Unstable horizons and cosmic censorship violation in holography

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to appear (with Luis Lehner)

$\Longrightarrow$ String theory/gauge theory correspondence is by now a mature framework to address interesting questions in strongly coupled gauge theories, that are often inaccessible with other theoretical tools

- In a nutshell, it establishes a *holographic correspondence* (a dictionary) between two objects:

$$\textbf{gauge theory} \qquad \Longleftrightarrow \qquad \textbf{higher-dimensional gravitational theory/string theory}$$

- A utility of the correspondence is in the fact that the strong coupling questions about the gauge theory are mapped into the questions in classical gravity

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  - $F_{\mu\nu}$ is a $U(1)$ field strength
  - ϕ is a scalar field dual to $\Delta = 2$ (a relevant operator) in CFT_3:

$$\phi = 0\ \frac{1}{r} + \mathcal{O}_2\ \frac{1}{r^2} + \cdots$$

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- Consider equilibrium states of a CFT at a finite temperature T and a chemical potential μ

- As well known, there are two phases of the model distinguished whether:

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  - exists for arbitrary temperature $T \geq 0$
  - gravitationally described by Reissner-Nordstrom AdS_4 black hole
  - with unbroken $\mathbb{Z}_2$ symmetry ($\phi \leftrightarrow -\phi$)

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- At low temperatures this $\mathbb{Z}_2$ symmetric phase becomes unstable:
  - on the gravity side this is a Gregory-Laflamme instability due to 'scalarization' of the Reissner-Nordstrom AdS_4 black hole horizon

- ϕ scalar is above the AdS_4 Breitenlohner-Freedman (BF) bound

$$m_\phi^2 = -2 \qquad > \qquad m^2_{BF[AdS_4]} = -\frac{(4-1)^2}{4L^2} = -\frac{9}{4}$$

- As Reissner-Nordstrom AdS_4 black hole becomes extremal ($\frac{T}{\mu} \to 0$), it develops $AdS_2 \times R^2$ near horizon geometry with the curvature radius

$$L_2^2 = \frac{L^2}{6}$$

- In this limit

$$m_\phi^2 = -2 \qquad < \qquad m^2_{BF[AdS_2]} = -\frac{(2-1)^2}{4L_2^2} = -\frac{3}{2}$$

becomes unstable (its quasinormal frequency has $\mathrm{Im}[\omega] > 0$)

- The condensation of the gravitational scalar ϕ at low temperatures
  - is dynamically "stopped" by nonlinear effects
  - spontaneously breaks $\mathbb{Z}_2$ symmetry, leading to a new equilibrium phase of the CFT with

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$\Longrightarrow$ There are lots of studies/generalizations of the described phenomena in holography

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  - we work in microcanonical ensemble
  - I will keep calling the instability "GL"
- There is an equilibrium phase with spontaneously broken symmetry, branching off the GL onset of the instability, yet, this phase:
  - does not exist **below** the critical energy
  - has **lower** entropy above the criticality than the symmetric phase
  - thus,

the horizon is unstable, but it is unknown what is the end point of its instability!

$\Longrightarrow$ To the best of my knowledge, this phenomena was first identified in bottom-up holographic model:

A. Buchel and C. Pagnutti, "Exotic Hairy Black Holes," Nucl. Phys. B 824, 85 (2010) doi:10.1016/j.nuclphysb.2009.08.017 [arXiv:0904.1716 [hep-th]]

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$\Longrightarrow$ Later, it was observed in top-down construction:

A. Donos and J. P. Gauntlett, "Superfluid black branes in $AdS_4 \times S^7$," JHEP 1106, 053 (2011) doi:10.1007/JHEP06(2011)053 [arXiv:1104.4478 [hep-th]]

$\Longrightarrow$ It is also the GL instability physics of small black holes in $AdS_5 \times S^5$:

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$\Longrightarrow$ I will report on progress in understanding this phenomena

Outline of the talk:

- "Exotic hairy black holes"
  - equilibrium states
  - GL instability
- Dynamics of the exotic unstable horizons
  - dynamical setup
  - validation of the GL instability
  - fully non-linear evolutions above/below critically
- Conclusions and comments

$\Longrightarrow$ The effective four-dimensional gravitational bulk action takes form (AdS_4 radius is set to 1)

$$S_4 = S_{CFT} + S_r + S_i = \frac{1}{2\kappa^2}\int dx^4 \sqrt{-\gamma}\,[\mathcal{L}_{CFT} + \mathcal{L}_r + \mathcal{L}_i]$$

with

- a CFT part

$$\mathcal{L}_{CFT} = R + 6$$

- its deformation by a relevant operator $\mathcal{O}_r$ (here $\Delta_r = 2$)

$$\mathcal{L}_r = -\frac{1}{2}\left(\nabla\phi\right)^2 + \phi^2$$

- a sector S_i involving an irrelevant operator $\mathcal{O}_i$ (here $\Delta_i = 4$) along with its mixing with $\mathcal{O}_r$ under the RG dynamics

$$\mathcal{L}_i = -\frac{1}{2}\left(\nabla\chi\right)^2 - 2\chi^2 - g\phi^2\chi^2$$

$\Longrightarrow$ Effective action S_4 has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ discrete symmetry that acts as a parity transformation on the scalar fields ϕ and χ.

