Quasinormal modes: from high order hydrodynamics to nonconformal plasma

Romuald A. Janik

Jagiellonian University
Kraków

Outline

Introduction to quasinormal modes
- Small excitations of a uniform static (strongly coupled) plasma

What is the physics of (the higher) quasinormal modes?
- Example: Homogeneous isotropization

Quasinormal modes as a kind of UV completion of hydrodynamics

Quasinormal modes on top of a hydrodynamic background

Quasinormal modes and the range of applicability of hydrodynamics

Quasinormal modes in other setups
- Nonconformal plasma
- External magnetic fields
- Nonrelativistic systems

Conclusions

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
   - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

   - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
   - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

   - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
    - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

    - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
   - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

   - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
   - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

   - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
   - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

   - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What are quasinormal modes?

1. Within the AdS/CFT correspondence quasinormal modes correspond to poles in the retarded Green's function of some local operator (like the energy momentum tensor) evaluated at finite temperature
2. This corresponds to analyzing what kind of (collective) modes may propagate in the system at the linearized level
   - Typically we are interested in plane wave excitations

$$e^{-i\omega(k)t+ikx}$$

   - ... and determining the dispersion relation $\omega(k)$
3. Using the above interpretation one can also ask a more general question about linearized excitations above some dynamical background...
4. In this case the plane wave form will have to be modified...

What kind of excitations propagate in a static uniform strongly coupled plasma system?

Answer 1: Hydrodynamics

- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3 \sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Consider small perturbations

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

Answer 1: Hydrodynamics

- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Consider small perturbations

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

Answer 1: Hydrodynamics

- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^{μ}
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^{\mu}u^{\nu})}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+\underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Consider small perturbations

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

Answer 1: Hydrodynamics

- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Consider small perturbations

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

Answer 1: Hydrodynamics

- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Consider small perturbations

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:

  shear modes:

$$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$

  sound modes:

$$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$

- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i\frac{\eta}{E+p}k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}}k - i\frac{2}{3}\frac{\eta}{E+p}k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:

  shear modes:

$$\omega_{shear} = -i\frac{\eta}{E+p}k^2$$

  sound modes:

$$\omega_{sound} = \frac{1}{\sqrt{3}}k - i\frac{2}{3}\frac{\eta}{E+p}k^2$$

- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 1: Hydrodynamics

- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma ≡ small perturbations of the black hole metric (≡ quasinormal modes (QNM))

$$g_{\alpha\beta}^{5D} = g_{\alpha\beta}^{5D,black\ hole} + \delta g_{\alpha\beta}^{5D}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g^{5D}_{\alpha\beta} = g^{5D,black\ \ hole}_{\alpha\beta} + \delta g^{5D}_{\alpha\beta}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

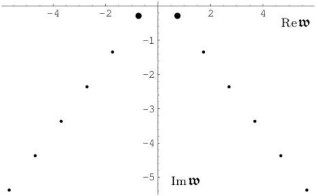
- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g_{\alpha\beta}^{5D} = g_{\alpha\beta}^{5D,black\ \ hole} + \delta g_{\alpha\beta}^{5D}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel



from Kovtun,Starinets hep-th/0506184

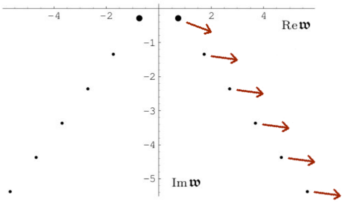
- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g^{5D}_{\alpha\beta} = g^{5D,black\ \ hole}_{\alpha\beta} + \delta g^{5D}_{\alpha\beta}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel



from Kovtun,Starinets hep-th/0506184

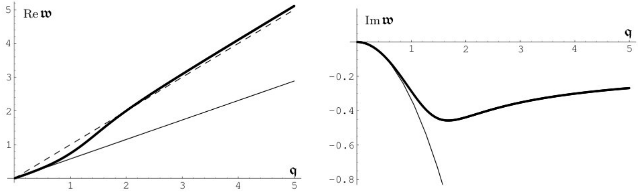
- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g^{5D}_{\alpha\beta} = g^{5D,black\ \ hole}_{\alpha\beta} + \delta g^{5D}_{\alpha\beta}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel



from Kovtun,Starinets hep-th/0506184

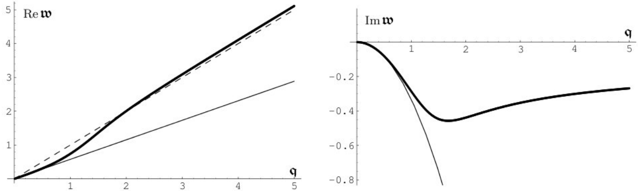
- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g_{\alpha\beta}^{5D} = g_{\alpha\beta}^{5D,black\ \ hole} + \delta g_{\alpha\beta}^{5D}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel



from Kovtun,Starinets hep-th/0506184

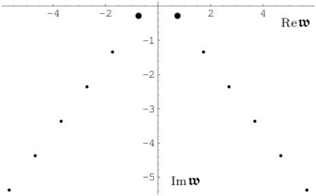
- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

Answer 2: AdS/CFT – gravitational description

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g^{5D}_{\alpha\beta} = g^{5D,black\ \ hole}_{\alpha\beta} + \delta g^{5D}_{\alpha\beta}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel



from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

What is the physics of (the higher) quasinormal modes?

Physics of higher QNMs

- QFT has much more degrees of freedom than only hydrodynamic ones...
- Suppose that we have a weakly coupled system with some distribution of momenta
- Hydrodynamics is essentially a description of the evolution of the local conserved charges of the system
- We can also ask questions about the evolution of the *shape* of the distribution — this will correspond to nonhydrodynamic quasinormal modes...

Simple system governed only by nonhydrodynamic modes...

Physics of higher QNMs

- QFT has much more degrees of freedom than only hydrodynamic ones...
- Suppose that we have a weakly coupled system with some distribution of momenta
- Hydrodynamics is essentially a description of the evolution of the local conserved charges of the system
- We can also ask questions about the evolution of the *shape* of the distribution — this will correspond to nonhydrodynamic quasinormal modes...

Simple system governed only by nonhydrodynamic modes...

Physics of higher QNMs

- QFT has much more degrees of freedom than only hydrodynamic ones...
- Suppose that we have a weakly coupled system with some distribution of momenta
- Hydrodynamics is essentially a description of the evolution of the local conserved charges of the system
- We can also ask questions about the evolution of the *shape* of the distribution — this will correspond to nonhydrodynamic quasinormal modes...

Simple system governed only by nonhydrodynamic modes...

Physics of higher QNMs

- QFT has much more degrees of freedom than only hydrodynamic ones...
- Suppose that we have a weakly coupled system with some distribution of momenta
- Hydrodynamics is essentially a description of the evolution of the local conserved charges of the system
- We can also ask questions about the evolution of the *shape* of the distribution — this will correspond to nonhydrodynamic quasinormal modes...

Simple system governed only by nonhydrodynamic modes...

Physics of higher QNMs

- QFT has much more degrees of freedom than only hydrodynamic ones...
- Suppose that we have a weakly coupled system with some distribution of momenta
- Hydrodynamics is essentially a description of the evolution of the local conserved charges of the system
- We can also ask questions about the evolution of the *shape* of the distribution — this will correspond to nonhydrodynamic quasinormal modes...

Simple system governed only by nonhydrodynamic modes...

Physics of higher QNMs

- QFT has much more degrees of freedom than only hydrodynamic ones...
- Suppose that we have a weakly coupled system with some distribution of momenta
- Hydrodynamics is essentially a description of the evolution of the local conserved charges of the system
- We can also ask questions about the evolution of the *shape* of the distribution — this will correspond to nonhydrodynamic quasinormal modes...

Simple system governed only by nonhydrodynamic modes...

Physics of higher QNMs — Homogeneous isotropization

- Consider spatially homogeneous plasma with anisotropic momentum distributions. This corresponds to a constant (in space) energy momentum tensor with anisotropic pressures
- The system will evolve in time with the energy-momentum tensor having the form

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Due to the symmetries of the system there is no flow — the setup is orthogonal to hydrodynamic degrees of freedom

The whole process of equilibration is due to degrees of freedom present in nonhydrodynamic quasinormal modes

- This has been first studied numerically in the context of a quench (time dependent gauge theory metric here) by Chesler, Yaffe.

Physics of higher QNMs — Homogeneous isotropization

- Consider spatially homogeneous plasma with anisotropic momentum distributions. This corresponds to a constant (in space) energy momentum tensor with anisotropic pressures
- The system will evolve in time with the energy-momentum tensor having the form

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Due to the symmetries of the system there is no flow — the setup is orthogonal to hydrodynamic degrees of freedom

  The whole process of equilibration is due to degrees of freedom present in nonhydrodynamic quasinormal modes

- This has been first studied numerically in the context of a quench (time dependent gauge theory metric here) by Chesler, Yaffe.

Physics of higher QNMs — Homogeneous isotropization

- Consider spatially homogeneous plasma with anisotropic momentum distributions. This corresponds to a constant (in space) energy momentum tensor with anisotropic pressures
- The system will evolve in time with the energy-momentum tensor having the form

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Due to the symmetries of the system there is no flow — the setup is orthogonal to hydrodynamic degrees of freedom

  The whole process of equilibration is due to degrees of freedom present in nonhydrodynamic quasinormal modes

- This has been first studied numerically in the context of a quench (time dependent gauge theory metric here) by Chesler, Yaffe.

Physics of higher QNMs — Homogeneous isotropization

- Consider spatially homogeneous plasma with anisotropic momentum distributions. This corresponds to a constant (in space) energy momentum tensor with anisotropic pressures
- The system will evolve in time with the energy-momentum tensor having the form

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Due to the symmetries of the system there is no flow — the setup is orthogonal to hydrodynamic degrees of freedom

The whole process of equilibration is due to degrees of freedom present in nonhydrodynamic quasinormal modes

- This has been first studied numerically in the context of a quench (time dependent gauge theory metric here) by Chesler, Yaffe.

Physics of higher QNMs — Homogeneous isotropization

- Consider spatially homogeneous plasma with anisotropic momentum distributions. This corresponds to a constant (in space) energy momentum tensor with anisotropic pressures
- The system will evolve in time with the energy-momentum tensor having the form

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Due to the symmetries of the system there is no flow — the setup is orthogonal to hydrodynamic degrees of freedom

The whole process of equilibration is due to degrees of freedom present in nonhydrodynamic quasinormal modes

- This has been first studied numerically in the context of a quench (time dependent gauge theory metric here) by Chesler, Yaffe.

Physics of higher QNMs — Homogeneous isotropization

- Consider spatially homogeneous plasma with anisotropic momentum distributions. This corresponds to a constant (in space) energy momentum tensor with anisotropic pressures
- The system will evolve in time with the energy-momentum tensor having the form

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p_{\parallel}(t) & 0 & 0 \\ 0 & 0 & p_{\perp}(t) & 0 \\ 0 & 0 & 0 & p_{\perp}(t) \end{pmatrix}$$

- Due to the symmetries of the system there is no flow — the setup is orthogonal to hydrodynamic degrees of freedom

  The whole process of equilibration is due to degrees of freedom present in nonhydrodynamic quasinormal modes

- This has been first studied numerically in the context of a quench (time dependent gauge theory metric here) by Chesler, Yaffe.

