Colliding Black Holes in AdS

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Outline

- Motivation
- Setup
- Simulations
- Summary

Motivation

Heavy ion collisions

- Why collisions?
  to probe the quark and gluon constituents of nuclei
- Why heavy ions?
  to get as many p^+ *and* n^0 *as possible to hit each other*

The STAR, PHENIX experiments at RHIC, the ALICE, ATLAS, CMS experiments at LHC

- Strip gold ($^{197}_{79}$Au) or lead ($^{208}_{82}$Pb) nuclei of electrons
- Accelerate to speeds close to c
- Arrange for a collision
- Collision energies of 200[GeV] per nucleon at RHIC, 2.76[TeV] per nucleon at LHC

Motivation

- A non-perturbative problem in QCD
- Lattice QCD has no access to real-time dynamics
- Experimental data are well described by relativistic viscous hydrodynamic simulations
- But, several competing models for the pre-equilibrium stage that yield different initial energy density and flow velocity profiles for matching onto the hydrodynamic stage
- Would be desirable to have a *single* model to describe both the pre-equilibrium stage and the hydrodynamic stage

Motivation

Pre-equilibrium stage

- Duration: 0.2-0.4 fm/c
- A model: classical Yang-Mills dynamics of gluons
- Resulting energy density and flow velocity profiles are used to match onto a hydrodynamic form of the stress tensor in subsequent hydrodynamic stage

Hydrodynamic stage

- Duration: 5-10 fm/c ($\uparrow$ for higher collision energies)
- A model: relativistic viscous hydrodynamics
- Resulting hydrodynamic output is used to match onto particle distributions in subsequent hadronic stage

Hadronic stage

- Duration: remaining evolution time
- A model: microscopic kinetic description

Motivation

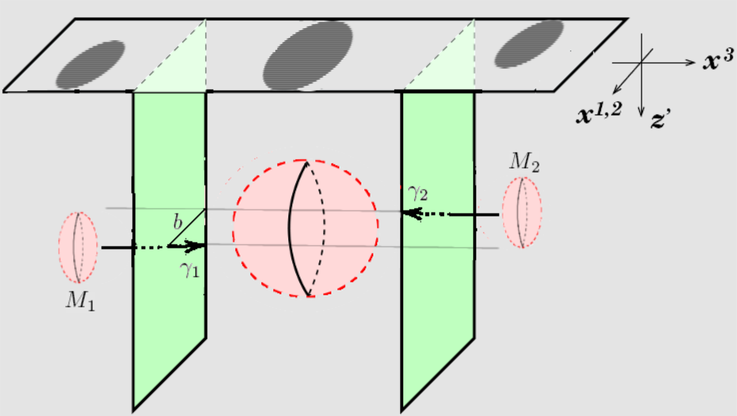

FIGURE: BH-BH collision in a Poincaré patch of AdS_5, with black hole masses M_1, M_2, boosts γ_1,γ_2, and impact parameter b.

Adapted from `hep-th/0805.1551`

Motivation

AdS/CFT correspondence

between an asymptotically AdS spacetime in $d+1$ dimensions and a CFT in d dimensions

Proposed use

to find a gravity description of non-perturbative problems in QCD

Major obstacle is the current lack of a gravity dual for QCD

Possible approach: try to capture some features of QCD with a CFT toy model for which there is a known gravity dual

$\mathcal{N} = 4$ SYM_4 at strong coupling $\longleftrightarrow$ AdS_5 classical gravity

Motivation

AdS/CFT correspondence

between an asymptotically AdS spacetime in $d+1$ dimensions and a **CFT** in d dimensions

Proposed use

to find a gravity description of non-perturbative problems in **QCD**

Major obstacle is the current lack of a gravity dual for QCD

Possible approach: try to capture some features of QCD with a CFT toy model for which there *is* a known gravity dual

$\mathcal{N} = 4$ SYM$_4$ at strong coupling $\longleftrightarrow$ AdS$_5$ classical gravity

Setup

Classical gravity in $d+1$ dimensions with cosmological constant $\Lambda = d(d-1)/(2L^2)$, coupled to real scalar field matter[1]:

$$S = \int dx^{d+1}\sqrt{-g}\left[\frac{1}{16\pi}(R-2\Lambda) - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - V(\varphi)\right]$$

The corresponding field equations take the local form[2]:

$$\begin{aligned}\Box\varphi &= \frac{dV}{d\varphi} \\ R_{\mu\nu} &= \frac{2\Lambda}{d-1}g_{\mu\nu} + 8\pi\left(T_{\mu\nu} - \frac{1}{d-1}T^{\alpha}{}_{\alpha}g_{\mu\nu}\right)\end{aligned}$$

[1] We will use scalar field collapse as a convenient mechanism to form BHs

[2] Real scalar field: $T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi + V(\varphi)\right)$

Setup

$\mu, \nu = 1, ..., d+1$

$$0 = R_{\mu\nu} - \frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right)$$

Setup

$\mu, \nu = 1, ..., d+1$

$$0 = -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right)$$

$R_{\mu\nu}$

Setup

$\mu, \nu = 1, ..., d+1$

$$0 = -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right)$$

$$-\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + g^{\alpha\beta} g_{\beta(\mu,\nu)\alpha} + \frac{1}{2} g^{\alpha\beta}{}_{,\alpha} \left(g_{\alpha\beta,\nu} - g_{\nu\mu,\beta} + g_{\beta\nu,\mu} \right)$$

$$- \left(\log \sqrt{-g} \right)_{,\mu\nu} + \left(\log \sqrt{-g} \right)_{,\beta} \Gamma^{\beta}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\nu\beta} \Gamma^{\beta}{}_{\alpha\nu}$$

