Holographic correlation functions out of equilibrium

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Why NumHol?

Because you can't solve analytically.

Do numerical experiments.

Simulate (real) time evolution.

Motivate experiments in laboratories/big machines.

But learn more from theoretical perspectives.

Motivations

Correlators in real-time AdS/CFT formalism:
Some complexity. Different methods available.
[e.g. Banks et al, Skenderis-van Rees, Keranen-Kleinert]

Numerical time dependent AdS/CFT:
Horizon formation, shockwave collision, QGP
[e.g. Chesler-Yaffe, Heller et al, Bantilan-Romatschke]

Holography and thermalization of nonequilibrium correlation functions/nonlocal operators
[e.g. Balasubramanian et al, Chesler-Teaney]

Plan

1. Basics from field theory

2. Holographic example

Linear response theory

Perturb a Hamiltonian by a tiny source J

$$H = H_0 + \delta H, \quad \delta H \sim J\,O$$

The source induces a response linearly

$$\langle O(x)\rangle_J - \underline{\langle O(x)\rangle_0} = -\int d^dx'\, G_R(x,x')J(x')$$

=0 for simplicity

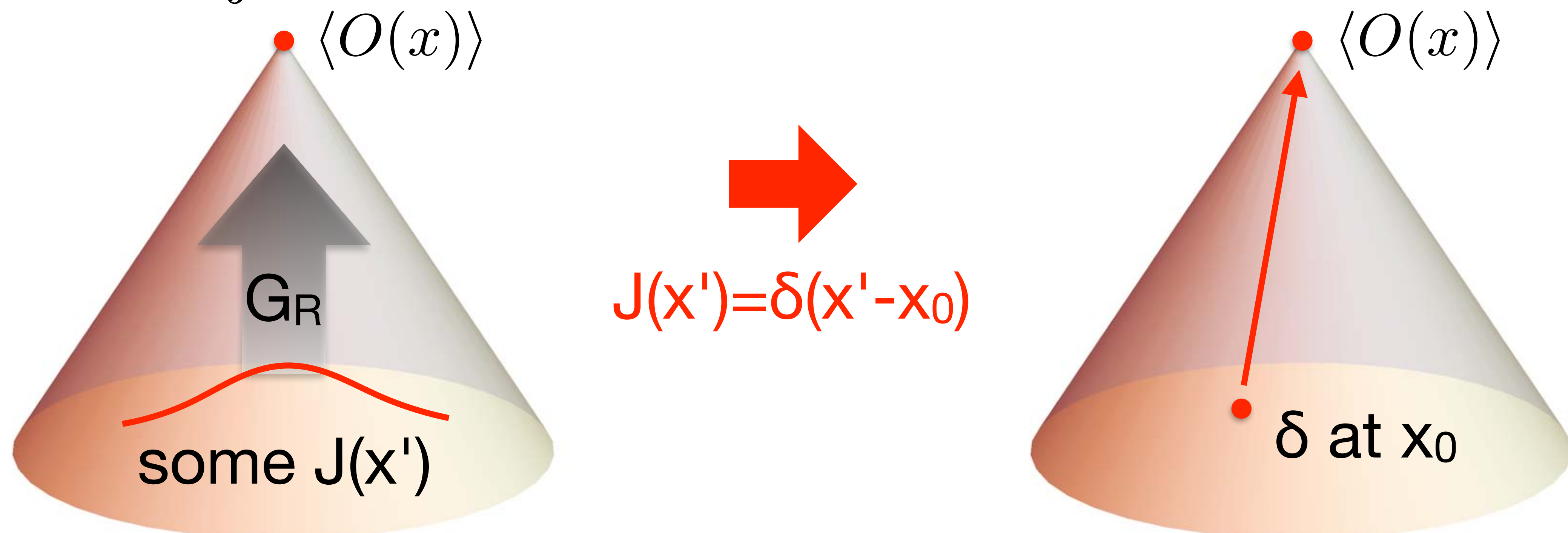
Retarded Green's function:

$$G_R(x,x') = -i\theta(t-t')\langle[O(x),O(x')]\rangle$$

Want to get G_R

Response: causal past determines the future.

$$\langle O(x)\rangle \sim \int G_R(x,x')J(x') \qquad \langle O(x)\rangle_\delta = -G_R(x,x_0)$$



This looks simple.

Use a δ-source and calculate the response.

Partial Fourier space

Can be a simplification if spatially translationally invariant: Consider each momentum sector.

$$\langle O(x) \rangle \sim \int G_R(x, x') J(x')$$

$$\Big\downarrow \quad \times \int d\boldsymbol{x}\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \,, \quad J(x') = f(t')\delta(\boldsymbol{x'} - \boldsymbol{x_0})$$

$$\langle O(t; \boldsymbol{k}) \rangle \sim \int G_R(t, t'; \boldsymbol{k}) f(t')$$

$$\Big\downarrow \quad f(t') = \delta(t' - t_0)$$

$$\langle O(t; \boldsymbol{k}) \rangle_\delta = -G_R(t, t_0; \boldsymbol{k})$$

Frequency analysis

Equilibrium: $G_R(t,t_0)$ depends only on $t-t_0$. Use Fourier transform.

$$\widetilde{G}_R(\omega;\boldsymbol{k}) = \int dt'\, e^{i\omega t'} G_R(t'=t-t_0;\boldsymbol{k})$$

Nonequilibrium: Convert one of the two times in $G_R(t,t_0)$ to frequency. Can use Wigner transform

$$\widetilde{G}_R(\omega,\bar{t};\boldsymbol{k}) = \int dt'\, e^{i\omega t'} G_R(\bar{t}+t'/2,\bar{t}-t'/2;\boldsymbol{k})$$

$t'=t-t_0$, $t=(\bar{t}+t_0)/2$

Is "G_R a ratio of source and VEV"?