Physics of higher QNMs — Homogeneous isotropization

- Heller, Mateos, van der Schee, Triana made a detailed investigation of homogeneous isotropization from various initial conditions in terms of linearized/QNM approximations
- Relatively good description of nonlinear evolution was found using lowest 10 QNMs as well as linearized Einstein equations
- However the homogeneous isotropization setup is quite special as the energy is constant and one knows which QNMs to use...

QNMs on top of a hydrodynamic background?
(see later)

Physics of higher QNMs — Homogeneous isotropization

- Heller, Mateos, van der Schee, Triana made a detailed investigation of homogeneous isotropization from various initial conditions in terms of linearized/QNM approximations
- Relatively good description of nonlinear evolution was found using lowest 10 QNMs as well as linearized Einstein equations
- However the homogeneous isotropization setup is quite special as the energy is constant and one knows which QNMs to use...

QNMs on top of a hydrodynamic background?

(see later)

Physics of higher QNMs — Homogeneous isotropization

- Heller, Mateos, van der Schee, Triana made a detailed investigation of homogeneous isotropization from various initial conditions in terms of linearized/QNM approximations
- Relatively good description of nonlinear evolution was found using lowest 10 QNMs as well as linearized Einstein equations
- However the homogeneous isotropization setup is quite special as the energy is constant and one knows which QNMs to use...

QNMs on top of a hydrodynamic background?
(see later)

- Heller, Mateos, van der Schee, Triana made a detailed investigation of homogeneous isotropization from various initial conditions in terms of linearized/QNM approximations
- Relatively good description of nonlinear evolution was found using lowest 10 QNMs as well as linearized Einstein equations
- However the homogeneous isotropization setup is quite special as the energy is constant and one knows which QNMs to use...

QNMs on top of a hydrodynamic background?
(see later)

Physics of higher QNMs — Homogeneous isotropization

- Heller, Mateos, van der Schee, Triana made a detailed investigation of homogeneous isotropization from various initial conditions in terms of linearized/QNM approximations
- Relatively good description of nonlinear evolution was found using lowest 10 QNMs as well as linearized Einstein equations
- However the homogeneous isotropization setup is quite special as the energy is constant and one knows which QNMs to use...

QNMs on top of a hydrodynamic background?

(see later)

Comments

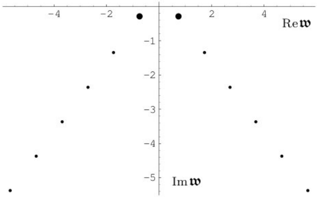
- A typical picture of the quasinormal spectrum of a black hole:

from Kovtun,Starinets hep-th/0506184

- A characteristic generic feature is the nonzero **real part**
- In contrast on the field theory side (weak coupling perturbation theory, the Boltzmann equation), the decaying modes are purely imaginary...
- Challenge for lattice QCD!
- Other systems: **cold atoms**: see talk by P. Romatschke

Comments

- A typical picture of the quasinormal spectrum of a black hole:



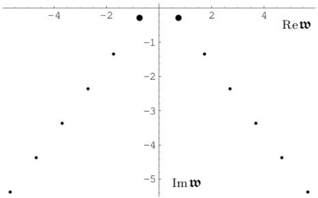
from Kovtun,Starinets hep-th/0506184

- A characteristic generic feature is the nonzero **real part**
- In contrast on the field theory side (weak coupling perturbation theory, the Boltzmann equation), the decaying modes are purely imaginary...
- Challenge for lattice QCD!
- Other systems: **cold atoms**: see talk by P. Romatschke

Physics of higher QNMs

Comments

- A typical picture of the quasinormal spectrum of a black hole:



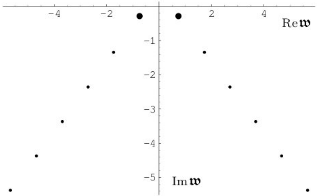
from Kovtun,Starinets hep-th/0506184

- A characteristic generic feature is the nonzero **real part**
- In contrast on the field theory side (weak coupling perturbation theory, the Boltzmann equation), the decaying modes are purely imaginary...
- Challenge for lattice QCD!
- Other systems: **cold atoms**: see talk by P. Romatschke

Physics of higher QNMs

Comments

- A typical picture of the quasinormal spectrum of a black hole:



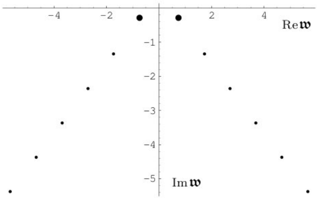
from Kovtun,Starinets hep-th/0506184

- A characteristic generic feature is the nonzero **real part**
- In contrast on the field theory side (weak coupling perturbation theory, the Boltzmann equation), the decaying modes are purely imaginary...
- Challenge for lattice QCD!
- Other systems: **cold atoms**: see talk by P. Romatschke

Physics of higher QNMs

Comments

- A typical picture of the quasinormal spectrum of a black hole:



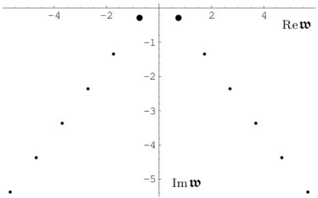
from Kovtun,Starinets hep-th/0506184

- A characteristic generic feature is the nonzero **real part**
- In contrast on the field theory side (weak coupling perturbation theory, the Boltzmann equation), the decaying modes are purely imaginary...
- Challenge for lattice QCD!
- Other systems: **cold atoms**: see talk by P. Romatschke

Physics of higher QNMs

Comments

- A typical picture of the quasinormal spectrum of a black hole:



from Kovtun,Starinets hep-th/0506184

- A characteristic generic feature is the nonzero **real part**
- In contrast on the field theory side (weak coupling perturbation theory, the Boltzmann equation), the decaying modes are purely imaginary...
- Challenge for lattice QCD!
- Other systems: **cold atoms**: see talk by P. Romatschke

Quasinormal modes as a kind of UV completion of hydrodynamics

Suppose that we know all high order hydrodynamic transport coefficients...

Are the nonhydrodynamic higher QNMs determined or arbitrary?

Quasinormal modes as a kind of UV completion of hydrodynamics

Suppose that we know all high order hydrodynamic transport coefficients...

Are the nonhydrodynamic higher QNMs determined or arbitrary?

Quasinormal modes as a kind of UV completion of hydrodynamics

Suppose that we know all high order hydrodynamic transport coefficients...

Are the nonhydrodynamic higher QNMs determined or arbitrary?

Quasinormal modes as a kind of UV completion of hydrodynamics

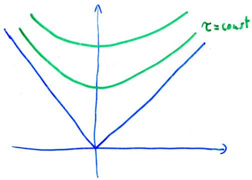
Suppose that we know all high order hydrodynamic transport coefficients...

Are the nonhydrodynamic higher QNMs determined or arbitrary?

Boost-invariant flow

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

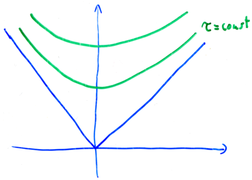
$$\frac{1}{\tau^{\frac{2}{3}}}$$

- A new combination of transport coefficients appears at each order in the above expansion

Boost-invariant flow

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



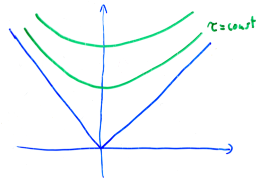
- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

$$\frac{1}{\tau^{\frac{2}{3}}}$$

- A new combination of transport coefficients appears at each order in the above expansion

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

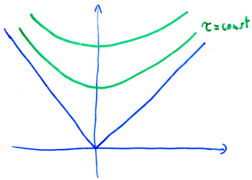
$$\frac{1}{\tau^{\frac{2}{3}}}$$

- A new combination of transport coefficients appears at each order in the above expansion

Boost-invariant flow

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



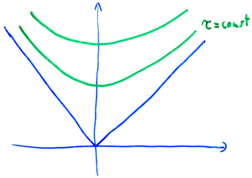
- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

$$\frac{1}{\tau^{\frac{2}{3}}}$$

- A new combination of transport coefficients appears at each order in the above expansion

Boost-invariant flow

Bjorken '83
Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_{\mu} T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

$$\frac{1}{\tau^{\frac{2}{3}}}$$

- A new combination of transport coefficients appears at each order in the above expansion

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24\log 2 - 24\log^2 2}{324 \cdot 2^{\frac{1}{2}} 3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}}\frac{1}{\tau^{2}} + \frac{1+2\log 2}{12\sqrt{3}}\frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}}\frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24\log 2 - 24\log^2 2}{324 \cdot 2^{\frac{1}{2}} 3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}}\frac{1}{\tau^{2}} + \frac{1+2\log 2}{12\sqrt{3}}\frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}}\frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24\log 2 - 24\log^2 2}{324 \cdot 2^{\frac{1}{2}} 3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24\log 2 - 24\log^2 2}{324 \cdot 2^{\frac{1}{2}} 3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1 + 2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3 + 2\pi^2 + 24\log 2 - 24\log^2 2}{324 \cdot 2^{\frac{1}{2}} 3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term — perfect fluid behaviour
  second term — 1^{st} order viscous hydrodynamics
  third term — 2^{nd} order viscous hydrodynamics
  fourth term — 3^{rd} order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

- At each order new transport coefficients start to contribute to ε_n...

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations

• to get to so high orders we need very high precision computations

• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

Large τ behaviour of $\varepsilon(\tau)$

- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

Heller, RJ, Witaszczyk

$$\varepsilon(\tau) = \sum_{n=2}^{242} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

• chief complication – generate the r.h.s. of the equations
• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

- Introduce $u \equiv 1/\tau^{2/3}$

$$\varepsilon(u) = \sum_{n=2}^{242} \varepsilon_n u^n$$

- The coefficients of this series are determined only by hydrodynamic transport coefficients and a single 'initial condition'

Question:

Can we infer anything about the nonhydrodynamic higher quasinormal modes just from this series?