Setup

$\mu, \nu = 1, ..., d+1$

$$
\begin{aligned}
0 \quad = \quad & -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right) \\
& -\nabla_{(\mu} C_{\nu)} \\
& -\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + g^{\alpha\beta} g_{\beta(\mu,\nu)\alpha} + \frac{1}{2} g^{\alpha\beta}{}_{,\alpha} \left(g_{\alpha\beta,\nu} - g_{\nu\mu,\beta} + g_{\beta\nu,\mu} \right) \\
& - \left(\log \sqrt{-g} \right)_{,\mu\nu} + \left(\log \sqrt{-g} \right)_{,\beta} \Gamma^{\beta}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\nu\beta} \Gamma^{\beta}{}_{\alpha\nu}
\end{aligned}
$$

$$C_\mu \equiv H_\mu - \Box x_\mu$$

(physical solutions satisfy $C_\mu = 0$)

Setup

$\mu, \nu = 1, ..., d+1$

$$
\begin{aligned}
0 \;=\; & -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right) \\
& -\nabla_{(\mu} H_{\nu)} + \nabla_{(\mu} \Box x_{\nu)} \\
& -\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + g^{\alpha\beta} g_{\beta(\mu,\nu)\alpha} + \frac{1}{2} g^{\alpha\beta}{}_{,\alpha} \left(g_{\alpha\beta,\nu} - g_{\nu\mu,\beta} + g_{\beta\nu,\mu} \right) \\
& - \left(\log \sqrt{-g} \right)_{,\mu\nu} + \left(\log \sqrt{-g} \right)_{,\beta} \Gamma^{\beta}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\nu\beta} \Gamma^{\beta}{}_{\alpha\nu}
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0 \quad = \quad & -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right) \\
& -\nabla_{(\mu} H_{\nu)} + \cancel{\nabla_{(\mu} \Box x_{\nu)}} \\
& -\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \cancel{g^{\alpha\beta} g_{\beta(\mu,\nu)\alpha}} + \cancel{\frac{1}{2} g^{\alpha\beta}{}_{,\alpha} \left(g_{\alpha\beta,\nu} - g_{\nu\mu,\beta} + g_{\beta\nu,\mu} \right)} \\
& -\cancel{\left(\log \sqrt{-g} \right)_{,\mu\nu}} + \cancel{\left(\log \sqrt{-g} \right)_{,\beta} \Gamma^{\beta}{}_{\mu\nu}} - \Gamma^{\alpha}{}_{\nu\beta} \Gamma^{\beta}{}_{\alpha\nu} - g^{\alpha\beta}{}_{,(\mu} g_{\nu)\alpha,\beta}
\end{aligned}
$$

$$C_\mu \equiv H_\mu - \Box x_\mu$$

(physical solutions satisfy $C_\mu = 0$)

Setup

$$\mu, \nu = 1, ..., d+1$$

$$\begin{aligned} 0 \quad = \quad & - \frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right) \\ & - \nabla_{(\mu} H_{\nu)} + \cancel{\nabla_{(\mu} \Box x_{\nu)}} \\ & - \frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \cancel{g^{\alpha\beta} g_{\beta(\mu,\nu)\alpha}} + \cancel{\frac{1}{2} g^{\alpha\beta}{}_{,\alpha} \left(g_{\alpha\beta,\nu} - g_{\nu\mu,\beta} + g_{\beta\nu,\mu} \right)} \\ & - \cancel{\left(\log \sqrt{-g}\right)_{,\mu\nu}} + \cancel{\left(\log \sqrt{-g}\right)_{,\beta} \Gamma^{\beta}{}_{\mu\nu}} - \Gamma^{\alpha}{}_{\nu\beta} \Gamma^{\beta}{}_{\alpha\nu} - g^{\alpha\beta}{}_{,(\mu} g_{\nu)\alpha,\beta} \end{aligned}$$

$$C_\mu \equiv H_\mu - \Box x_\mu$$

(physical solutions satisfy $C_\mu = 0$)

choose some $H_\mu = f_\mu(g)$

(this sets $\Box x_\mu = f_\mu(g)$ as long as $C_\mu = 0$)

Setup

$\mu, \nu = 1, ..., d+1$

$$
\begin{aligned}
0 \quad = \quad & -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^{\alpha}{}_{\alpha} g_{\mu\nu} \right) \\
& -\nabla_{(\mu} H_{\nu)} + \cancel{\nabla_{(\mu} \Box x_{\nu)}} - \kappa_1 \left(2 n_{(\mu} C_{\nu)} - (1+\kappa_2) g_{\mu\nu} n^{\alpha} C_{\alpha} \right) \\
& -\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \cancel{g^{\alpha\beta} g_{\beta(\mu,\nu)\alpha}} + \cancel{\frac{1}{2} g^{\alpha\beta}{}_{,\alpha} \left(g_{\alpha\beta,\nu} - g_{\nu\mu,\beta} + g_{\beta\nu,\mu} \right)} \\
& \cancel{-\left(\log\sqrt{-g} \right)_{,\mu\nu}} + \cancel{\left(\log\sqrt{-g} \right)_{,\beta} \Gamma^{\beta}{}_{\mu\nu}} - \Gamma^{\alpha}{}_{\nu\beta} \Gamma^{\beta}{}_{\alpha\nu} - g^{\alpha\beta}{}_{,(\mu} g_{\nu)\alpha,\beta}
\end{aligned}
$$

$C_\mu \equiv H_\mu - \Box x_\mu$

(physical solutions satisfy $C_\mu = 0$)

choose some $H_\mu = f_\mu(g)$

(this sets $\Box x_\mu = f_\mu(g)$ as long as $C_\mu = 0$)