Coordinate space

$$\langle O(t)\rangle_\delta \sim \int G_R(t,t')\delta(t'-t_0) \sim G_R(t,t_0)$$

$G_R(t,t_0)$: You don't divide <O> by δ(t).

Frequency space

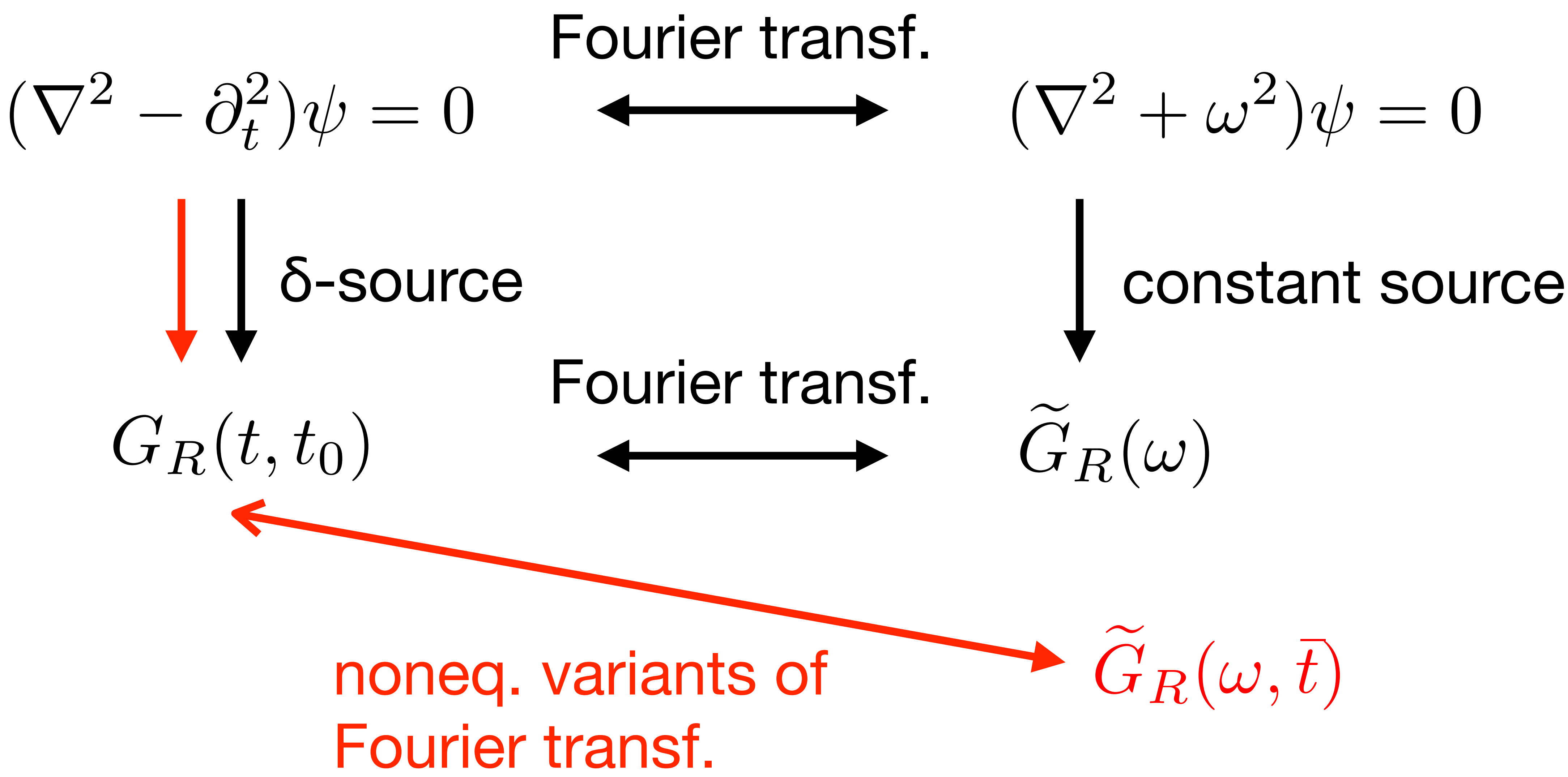
$$\int dt\, e^{i\omega t}\delta(t) = 1$$

$$\langle \widetilde{O}(\omega)\rangle_\delta \sim \widetilde{G}_R(\omega) \times 1 \quad \Rightarrow \quad \widetilde{G}_R(\omega) \sim \langle \widetilde{O}(\omega)\rangle_\delta / 1$$

$G_R(\omega)$: You may divide <O> by 1.

