\theta)\,d\tau^2 + \frac{(1+r^2)^2G(r,\theta)dr^2}{f(r)} + r^2C(r,\theta)(d\theta + \frac{1}{r}H(r,\theta)dr)^2 + r^2d\phi^2B(r,\theta)\sin^2\theta\right)$$

$$f(r) = 1 - r^2 + r^4$$

$$\tilde{A}^{sol}_\tau = irD(r,\theta)$$

$$\Phi(r,\theta) = (1-r^2)\varphi(r,\theta)$$

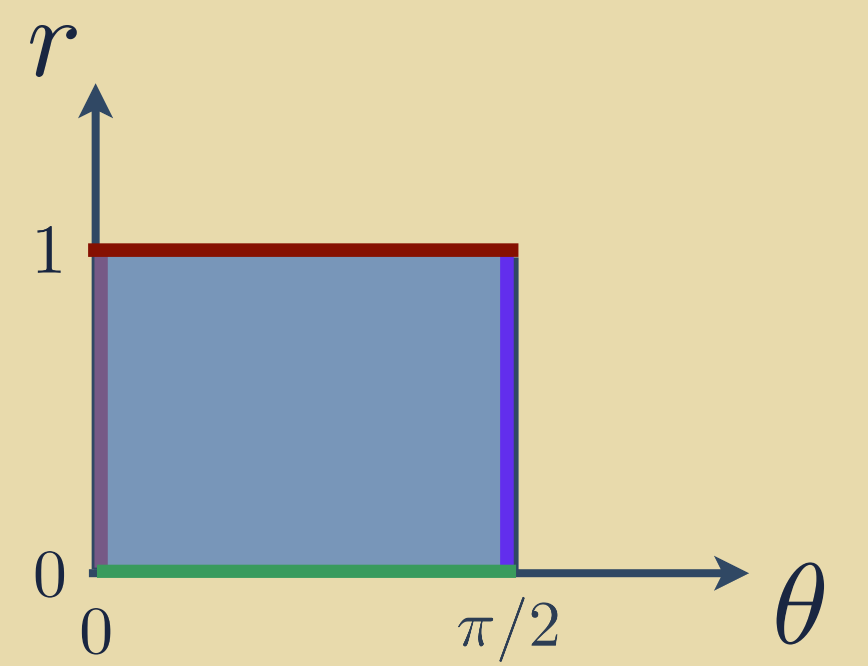
$$ds^2_{BH} = \frac{1}{(1-r^2)^2}\left(d\tau^2r^2A(r,\theta)f_{BH}(r) + R^2\left(\frac{4dr^2G(r,\theta)}{f_{BH}(r)} + C(r,\theta)(d\theta + 2rdrH(r,\theta))^2 + d\phi^2B(r,\theta)\sin^2\theta\right)\right)$$

$$f_{BH}(r) = (1-r^2)^2 - Q^2(1-r^2)^3 + R^2(3-3r^2+r^4)$$

$$\tilde{A}^{BH}_\tau = ir^2D(r,\theta)$$

Boundary Conditions:

- Regularity at Axes of Symmetry { horizon, equator, pole }




- At infinity $A = G = C = B = 1$

$$H = 0$$

$$D = \mathcal{E}\cos\theta$$ ← Non-normalizable mode for the gauge field

$$\nabla^2_z\Phi = 0$$ ← No scalar source

# Einstein-deTurck Equations

[Headrick, Kitchen, Wiseman '09]

$$d\left(e^{\Phi}\star F\right)=0$$

$$\nabla^2\Phi+\frac{2}{l^2}\sinh\Phi-\frac{1}{2}e^{\Phi}F^2=0$$

$$R_{\mu\nu}=T_{\mu\nu}$$

- diffeomorphism invariance (underdetermined)
  - numerics: pure gauge fluctuations cause convergence problems

$\theta$

# Einstein-deTurck Equations

[Headrick, Kitchen, Wiseman '09]

$$d\left(e^{\Phi}\star F\right)=0$$

$$\nabla^2\Phi+\frac{2}{l^2}\sinh\Phi-\frac{1}{2}e^{\Phi}F^2=0$$

$$R_{\mu\nu}-\nabla_{(\mu}\xi_{\nu)}+\frac{1}{l^2}(\cosh\Phi+2)g_{\mu\nu}-\frac{1}{2}\nabla_\mu\Phi\nabla_\nu\Phi+e^{\Phi}\left(\frac{1}{4}F^2g_{\mu\nu}-F_{\mu\alpha}F_\nu^\alpha\right)=0$$

DeTurck term that makes Einstein equations elliptic

$$\xi^\lambda\equiv g^{\mu\nu}\left(\Gamma^\lambda_{\mu\nu}-\bar{\Gamma}^\lambda_{\mu\nu}\right)$$

Reference metric: (AdS)

DeTurck vector: vanishes on numerical solution

$\theta$

# Einstein-deTurck Equations

[Headrick, Kitchen, Wiseman '09]

$$d\left(e^{\Phi} \star F\right) = 0$$

$$\nabla^2\Phi + \frac{2}{l^2}\sinh\Phi - \frac{1}{2}e^{\Phi}F^2 = 0$$

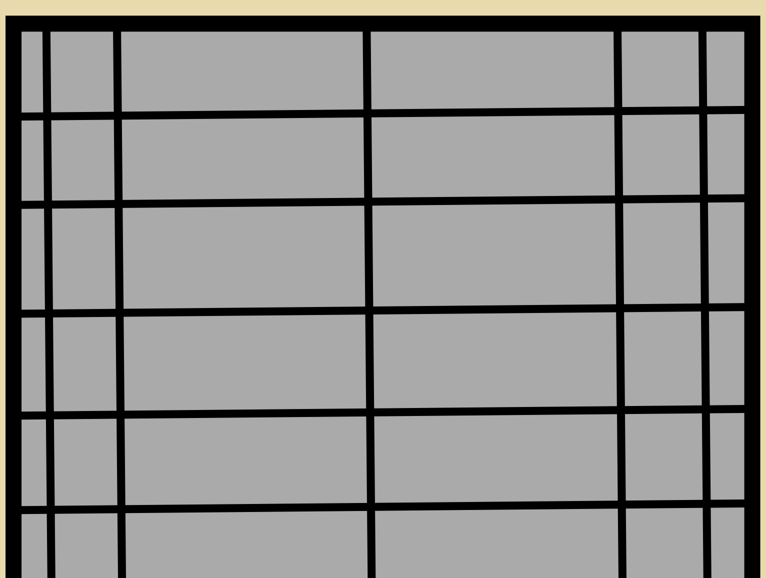
$$R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} + \frac{1}{l^2}(\cosh\Phi + 2)g_{\mu\nu} - \frac{1}{2}\nabla_\mu\Phi\nabla_\nu\Phi + e^{\Phi}\left(\frac{1}{4}F^2 g_{\mu\nu} - F_{\mu\alpha}F^{\alpha}_{\nu}\right) = 0$$

DeTurck term that makes Einstein equations elliptic

$$\xi^\lambda \equiv g^{\mu\nu}\left(\Gamma^\lambda_{\mu\nu} - \bar{\Gamma}^\lambda_{\mu\nu}\right)$$

Reference metric: (AdS)

DeTurck vector:
vanishes on numerical solution



- Descretize PDEs with a chebyshev grid in r and fourier in $\theta$
- Solve with Spectral Methods (exponential convergence)

  derivatives estimated using
  a polynomial approximation
  that includes all points on the grid.