Setup

Ingredients

- Evolution Equations
- Initial Data
- Boundary Conditions
- Gauge Choice

Setup

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Evolution Equations

$$
\begin{aligned}
0 &= -\frac{1}{2} g^{\alpha\beta} g_{\mu\nu,\alpha\beta} - g^{\alpha\beta}{}_{,(\mu} g_{\nu)\alpha,\beta} \\
&\quad -H_{(\mu,\nu)} + H_\alpha \Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\beta\mu} \Gamma^\beta{}_{\alpha\nu} \\
&\quad -\kappa_1 \left(2 n_{(\mu} C_{\nu)} - (1+\kappa_2) g_{\mu\nu} n^\alpha C_\alpha \right) \\
&\quad -\frac{2\Lambda}{d-1} g_{\mu\nu} - 8\pi \left(T_{\mu\nu} - \frac{1}{d-1} T^\alpha{}_\alpha g_{\mu\nu} \right) \\
&\quad \downarrow \\
0 &= E_{(g_{\mu\nu})} \qquad (d+2)(d+1)/2 \text{ such equations, one for each } g_{\mu\nu}
\end{aligned}
$$

$$
H_\mu = f_\mu(g) \qquad \text{constraint damping terms} \sim \kappa_1, \text{ designed to damp towards } C^\mu = 0
$$

Setup

$$
\begin{aligned}
g_{\mu\nu}dx^{\mu}dx^{\nu} &= g_{tt}dt^2 + 2g_{tz}dtdz + 2g_{tx_1}dtdx_1 + 2g_{tx_2}dtdx_2 + \\
&\quad g_{zz}dz^2 + 2g_{zx_1}dzdx_1 + 2g_{zx_2}dzdx_2 + \\
&\quad g_{x_1x_1}dx_1^2 + 2g_{x_1x_2}dx_1dx_2 + \\
&\quad g_{x_2x_2}dx_2^2
\end{aligned}
$$

$$
g_{\mu\nu} = g_{\mu\nu}(t, z, x_1, x_2)
$$

Setup

$$
\begin{aligned}
g_{\mu\nu}dx^{\mu}dx^{\nu} &= g_{tt}dt^2 + 2g_{tz}dtdz + 2g_{tx_1}dtdx_1 + 2g_{tx_2}dtdx_2 + 2g_{tx_3}dtdx_3 + \\
&\quad g_{zz}dz^2 + 2g_{zx_1}dzdx_1 + 2g_{zx_2}dzdx_2 + 2g_{zx_3}dzdx_3 + \\
&\quad g_{x_1x_1}dx_1^2 + 2g_{x_1x_2}dx_1dx_2 + 2g_{x_1x_3}dx_1dx_3 + \\
&\quad g_{x_2x_2}dx_2^2 + g_{x_2x_3}dx_2dx_3 + \\
&\quad g_{x_3x_3}dx_3^2 + \\
\\
g_{\mu\nu} &= g_{\mu\nu}(t, z, x_1, x_2, x_3)
\end{aligned}
$$

Setup

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= g_{tt}dt^2 + 2g_{tz}dtdz + 2g_{tx_1}dtdx_1 + 2g_{tx_2}dtdx_2 + 2g_{tx_3}dtdx_3 + \\
&\quad g_{zz}dz^2 + 2g_{zx_1}dzdx_1 + 2g_{zx_2}dzdx_2 + 2g_{zx_3}dzdx_3 + \\
&\quad g_{x_1x_1}dx_1^2 + 2g_{x_1x_2}dx_1dx_2 + 2g_{x_1x_3}dx_1dx_3 + \\
&\quad g_{x_2x_2}dx_2^2 + g_{x_2x_3}dx_2dx_3 + \\
&\quad g_{x_3x_3}dx_3^2 +
\end{aligned}
$$

$$
g_{\mu\nu} = g_{\mu\nu}(t, z, x_1 = 0, x_2, x_3) \;\; \text{with } SO(2) \text{ in } x_1, x_2
$$

Setup

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= g_{tt}dt^2 + 2g_{tz}dtdz + 2g_{tx_1}dtdx_1 + 2g_{tx_2}dtdx_2 + 2g_{tx_3}dtdx_3 + \\
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&\quad g_{x_2x_2}dx_2^2 + g_{x_2x_3}dx_2dx_3 + \\
&\quad g_{x_3x_3}dx_3^2 +
\end{aligned}
$$

$$
g_{\mu\nu} = g_{\mu\nu}(t, z, x_1 = 0, x_2, x_3) \text{ with } SO(2) \text{ in } x_1, x_2
$$

$$
\mathcal{L}_\xi g_{\mu\nu} = 0
$$

$$
\mathcal{L}_\xi H_\mu = 0 \qquad \xi = x_2 \frac{\partial}{\partial x_1} - x_1 \frac{\partial}{\partial x_2}
$$

$$
\mathcal{L}_\xi \varphi = 0
$$

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Initial Data

$$
\begin{aligned}
0 &= {}^{(d)}R + K^2 - K_{ij}K^{ij} - 2\Lambda - 16\pi\rho \\
0 &= D_j K^j{}_i - D_i K - 8\pi j_i \\
&\downarrow \\
0 &= E_{(\zeta_\mu)} \qquad\qquad (d+1) \text{ such equations, one for each } \zeta_\mu
\end{aligned}
$$

where[1] $n_\mu = -\alpha \partial_\mu t$,

$\rho = n_\mu n_\nu T^{\mu\nu}$,

$j_i = -g_{\mu i} n_\nu T^{\mu\nu}$,

$K_{ij} = -\frac{1}{2}\mathcal{L}_n g_{ij} = -\frac{1}{2\alpha}\left(-\partial_t g_{ij} + D_i\beta_j + D_j\beta_i\right)$