Summary: Equilibrium or nonequilibrium

No time translation invariance in nonequilibrium problems.



To use holography

Typically: Prepare bulk initial data by the δ-source.

This will be some bulk Green's function.

And let it evolve in time.

May be sometimes analytic, sometimes tricky.

Here: Try to avoid technically involved steps and find a way that will be generically applicable.

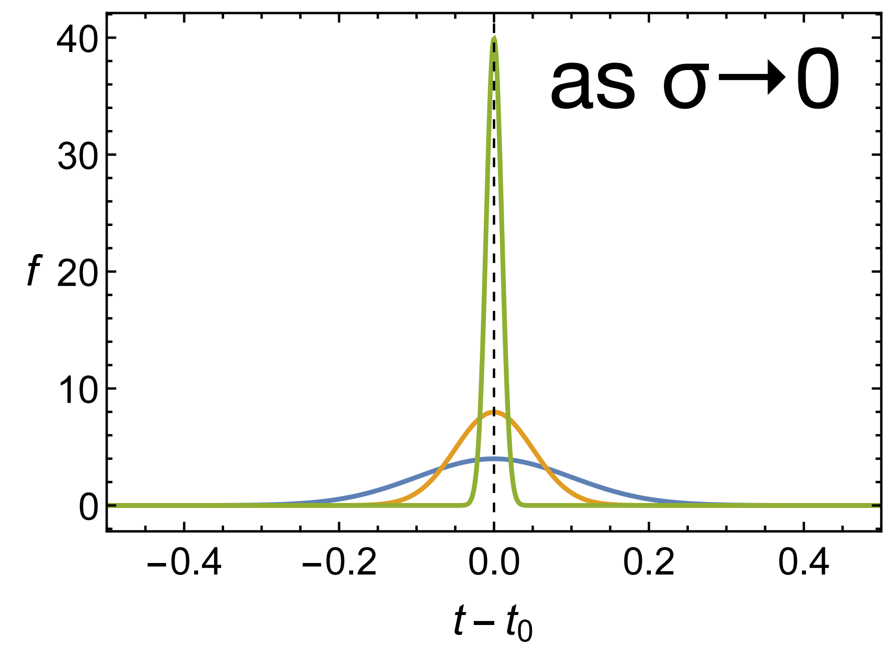
Using δ($t-t_0$) numerically

Use a preferred source whose limit is δ($t-t_0$).

e.g.) Normalized Gaussian function

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(t-t_0)^2/2\sigma^2}$$

$$\int_{-\infty}^{\infty} dt\, f(t) = 1$$



Practically, it works some small σ that gives smaller systematic errors than other errors.

We may extrapolate to σ→0.

Plan

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Simple holographic case

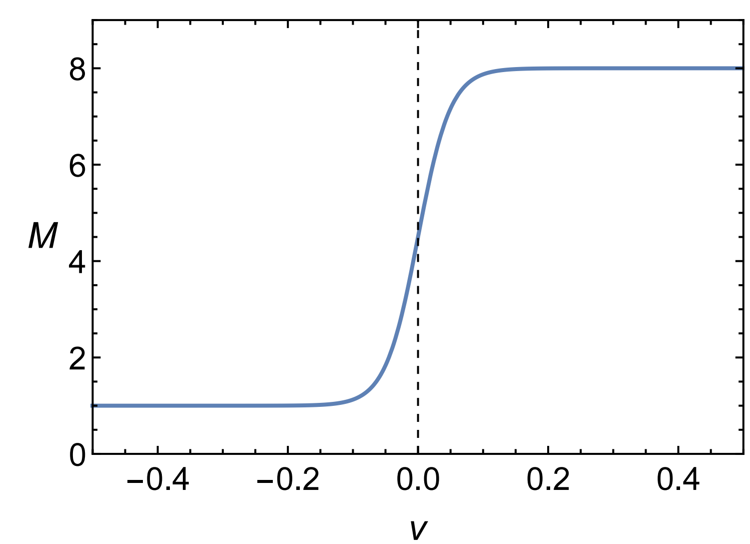
Probe scalar in Vaidya-AdS$_4$ (m^2=-2 → Δ=2)

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)$$

$$ds^2 = \frac{1}{z^2}\left(-F(z,v)dv^2 - 2dvdz + d\boldsymbol{x}^2\right)$$

$$F(z,v) = 1 - M(v)z^3$$

M(v) changes as tanh(v/Δv).



Choose initial and final temperatures twice different.