# Observables- Soliton

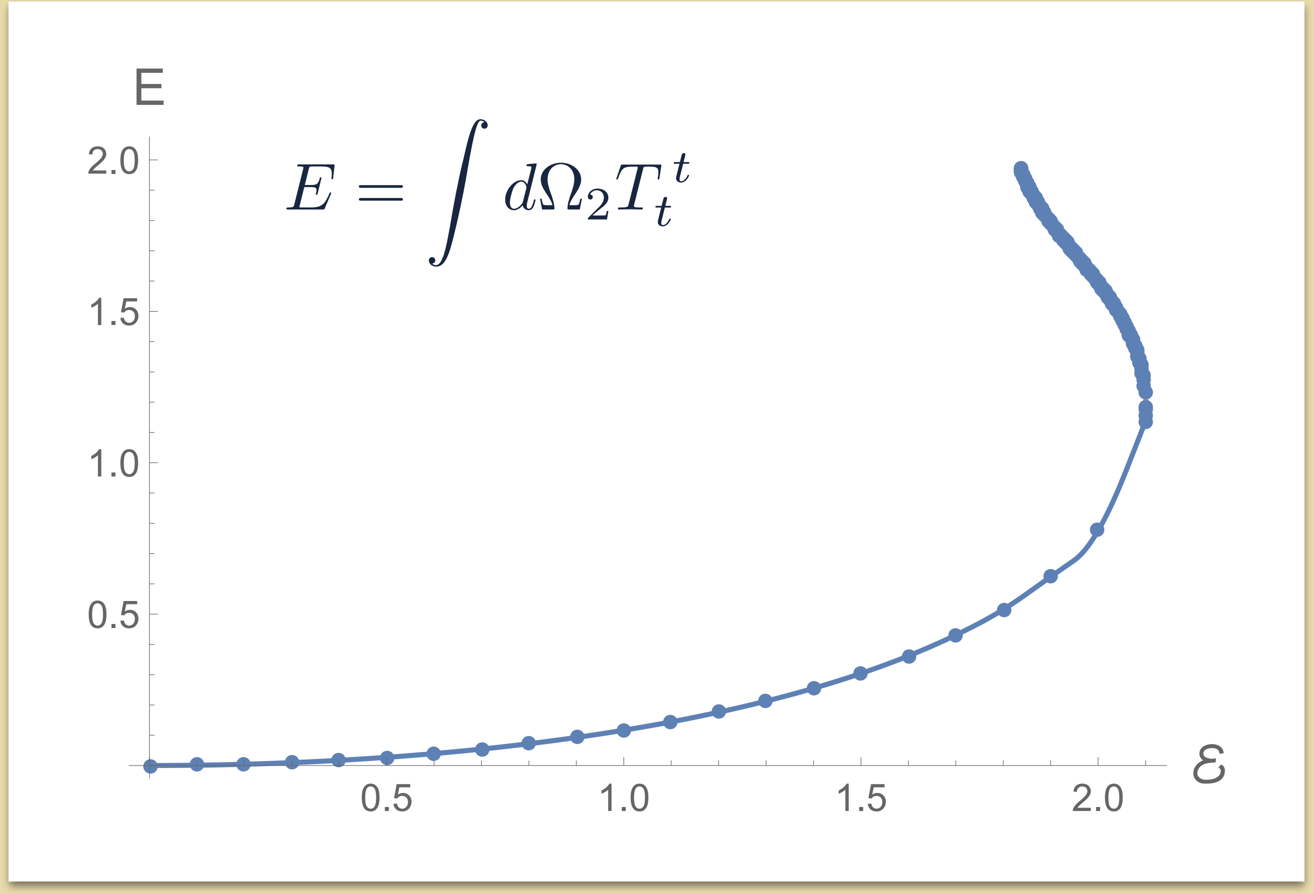
$$T_t^{\ t} = \frac{3}{256\pi G_N}\left(2\alpha_3 + \phi_0^2\right)$$

$$T_\theta^{\ \theta} = \frac{3}{256\pi G_N}\left(2\chi_3 - \phi_0^2\right)$$

$$T_\phi^{\ \phi} = \frac{-3}{128\pi G_N}\left(\chi_3 + \alpha_3 + \phi_0^2\right)$$

traceless, conserved

$$A(r,\theta) = \sum_{i=1}^{n} \alpha_i(\theta)(1-r)^i$$



# Observables- Soliton

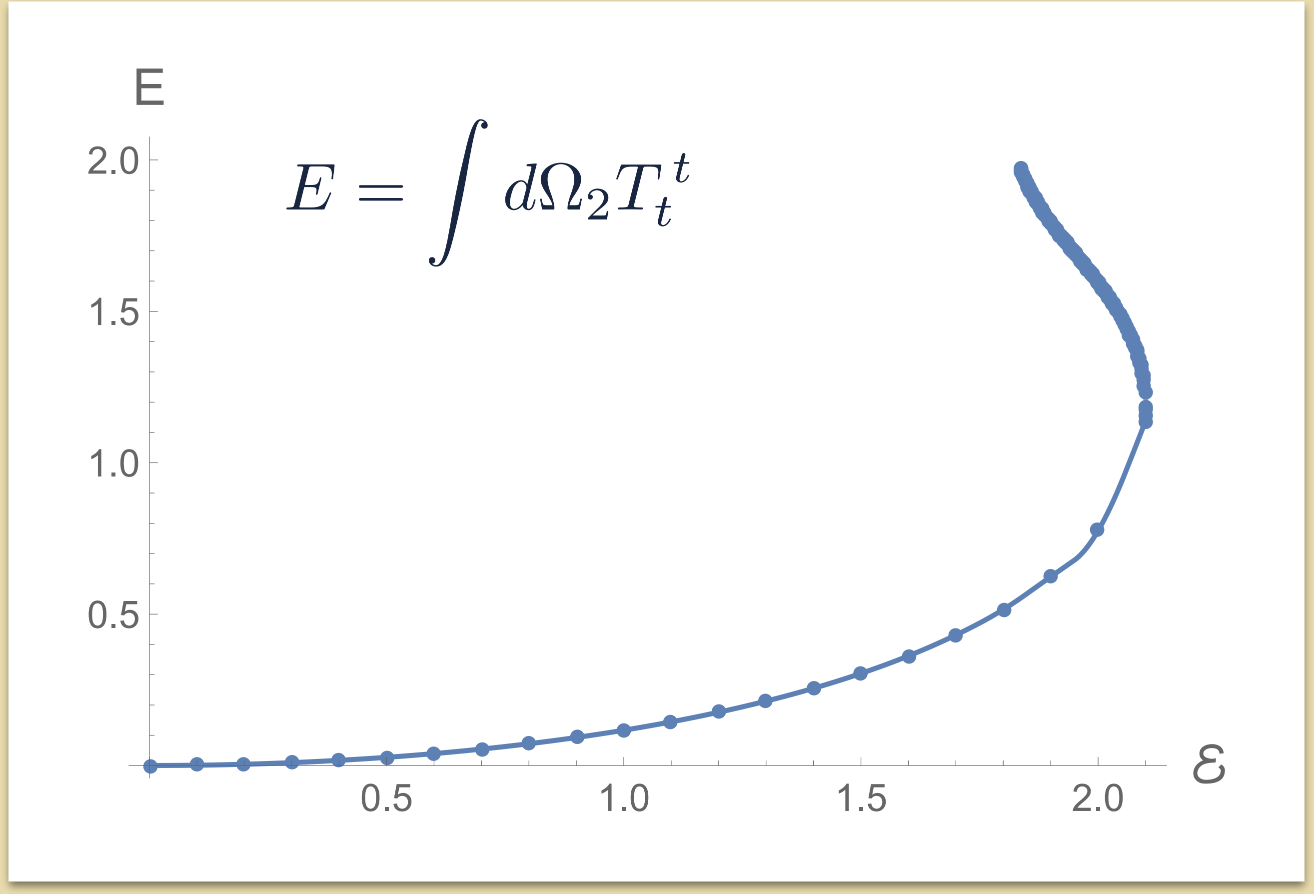
$$T_t^{\ t} = \frac{3}{256\pi G_N}\left(2\alpha_3 + \phi_0^2\right)$$

$$T_\theta^{\ \theta} = \frac{3}{256\pi G_N}\left(2\chi_3 - \phi_0^2\right)$$

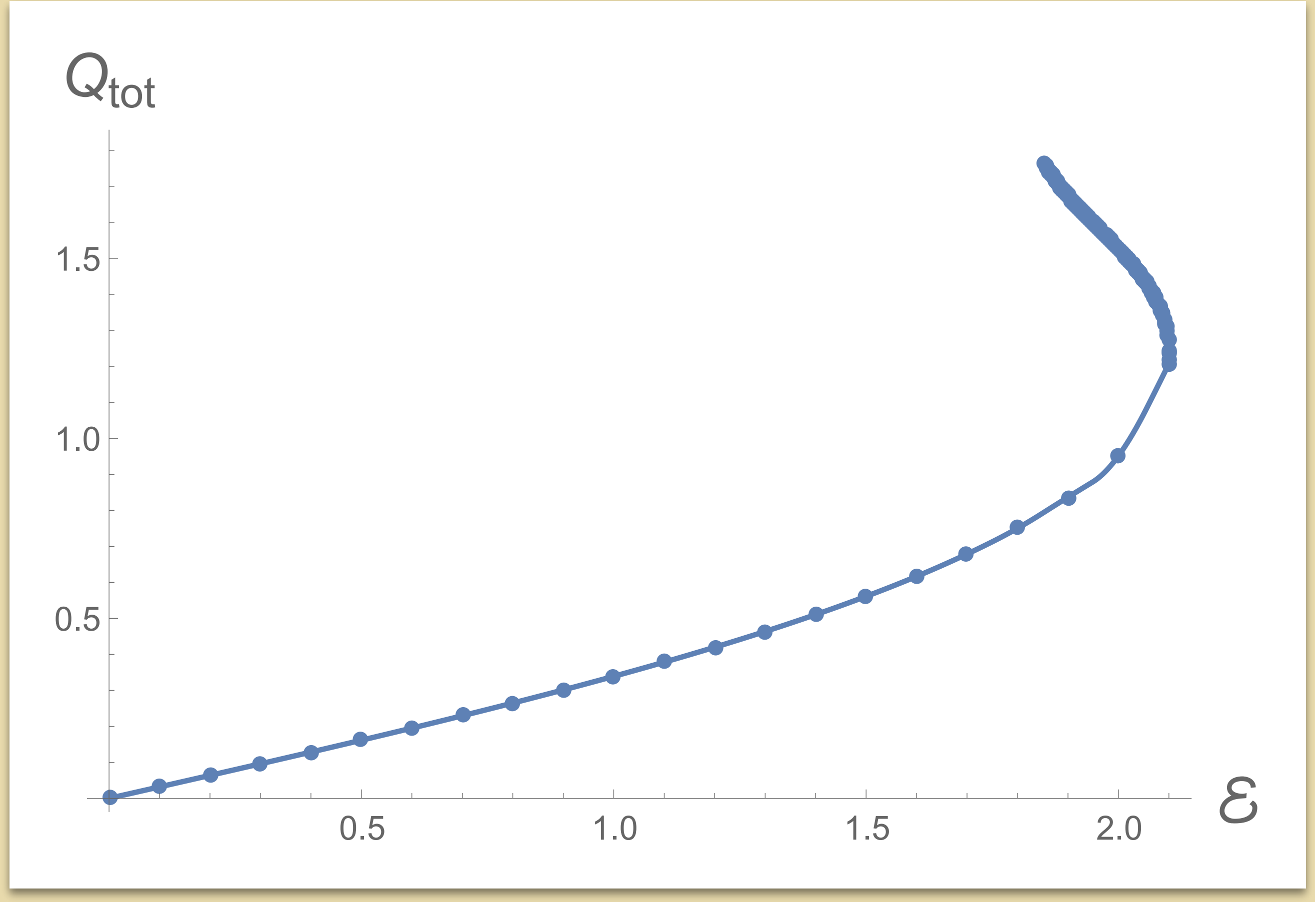
$$T_\phi^{\ \phi} = \frac{-3}{128\pi G_N}\left(\chi_3 + \alpha_3 + \phi_0^2\right)$$