[1] Here, α is the lapse function and β_i is the shift vector

Initial Data (At a Moment of Time Symmetry)

$$
\begin{aligned}
0 &= {}^{(d)}R + \qquad\qquad - 2\Lambda - 16\pi\rho \\
0 &= \qquad\qquad - 8\pi j_i \\
& \downarrow \\
0 &= E_{(\zeta)} \qquad\qquad \text{1 equation, for } g_{ij} = \zeta^2 g_{ij}^{AdS}
\end{aligned}
$$

where[1] $n_\mu = -\alpha\partial_\mu t$,

$\rho = n_\mu n_\nu T^{\mu\nu}$,

$j_i = 0$,

$$K_{ij} = 0 = -\frac{1}{2\alpha}\left(-\partial_t g_{ij} + D_i\beta_j + D_j\beta_i\right)$$

[1] Here, α is the lapse function and β_i is the shift vector

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Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, z, x_1, ..., x_{d-1})$ with $z \in [0, \infty)$, $x_i \in (-\infty, \infty)$:

$$\frac{L^2}{z^2}\left(-dt^2 + dz^2 + dx_1^2 + ... + dx_{d-1}^2\right)$$

Boundary conditions at $z = 0$:

$$\begin{aligned} h_{zz} &= z^{d-2} f_{zz}(t, x_1, ..., x_{d-1}) + ... \\ h_{zm} &= z^{d-1} f_{zm}(t, x_1, ..., x_{d-1}) + ... \\ h_{mn} &= z^{d-2} f_{mn}(t, x_1, ..., x_{d-1}) + ... \\ \varphi &= z^{d} f_{\varphi}(t, x_1, ..., x_{d-1}) + ... \end{aligned}$$

Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, z, x_1, ..., x_{d-1})$, $z = l_1(l_1^2 - x^2)/x^2$, $x_i = \tan((y_i/l_2)(\pi/2))$:

$$\frac{L^2}{z^2}\left(-dt^2 + dz^2 + dx_1^2 + ... + dx_{d-1}^2\right)$$

Boundary conditions at $z = 0$:

$$\begin{aligned} h_{zz} &= z^{d-2} f_{zz}(t, x_1, ..., x_{d-1}) + ... \\ h_{zm} &= z^{d-1} f_{zm}(t, x_1, ..., x_{d-1}) + ... \\ h_{mn} &= z^{d-2} f_{mn}(t, x_1, ..., x_{d-1}) + ... \\ \varphi &= z^{d} f_{\varphi}(t, x_1, ..., x_{d-1}) + ... \end{aligned}$$

Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, z, x_1, ..., x_{d-1})$, $z = (1 - x^2)/x^2$, $x_i = \tan(y_i \pi/2)$:

$$\frac{L^2}{z^2}\left(-dt^2 + dz^2 + dx_1^2 + ... + dx_{d-1}^2\right)$$

Boundary conditions at $z = 0$:

$$\begin{aligned}
h_{zz} &= z^{d-2} f_{zz}(t, x_1, ..., x_{d-1}) + ... \\
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\varphi &= z^{d} f_{\varphi}(t, x_1, ..., x_{d-1}) + ...
\end{aligned}$$

Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, x, y_1, ..., y_{d-1})$ with $x \in [0, 1]$, $y_i \in [-1, 1]$:

$$\frac{L^2}{z^2}\left(-dt^2 + dz^2 + dx_1^2 + ... + dx_{d-1}^2\right)$$

Boundary conditions at $z = 0$:

$$\begin{aligned} h_{zz} &= z^{d-2} f_{zz}(t, x_1, ..., x_{d-1}) + ... \\ h_{zm} &= z^{d-1} f_{zm}(t, x_1, ..., x_{d-1}) + ... \\ h_{mn} &= z^{d-2} f_{mn}(t, x_1, ..., x_{d-1}) + ... \\ \varphi &= z^{d} f_{\varphi}(t, x_1, ..., x_{d-1}) + ... \end{aligned}$$

Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, x, y_1, ..., y_{d-1})$ with $x \in [0,1]$, $y_i \in (-1,1)$:

$$\frac{L^2}{(1-x^2)^2}\left(-dt^2/x^4 + 4dx^2/x^2 + ... + (\pi/2)^2 x^4 \cos^4(y_1\pi/2) dy_{d-1}^2\right)$$

Boundary conditions at $x = 1$:

$$\begin{aligned} h_{xx} &= (1-x)^{d-2} f_{xx}(t, y_1, ..., y_{d-1}) + ... \\ h_{xm} &= (1-x)^{d-1} f_{xm}(t, y_1, ..., y_{d-1}) + ... \\ h_{mn} &= (1-x)^{d-2} f_{mn}(t, y_1, ..., y_{d-1}) + ... \\ \varphi &= (1-x)^{d} f_{\varphi}(t, y_1, ..., y_{d-1}) + ... \end{aligned}$$

Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, x, y_1, ..., y_{d-1})$ with $x \in [0,1]$, $y_i \in (-1,1)$:

$$\frac{L^2}{(1-x^2)^2}\left(-dt^2/x^4 + 4dx^2/x^2 + ... + (\pi/2)^2 x^4 \cos^4(y_1\pi/2) dy_{d-1}^2\right)$$

Boundary conditions at $x = 1$:

$$\begin{aligned}
h_{xx} &= (1-x)^{d-3}\left[f_{xx}(t, y_1, ..., y_{d-1})(1-x) + ...\right] \\
h_{xm} &= (1-x)^{d-2}\left[f_{xm}(t, y_1, ..., y_{d-1})(1-x) + ...\right] \\
h_{mn} &= (1-x)^{d-3}\left[f_{mn}(t, y_1, ..., y_{d-1})(1-x) + ...\right] \\
\varphi &= (1-x)^{d-1}\left[f_{\varphi}(t, y_1, ..., y_{d-1})(1-x) + ...\right]
\end{aligned}$$