Holographic dictionary

Solve the bulk EoM using any methods you like.

$$(\mathrm{d}_+\phi)' - \frac{\mathrm{d}_+\phi}{z} + \frac{F}{2z}\phi' + \left(\frac{m^2}{2z^2} + \frac{k^2}{2}\right)\phi = 0$$

Obtain the field theory VEV from the AdS boundary

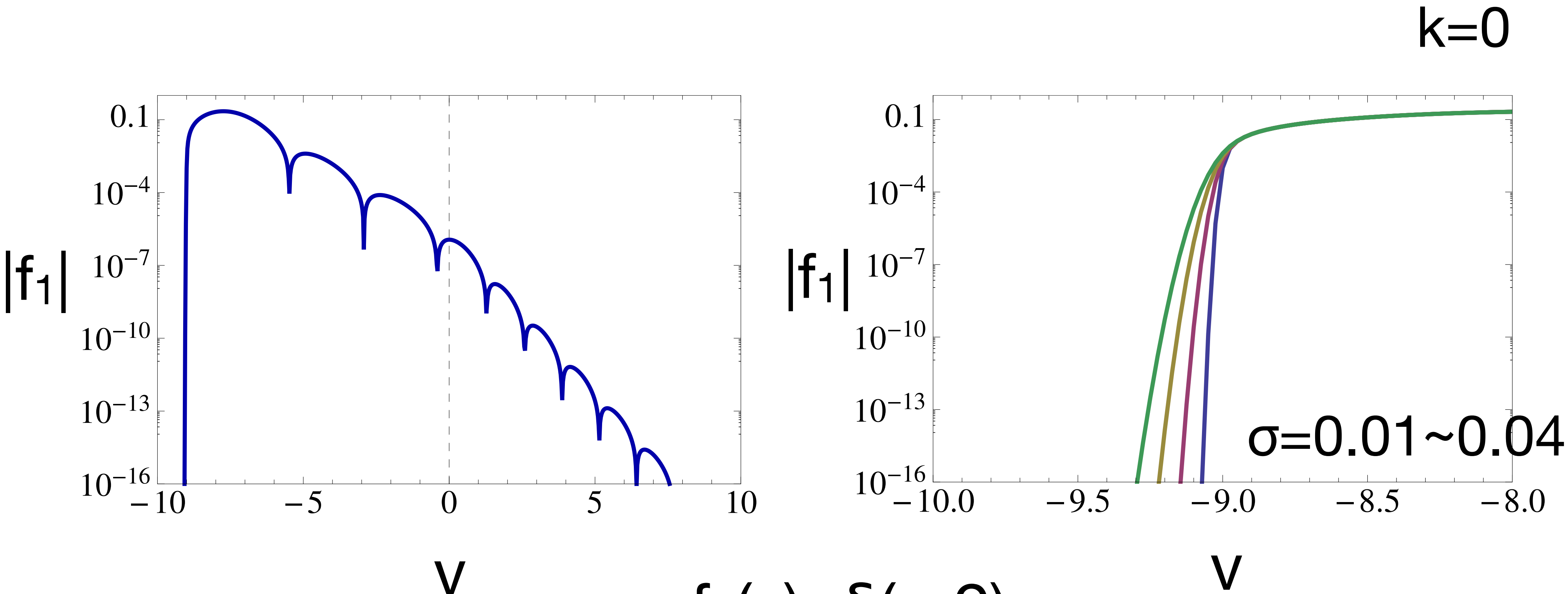
$$\phi = f_0(v)z + f_1(v)z^2 + \cdots$$

$$\langle O(v)\rangle = f_1(v) - \dot{f}_0(v)$$

$$G_R(v, v_0) = -\langle O(v)\rangle_\delta = -f_1(v) + \underline{\dot{\delta}(v - v_0)}$$