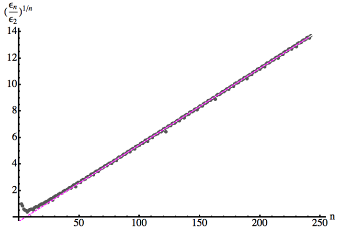
- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$



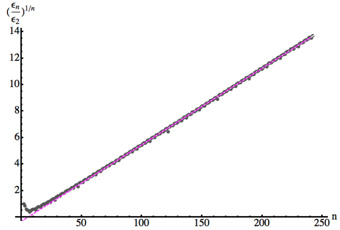
- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$



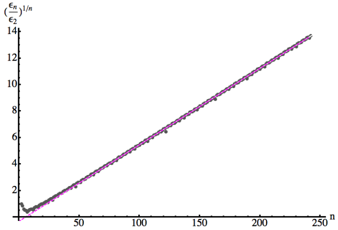
- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$



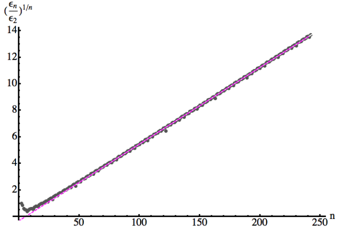
- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$



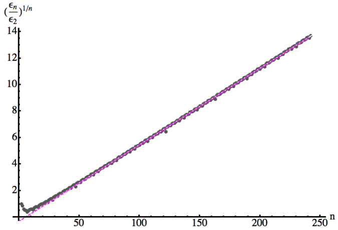
- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$



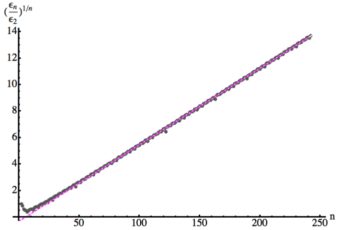
- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$



- The coefficients grow factorially $\varepsilon_n \sim n!$
- Zero radius of convergence $\longrightarrow$ asymptotic series...

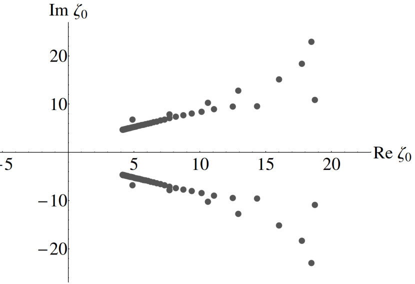
1. Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

2. Identify singularities of the Borel transform $\tilde{\varepsilon}(u)$ (Pade approximant)
3. Interpret the meaning of the singularities through the inverse Borel transform

$$\varepsilon_{inverse\ Borel}(u) = \int_C e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$

Singularities in the Borel plane

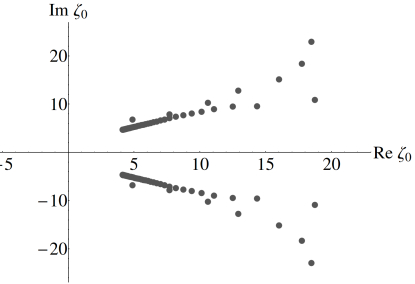


- Branch cuts on the Borel plane
- Branch points set the radius of convergence of the Borel transform

Question:

What is the physical interpretation of the branch cut singularities?

Singularities in the Borel plane

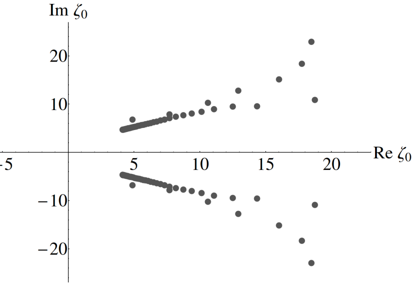


- Branch cuts on the Borel plane
- Branch points set the radius of convergence of the Borel transform

Question:

What is the physical interpretation of the branch cut singularities?

Singularities in the Borel plane

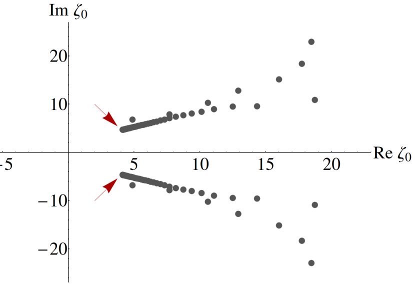


- Branch cuts on the Borel plane
- Branch points set the radius of convergence of the Borel transform

Question:

What is the physical interpretation of the branch cut singularities?

Singularities in the Borel plane

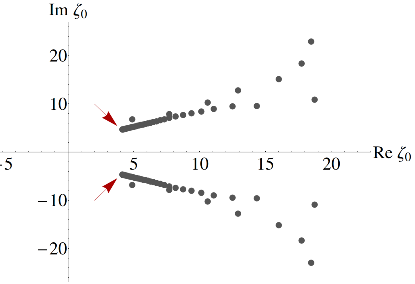


- Branch cuts on the Borel plane
- Branch points set the radius of convergence of the Borel transform

Question:

What is the physical interpretation of the branch cut singularities?

Singularities in the Borel plane

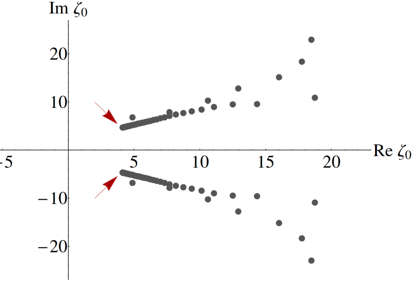


- Branch cuts on the Borel plane
- Branch points set the radius of convergence of the Borel transform

Question:

What is the physical interpretation of the branch cut singularities?

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

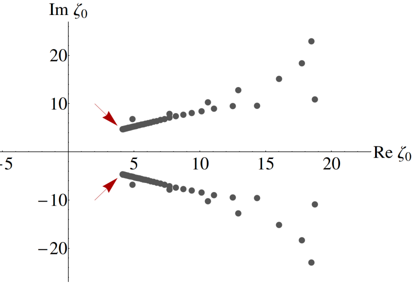
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\, \tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const \cdot \pi T t}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i \underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM} \int \underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}} d\tau = \underbrace{-i \frac{3}{2}(3.1195 - 2.7467\, i)\, \tau^{\frac{2}{3}}}_{-4.12005-4.67925\, i}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^{\infty} e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

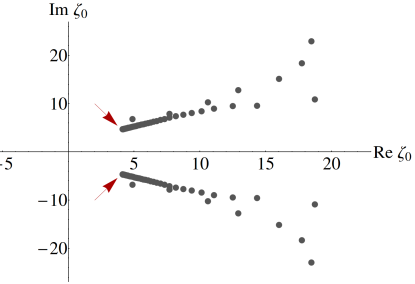
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\, \tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const\cdot \pi T t}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i \underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM} \int \underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}} d\tau = \underbrace{-i \frac{3}{2} (3.1195 - 2.7467\, i)}_{-4.12005-4.67925\, i} \tau^{\frac{2}{3}}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

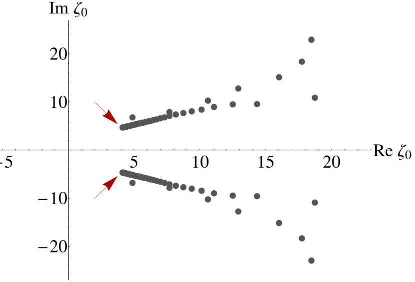
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\, \tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const \cdot \pi T t}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i \underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM} \int \underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}} d\tau = \underbrace{-i \frac{3}{2}(3.1195 - 2.7467\, i)\, \tau^{\frac{2}{3}}}_{-4.12005-4.67925\, i}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

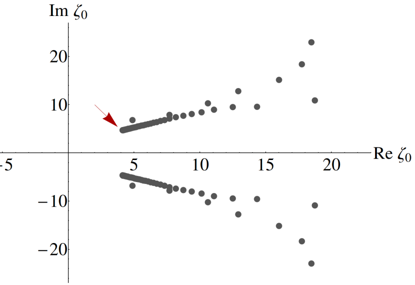
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\,i$) will contribute as

$$e^{-(4.12065+4.67895\,i)\,\tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const\cdot\pi Tt}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i\underbrace{(3.1195 - 2.7467\,i)}_{planar\ BH\ QNM}\int\underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}}d\tau = \underbrace{-i\frac{3}{2}(3.1195 - 2.7467\,i)}_{-4.12005-4.67925\,i}\tau^{\frac{2}{3}}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

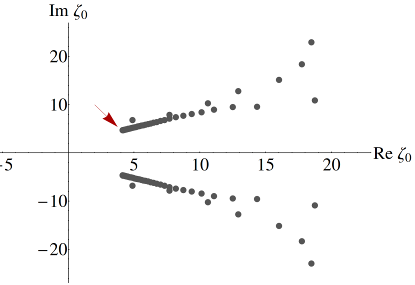
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\, \tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const\cdot \pi T t}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i \underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM} \int \underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}} d\tau = \underbrace{-i\frac{3}{2}(3.1195 - 2.7467\, i)\, \tau^{\frac{2}{3}}}_{-4.12005-4.67925\, i}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta / \tau^{\frac{2}{3}}\right)\, d\zeta$$

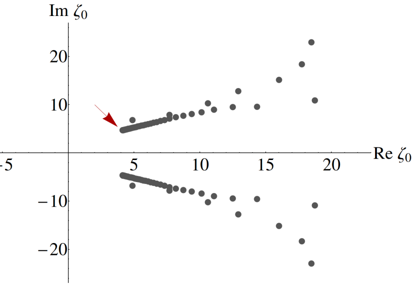
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\, \tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const \cdot \pi\, Tt}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i \underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM} \int \underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}} d\tau = \underbrace{-i \frac{3}{2}(3.1195 - 2.7467\, i)}_{-4.12005 - 4.67925\, i} \tau^{\frac{2}{3}}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta}\tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right)\, d\zeta$$

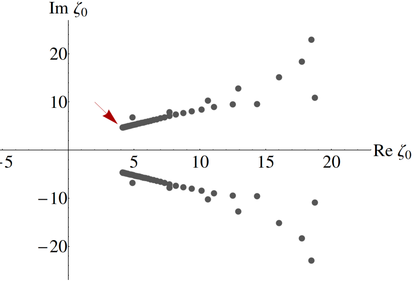
- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\,\tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const\cdot\pi\, Tt}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i\underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM}\int\underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}}d\tau = \underbrace{-i\frac{3}{2}(3.1195 - 2.7467\, i)}_{-4.12005-4.67925\, i}\tau^{\frac{2}{3}}$$

Singularities in the Borel plane



- Deform the contour of the inverse Borel transform

$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta}\tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right)\, d\zeta$$

- The pole at the edge of the cut ($\zeta_0 = 4.12065 + 4.67895\, i$) will contribute as

$$e^{-(4.12065+4.67895\, i)\,\tau^{\frac{2}{3}}}$$

- This is exactly the first lowest non-hydrodynamic quasi-normal mode! but recall $e^{-const\cdot\pi Tt}$
- It is simply related to the scalar QNM of the planar black hole through RJ, Peschanski

$$-i\underbrace{(3.1195 - 2.7467\, i)}_{planar\ BH\ QNM}\int\underbrace{\pi T(\tau)}_{1/\tau^{\frac{1}{3}}}\, d\tau = \underbrace{-i\frac{3}{2}(3.1195 - 2.7467\, i)\,\tau^{\frac{2}{3}}}_{-4.12005-4.67925\, i}$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?

- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

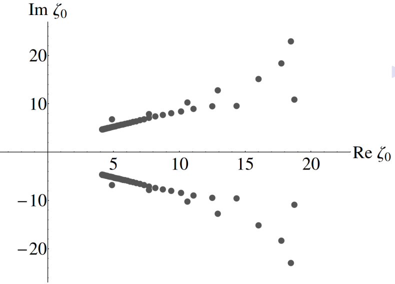
$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right) \longrightarrow \int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2} e^{-i\omega_{QNM}\int \pi T(\tau) d\tau} \longrightarrow \quad -2 - i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an ‘adiabatic’ approximation

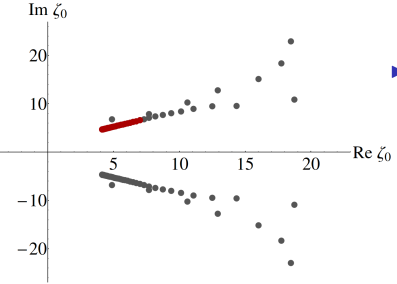
$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right) \longrightarrow \int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2} e^{-i\omega_{QNM}\int \pi T(\tau)d\tau} \longrightarrow \quad -2 - i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i}\cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

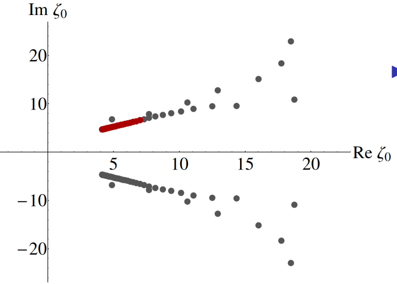
$$\pi T(\tau)=\frac{1}{\tau^{\frac{1}{3}}}\left(1-\frac{1}{6\tau^{\frac{2}{3}}}+\ldots\right)\longrightarrow\int\pi T(\tau)d\tau\sim\frac{3}{2}\tau^{\frac{2}{3}}-\frac{1}{6}\log\tau+\ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2}e^{-i\omega_{QNM}\int\pi T(\tau)d\tau}\longrightarrow\quad -2-i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right)=-1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i}\cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

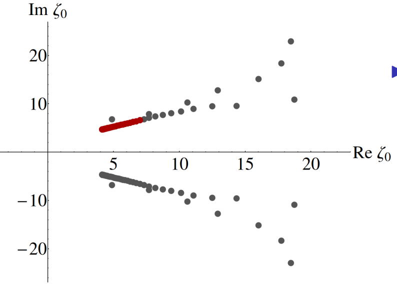
$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \dots\right) \longrightarrow \int \pi T(\tau)d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \dots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2}e^{-i\omega_{QNM}\int \pi T(\tau)d\tau} \longrightarrow -2 - i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

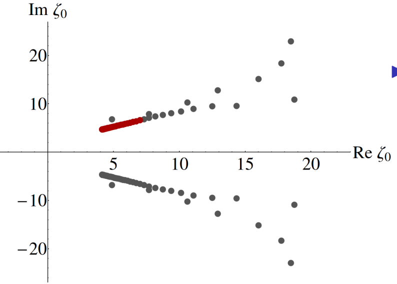
$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right) \longrightarrow \int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2} e^{-i\omega_{QNM}\int \pi T(\tau)d\tau} \longrightarrow \quad -2-i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

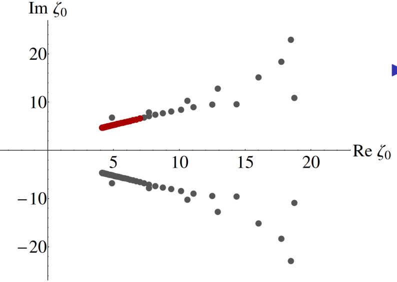
$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right) \longrightarrow \int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2} e^{-i\omega_{QNM}\int \pi T(\tau) d\tau} \longrightarrow \quad -2-i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

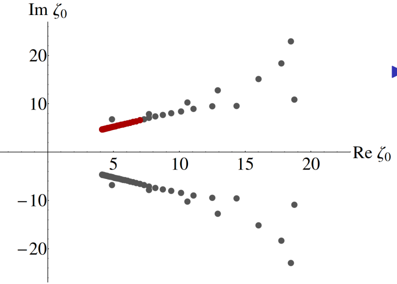
$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right) \longrightarrow \int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2} e^{-i\omega_{QNM}\int \pi T(\tau) d\tau} \longrightarrow \quad -2 - i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Singularities in the Borel plane

What is the interpretation of the whole branch cut?



- From the marked cut we obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The preexponential power law can still be understood in terms of an 'adiabatic' approximation

$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right) \longrightarrow \int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

- The QNM including viscous corrections follows from the simple formula

$$\tau^{-2} e^{-i\omega_{QNM}\int \pi T(\tau)d\tau} \rightarrow \quad -2-i\underbrace{(3.1195-2.7467i)}_{\omega_{QNM}}\cdot\left(-\tfrac{1}{6}\right) = -1.54222+0.519917i$$

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Comments

- The explicit (high order) hydrodynamic transport coefficients in $\mathcal{N} = 4$ SYM are linked with **nonhydrodynamic** degrees of freedom
- Once we include hydrodynamics, the higher quasinormal modes/degrees of freedom have to be included for consistency
- The setup is surprisingly akin to the relation of perturbation theory and instantons in QFT...
- People started to analyze resurgence properties of (phenomenological) partially resummed hydrodynamic models

Heller, Spaliński; Aniceto, Spaliński

- The hydrodynamic QNMs appear here as branch cuts
- This makes it difficult to see the next highest QNMs

There exists another setup where this difficulty is not present...

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

from 1603.05344

- Agrees very well with the first 10 QNM!

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane

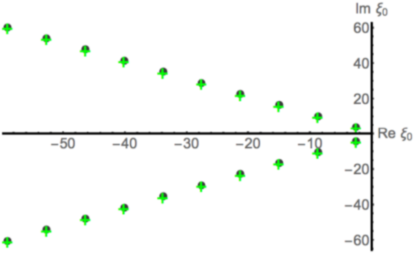
from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane



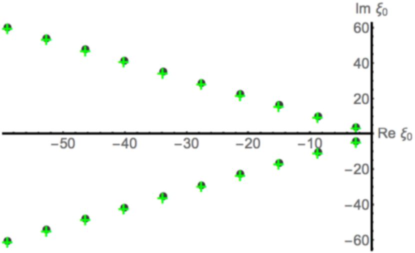
from 1603.05344

- Agrees very well with the first 10 QNM!

Plasma in a FRW background

Buchel, Heller, Noronha

- The authors considered $\mathcal{N} = 2^*$ plasma in an expanding FRW universe
- Hydrodynamic entropy production in terms of scale factor
- The obtained series is asymptotic $\longrightarrow$ Borel transform $\longrightarrow$ Pade approximant $\longrightarrow$ singularities in the Borel plane



from 1603.05344

- Agrees very well with the first 10 QNM!

Quasinormal modes on top of a hydrodynamic background

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

Quasinormal modes on top of a hydrodynamic background

- In the boost invariant setting we saw that the 4D spacetime dependence of a QNM is no longer a plane wave...
- It is interesting to understand QNMs in various time-dependent settings — of course QNM's have to be understood in a slightly more general sense than usual...
- A classic example would be QNM on top of a generic hydrodynamic fluid background
- This may be important for practical applications as the typical configurations of plasma produced in experiments are very far from being uniform and static
- In this sense homogenous isotropization is a very much nongeneric setup which can be very cleanly understood in terms of QNMs on top of a static black hole — typically we have to deal with a mixture of hydrodynamic and nonhydrodynamic modes
- In general time-dependent backgrounds very little is known about QNMs...

A simple scalar example

Heller, RJ, Spaliński, Witaszczyk

- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$)
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by *pairs* of quasinormal modes:

from Kovtun,Starinets hep-th/0506184

- We will focus on the lowest (least damped) modes characterized by

$$\omega = \pm \omega_R(k) T - i \omega_I(k) T$$

where we extracted a trivial scaling with the temperature

A simple scalar example

Heller, RJ, Spaliński, Witaszczyk

- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$)
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by *pairs* of quasinormal modes:

from Kovtun,Starinets hep-th/0506184

- We will focus on the lowest (least damped) modes characterized by

$$\omega = \pm\omega_R(k)T - i\omega_I(k)T$$

where we extracted a trivial scaling with the temperature

A simple scalar example

Heller, RJ, Spaliński, Witaszczyk

- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$)
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by *pairs* of quasinormal modes:

from Kovtun,Starinets hep-th/0506184

- We will focus on the lowest (least damped) modes characterized by

$$\omega = \pm\omega_R(k)T - i\omega_I(k)T$$

where we extracted a trivial scaling with the temperature

A simple scalar example

Heller, RJ, Spaliński, Witaszczyk

- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$)
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by *pairs* of quasinormal modes:

from Kovtun,Starinets hep-th/0506184

- We will focus on the lowest (least damped) modes characterized by

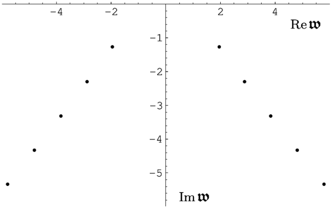
$$\omega = \pm\omega_R(k)T - i\omega_I(k)T$$

where we extracted a trivial scaling with the temperature

A simple scalar example

Heller, RJ, Spaliński, Witaszczyk

- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$)
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by *pairs* of quasinormal modes:



from Kovtun,Starinets hep-th/0506184

- We will focus on the lowest (least damped) modes characterized by

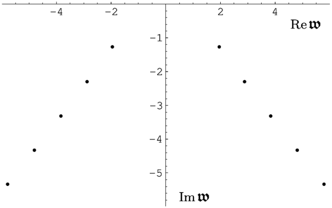
$$\omega = \pm\omega_R(k)T - i\omega_I(k)T$$

where we extracted a trivial scaling with the temperature

A simple scalar example

Heller, RJ, Spaliński, Witaszczyk

- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$)
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by *pairs* of quasinormal modes:



from Kovtun,Starinets hep-th/0506184

- We will focus on the lowest (least damped) modes characterized by

$$\omega = \pm\omega_R(k)T - i\omega_I(k)T$$

where we extracted a trivial scaling with the temperature

A simple scalar example

- The general 4D behaviour of the scalar expectation value is

$$\phi(t,x) = \int d^3k\, \alpha(k)\, e^{-\omega_I Tt} \cos(\omega_R Tt + \beta(k))\, e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- Despite appearances this is really a PDE — the dependence on spatial coordinates sits in $\omega_{QNM}(k)$ with $k^2 \to -\Delta_\perp$
- E.g. $\omega_I = 0$ and $\omega_R(k) = \frac{1}{T}\sqrt{m^2 + k^2}$ would give the standard Klein-Gordon equation

A simple scalar example

- The general 4D behaviour of the scalar expectation value is

$$\phi(t,x) = \int d^3k\, \alpha(k)\, e^{-\omega_I Tt} \cos(\omega_R Tt + \beta(k))\, e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- Despite appearances this is really a PDE — the dependence on spatial coordinates sits in $\omega_{QNM}(k)$ with $k^2 \to -\Delta_\perp$
- E.g. $\omega_I = 0$ and $\omega_R(k) = \frac{1}{T}\sqrt{m^2 + k^2}$ would give the standard Klein-Gordon equation