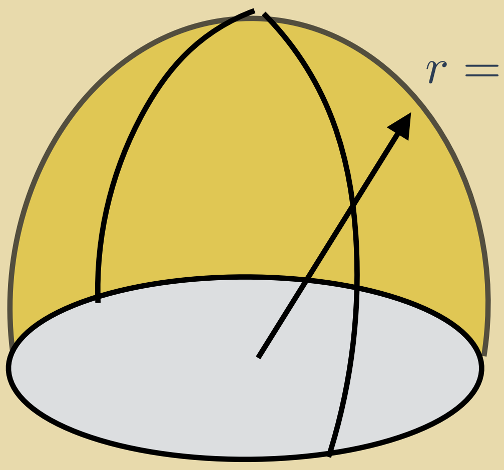
traceless, conserved

$$A(r,\theta) = \sum_{i=1}^{n} \alpha_i(\theta)(1-r)^i$$

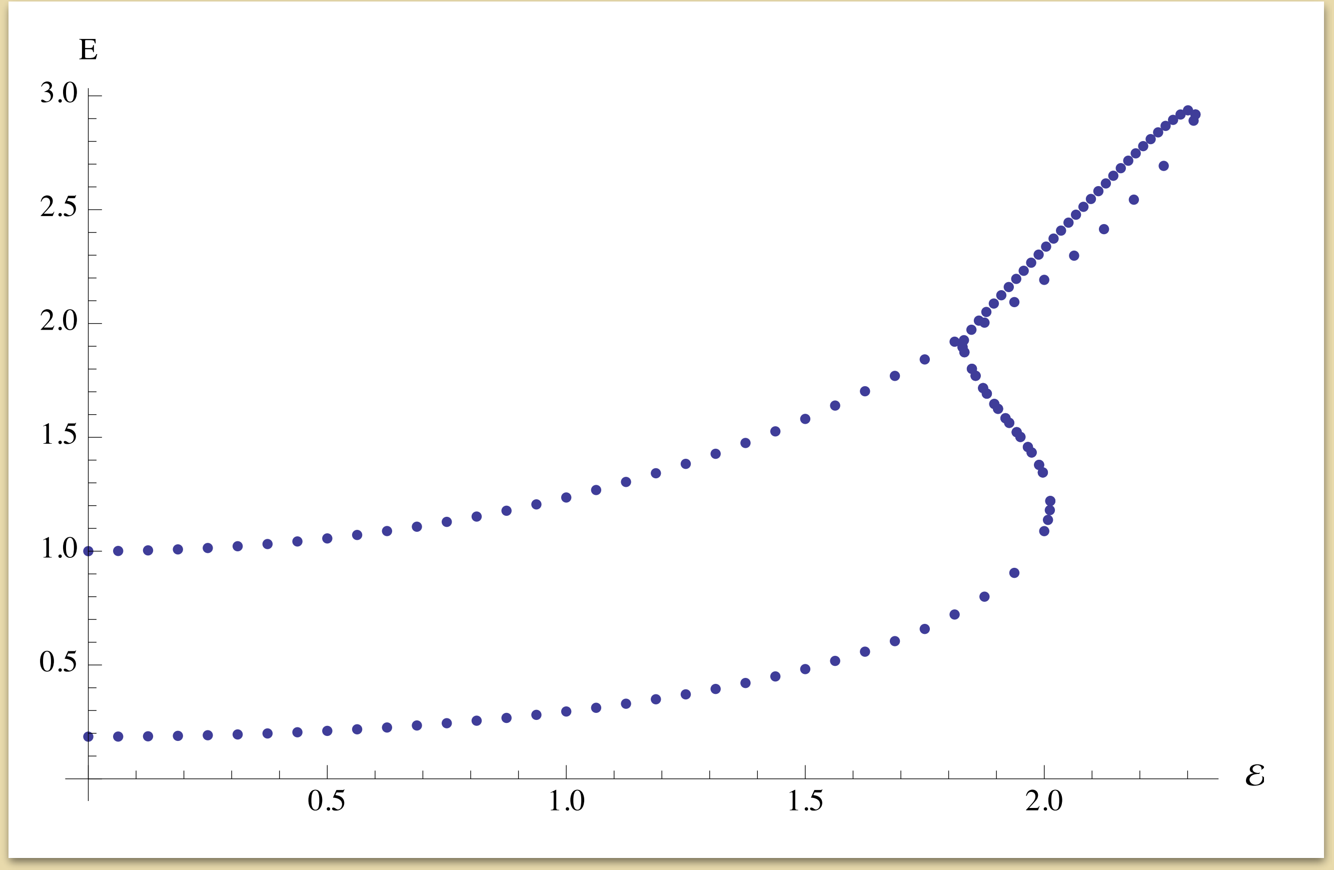


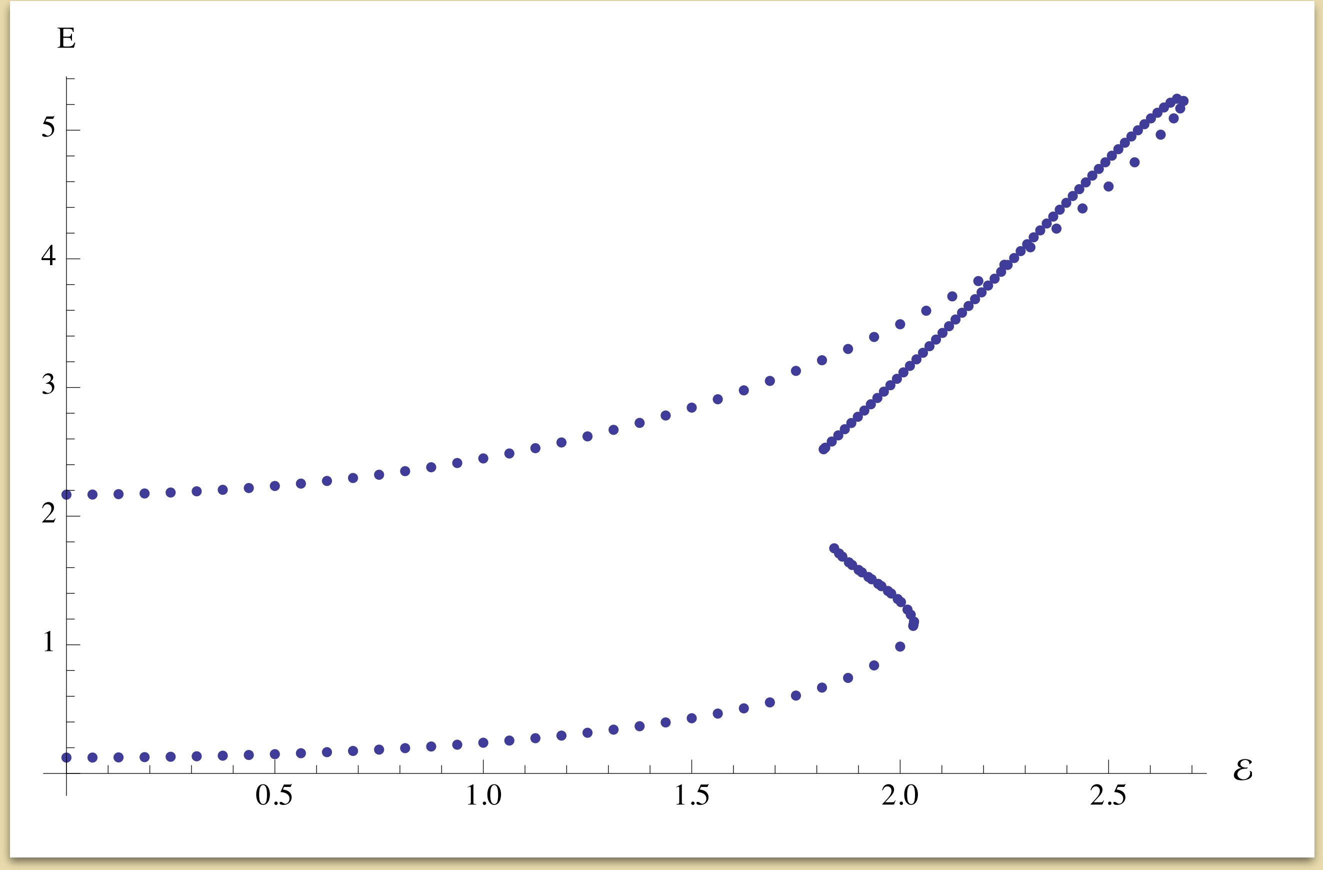

charge in one hemisphere



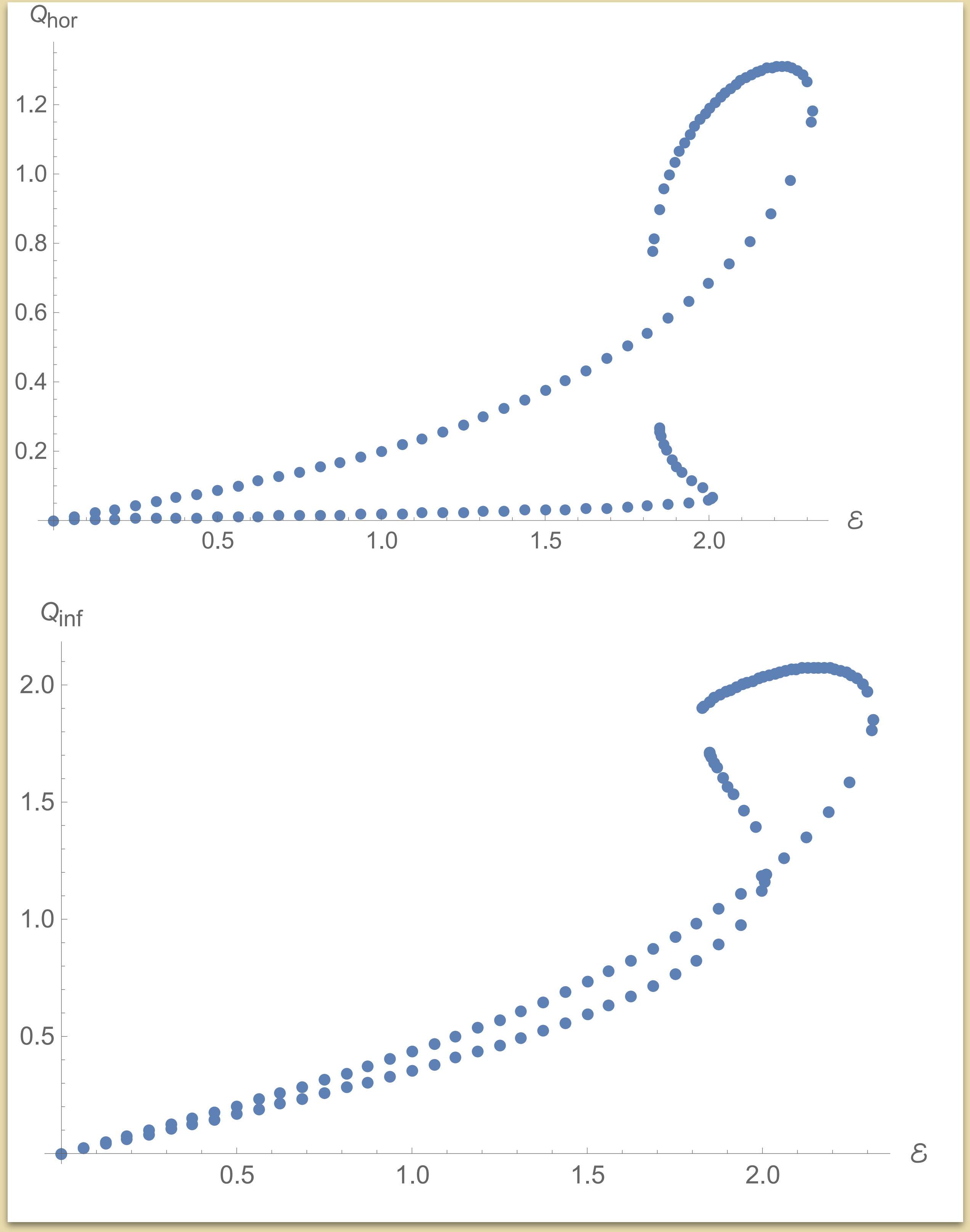