ultralocal

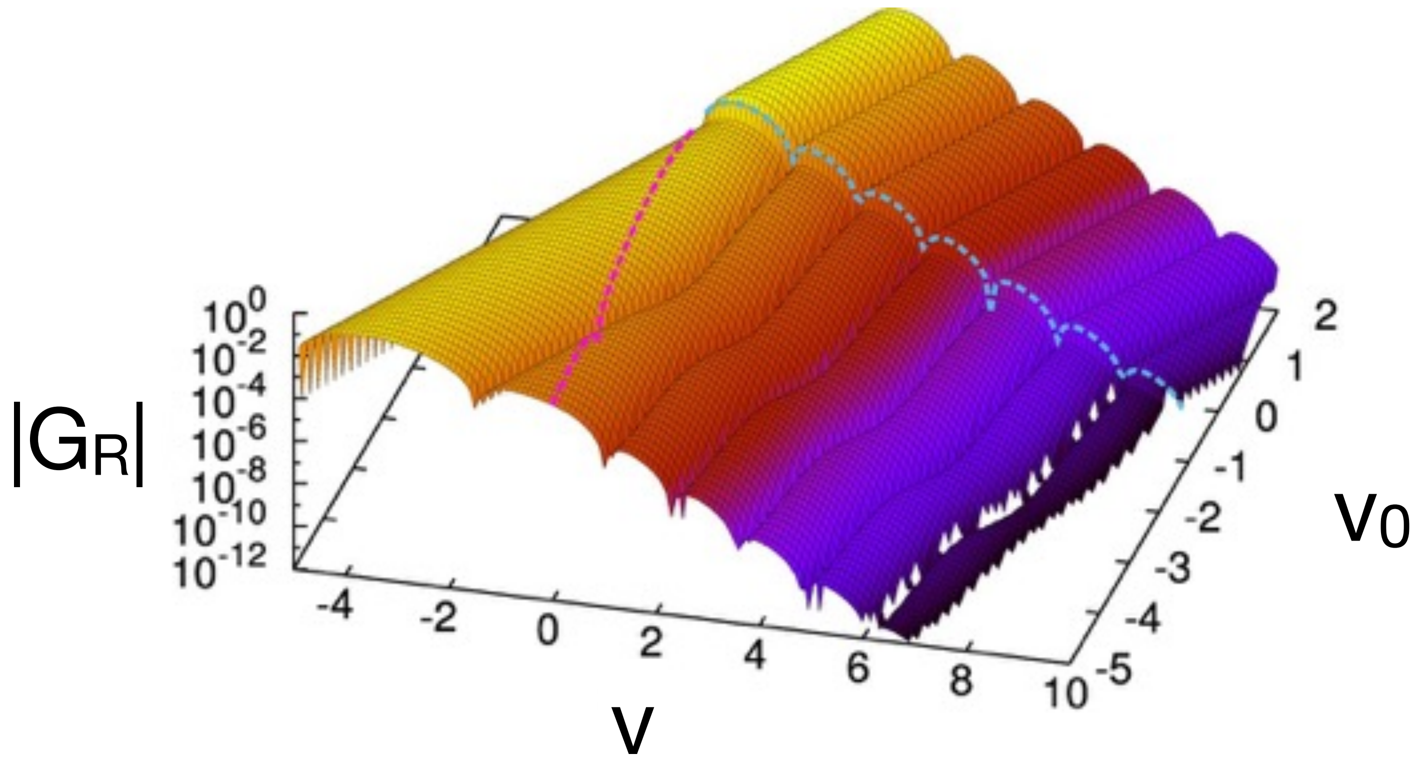
$G_R(v,v_0)$ for a given v_0



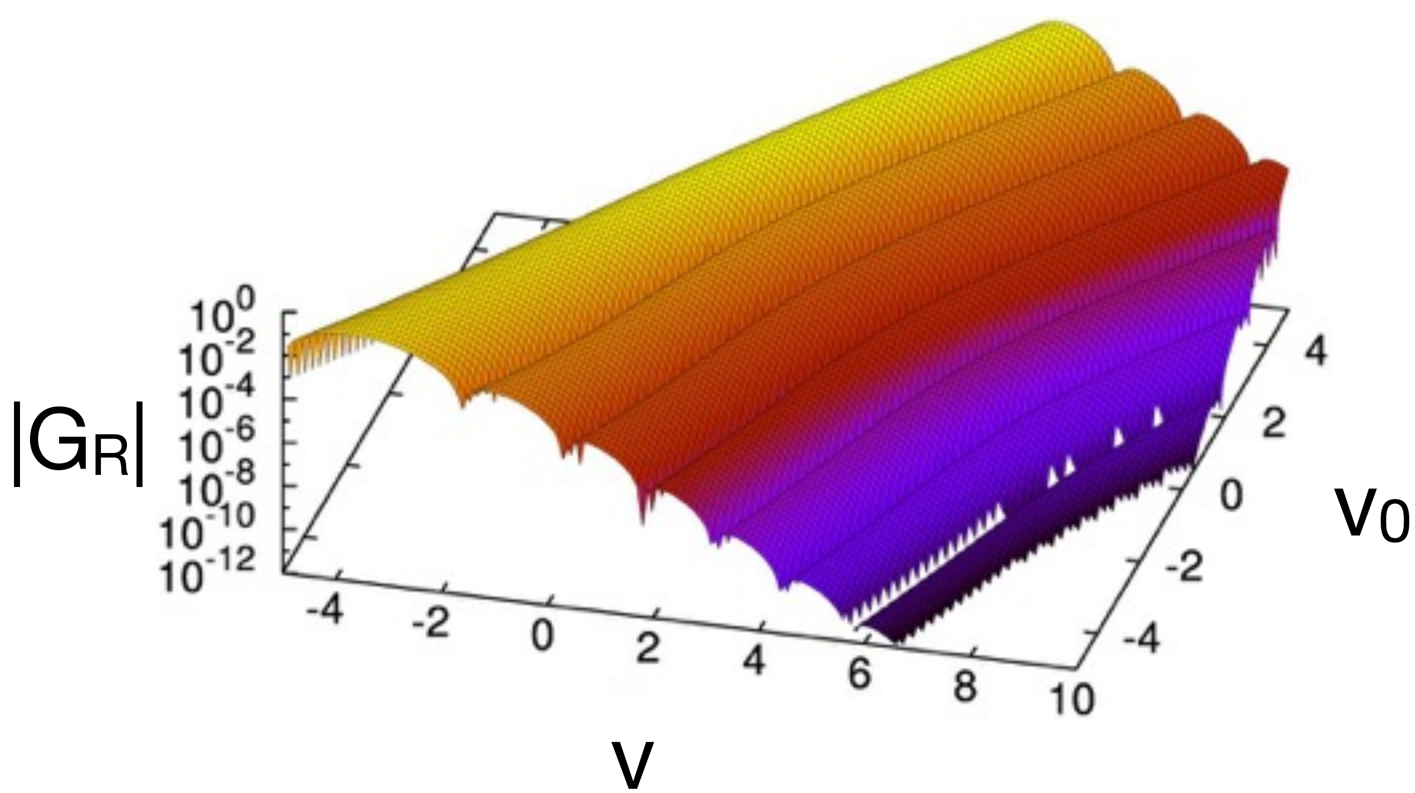
$f_0(v)=\delta(v-9)$

The v-dependence in $G_R(v,v_0)$ follows the lowest quasinormal mode.