- The discrete symmetry

$$\phi \leftrightarrow -\phi$$

is explicitly broken by a relevant deformation of the AdS_4 CFT;

- while as we will see, the

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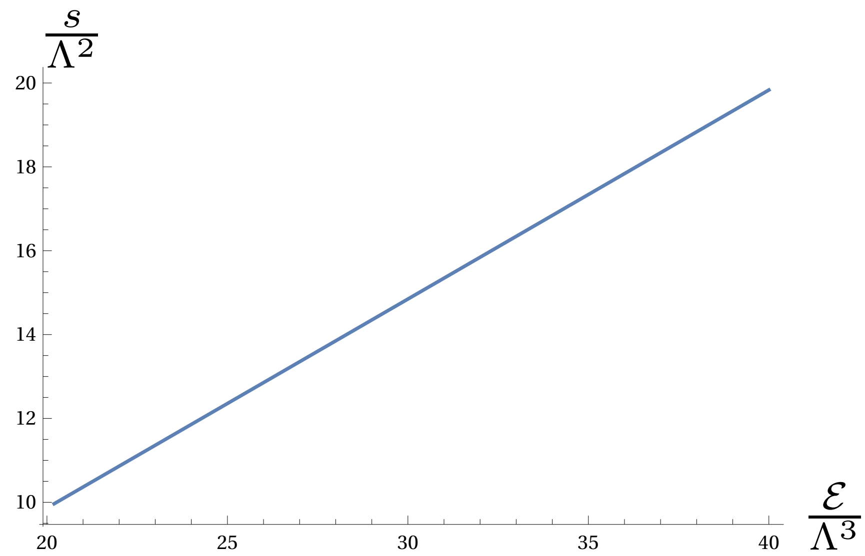
$\Longrightarrow$ This mechanism for the GL instability has been proposed by Gubser

Thermal equilibrium phase with $\langle \mathcal{O}_i \rangle = 0$:

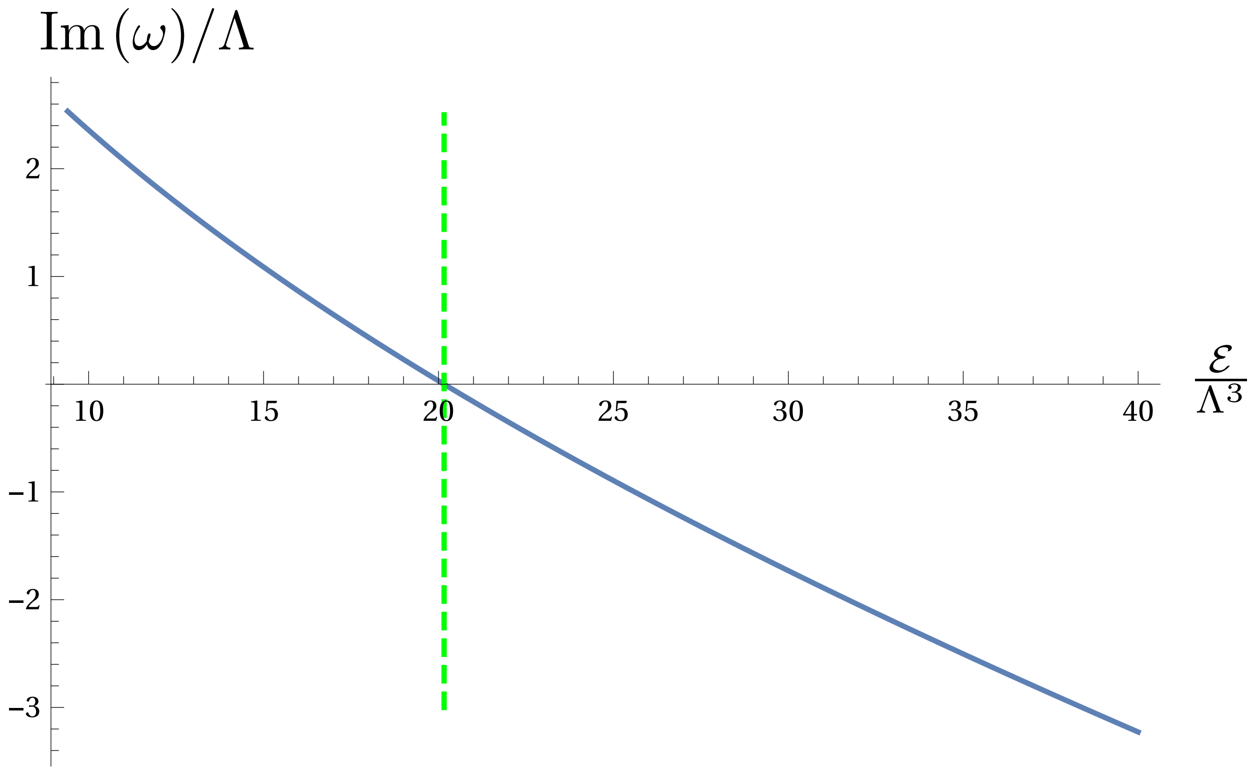


- Λ — the mass scale associates with the relevant deformation of the CFT:

$$\mathcal{H}_{CFT} \to \mathcal{H}_{CFT} + \Lambda\ \mathcal{O}_r$$

- $\mathcal{E}$ — equilibrium energy density
- s — equilibrium entropy density

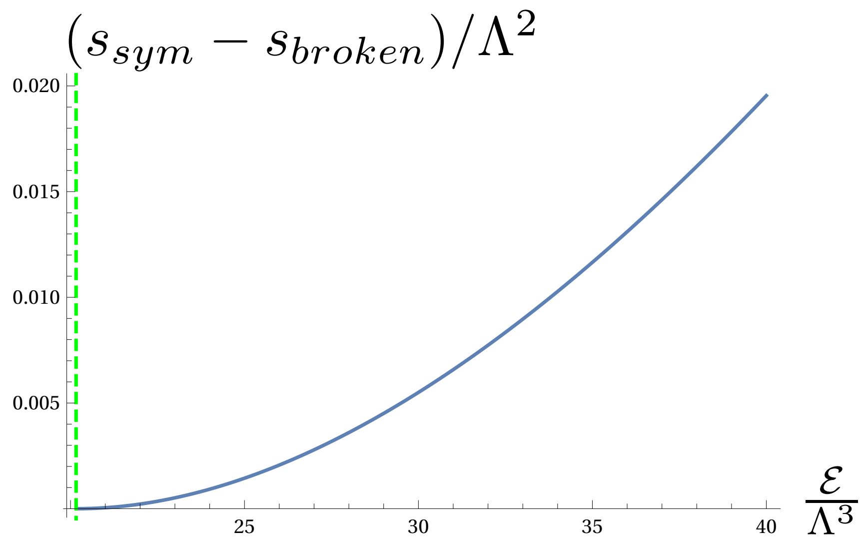
Lowest QMN of χ scalar (dual to $\mathcal{O}_i$ operator) in $\langle \mathcal{O}_i \rangle = 0$ phase:

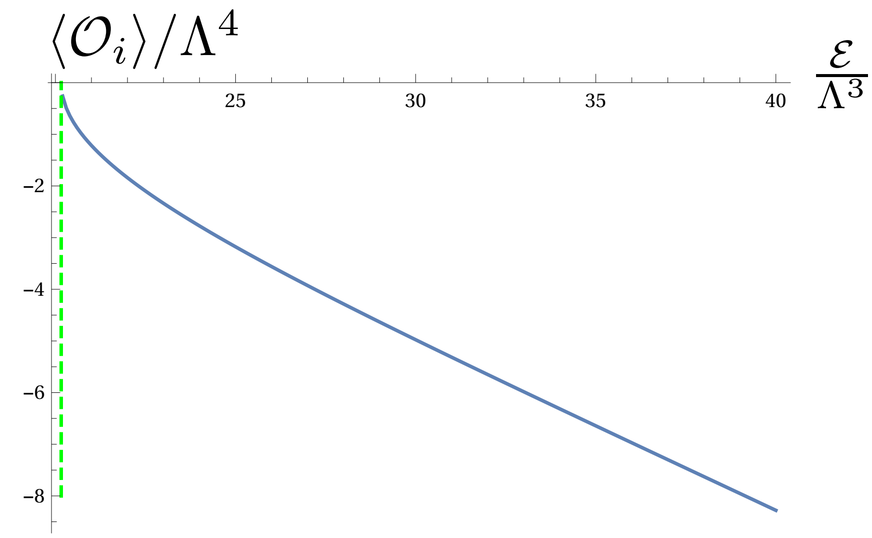


- Vertical green line indicates instability below the critical energy density $\mathcal{E}_c$:

$$\frac{\mathcal{E}_c}{\Lambda^3} = 20.16021(8)$$

The phase with $\langle \mathcal{O}_i \rangle \neq 0$ (symmetry broken phase) exists only for $\mathcal{E} > \mathcal{E}_c$:





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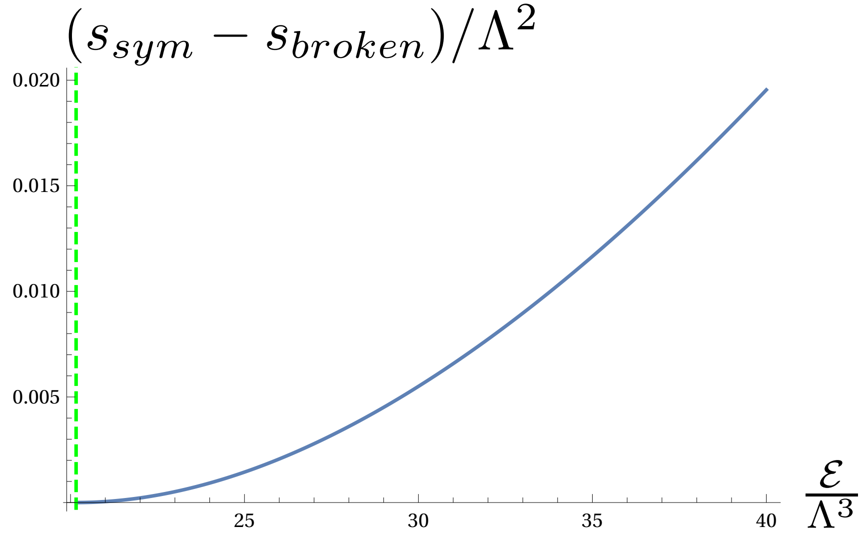

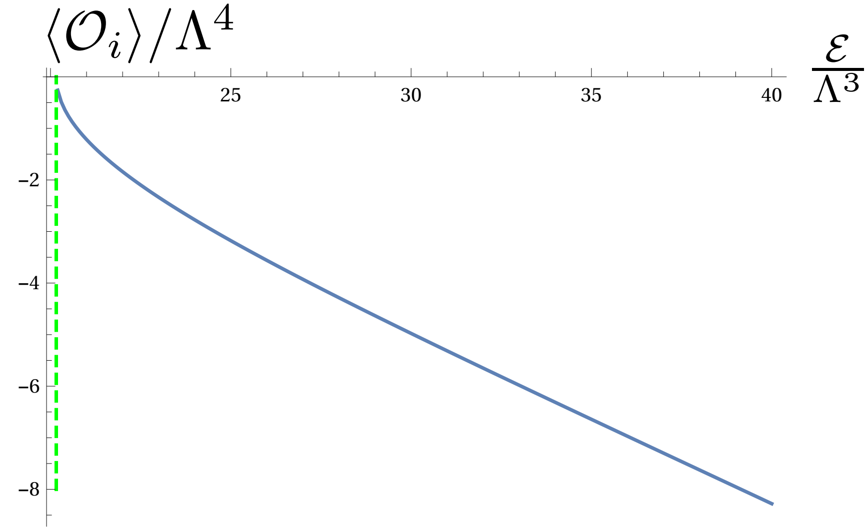

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What is the end-point of the symmetric phase instability for $\mathcal{E} < \mathcal{E}_c$?