# Observables- BH energy

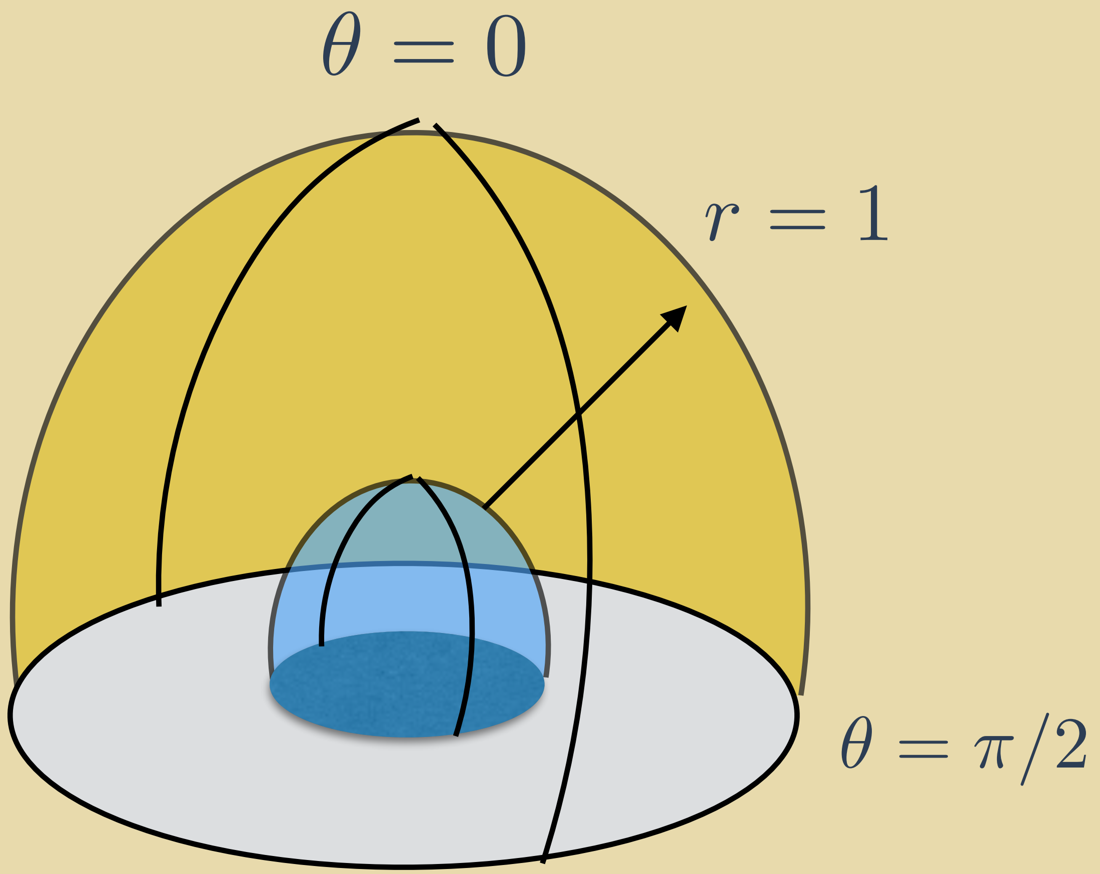



temp →

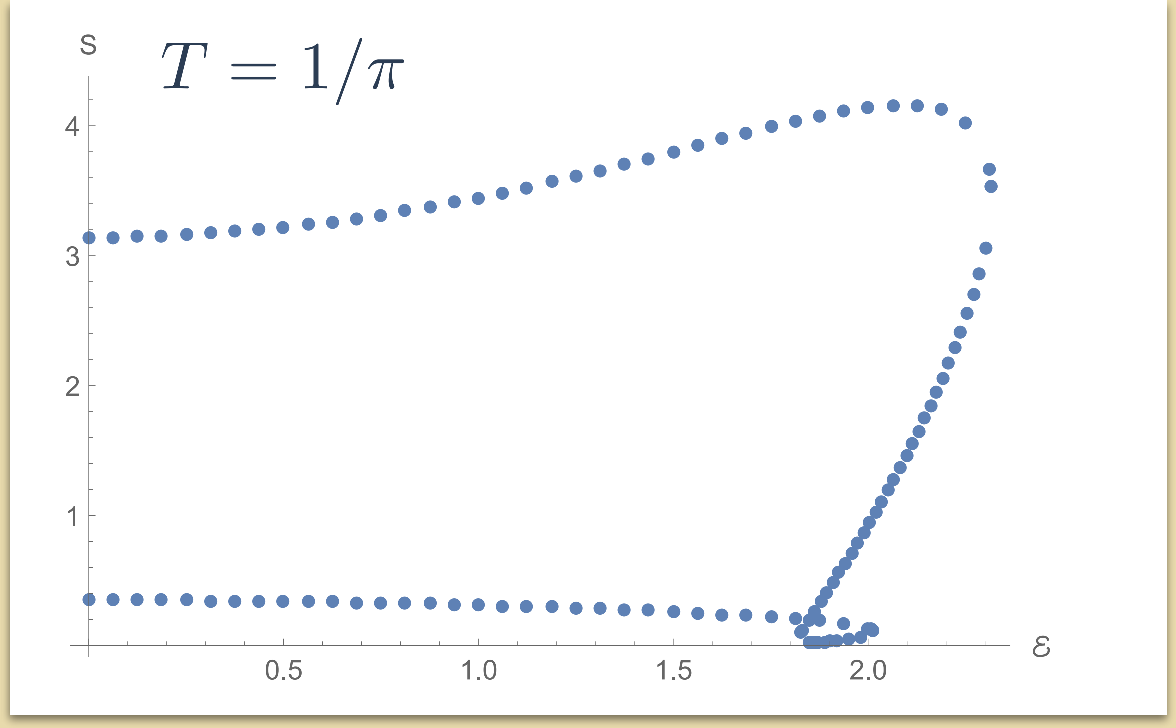
# Observables- BH charge density



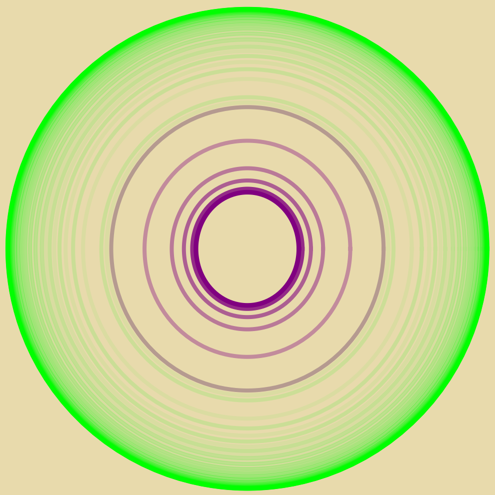
# Observables- BH entropy

$$S = \frac{\mathcal{A}}{4G_N} = \frac{\pi