Full $G_R(v,v_0)$



Fast background change: $\Delta v=0.05$

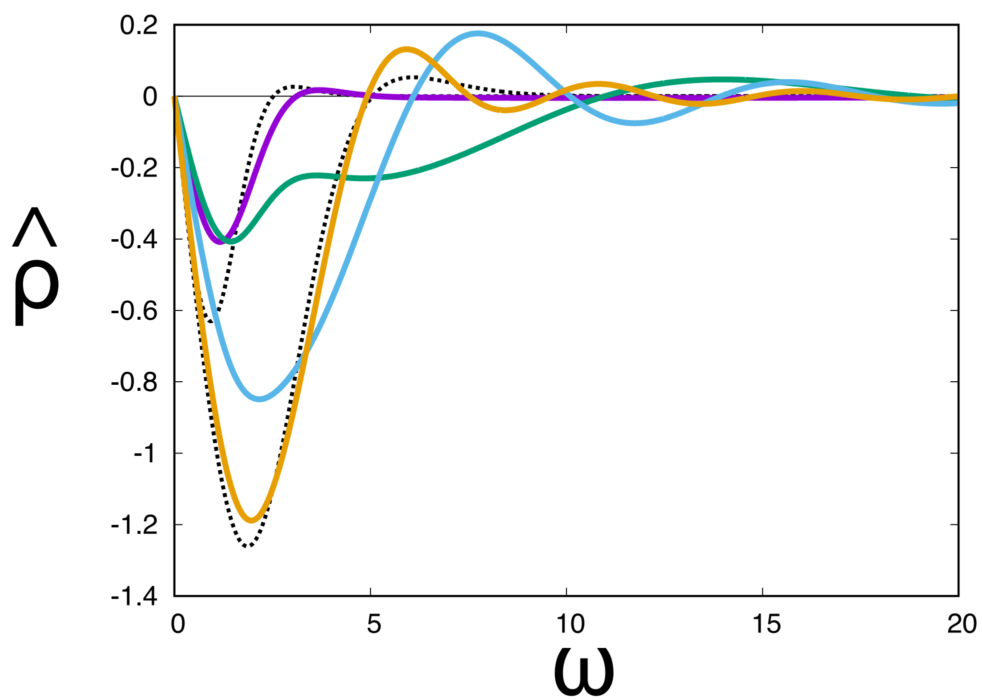


Slow background change: $\Delta v=2$

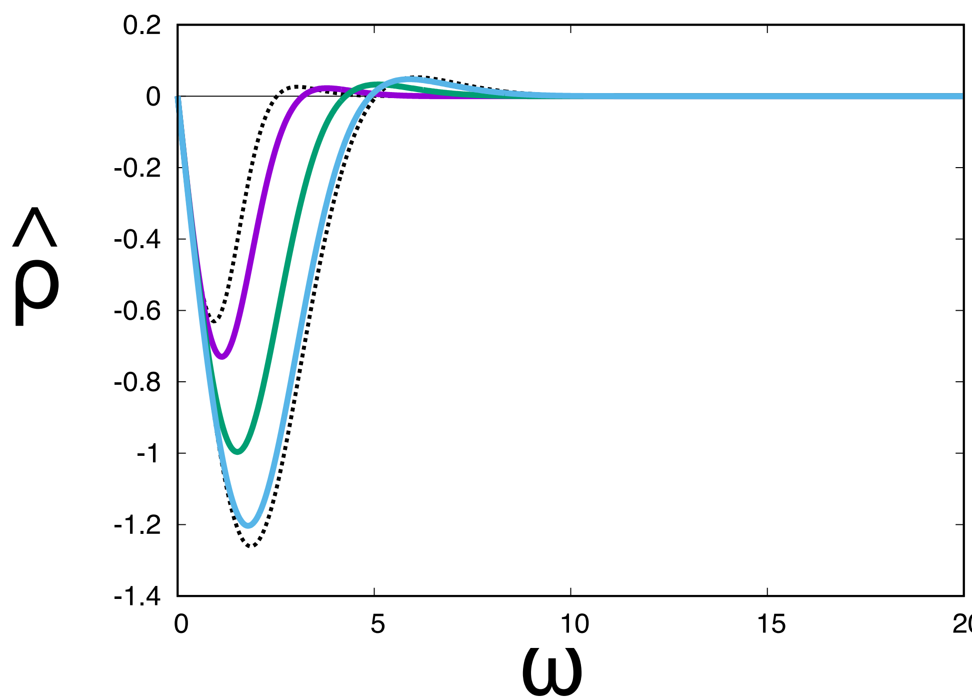
Spectral function

Spectral function: $\rho = -2\,\mathrm{Im}\,\widetilde{G}_R$

Plotted: $\hat{\rho} \equiv \rho - \rho_{\mathrm{AdS}} = \rho - 2\theta(\omega - k)\sqrt{\omega^2 - k^2}$