A simple scalar example

- The general 4D behaviour of the scalar expectation value is

$$\phi(t,x) = \int d^3k\, \alpha(k)\, e^{-\omega_I T t} \cos(\omega_R T t + \beta(k))\, e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- Despite appearances this is really a PDE — the dependence on spatial coordinates sits in $\omega_{QNM}(k)$ with $k^2 \to -\Delta_\perp$
- E.g. $\omega_I = 0$ and $\omega_R(k) = \frac{1}{T}\sqrt{m^2+k^2}$ would give the standard Klein-Gordon equation

A simple scalar example

- The general 4D behaviour of the scalar expectation value is

$$\phi(t,x) = \int d^3k\, \alpha(k)\, e^{-\omega_I Tt} \cos(\omega_R Tt + \beta(k))\, e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- Despite appearances this is really a PDE — the dependence on spatial coordinates sits in $\omega_{QNM}(k)$ with $k^2 \to -\Delta_\perp$
- E.g. $\omega_I = 0$ and $\omega_R(k) = \frac{1}{T}\sqrt{m^2 + k^2}$ would give the standard Klein-Gordon equation

A simple scalar example

- The general 4D behaviour of the scalar expectation value is

$$\phi(t,x) = \int d^3k\, \alpha(k)\, e^{-\omega_I Tt} \cos(\omega_R Tt + \beta(k))\, e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- Despite appearances this is really a PDE — the dependence on spatial coordinates sits in $\omega_{QNM}(k)$ with $k^2 \to -\Delta_\perp$
- E.g. $\omega_I = 0$ and $\omega_R(k) = \frac{1}{T}\sqrt{m^2 + k^2}$ would give the standard Klein-Gordon equation

A simple scalar example

- The general 4D behaviour of the scalar expectation value is

$$\phi(t,x) = \int d^3k\,\alpha(k)\,e^{-\omega_I Tt}\cos(\omega_R Tt + \beta(k))\,e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I\frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- Despite appearances this is really a PDE — the dependence on spatial coordinates sits in $\omega_{QNM}(k)$ with $k^2 \to -\Delta_\perp$
- E.g. $\omega_I = 0$ and $\omega_R(k) = \frac{1}{T}\sqrt{m^2 + k^2}$ would give the standard Klein-Gordon equation

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):

from Kovtun,Starinets hep-th/0506184

- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):

from Kovtun,Starinets hep-th/0506184

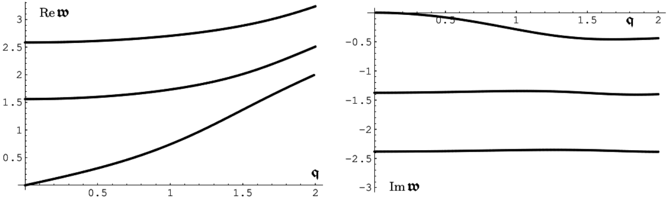
- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):



from Kovtun,Starinets hep-th/0506184

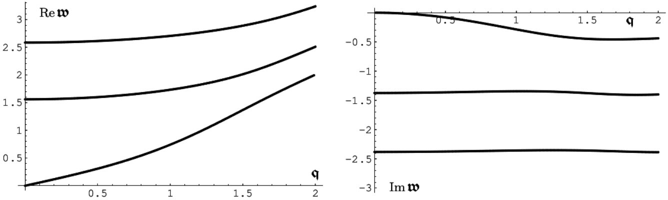
- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I\frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):



from Kovtun,Starinets hep-th/0506184

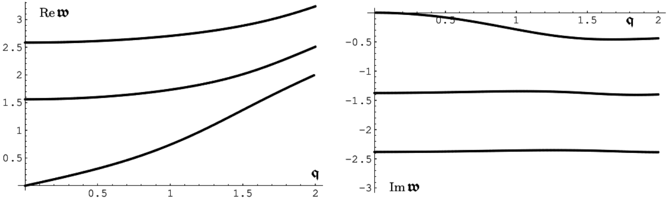
- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):



from Kovtun,Starinets hep-th/0506184

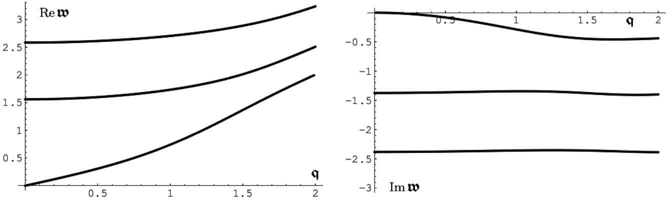
- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):



from Kovtun,Starinets hep-th/0506184

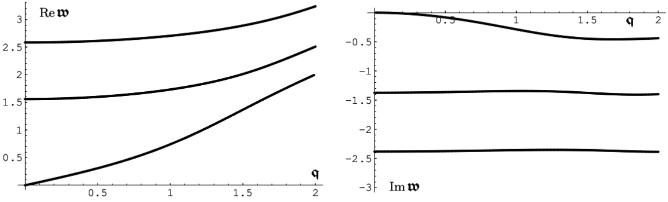
- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

Dispersion relation (in sound channel):



from Kovtun,Starinets hep-th/0506184

- **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics becomes **'ultralocal'**
- Damping occurs locally (and independently) in space...

Proceed to general hydrodynamic flows...

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}^W_\mu \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w \mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \tfrac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}^W_\mu \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w \mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \tfrac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}_\mu^W \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w \mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \frac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}^W_\mu \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w\mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \frac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}^W_\mu \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w \mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \tfrac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}^W_\mu \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w \mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \frac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

A simple scalar example

$$\frac{1}{T}\frac{d}{dt}\frac{1}{T}\frac{d}{dt}\phi + 2\omega_I \frac{1}{T}\frac{d}{dt}\phi + \left(\omega_I^2 + \omega_R^2\right)\phi = 0$$

- The equation in a generic hydrodynamic background should *locally* look like the equation above...
- We should 'covariantize' the time derivative...

$$\frac{1}{T}\frac{d}{dt} \longrightarrow \mathcal{D} \equiv \frac{1}{T} u^\mu \partial_\mu$$

- This should be true only up to terms involving $\partial_\mu u^\nu$ which are not fixed by this argument...
- We propose to include first gradient corrections by imposing Weyl covariance... (hence we asssume a conformal theory)

c.f. Loganayagam

- We define $\mathcal{D}$ in terms of the 'Weyl derivative':

$$\mathcal{D}\phi \equiv \frac{1}{T} u^\mu \mathcal{D}^W_\mu \phi = \frac{1}{T} u^\mu (\partial_\mu \phi + w \mathcal{A}_\mu \phi)$$

where w is the Weyl weight of the field and

$$\mathcal{A}_\mu = u^\lambda \partial_\lambda u_\mu - \tfrac{1}{3}(\partial_\lambda u^\lambda) u_\mu$$

Metric QNM

- For the transverse traceless tensor $\Pi^{\mu\nu}_{noneq}$, we proposed the following equation

$$\frac{1}{T}\mathcal{D}^W\frac{1}{T}\mathcal{D}^W\Pi^{\mu\nu}_{noneq} + 2\omega_I\frac{1}{T}\mathcal{D}^W\Pi^{\mu\nu}_{noneq} + \left(\omega_I^2 + \omega_R^2\right)\Pi^{\mu\nu}_{noneq} = 0$$

- We checked that the new terms introduced by the Weyl derivative were necessary for correctly reproducing the linear perturbations on top of the *viscous* boost-invariant geometry
- We proposed a phenomenological way to couple the above equation to hydrodynamics in order to incorporate the lowest nonhydrodynamic degrees of freedom into a common 4D description... Heller, RJ, Spaliński, Witaszczyk

Challenge:

A general theory of QNMs over fluid backgrounds is still needed...

Metric QNM

- For the transverse traceless tensor $\Pi^{\mu\nu}_{noneq}$, we proposed the following equation

$$\frac{1}{T}\mathcal{D}^W \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + 2\omega_I \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + \left(\omega_I^2 + \omega_R^2\right) \Pi^{\mu\nu}_{noneq} = 0$$

- We checked that the new terms introduced by the Weyl derivative were necessary for correctly reproducing the linear perturbations on top of the *viscous* boost-invariant geometry
- We proposed a phenomenological way to couple the above equation to hydrodynamics in order to incorporate the lowest nonhydrodynamic degrees of freedom into a common 4D description... Heller, RJ, Spaliński, Witaszczyk

Challenge:

A general theory of QNMs over fluid backgrounds is still needed...

Metric QNM

- For the transverse traceless tensor $\Pi^{\mu\nu}_{noneq}$, we proposed the following equation

$$\frac{1}{T}\mathcal{D}^W \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + 2\omega_I \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + \left(\omega_I^2 + \omega_R^2\right) \Pi^{\mu\nu}_{noneq} = 0$$

- We checked that the new terms introduced by the Weyl derivative were necessary for correctly reproducing the linear perturbations on top of the *viscous* boost-invariant geometry
- We proposed a phenomenological way to couple the above equation to hydrodynamics in order to incorporate the lowest nonhydrodynamic degrees of freedom into a common 4D description... Heller, RJ, Spaliński, Witaszczyk

Challenge:

A general theory of QNMs over fluid backgrounds is still needed...

Metric QNM

- For the transverse traceless tensor $\Pi^{\mu\nu}_{noneq}$, we proposed the following equation

$$\frac{1}{T}\mathcal{D}^W \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + 2\omega_I \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + \left(\omega_I^2 + \omega_R^2\right) \Pi^{\mu\nu}_{noneq} = 0$$

- We checked that the new terms introduced by the Weyl derivative were necessary for correctly reproducing the linear perturbations on top of the *viscous* boost-invariant geometry
- We proposed a phenomenological way to couple the above equation to hydrodynamics in order to incorporate the lowest nonhydrodynamic degrees of freedom into a common 4D description... Heller, RJ, Spaliński, Witaszczyk

Challenge:

A general theory of QNMs over fluid backgrounds is still needed...

Metric QNM

- For the transverse traceless tensor $\Pi^{\mu\nu}_{noneq}$, we proposed the following equation

$$\frac{1}{T}\mathcal{D}^W\frac{1}{T}\mathcal{D}^W\Pi^{\mu\nu}_{noneq} + 2\omega_I\frac{1}{T}\mathcal{D}^W\Pi^{\mu\nu}_{noneq} + \left(\omega_I^2+\omega_R^2\right)\Pi^{\mu\nu}_{noneq} = 0$$

- We checked that the new terms introduced by the Weyl derivative were necessary for correctly reproducing the linear perturbations on top of the *viscous* boost-invariant geometry
- We proposed a phenomenological way to couple the above equation to hydrodynamics in order to incorporate the lowest nonhydrodynamic degrees of freedom into a common 4D description... Heller, RJ, Spaliński, Witaszczyk

Challenge:

A general theory of QNMs over fluid backgrounds is still needed...