$\Longrightarrow$ Dynamical set-up:

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- metric ansatz

$$ds_4^2 = 2dt\,(dr - A(t,r)\ dt) + \Sigma(t,r)^2\ \left[dx_1^2 + dx_2^2\right]$$

$$\phi = \phi(t,r)\,, \qquad \psi = \psi(t,r)$$

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- boundary asymptotic expansions ($r \to \infty$):

$$\Sigma = r + \lambda(t) - \frac{1}{8}p_1^2\ \frac{1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$A = \frac{r^2}{2} + \lambda(t)\ r - \frac{1}{8}p_1^2 + \frac{1}{2}\lambda(t)^2 - \dot{\lambda}(t) + \left(\mu - \frac{1}{4}p_1 p_2(t) - \frac{1}{4}p_1^2\lambda(t)\right)\frac{1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

$$\phi = \frac{p_1}{r} + \frac{p_2(t)}{r^2} + \mathcal{O}\left(\frac{1}{r^3}\right)\,, \qquad \chi = \frac{q_4(t)}{r^4} + \mathcal{O}\left(\frac{1}{r^5}\right)$$

$\Longrightarrow$ Parameters of the asymptotic expansion:

$$p_1 \ (\text{const}) \,, \qquad p_2(t) \,, \qquad q_4(t) \,, \qquad \mu \ (\text{const}) \,, \qquad \lambda(t)$$

Interpretation:

- $$p_1 = \Lambda \,, \qquad p_2(t) = \langle \mathcal{O}_r(t) \rangle$$
- $$q_4(t) = \langle \mathcal{O}_i(t) \rangle$$
- $$-4\mu = \frac{\mathcal{E}}{\Lambda^3}$$
- $\lambda(t)$ — residual radial coordinate diffeomorphism parameter

  $$r \to r + \lambda(t)$$

  adjusted to keep the location of the apparent horizon fixed:

  $$\left(\partial_t + A(t,r)\partial_r \right) \Sigma(t,r) \ \equiv \ d_+\Sigma(t,r) \Big|_{r=1} = 0$$

$\Longrightarrow$ Brief comment on numerical simulations:

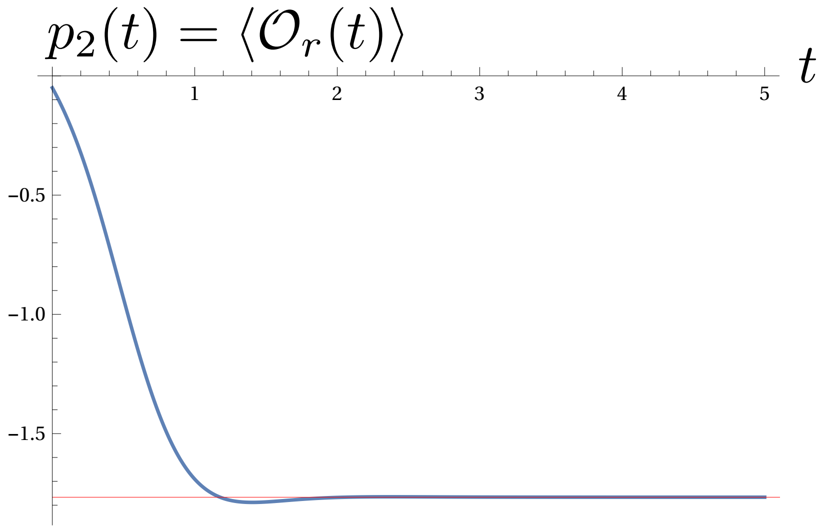
- Use Chesler-Yaffe characteristic formulation as in 1309.1439
- There are subtleties with field redefinitions (scalar ϕ with a non-normalizable coefficient turned on)
- Subtleties in adjusting $\lambda(t)$ to keep apparent horizon location at $r = 1$.
- Initial conditions are specified with constant amplitudes $\{\mathcal{A}_\phi, \mathcal{A}_\chi\}$:

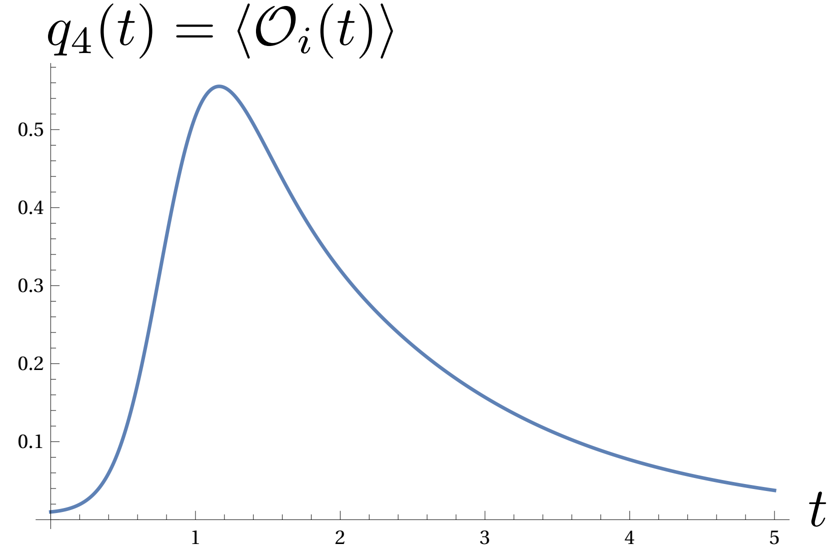
$$\phi(t=0,r) = \frac{p_1}{r} + \mathcal{A}_\phi \, \frac{e^{-1/r}}{r^2}$$

$$\chi(t=0,r) = \mathcal{A}_\chi \, \frac{e^{-1/r}}{r^4}$$

$\Longrightarrow$ We did convergence tests and computed residuals; the code is stable, when the final state is known, it equilibrates to it; many more tests...