R^2}{G_N}\int_0^{\frac{\pi}{2}} d\theta\,\sin\theta\sqrt{C(0,\theta)B(0,\theta)}$$

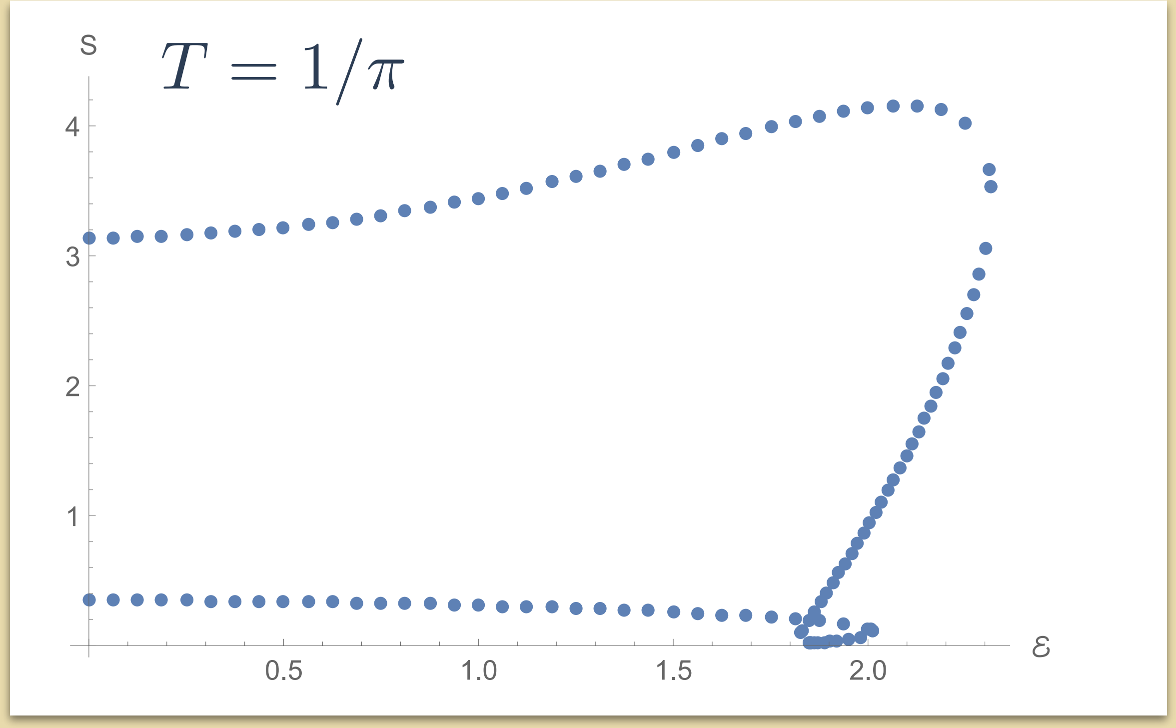


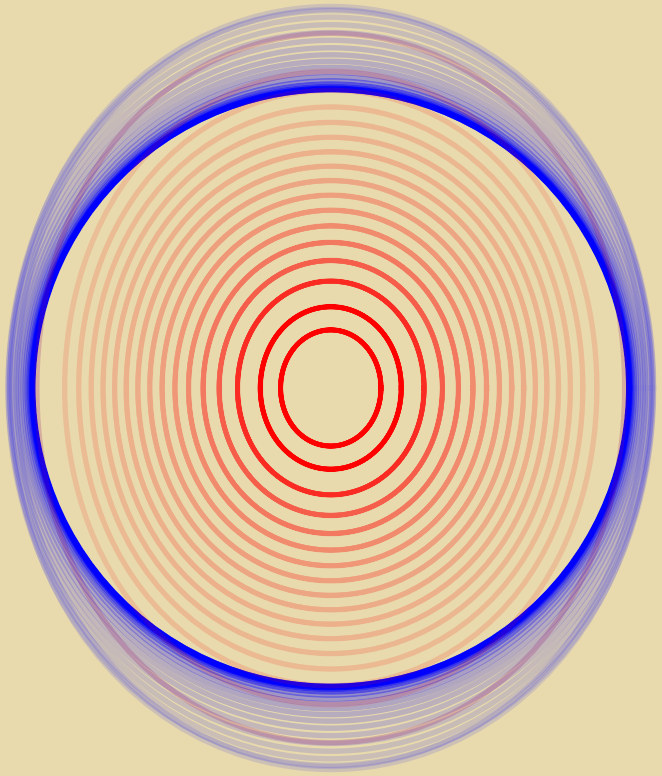
# Observables- BH entropy

$$S = \frac{\mathcal{A}}{4G_N} = \frac{\pi R^2}{G_N}\int_0^{\frac{\pi}{2}} d\theta\ \sin\theta\sqrt{C(0,\theta)B(0,\theta)}$$