Setup

Boundary Conditions

Decompose metric into a pure AdS piece and a deviation:

$$g_{\mu\nu} = g_{\mu\nu}^{AdS} + (1-x)^{\text{``power''}}\bar{g}_{\mu\nu}$$

A Poincaré patch of pure AdS in coordinates $(t, x, y_1, ..., y_{d-1})$ with $x \in [0,1]$, $y_i \in (-1,1)$:

$$\frac{L^2}{(1-x^2)^2}\left(-dt^2/x^4 + 4dx^2/x^2 + ... + (\pi/2)^2 x^4 \cos^4(y_1\pi/2) dy_{d-1}^2\right)$$

Boundary conditions at $x=1$:

$$\begin{aligned}
g_{xx} &= g_{xx}^{AdS} + (1-x)^{d-3}\bar{g}_{xx}(t,x,y_1,...,y_{d-1}) & \bar{g}_{xx}|_{x=1} = 0 \\
g_{xm} &= g_{xm}^{AdS} + (1-x)^{d-2}\bar{g}_{xm}(t,x,y_1,...,y_{d-1}) & \bar{g}_{xm}|_{x=1} = 0 \\
g_{mn} &= g_{xx}^{AdS} + (1-x)^{d-3}\bar{g}_{mn}(t,x,y_1,...,y_{d-1}) & \bar{g}_{mn}|_{x=1} = 0 \\
\varphi &= (1-x)^{d-1}\bar{\varphi}(t,x,y_1,...,y_{d-1}) & \bar{\varphi}|_{x=1} = 0
\end{aligned}$$

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Expand metric variables in power series near x=1:

$$\bar{g}_{\mu\nu} = (1-x)\bar{g}_{(1)\mu\nu} + (1-x)^2\bar{g}_{(2)\mu\nu} + ...$$

Expand field equations in power series near x=1:

$$\tilde{\Box}\bar{g}_{(1)tt} = (-2\bar{g}_{(1)xx} + \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)xx} = (4\bar{g}_{(1)tt} + 3\bar{g}_{(1)xx} - 4\Sigma_i\bar{g}_{(1)y_iy_i} - 2\bar{H}_{(1)x})(1-x)^{-2} +$$
$$\tilde{\Box}\bar{g}_{(1)y_1y_1} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)y_2y_2} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)y_3y_3} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$

...

Expand $C_\mu \equiv H_\mu - \Box x_\mu = 0$ in power series near x=1:

$$\bar{C}_{(1)x} \equiv (-4\bar{g}_{(1)tt} - \bar{g}_{(1)xx} + 4\Sigma_i\bar{g}_{(1)y_iy_i} + \bar{H}_{(1)x}) + ... = 0$$

...

Setup

Gauge Choice

Expand metric variables in power series near x=1:

$$\bar{g}_{\mu\nu} = (1-x)\bar{g}_{(1)\mu\nu} + (1-x)^2\bar{g}_{(2)\mu\nu} + ...$$

Expand field equations, with $C_\mu = 0$, near x=1:

$$\tilde{\Box}\bar{g}_{(1)tt} = (-2\bar{g}_{(1)xx} + \bar{H}_{(1)x})(1-x)^{-2} + ...$$

$$\tilde{\Box}\bar{g}_{(1)xx} = (\quad + 2\bar{g}_{(1)xx} \quad - \bar{H}_{(1)x})(1-x)^{-2} + ...$$

$$\tilde{\Box}\bar{g}_{(1)y_1y_1} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$

$$\tilde{\Box}\bar{g}_{(1)y_2y_2} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$

$$\tilde{\Box}\bar{g}_{(1)y_3y_3} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$

...

Gauge Choice

Expand metric variables in power series near x=1:

$$\bar{g}_{\mu\nu} = (1-x)\bar{g}_{(1)\mu\nu} + (1-x)^2\bar{g}_{(2)\mu\nu} + ...$$

Expand field equations, with $C_\mu = 0$, near x=1:

$$\tilde{\Box}\bar{g}_{(1)tt} = (-2\bar{g}_{(1)xx} + \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)xx} = (\qquad + 2\bar{g}_{(1)xx} \qquad - \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)y_1y_1} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)y_2y_2} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$
$$\tilde{\Box}\bar{g}_{(1)y_3y_3} = (2\bar{g}_{(1)xx} - \bar{H}_{(1)x})(1-x)^{-2} + ...$$

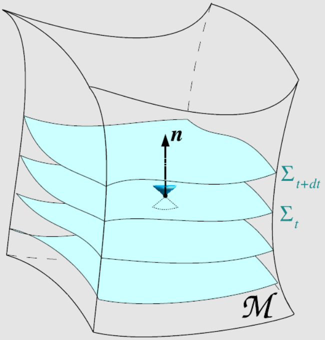
...

Gauge choice near x=1:

$$\bar{H}_{(1)t} = 5/2\bar{g}_{(1)tx} \quad \bar{H}_{(1)x} = 2\bar{g}_{(1)xx} \quad \bar{H}_{(1)y_i} = 5/2\bar{g}_{(1)xy_i}$$

Summary

1. Find $\bar{g}_{\mu\nu}|_{t=0}$, $\partial_t \bar{g}_{\mu\nu}|_{t=0}$ on some initial spatial slice $\Sigma_{t=0}$, given some initial matter distribution $\bar{\varphi}$ on $\Sigma_{t=0}$
2. Update $\bar{g}_{\mu\nu}$ from Σ_t to Σ_{t+dt}, subject to boundary conditions $\bar{g}_{\mu\nu}|_{x=1} = 0$, $\bar{\varphi}|_{x=1} = 0$ and a gauge choice $\bar{H}_\mu = f_\mu(\bar{g})$



Adapted from `gr-qc/0703035`

Strategy, in General

3+1 Evolution on Poincaré AdS_4:
with no symmetry assumptions in 4D
(built on top of PAMR/AMRD)

$\downarrow$

3+1 Evolution on Poincaré AdS_5:
with $SO(2)$ symmetry in 5D via "modified cartoon"
(built on top of PAMR/AMRD)