$\Longrightarrow$ Results $\mathcal{E} > \mathcal{E}_c$ (stable regime)





- **Left panel**: expectation value of the relevant operator; the red thin line corresponds to equilibrium value $\langle \mathcal{O}_r \rangle^{equilibrium}$, given Λ

- **Right panel**: expectation value of the irrelevant operator; note that $\langle \mathcal{O}_i(t) \rangle \ \propto e^{-\text{const} \cdot t}$ at late times.

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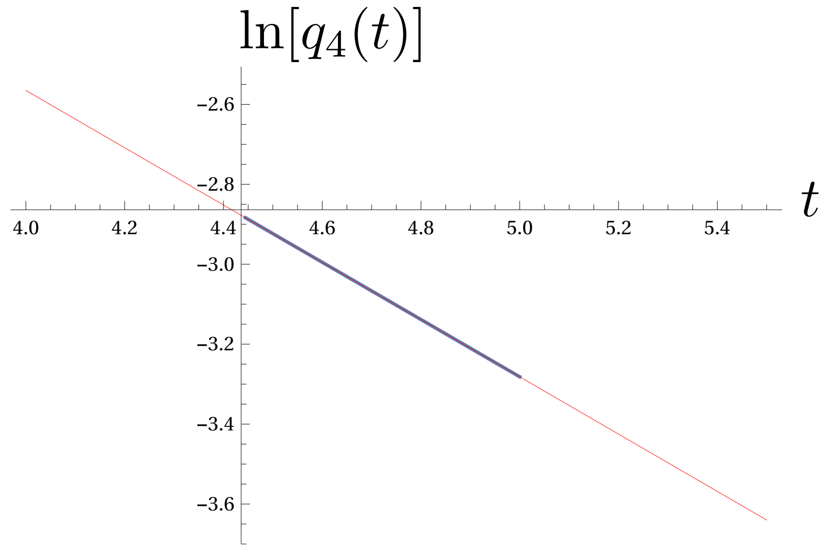

- **Blue curve**: late time behaviour of $\ln[q_4]$
- **Thin red curve**: a linear fit to the late time behaviour,

$$\ln[q_4]\Big|_{fit} = 0.301112 - 0.716569\ t$$

- χ-scalar quasinormal frequency at given $\mathcal{E}$:

$$\mathrm{Im}\,[\omega] = -0.71707580318909458115141509922001721094\,1$$

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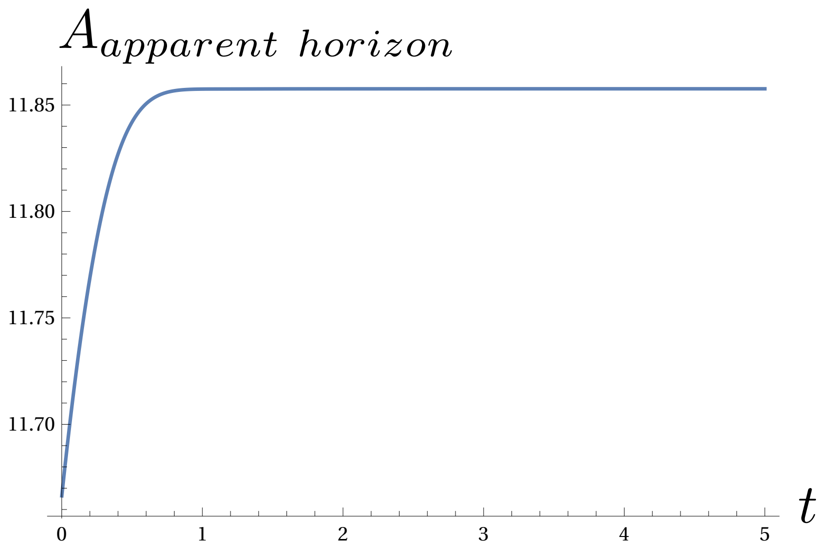

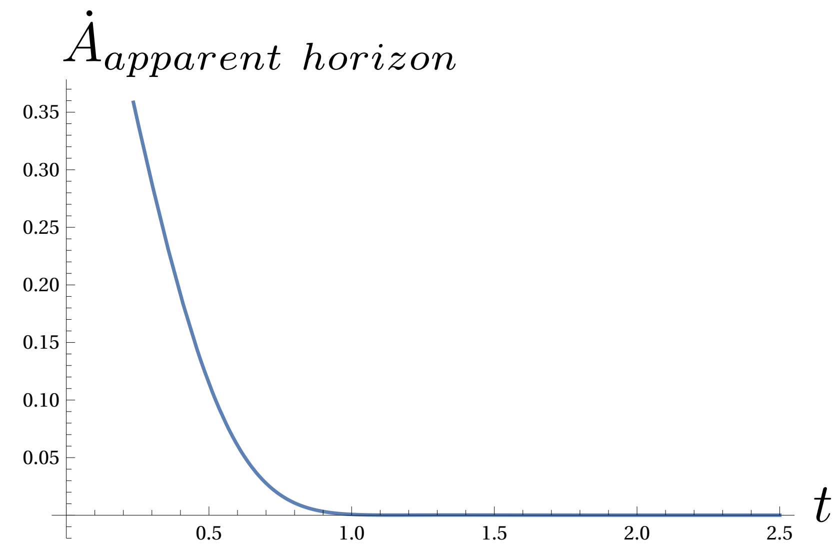

- **Left panel**: area of apparent horizon
- **Right panel**: growth rate of apparent horizon area;