Metric QNM

- For the transverse traceless tensor $\Pi^{\mu\nu}_{noneq}$, we proposed the following equation

$$\frac{1}{T}\mathcal{D}^W \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + 2\omega_I \frac{1}{T}\mathcal{D}^W \Pi^{\mu\nu}_{noneq} + \left(\omega_I^2 + \omega_R^2\right) \Pi^{\mu\nu}_{noneq} = 0$$

- We checked that the new terms introduced by the Weyl derivative were necessary for correctly reproducing the linear perturbations on top of the *viscous* boost-invariant geometry
- We proposed a phenomenological way to couple the above equation to hydrodynamics in order to incorporate the lowest nonhydrodynamic degrees of freedom into a common 4D description... Heller, RJ, Spaliński, Witaszczyk

Challenge:

A general theory of QNMs over fluid backgrounds is still needed...

Quasinormal modes and the range of applicability of hydrodynamics

The range of applicability of hydrodynamics

- Once we know the (complex) dispersion relation of all modes we can ask whether for all momenta k, the hydrodynamic modes are less damped than the higher QNM's
- In the **conformal** case in the sound channel this is always the case:

from Kovtun,Starinets hep-th/0506184

- However Landsteiner discovered that in the shear channel, the hydrodynamic mode becomes more damped than the nonhydro mode for $q = \frac{k}{2\pi T} > 1.3$

from Landsteiner 1202.3550

The range of applicability of hydrodynamics

- Once we know the (complex) dispersion relation of all modes we can ask whether for all momenta k, the hydrodynamic modes are less damped than the higher QNM's
- In the **conformal** case in the sound channel this is always the case:

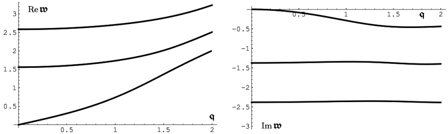
from Kovtun,Starinets hep-th/0506184

- However Landsteiner discovered that in the shear channel, the hydrodynamic mode becomes more damped than the nonhydro mode for $q = \frac{k}{2\pi T} > 1.3$

from Landsteiner 1202.3550

The range of applicability of hydrodynamics

- Once we know the (complex) dispersion relation of all modes we can ask whether for all momenta k, the hydrodynamic modes are less damped than the higher QNM's
- In the **conformal** case in the sound channel this is always the case:



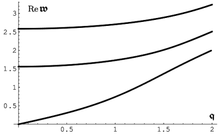
from Kovtun,Starinets hep-th/0506184

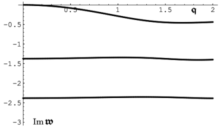
- However Landsteiner discovered that in the shear channel, the hydrodynamic mode becomes more damped than the nonhydro mode for $q = \frac{k}{2\pi T} > 1.3$

from Landsteiner 1202.3550

The range of applicability of hydrodynamics

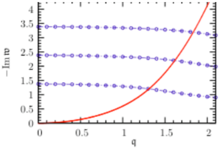
- Once we know the (complex) dispersion relation of all modes we can ask whether for all momenta k, the hydrodynamic modes are less damped than the higher QNM's
- In the **conformal** case in the sound channel this is always the case:





from Kovtun,Starinets hep-th/0506184

- However Landsteiner discovered that in the shear channel, the hydrodynamic mode becomes more damped than the nonhydro mode for $q = \frac{k}{2\pi T} > 1.3$



from Landsteiner 1202.3550

Coupling dependence

Stricker
Waeber, Schafer, Vuorinen, Yaffe
Grozdanov, Kaplis, Starinets

- If one decreases the coupling, on the gravity side one has to add higher derivative corrections to Einstein gravity
- The imaginary parts of the higher quasinormal modes **decrease**
- The range of applicability of a hydrodynamic description goes down as we decrease the coupling...

from Grozdanov, Kaplis, Starinets 1605.02173

Coupling dependence

Stricker
Waeber, Schafer, Vuorinen, Yaffe
Grozdanov, Kaplis, Starinets

- If one decreases the coupling, on the gravity side one has to add higher derivative corrections to Einstein gravity
- The imaginary parts of the higher quasinormal modes **decrease**
- The range of applicability of a hydrodynamic description goes down as we decrease the coupling...

from Grozdanov, Kaplis, Starinets 1605.02173

Coupling dependence

Stricker
Waeber, Schafer, Vuorinen, Yaffe
Grozdanov, Kaplis, Starinets

- If one decreases the coupling, on the gravity side one has to add higher derivative corrections to Einstein gravity
- The imaginary parts of the higher quasinormal modes **decrease**
- The range of applicability of a hydrodynamic description goes down as we decrease the coupling...

from Grozdanov, Kaplis, Starinets 1605.02173

Coupling dependence

Stricker
Waeber, Schafer, Vuorinen, Yaffe
Grozdanov, Kaplis, Starinets

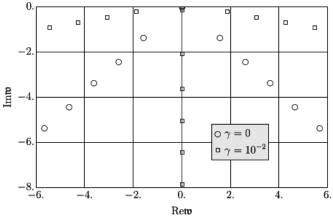
- If one decreases the coupling, on the gravity side one has to add higher derivative corrections to Einstein gravity
- The imaginary parts of the higher quasinormal modes **decrease**
- The range of applicability of a hydrodynamic description goes down as we decrease the coupling...

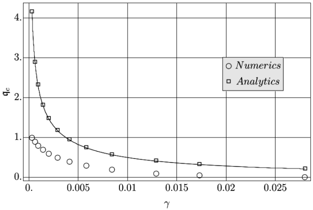
from Grozdanov, Kaplis, Starinets 1605.02173

Coupling dependence

Stricker
Waeber, Schafer, Vuorinen, Yaffe
Grozdanov, Kaplis, Starinets

- If one decreases the coupling, on the gravity side one has to add higher derivative corrections to Einstein gravity
- The imaginary parts of the higher quasinormal modes **decrease**
- The range of applicability of a hydrodynamic description goes down as we decrease the coupling...



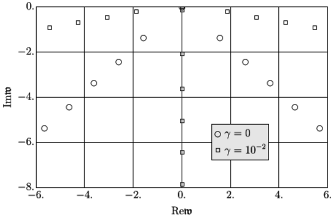


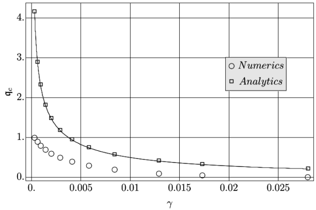
from Grozdanov, Kaplis, Starinets 1605.02173

Coupling dependence

Stricker
Waeber, Schafer, Vuorinen, Yaffe
Grozdanov, Kaplis, Starinets

- If one decreases the coupling, on the gravity side one has to add higher derivative corrections to Einstein gravity
- The imaginary parts of the higher quasinormal modes **decrease**
- The range of applicability of a hydrodynamic description goes down as we decrease the coupling...





from Grozdanov, Kaplis, Starinets 1605.02173

Nonconformality – phase transitions

RJ, Jankowski, Soltanpanahi
see talk by J. Jankowski

- In the vicinity of a phase transition (or even a sharp crossover) we get an additional constraint in **the sound channel**
- For certain momenta the lowest nonhydrodynamic mode in the sound channel becomes less damped than the ordinary hydrodynamic sound mode...

Nonconformality – phase transitions

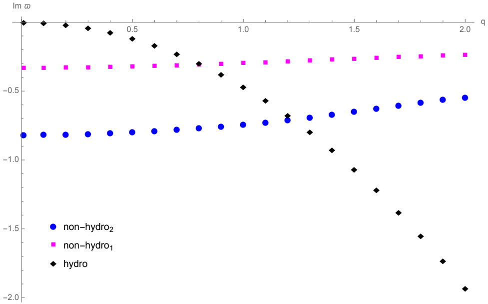
RJ, Jankowski, Soltanpanahi
see talk by J. Jankowski

- In the vicinity of a phase transition (or even a sharp crossover) we get an additional constraint in **the sound channel**
- For certain momenta the lowest nonhydrodynamic mode in the sound channel becomes less damped than the ordinary hydrodynamic sound mode...

Nonconformality – phase transitions

RJ, Jankowski, Soltanpanahi
see talk by J. Jankowski

- In the vicinity of a phase transition (or even a sharp crossover) we get an additional constraint in **the sound channel**
- For certain momenta the lowest nonhydrodynamic mode in the sound channel becomes less damped than the ordinary hydrodynamic sound mode...



Quasinormal modes in other setups

1. Nonconformal plasma
2. Magnetic fields
3. Nonrelativistic systems

Quasinormal modes in other setups

1. Nonconformal plasma
2. Magnetic fields
3. Nonrelativistic systems

Quasinormal modes in other setups

1. Nonconformal plasma
2. Magnetic fields
3. Nonrelativistic systems

Quasinormal modes in other setups

1. Nonconformal plasma
2. Magnetic fields
3. Nonrelativistic systems

Quasinormal modes in other setups

1. Nonconformal plasma
2. Magnetic fields
3. Nonrelativistic systems

How to model nonconformal plasma?

Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N} = 4$ SYM – explicitly known (but rather complicated) gravitational background

   $\mathcal{N} = 2^*$ theory

2. **Bottom-up approach: AdS/QCD**

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to approach as closely as possible to QCD physics (or other physics of interest)

How to model nonconformal plasma?

Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N}=4$ SYM – explicitly known (but rather complicated) gravitational background

   $\mathcal{N}=2^*$ theory

2. **Bottom-up approach: AdS/QCD**

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to approach as closely as possible to QCD physics (or other physics of interest)

How to model nonconformal plasma?

Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N} = 4$ SYM – explicitly known (but rather complicated) gravitational background

   $\mathcal{N} = 2^*$ theory

2. **Bottom-up approach: AdS/QCD**

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to approach as closely as possible to QCD physics (or other physics of interest)

How to model nonconformal plasma?

Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N} = 4$ SYM – explicitly known (but rather complicated) gravitational background

   $\mathcal{N} = 2^*$ theory

2. **Bottom-up approach: AdS/QCD**

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to approach as closely as possible to QCD physics (or other physics of interest)

How to model nonconformal plasma?

Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N} = 4$ SYM – explicitly known (but rather complicated) gravitational background

   $\mathcal{N} = 2^*$ theory

2. **Bottom-up approach: AdS/QCD**

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to approach as closely as possible to QCD physics (or other physics of interest)

How to model nonconformal plasma?