$\downarrow$

4+1 Evolution on Poincaré AdS_5:
with no symmetry assumptions in 5D
(built on top of GRChombo)

Poincaré Patch

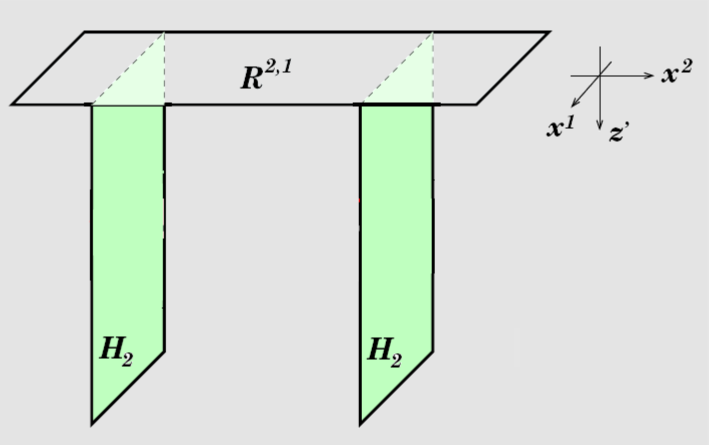


Figure: The Poincaré patch of AdS_4 drawn in coordinates adapted to the $\mathbb{R}^{2,1}$ boundary; constant-x_2 slices are copies of the hyperbolic plane H_2.

Adapted from `hep-th/0805.1551`

Massless Scalar Propagating in AdS (Color Scale: Metric) (Slice: $z = 0.5$)

Loading ...

Massless Scalar Propagating in AdS (Color Scale: Metric) (Slice: $x_1 = 0$)

Loading ...

Poincaré Patch with Non-Compact Horizon

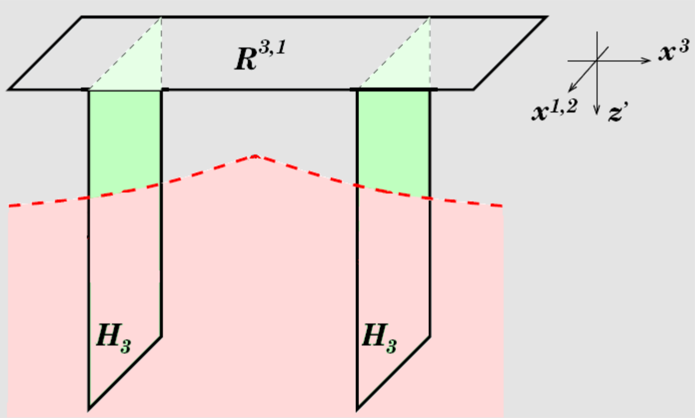


Figure: The Poincaré patch of AdS_5 drawn in coordinates adapted to the $\mathbb{R}^{3,1}$ boundary; constant-x_3 slices are copies of the hyperbolic plane H_3.

Adapted from `hep-th/0805.1551`

Collapse to BH with Non-Compact Horizon (Color Scale: Scalar Field) (Slice: $x_{1,2} = 0$)

Loading ...

Poincaré Patch with Compact Horizon

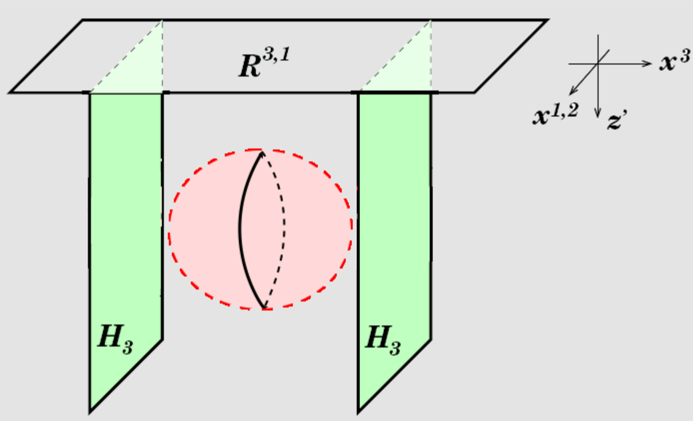


Figure: The Poincaré patch of AdS_5 drawn in coordinates adapted to the $\mathbb{R}^{3,1}$ boundary; constant-x_3 slices are copies of the hyperbolic plane H_3.

Adapted from `hep-th/0805.1551`

Collapse to BH with Compact Horizon (Color Scale: Scalar Field) (Slice: $x_{1,2} = 0$)

Loading ...

Strategy, for Stable BH-BH Mergers

Non-Compact Apparent Horizon:
add compactly-supported matter, then add non-compact sheet to ensure collapse to horizon with planar topology

↓

No Apparent Horizon:
keep compactly-supported matter, and remove non-compact sheet to arrange for zero background temperature

↓

Two Disjoint Compact Apparent Horizons:
increase strength of compactly-supported matter to arrange for two disjoint horizons each with spherical topology

↓

Merger to Form Single Compact Apparent Horizon:
find common horizon as the two disjoint horizons merge

Collapse to Non-Compact BH: 5% Background (Color Scale: Metric) (Slice: $x_{1,2} = 0$)

Loading ...

Remove Non-Compact BH: 0% Background (Color Scale: Metric) (Slice: $x_{1,2} = 0$)

Loading ...

Energy Density on $\mathbb{R}^{3,1}$: 1% Background

$x_1 \uparrow$

Loading ...

$\longrightarrow$
x_3

Energy Density on $\mathbb{R}^{3,1}$: 0% Background

$x_1 \uparrow$

Loading ...