$$\lim_{t\to\infty} \dot{A}_{apparent\ horizon} = 0$$

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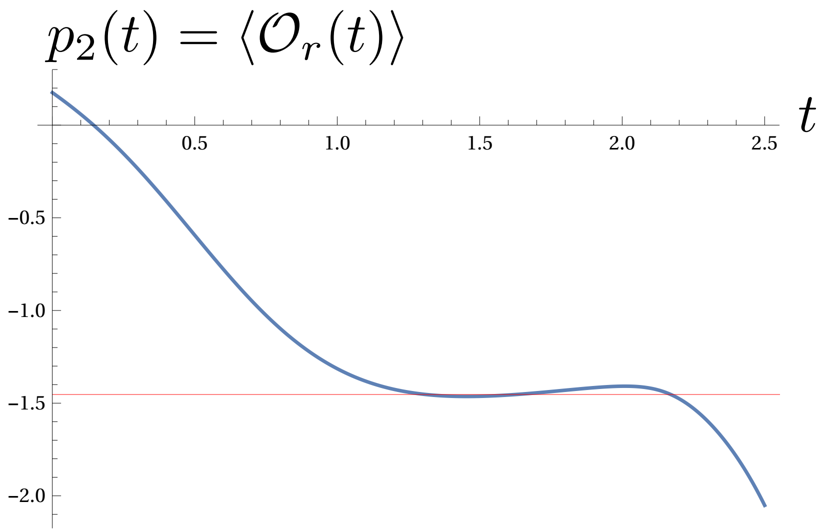

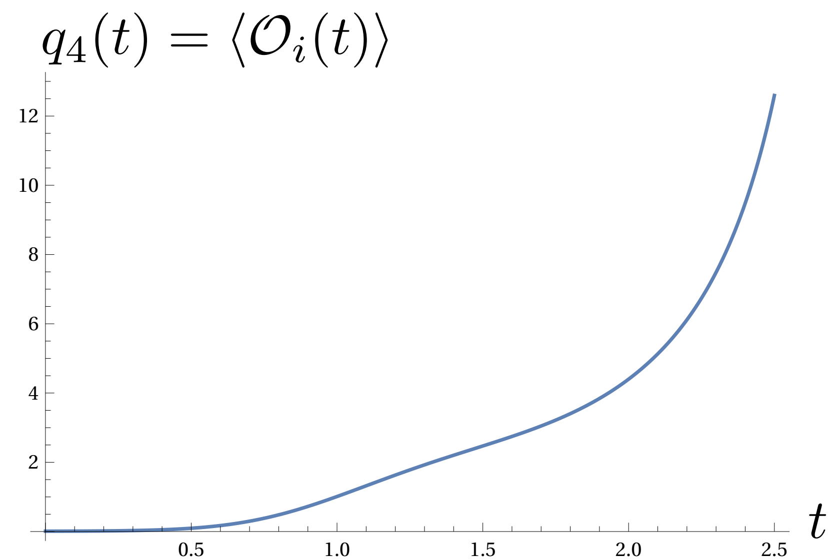

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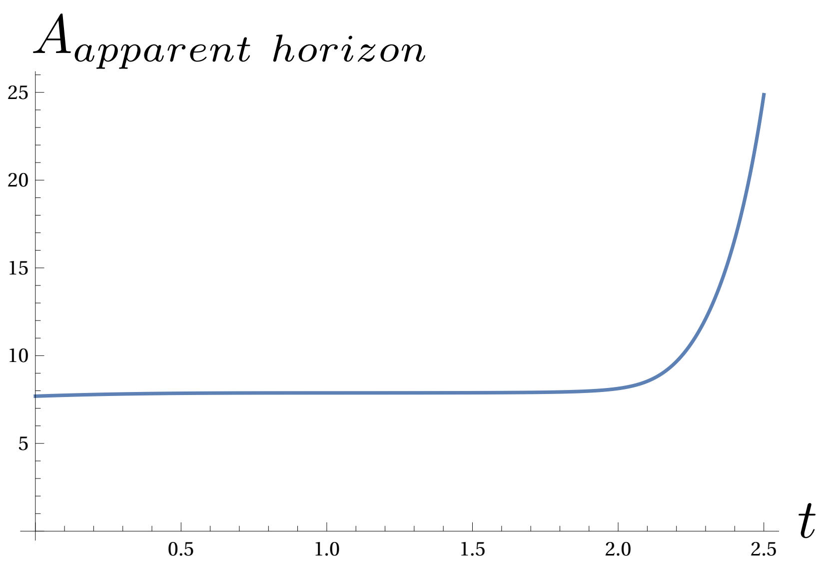
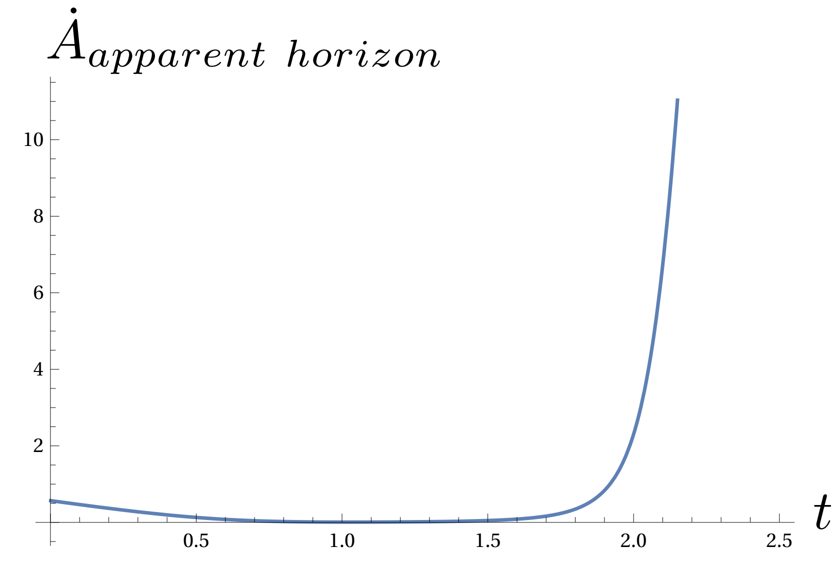
- **Right panel**: expectation value of the irrelevant operator; note that $\langle \mathcal{O}_i(t) \rangle$ exponentially growth with time (excluding the backreaction the growth rate matches perfectly the QNM prediction)