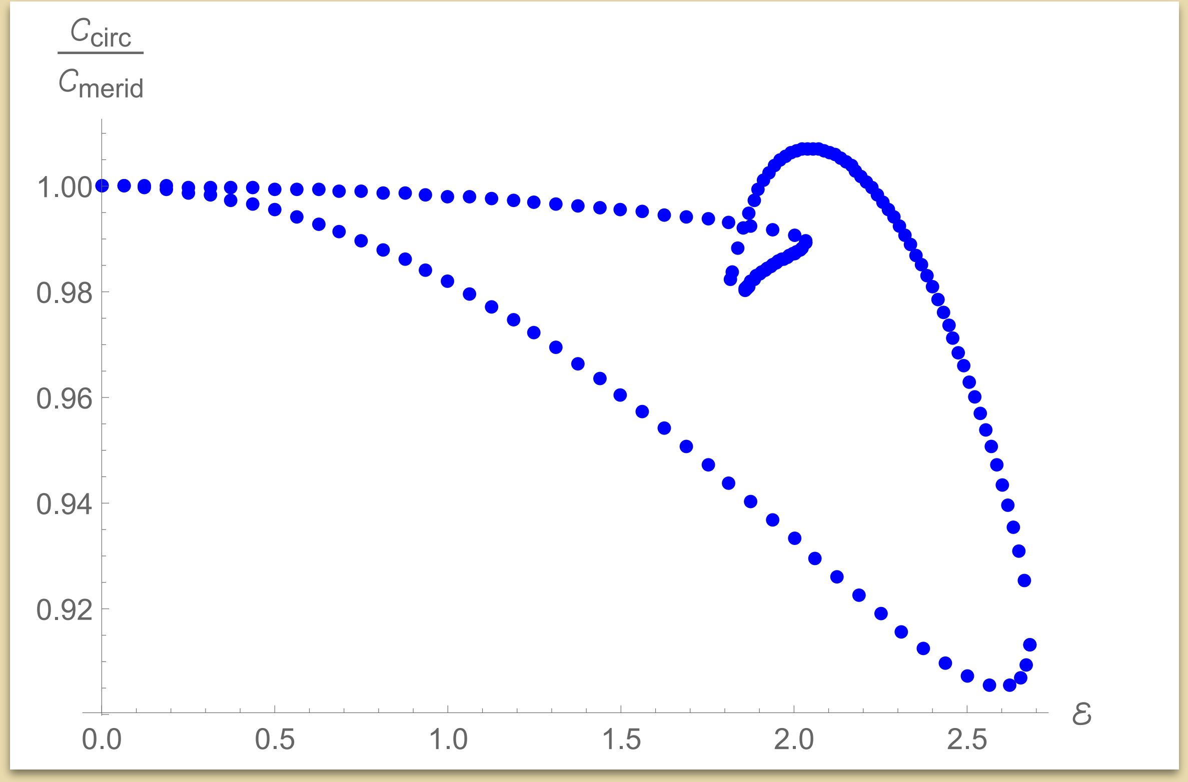


"small"



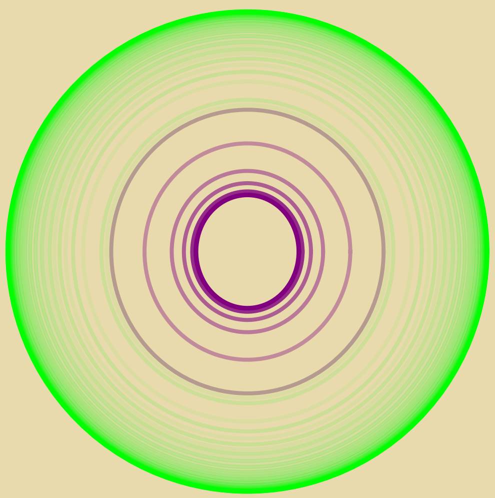



"large"

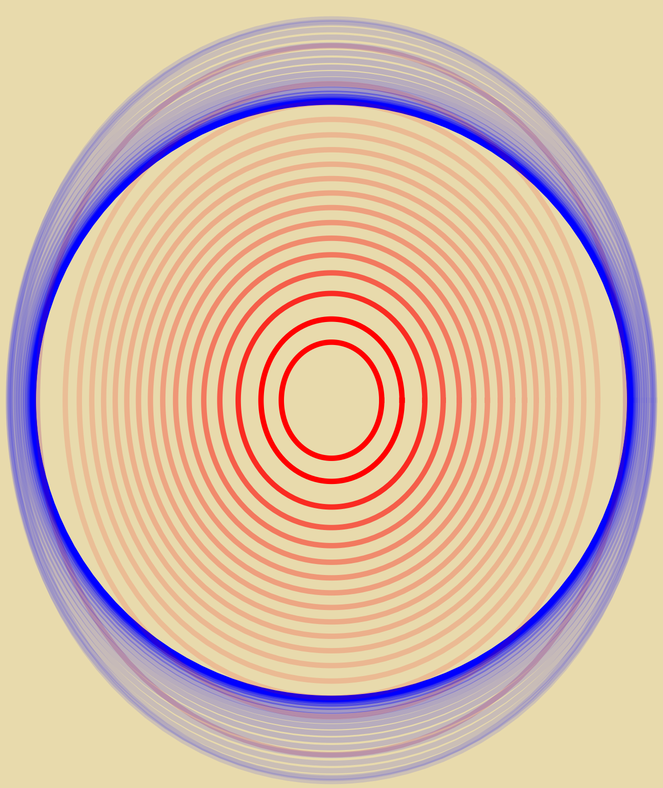


# Observables- BH entropy

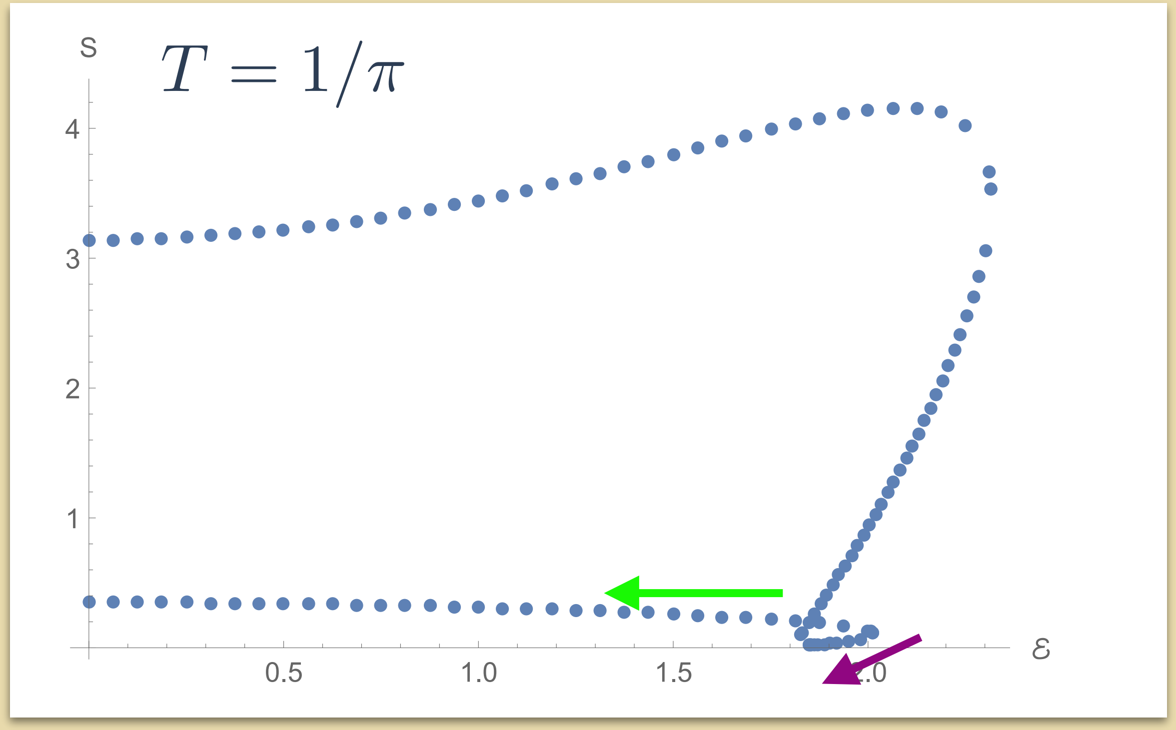
$$S = \frac{\mathcal{A}}{4G_N} = \frac{\pi R^2}{G_N}\int_0^{\frac{\pi}{2}} d\theta \, \sin\theta \sqrt{C(0,\theta)B(0,\theta)}$$

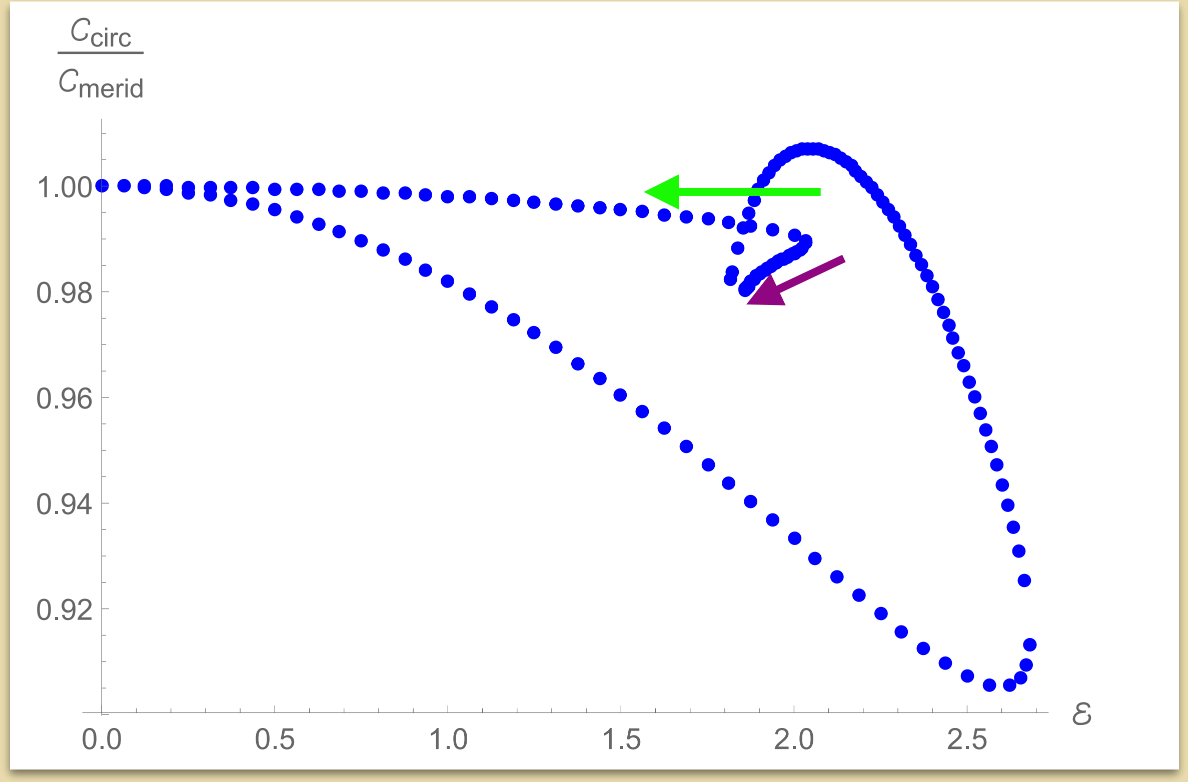


“small”