Fast: Δv=0.05



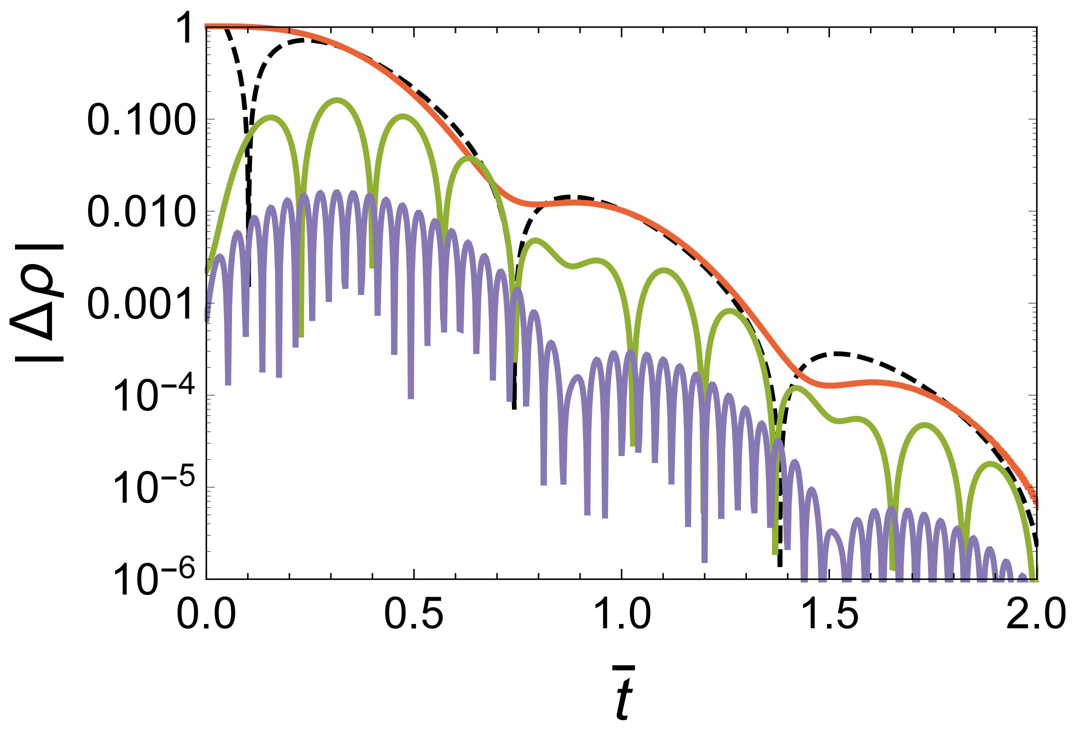
k=0

Near adiabatic: Δv=5

$\bar{t}$-dependence

How ρ approach the final state

$$\Delta\rho(\omega, \bar{t}; \boldsymbol{k}) = \rho(\omega, \bar{t}; \boldsymbol{k}) - \rho(\omega, \infty; \boldsymbol{k})$$