$\Longrightarrow$ the code crashed shortly after the time presented

$\Longrightarrow$ Results $\mathcal{E} < \mathcal{E}_c$ (unstable regime)





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$$\min_t \dot{A}_{apparent\ horizon} \gtrsim 0.007$$

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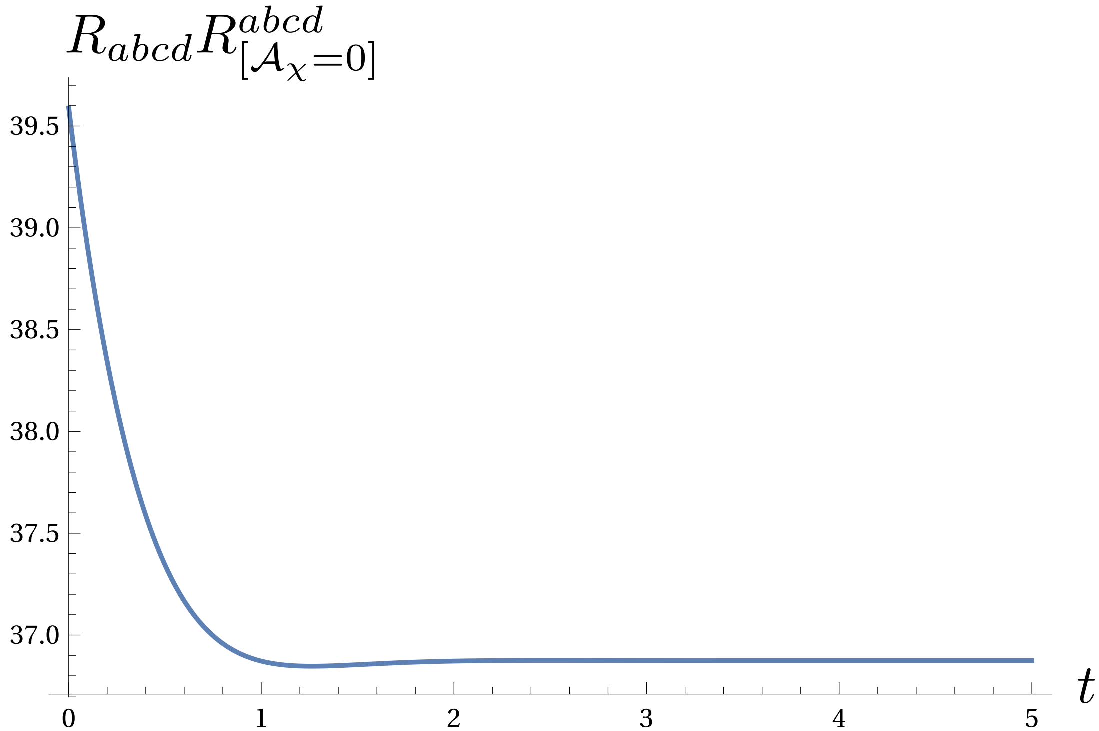

- Simulation below GL instability, but with $\mathcal{A}_\chi = 0$ (the instability is suppressed):

$$\lim_{t\to\infty} R_{abcd}R^{abcd}_{[\mathcal{A}_\chi=0]} = 36.874569568242876$$