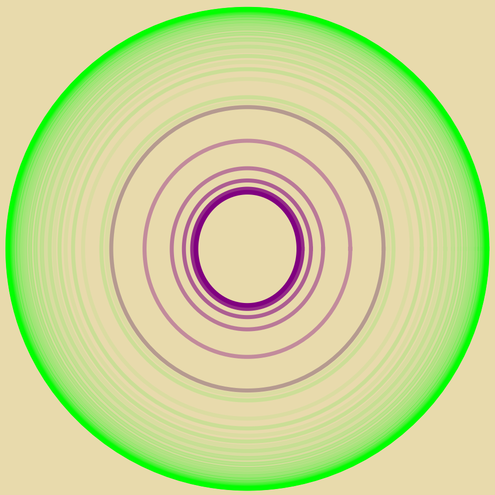
“large”



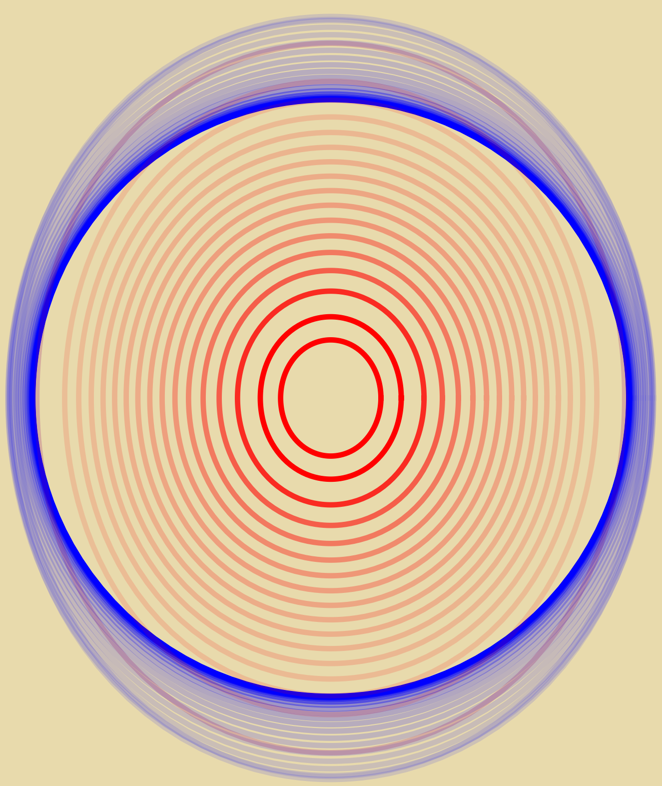



# Observables- BH entropy

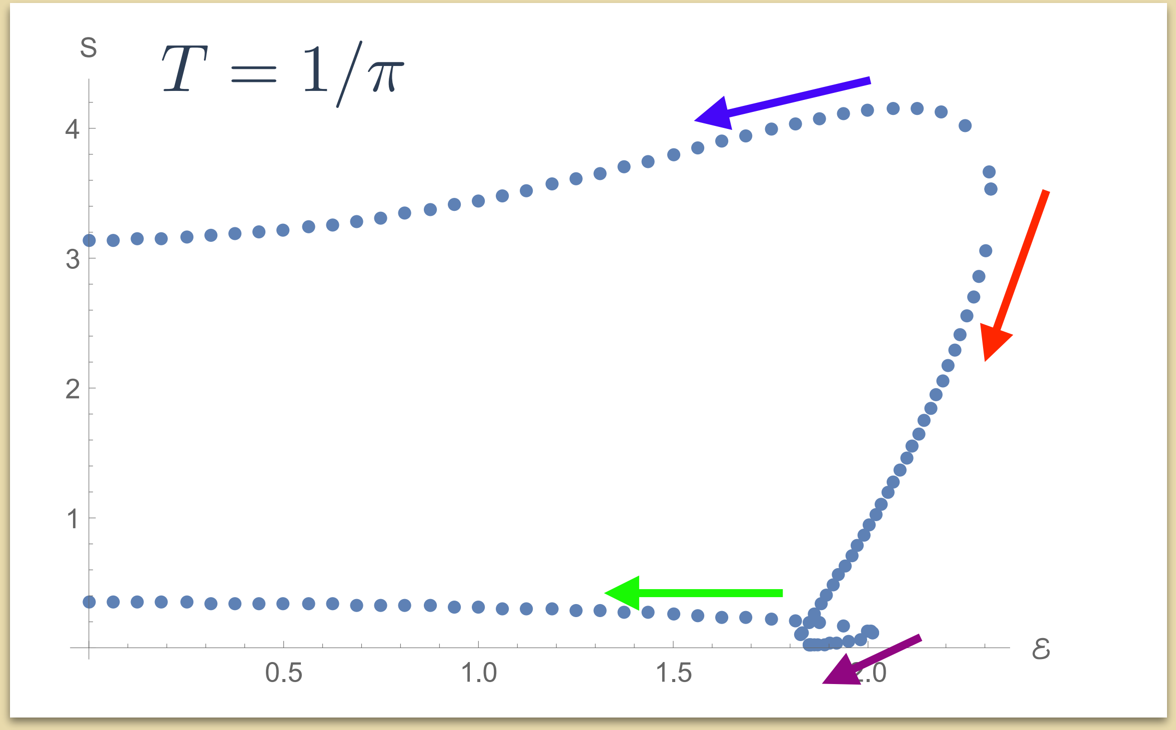
$$S = \frac{\mathcal{A}}{4G_N} = \frac{\pi R^2}{G_N}\int_0^{\frac{\pi}{2}} d\theta\,\sin\theta\sqrt{C(0,\theta)B(0,\theta)}$$

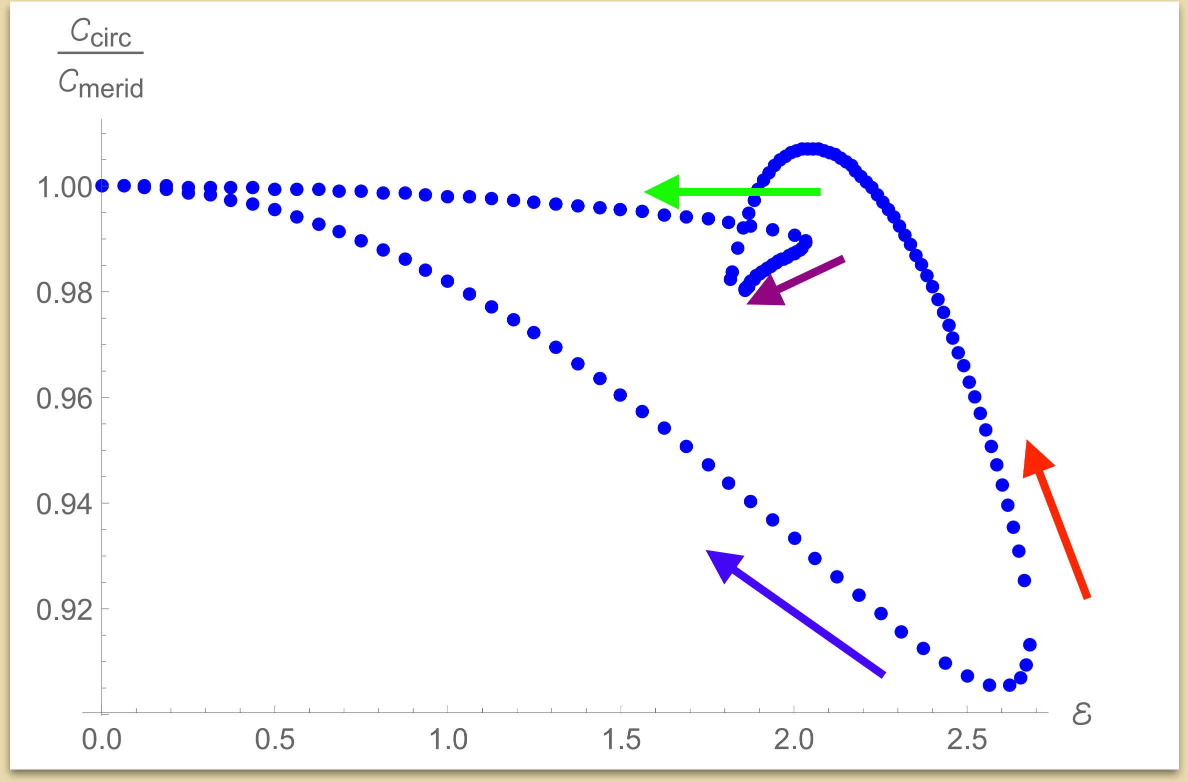


"small"