$\longrightarrow$

x_3

Poincaré Patch with Merger of Disjoint Compact Horizons

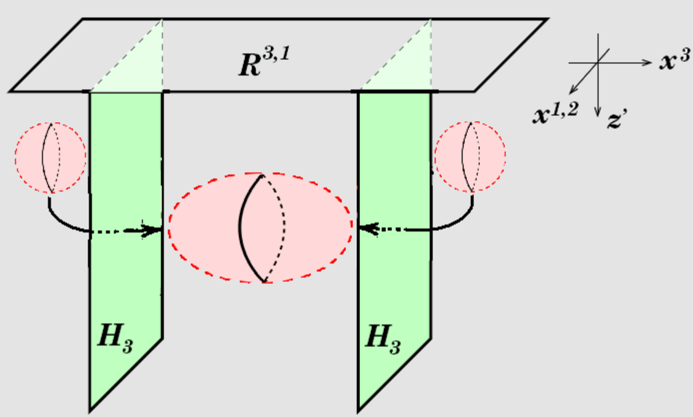


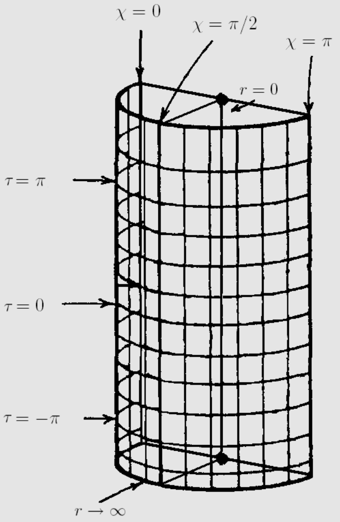
Figure: The Poincaré patch of AdS_5 drawn in coordinates adapted to the $\mathbb{R}^{3,1}$ boundary; constant-x_3 slices are copies of the hyperbolic plane H_3.

Adapted from `hep-th/0805.1551`

Collision of Two Compact BHs: Pre-Merger (Color Scale: Scalar Field) (Slice: $x_{1,2} = 0$)

Loading ...

Global AdS



Global AdS

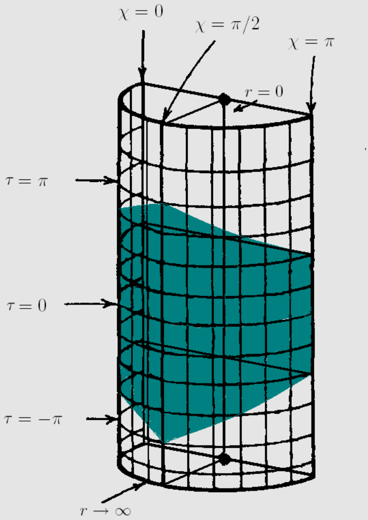
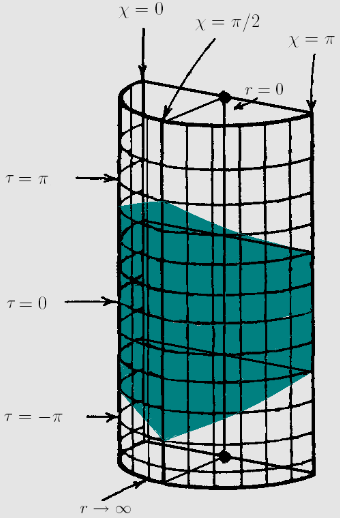
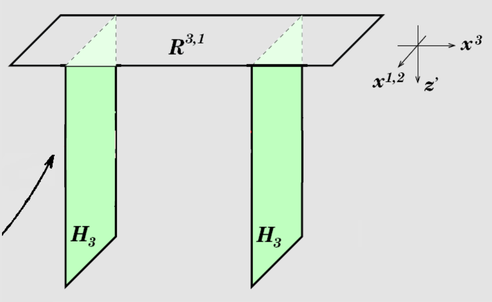


FIGURE: Shaded region depicting the Poincaré patch, defined by $\sqrt{1 + r^2/L^2}\cos(\tau/L) + r/L \sin\chi\cos\theta > 0$.

Global AdS



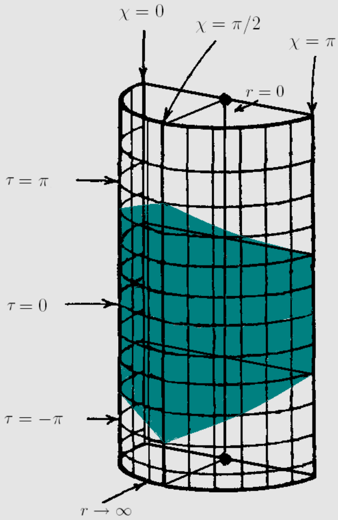


Poincaré coordinates (t, z, x_1, x_2, x_3)

global coordinates $(\tau, r, \chi, \theta, \phi)$

Figure: Shaded region depicting the Poincaré patch, defined by $\sqrt{1 + r^2/L^2}\cos(\tau/L) + r/L \sin\chi\cos\theta > 0$.

Global AdS



$$\frac{t}{L} = \frac{\sqrt{1 + r^2/L^2}\,\sin(\tau/L)}{\sqrt{1 + r^2/L^2}\,\cos(\tau/L) + r/L\,\sin\chi\cos\theta}$$

$$\frac{z}{L} = \frac{1}{\sqrt{1 + r^2/L^2}\,\cos(\tau/L) + r/L\,\sin\chi\cos\theta}$$

$$\frac{x_1}{L} = \frac{r/L\,\sin\chi\sin\theta\cos\phi}{\sqrt{1 + r^2/L^2}\,\cos(\tau/L) + r/L\,\sin\chi\cos\theta}$$

$$\frac{x_2}{L} = \frac{r/L\,\sin\chi\sin\theta\sin\phi}{\sqrt{1 + r^2/L^2}\,\cos(\tau/L) + r/L\,\sin\chi\cos\theta}$$

$$\frac{x_3}{L} = \frac{r/L\,\cos\chi}{\sqrt{1 + r^2/L^2}\,\cos(\tau/L) + r/L\,\sin\chi\cos\theta}$$

Figure: Shaded region depicting the Poincaré patch, defined by $\sqrt{1 + r^2/L^2}\,\cos(\tau/L) + r/L\,\sin\chi\cos\theta > 0$.