- In AdS_4: $R_{abcd}R^{abcd} = 24$

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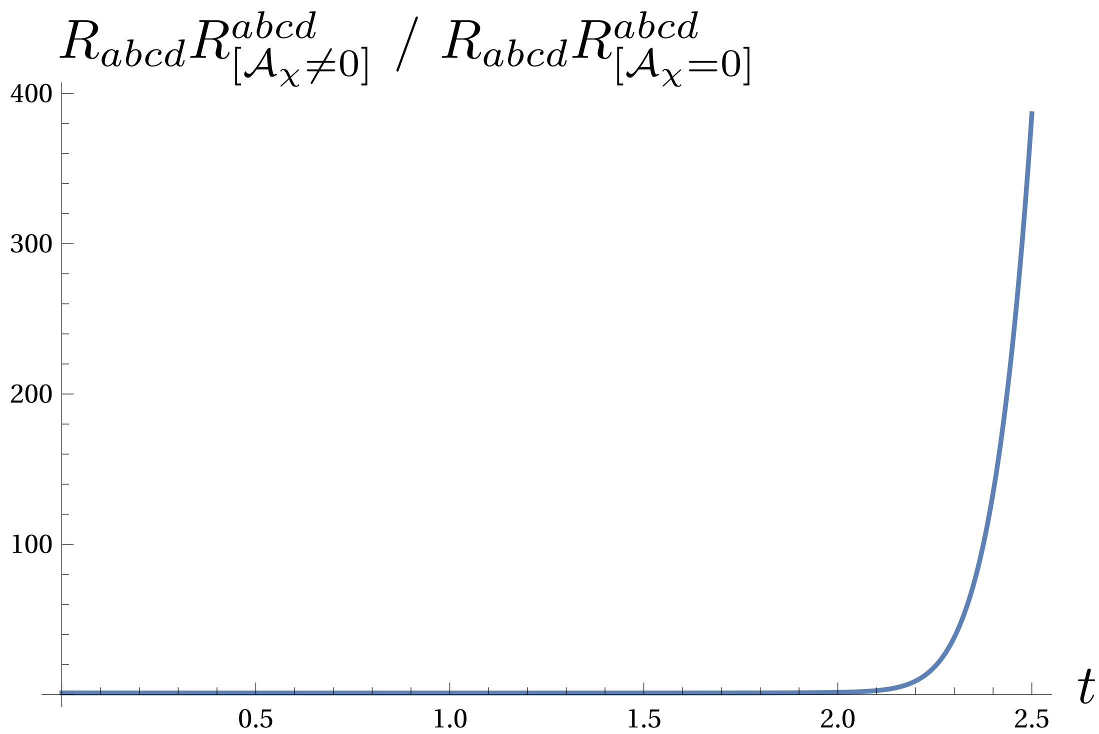

- Simulation below GL instability, with $\mathcal{A}_\chi = 0.01$ (same as previous plots for $\{\phi, \chi\}$)
- In AdS_4: $R_{abcd}R^{abcd} = 24$

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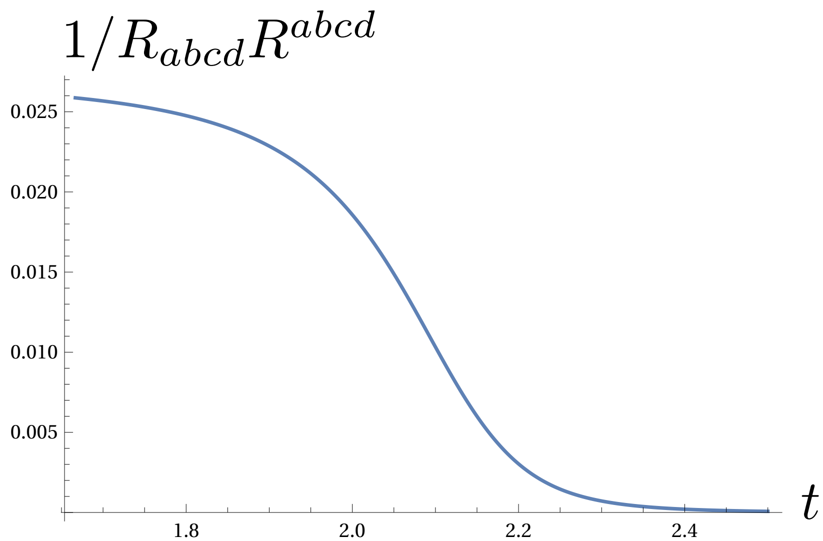
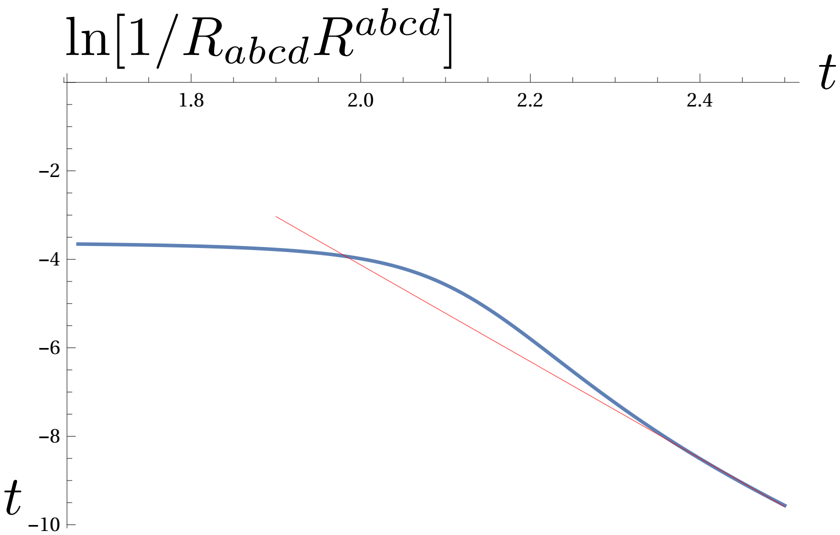
- Is a curvature singularity reached in finite time?





- It appears it takes infinite time to reach the curvature singularity
- The red line represents a linear fit to the data tail:

$$fit = 17.7397 - 10.9328\ t$$

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- This phenomena occurs in top-down/bottom-up up holographic models; in particular it is relevant for the small black hole instability in $AdS_5 \times S^5$
- This raises a question: *what is the fate of the instability?*
  - I argued, based on a numerical simulation, that the endpoint of the evolution is a naked singularity
  - It appears the singularity does not form in finite boundary time

$\Longrightarrow$ More studies are needed...