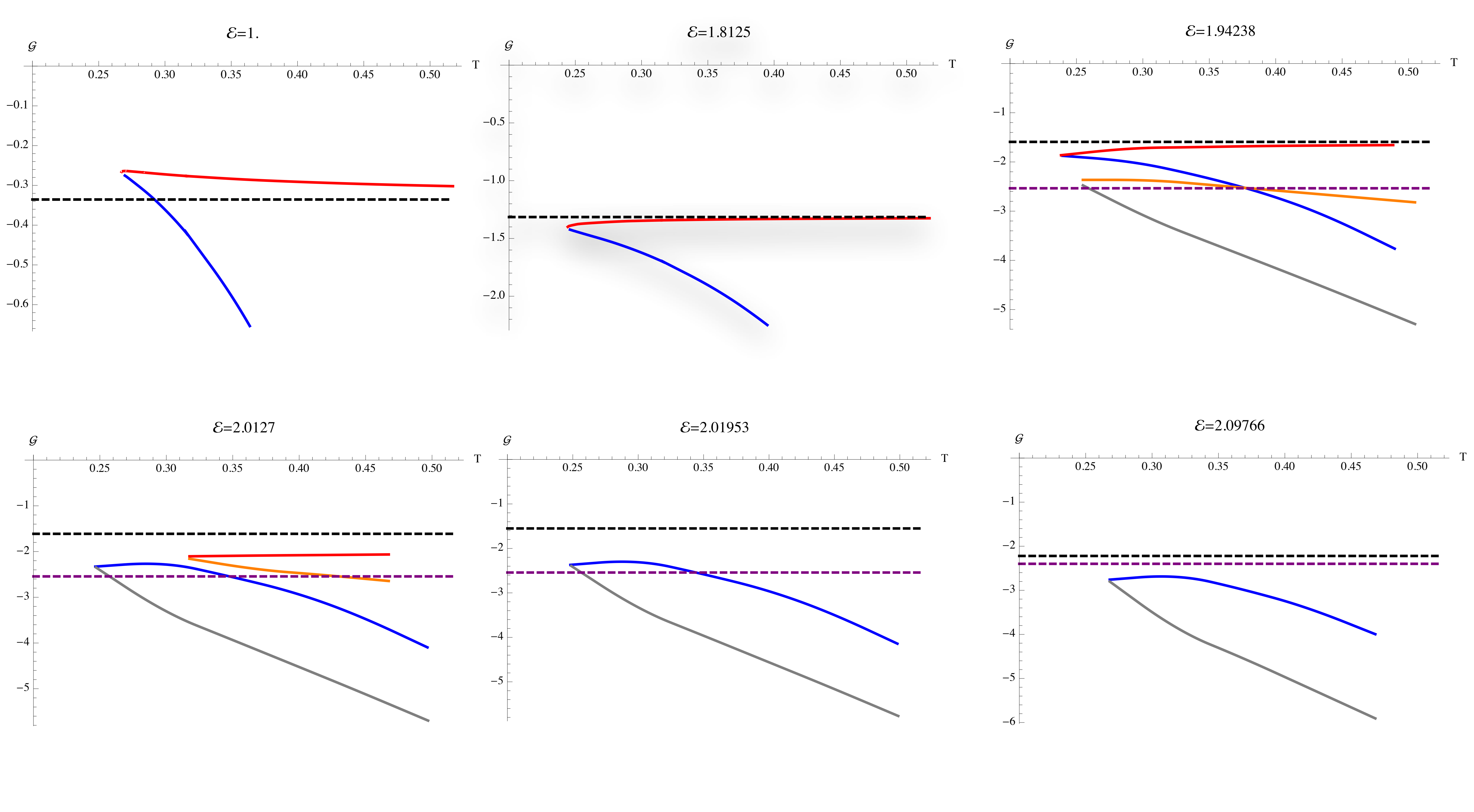
"large"



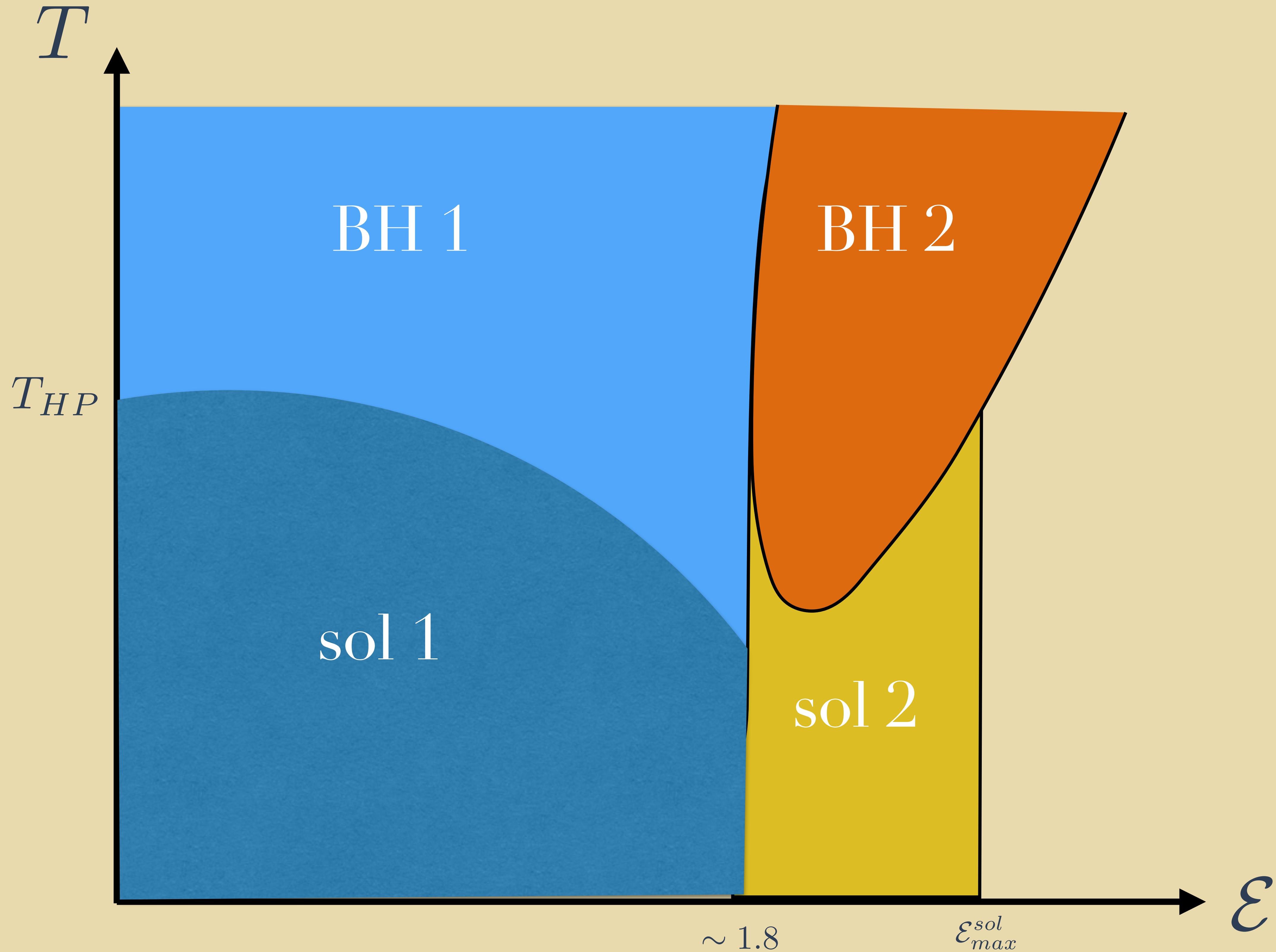



# Free Energy

$$\mathcal{G} = E - TS - 4\pi \int_0^{\pi/2} d\theta \sin\theta \, \rho(\theta) \, \mathcal{E} \cos\theta$$



# Phase diagram at strong Coupling
(a cartoon)



# Future Work

- study dynamical stability of BH.
- Can deformation of ABJM be simulated on a computer?
- Condensed Matter…

# Future Work

- study dynamical stability of BH.
- Can deformation of ABJM be simulated on a computer?
- Condensed Matter…

# Thank You

# Numerical Solutions

- 2 branches of soliton solutions varying electric field up to a maximum value
- 4 branches of black hole solutions varying electric field and temperature with maximum electric field and minimum temperature