Two approaches:

1. **Top-down approach:**

   Deform $\mathcal{N} = 4$ SYM – explicitly known (but rather complicated) gravitational background

   $\mathcal{N} = 2^*$ theory

2. **Bottom-up approach: AdS/QCD**

   Assume AdS/CFT dictionary but try to model the gravity+matter background so as to approach as closely as possible to QCD physics (or other physics of interest)

Buchel, Heller, Myers

QNM of scalar operators

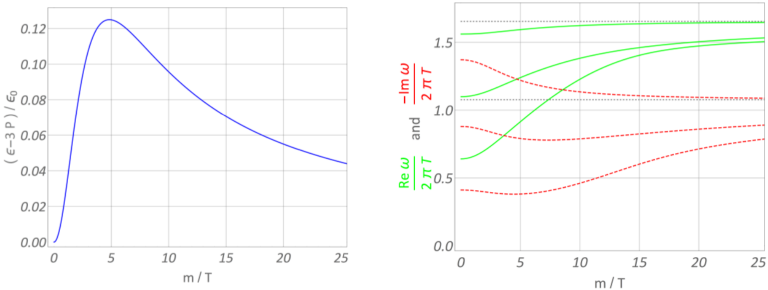
from 1503.07114

- The authors observed only mild modifications due to nonconformality in the scalar sector
- As far as thermalization is concerned (i.e. the approach to hydrodynamics from the high temperature regime) the current estimates from $\mathcal{N} = 4$ SYM theory should be quite robust

QNMs in $\mathcal{N} = 2^*$ theory

Buchel, Heller, Myers

QNM of scalar operators



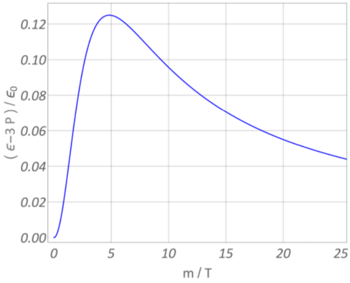
from 1503.07114

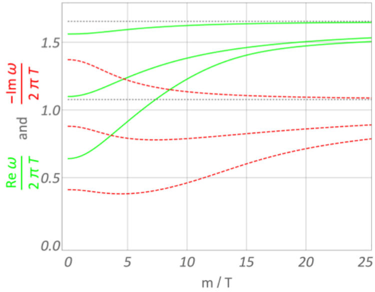
- The authors observed only mild modifications due to nonconformality in the scalar sector
- As far as thermalization is concerned (i.e. the approach to hydrodynamics from the high temperature regime) the current estimates from $\mathcal{N} = 4$ SYM theory should be quite robust

QNMs in $\mathcal{N} = 2^*$ theory

Buchel, Heller, Myers

QNM of scalar operators





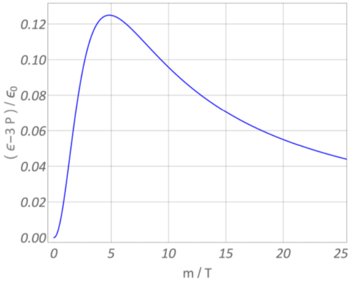
from 1503.07114

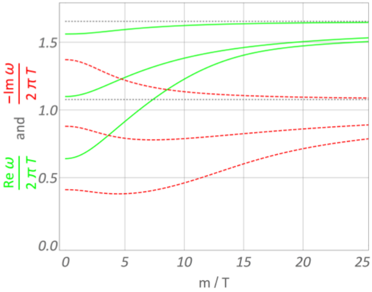
- The authors observed only mild modifications due to nonconformality in the scalar sector
- As far as thermalization is concerned (i.e. the approach to hydrodynamics from the high temperature regime) the current estimates from $\mathcal{N} = 4$ SYM theory should be quite robust

QNMs in $\mathcal{N} = 2^*$ theory

Buchel, Heller, Myers

QNM of scalar operators





from 1503.07114

- The authors observed only mild modifications due to nonconformality in the scalar sector
- As far as thermalization is concerned (i.e. the approach to hydrodynamics from the high temperature regime) the current estimates from $\mathcal{N} = 4$ SYM theory should be quite robust

QNMs for various nonconformal plasmas

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}\left[R - \frac{1}{2}\left(\partial\phi\right)^2 - V(\phi)\right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}}\cosh(\gamma\phi) + b_2\phi^2$$

QNMs for various nonconformal plasmas

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}} \cosh(\gamma\phi) + b_2\phi^2$$

QNMs for various nonconformal plasmas

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}} \cosh(\gamma\phi) + b_2\phi^2$$

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}} \cosh(\gamma\phi) + b_2\phi^2$$

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}} \cosh(\gamma\phi) + b_2\phi^2$$

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12\cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2}m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}} \cosh(\gamma\phi) + b_2\phi^2$$

QNMs for various nonconformal plasmas

RJ, Plewa, Soltanpanahi, Spaliński
RJ, Soltanpanahi, Jankowski x2
see talk by J. Jankowski

A bottom-up model

- Following Gubser et. al. we consider a gravity+scalar field system:

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] ,$$

- Here $V(\phi)$ is a self-interaction potential which we choose to reproduce the physics of interest (like lattice QCD equation of state, or a 1^{st} or 2^{nd} order transition)
- We choose the following parametrization for $V(\phi)$:

$$V(\phi) = 12 \cosh(\gamma\phi) + b_2\phi^2 + b_4\phi^4 + b_6\phi^6 \sim -12 + \frac{1}{2} m^2\phi^2 + O(\phi^4)$$

or (in the case of IHQCD)

$$V(\phi) = -12(1+\phi^2)^{\frac{1}{4}} \cosh(\gamma\phi) + b_2\phi^2$$

QNMs for various nonconformal plasmas

RJ, G. Plewa. H. Soltanpanahi, M. Spaliński:

- Choose $V(\Phi)$ to agree with lattice QCD equation of state (more precisely speed of sound $c_s(T)^2$) Gubser, Noronha...

QNMs for various nonconformal plasmas

RJ, G. Plewa. H. Soltanpanahi, M. Spaliński:

- Choose $V(\Phi)$ to agree with lattice QCD equation of state (more precisely speed of sound $c_s(T)^2$) Gubser, Noronha...

QNMs for various nonconformal plasmas

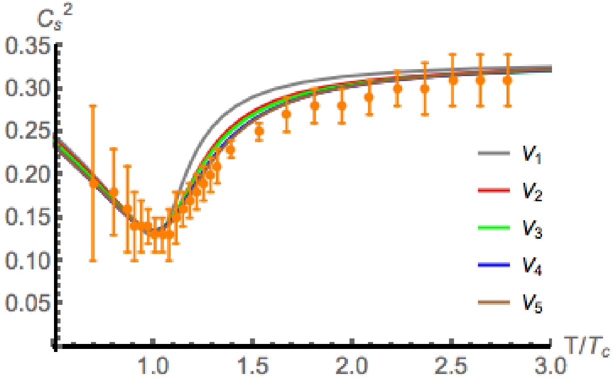
RJ, G. Plewa. H. Soltanpanahi, M. Spaliński:

- Choose $V(\Phi)$ to agree with lattice QCD equation of state (more precisely speed of sound $c_s(T)^2$) Gubser, Noronha...

QNMs for various nonconformal plasmas

RJ, G. Plewa. H. Soltanpanahi, M. Spaliński:

- Choose $V(\Phi)$ to agree with lattice QCD equation of state (more precisely speed of sound $c_s(T)^2$) Gubser, Noronha...



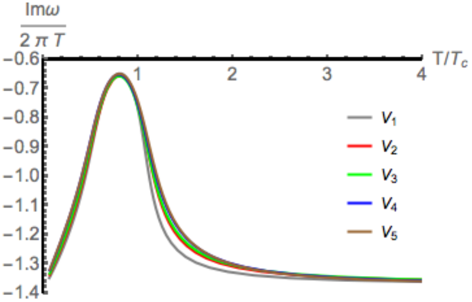
Massless scalar QNM's – QCD crossover potential

The damping of quasinormal modes decreases by a factor of two around T_c:

- The damping is essentially insensitive to differences in the UV
- The change in the damping seems to be correlated with deviations of the speed of sound from conformality
- **Message 1:** Approach to hydrodynamics is just as in the conformal case
- **Message 2:** We may expect the relevance of nonhydrodynamic modes close to the phase transition

Massless scalar QNM's – QCD crossover potential

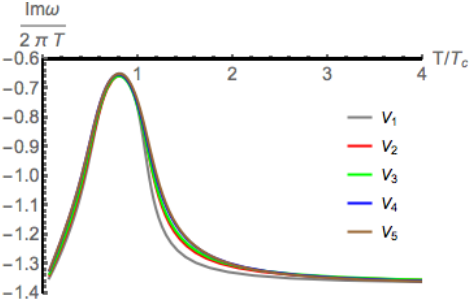
The damping of quasinormal modes decreases by a factor of two around T_c:



- The damping is essentially insensitive to differences in the UV
- The change in the damping seems to be correlated with deviations of the speed of sound from conformality
- **Message 1:** Approach to hydrodynamics is just as in the conformal case
- **Message 2:** We may expect the relevance of nonhydrodynamic modes close to the phase transition

Massless scalar QNM's – QCD crossover potential

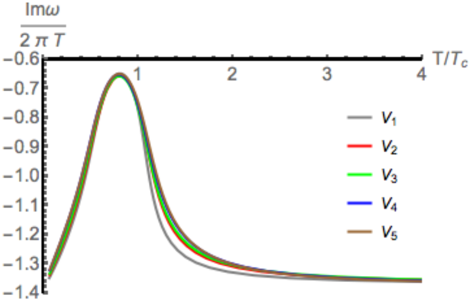
The damping of quasinormal modes decreases by a factor of two around T_c:



- The damping is essentially insensitive to differences in the UV
- The change in the damping seems to be correlated with deviations of the speed of sound from conformality
- **Message 1:** Approach to hydrodynamics is just as in the conformal case
- **Message 2:** We may expect the relevance of nonhydrodynamic modes close to the phase transition

Massless scalar QNM's – QCD crossover potential

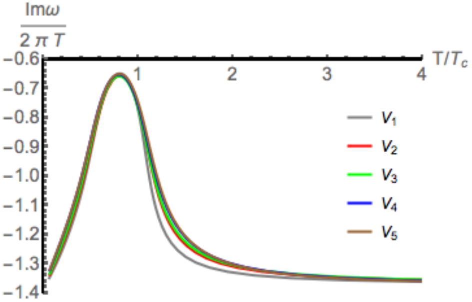
The damping of quasinormal modes decreases by a factor of two around T_c:



- The damping is essentially insensitive to differences in the UV
- The change in the damping seems to be correlated with deviations of the speed of sound from conformality
- **Message 1:** Approach to hydrodynamics is just as in the conformal case
- **Message 2:** We may expect the relevance of nonhydrodynamic modes close to the phase transition

Massless scalar QNM's – QCD crossover potential

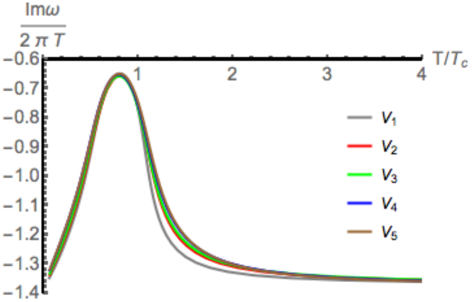
The damping of quasinormal modes decreases by a factor of two around T_c:



- The damping is essentially insensitive to differences in the UV
- The change in the damping seems to be correlated with deviations of the speed of sound from conformality
- **Message 1:** Approach to hydrodynamics is just as in the conformal case
- **Message 2:** We may expect the relevance of nonhydrodynamic modes close to the phase transition

Massless scalar QNM's – QCD crossover potential

The damping of quasinormal modes decreases by a factor of two around T_c:



- The damping is essentially insensitive to differences in the UV
- The change in the damping seems to be correlated with deviations of the speed of sound from conformality
- **Message 1:** Approach to hydrodynamics is just as in the conformal case
- **Message 2:** We may expect the relevance of nonhydrodynamic modes close to the phase transition

Metric QNM's – potentials with phase transition

We observe a rich variety of behaviours especially in the sound channel...

see talk by Jankowski

1^{st} order phase transition example

Metric QNM's – potentials with phase transition

We observe a rich variety of behaviours especially in the sound channel...

see talk by Jankowski

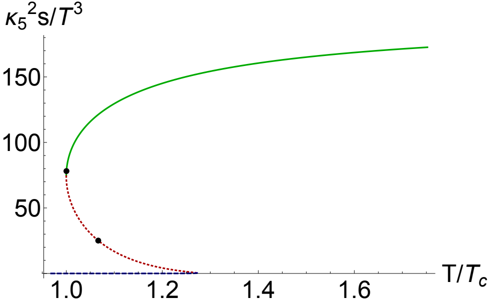
1^{st} order phase transition example

Metric QNM's – potentials with phase transition

We observe a rich variety of behaviours especially in the sound channel...

see talk by Jankowski

1^{st} order phase transition example



1^{st} **order phase transition** potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004 T_{min}$:

- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} **order phase transition** potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004 T_{min}$:

- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} **order phase transition** potential

see talk by Jankowski

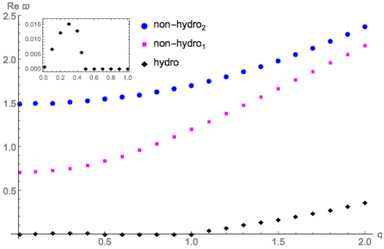
Overcooled branch $T \sim 1.00004\, T_{min}$:

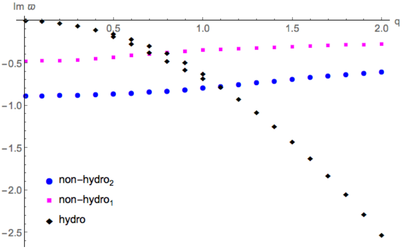
- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} order phase transition potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004\, T_{min}$:



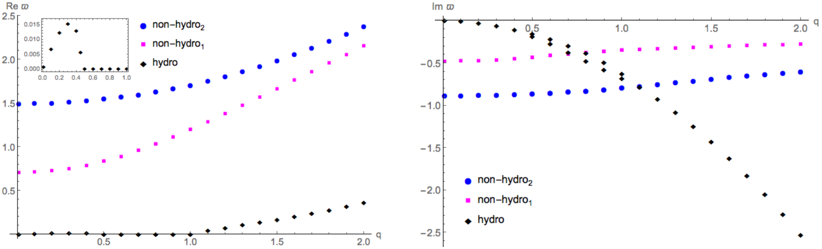


- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} order phase transition potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004\, T_{min}$:

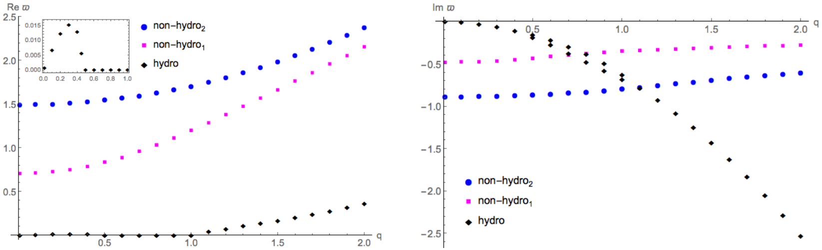


- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} order phase transition potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004\,T_{min}$:

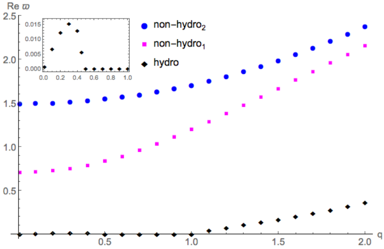


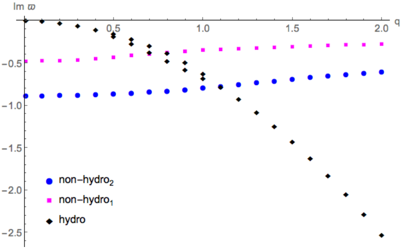
- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} order phase transition potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004\, T_{min}$:



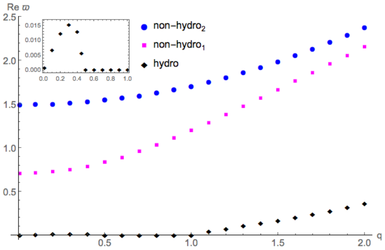


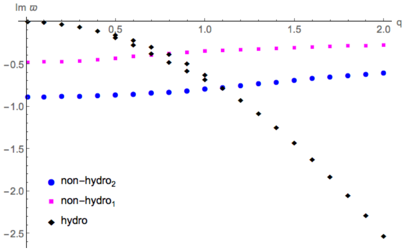
- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} order phase transition potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004\, T_{min}$:



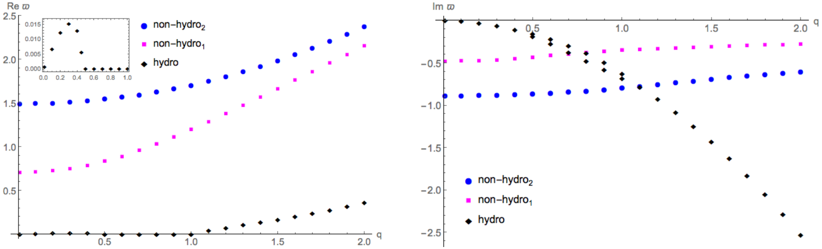


- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

1^{st} **order phase transition** potential

see talk by Jankowski

Overcooled branch $T \sim 1.00004\, T_{min}$:



- Speed of sound is very small
- Real part of the hydrodynamic sound mode vanishes for a range of momenta (here approximately $0.5 < q < 1$)
- The sound mode becomes nonpropagating for a range of length scales
- The onset of such a behaviour was also seen in [Gursoy, Shu, Shuryak]
- We see a crossing between hydro sound mode and nonhydrodynamic mode! (limits applicability of hydrodynamics)

QNMs in magnetic fields

Janiszewski, Kaminski 1508.06993
Demircik, Gürsoy 1605.08118
see also talk by Koirala

- Janiszewski, Kaminski studied magnetic black branes (top-down)
- Demircik, Gürsoy studied an improved holographic QCD model (IHQCD) with external magnetic field
- Surprisingly complex pattern of behaviours

from 1605.08118

QNMs in magnetic fields

Janiszewski, Kaminski 1508.06993
Demircik, Gürsoy 1605.08118
see also talk by Koirala

- Janiszewski, Kaminski studied magnetic black branes (top-down)
- Demircik, Gürsoy studied an improved holographic QCD model (IHQCD) with external magnetic field
- Surprisingly complex pattern of behaviours

from 1605.08118

QNMs in magnetic fields

Janiszewski, Kaminski 1508.06993
Demircik, Gürsoy 1605.08118
see also talk by Koirala

- Janiszewski, Kaminski studied magnetic black branes (top-down)
- Demircik, Gürsoy studied an improved holographic QCD model (IHQCD) with external magnetic field
- Surprisingly complex pattern of behaviours

from 1605.08118

QNMs in magnetic fields

Janiszewski, Kaminski 1508.06993
Demircik, Gürsoy 1605.08118
see also talk by Koirala

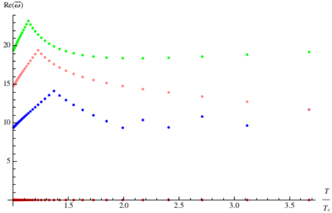
- Janiszewski, Kaminski studied magnetic black branes (top-down)
- Demircik, Gürsoy studied an improved holographic QCD model (IHQCD) with external magnetic field
- Surprisingly complex pattern of behaviours

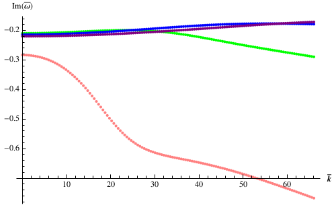
from 1605.08118

QNMs in magnetic fields

Janiszewski, Kaminski 1508.06993
Demircik, Gürsoy 1605.08118
see also talk by Koirala

- Janiszewski, Kaminski studied magnetic black branes (top-down)
- Demircik, Gürsoy studied an improved holographic QCD model (IHQCD) with external magnetic field
- Surprisingly complex pattern of behaviours





from 1605.08118

QNMs in nonrelativistic systems

Gürsoy, Jansen, Sybesma, Vandoren

- See talk by Jansen for details...
- QNMs for pressure anisotropy depend on the ratio

$$\alpha = \frac{z}{d-1}$$

- Again quasinormal modes give a good ballpark for understanding nonlinear homogeneous isotropization...

QNMs in nonrelativistic systems

Gürsoy, Jansen, Sybesma, Vandoren

- See talk by Jansen for details...
- QNMs for pressure anisotropy depend on the ratio

$$\alpha = \frac{z}{d-1}$$

- Again quasinormal modes give a good ballpark for understanding nonlinear homogeneous isotropization...

QNMs in nonrelativistic systems

Gürsoy, Jansen, Sybesma, Vandoren

- See talk by Jansen for details...
- QNMs for pressure anisotropy depend on the ratio

$$\alpha = \frac{z}{d-1}$$

- Again quasinormal modes give a good ballpark for understanding nonlinear homogeneous isotropization...

QNMs in nonrelativistic systems

Gürsoy, Jansen, Sybesma, Vandoren

- See talk by Jansen for details...
- QNMs for pressure anisotropy depend on the ratio

$$\alpha = \frac{z}{d-1}$$

- Again quasinormal modes give a good ballpark for understanding nonlinear homogeneous isotropization...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...

Conclusions

- (Higher) Quasinormal mode represent important field theory degrees of freedom responsible for generic equilibration
- They are intrinsically linked with high order hydrodynamics akin to the relation of perturbation theory and nonperturbative instantons
- This holds not only qualitatively but also quantitatively
- One can study QNM on top of fluid/gravity backgrounds – ultralocality – but still a lot to understand
- One can use them for constraining the range of applicability of hydrodynamics
- They exhibit quite intricate behaviours in the nonconformal and/or magnetic field case
- Despite all that one has to keep in mind that there is important physics which makes its appearance only in the nonlinear regime...