Holographic Heavy Ion Collisions in Confining Theories

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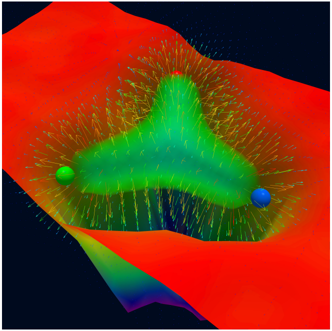
QCD

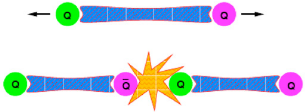
A non-Abelian gauge field theory with Lagrangian

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$



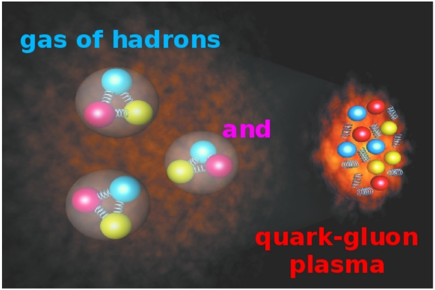
- The theory of strong interactions
- Very difficult to study
- Peculiar properties:
  - Confinement
  - Asymptotic freedom

Confinement



- Force between quarks does not diminish as they are separated
- When one separates a quark from other quarks, the energy in the gluon field is enough to create another quark pair
- Quarks are forever bound into hadrons
- Analytically unproven

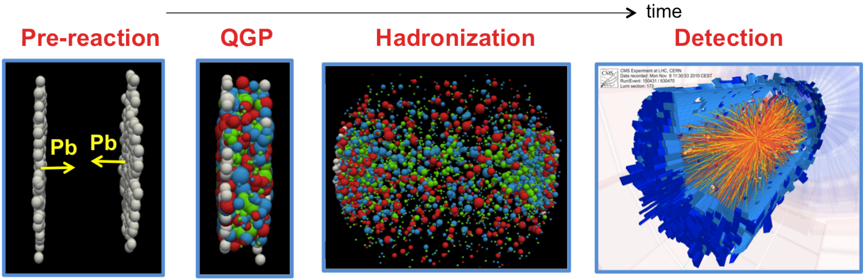
A new phase: Quark-Gluon Plasma



- $T \ll T_c$: Hadron Gas. Colour is confined;
- $T \sim T_c \sim 10^{12} K$: Rapid crossover;
- $T \gg T_c$: Quark-Gluon Plasma. Gas of quarks and gluons; colour is liberated;

In today's large accelerators, QGP can be created in a heavy-ion collision

Ultra-relativistic heavy-ion collisions

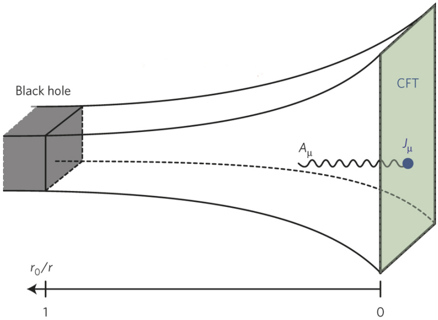


two nuclei approach, collide, form a QGP, the QGP expands and hadronizes, finally hadrons rescatter and freeze out

AdS/CFT

$\mathcal{N} = 4$ super-Yang-Mills is dual to IIB string theory on $AdS_5 \times S_5$

[Maldacena, Gubser, Klebanov, Polyakov, Witten 1998]



- We can learn about strongly coupled phenomena through gravity computations

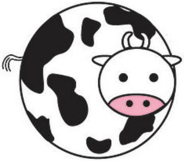
AdS/CFT

QCD

- non-conformal
- confinement
- not supersymmetric

$\mathcal{N} = 4$ SYM

- conformally invariant
- no confinement
- supersymmetric



Holography in confining theories

- QCD is not $\mathcal{N} = 4$ SYM ...
  - In a real heavy ion collision at RHIC or LHC, the temperature of the QGP is $2T_c \lesssim T \lesssim 4T_c$.
  - Since there is no clear separation of scales between the temperature of the fireball and the confinement scale, the latter may play a role in the dynamics.

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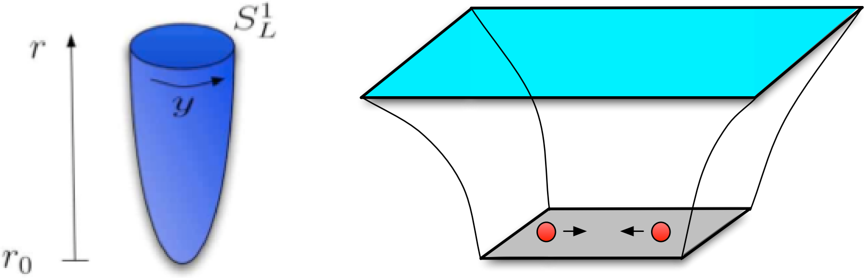
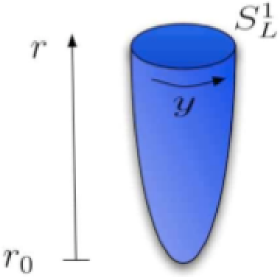
AdS-soliton background