Massless Scalar Propagating in AdS (Color Scale: Scalar Field)

Loading ...

Collision of Two Compact BHs: Pre-Merger (Color Scale: Scalar Field)

Loading ...

Collision of Two Compact BHs: Post-Merger (Color Scale: Scalar Field)

Loading ...

ENERGY DENSITY ON $\mathbb{R}^{3,1}$ BOUNDARY

$$\epsilon_{\mathbb{R}^{3,1}} = W^{-4}\epsilon_{\mathbb{R}\times S^3}$$

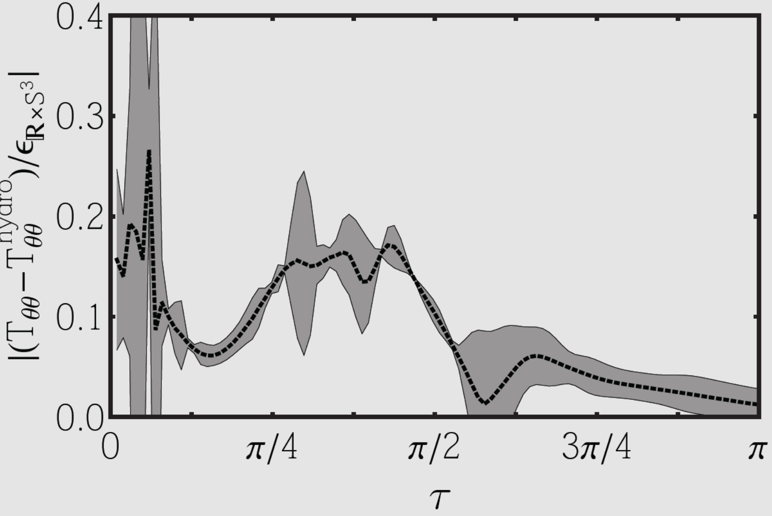
$x_3 \uparrow$

Loading ...

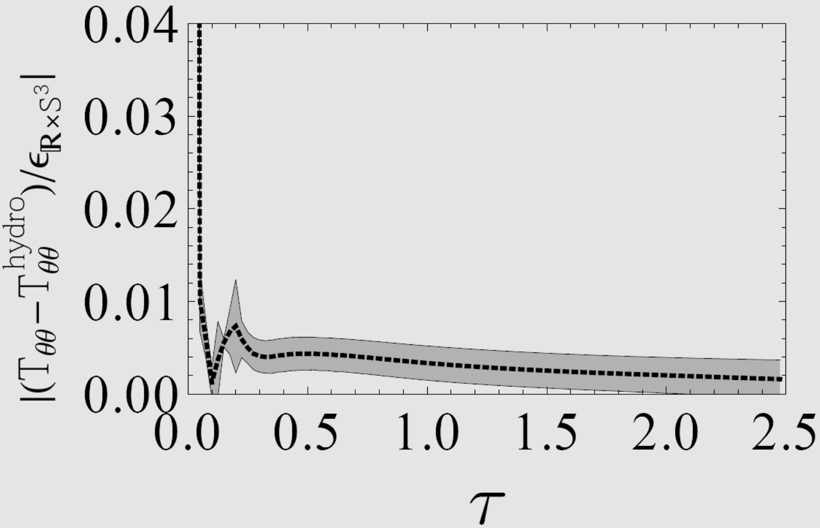
$\longrightarrow$
x_1

$$W = \sqrt{(t)^2 + (1 + x_1^2 + x_2^2 + x_3^2 - (t)^2)^2/4}$$

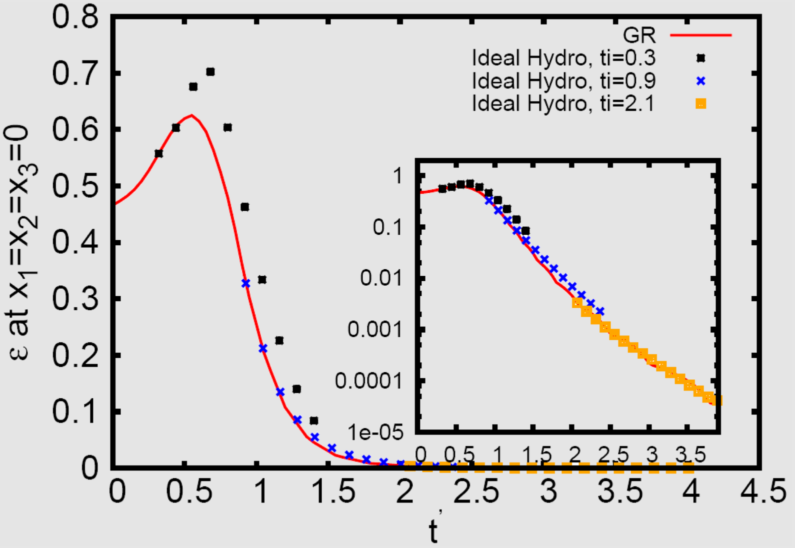
Comparison to Hydrodynamics: From Merger of Two BHs



Comparison to Hydrodynamics: From Single Deformed BH



Comparison to Hydrodynamics: Merger of Two BHs



Summary

What physics can we hope to extract from these simulations?

- dynamics of $\langle T_{\mu\nu} \rangle_{CFT}$ far from equilibrium, relevant to head-on heavy ion collisions

What has been done?

- BH-BH collisions in global AdS and on the Poincaré patch

What remains to be done?

- Post-merger stability
- Boosted black hole initial data
- GRChombo implementation (4+1, AMR, optimized)