Red: ω=2 ~ $\omega_{(1st\ QNM)}$

Green: ω=10

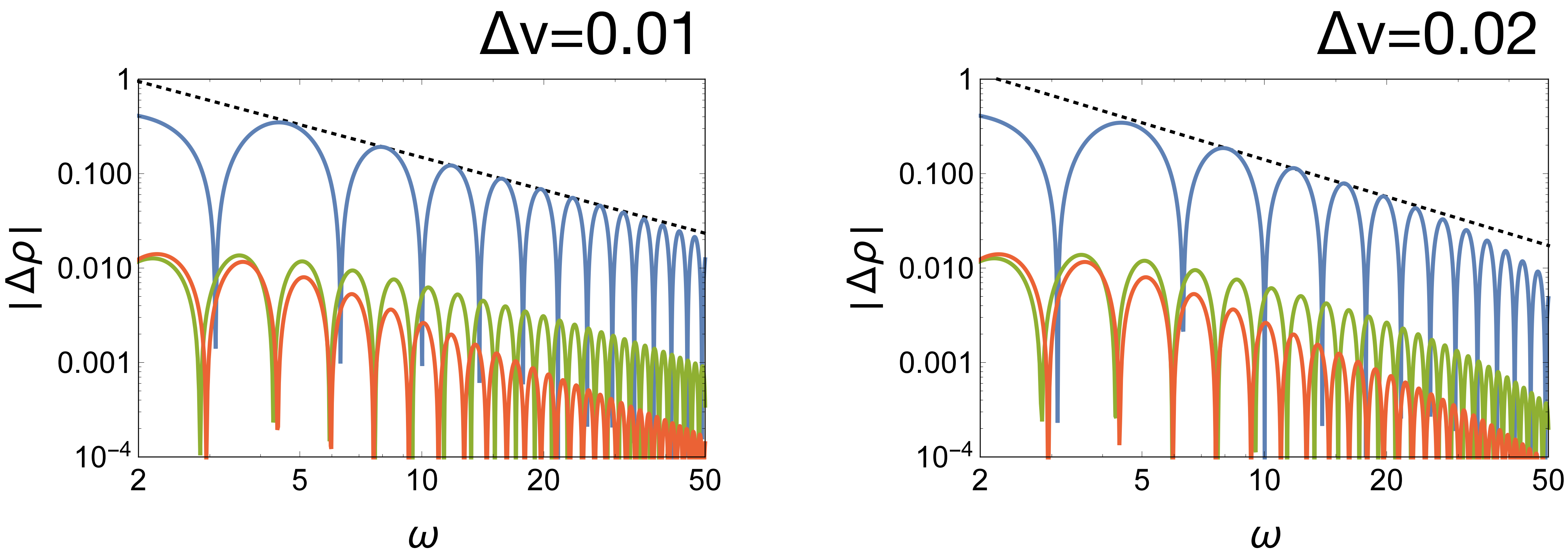
Purple: ω=40

(Black: final state QNM)

Early time: (higher) mode excitations

Late time: BH QNM damping with some ω-oscillations

ω-dependence



Blue: t=0.4

Green: k=0.8

Red: k=0.9

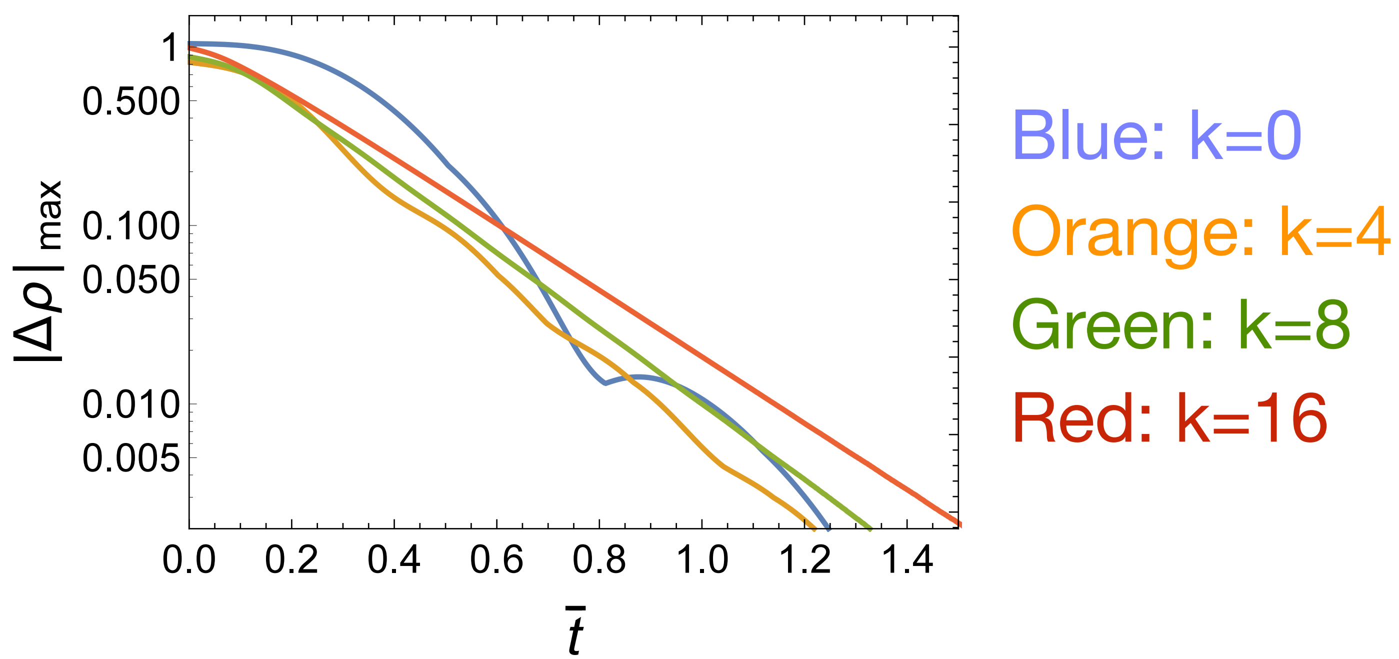
Power law region is bigger as $\Delta v \to 0$.

Higher ω is needed to probe the short timescale of Δv.

k-dependence

Introduce a measure for the difference from final state

$$|\Delta\rho|_{\max} \equiv \max_{\omega>0}\left(|\Delta\rho|\right)$$



QNM decay is more relevant in the late time.

Summary

To calculate $G_R(t,t')$, we introduced the δ-function source numerically by a Gaussian pulse.

Source-response is natural in holography.

Perhaps this brute-force way is generically useful in other setups, while it may be numerically costly.

We obtained the nonequilibrium spectral function.