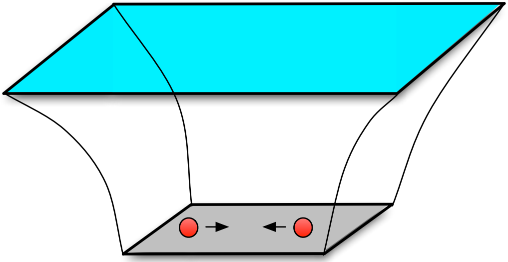


$$ds^2 = \frac{r^2}{L^2}\left(-dt^2 + dx^2_{(3)}\right) + \frac{dr^2}{F(r)} + F(r)dy^2$$

where $F(r) = \frac{r^2}{L^2}\left(1 - \frac{r_0^5}{r^5}\right)$ and $\Delta y = \frac{4\pi L^2}{5r_0}$

AdS-soliton background





- Gauge theory lives on $\mathbb{M}_{1,3} \times S^1$;
- the S^1 shrinks smoothly to zero size at $r = r_0$;

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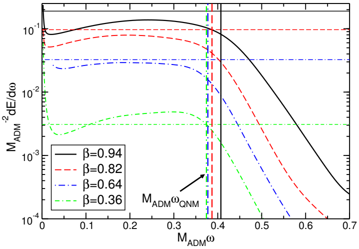
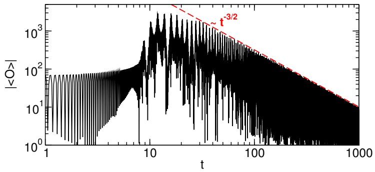
The ZFL framework



- Models colliding objects as point particles moving along geodesics in a background spacetime, colliding instantaneously.
- A stress-energy tensor for point particles is specified, used as a linearized perturbation of the background.
- Describes very well main features of high-energy collisions of equal-mass black holes.
- A cutoff scale needs to be introduced by hand at high frequencies.

Dual scalar operator



time-domain dual scalar operator as function of t for fixed distance and ω_{cutoff}

$$\langle \mathcal{O} \rangle - \langle \mathcal{O}_{\text{static}} \rangle \sim \sum_n \frac{\sin\left(\tilde{\omega}_n\, t\, r_0 / L^2\right)}{t^{3/2}}$$

$t^{-3/2}$ fall-off behaviour generic for massive theories

High energy collision of particles

[Cardoso et al 2014]

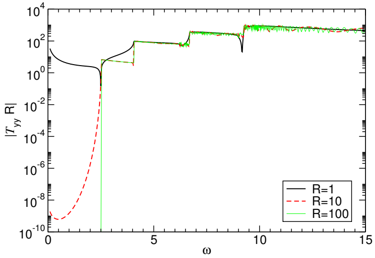
Stress-energy tensor for head-on collision of point-particles with mass m and speed v

$$T^{\mu\nu} = \frac{mL^3 r_0}{r^4\gamma}\delta(x_2)\delta(x_3)\delta(r-r_0)\Theta(-t) \times \left[u^{\mu}_{(1)}u^{\nu}_{(1)}\delta(x-vt) + u^{\mu}_{(2)}u^{\nu}_{(2)}\delta(x+vt)\right] + \frac{\Theta(t)ML^4}{r^4}u^{\mu}_{(3)}u^{\nu}_{(3)}\delta(x)\delta(x_2)\delta(x_3)\delta(r-r_0)$$

and metric

$$ds^2 = ds^2_{\text{soliton}} + \epsilon h_{\mu\nu}dx^{\mu}dx^{\nu}$$

Stress-energy tensor of the dual theory



- Operator $|\mathcal{T}_{yy}(\omega, R)|$ (spherically symmetric).
- Spectrum suppressed for frequencies smaller than the fundamental mode of the AdS-soliton.

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Action and Equations of Motion

$d+1$ dimensional action

$$\mathcal{S} = \int d^{d+1}\sqrt{-\bar{g}}\,(\bar{R} - 2\Lambda)$$

$$d\bar{s}^2 = \bar{g}_{AB}dx^A dx^B = e^{-\frac{2}{d-2}\phi} g_{\mu\nu}dx^\mu dx^\nu + e^{2\phi}dy^2\,,$$

$\mu = 0, \ldots, d-1$

EoMs

$$\frac{3}{d-2}\nabla_\mu\partial^\mu\phi + \Lambda e^{-\frac{2}{d-2}\phi} = 0\,,$$

$$R_{\mu\nu} = \frac{6}{(d-2)^2}\partial_\mu\phi\partial_\nu\phi + \frac{2}{d-2}\Lambda e^{-\frac{2}{d-2}\phi} g_{\mu\nu}\,.$$

Characteristic formulation

Bondi-Sachs form

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -e^{\frac{2}{d-2}\phi}\left(A - \frac{F^2}{\Sigma^2}\right)dt^2 + \Sigma\left(e^{B}d\boldsymbol{x}_{\perp}^2 + e^{-(d-3)B}dz^2\right)$$
$$+ 2dt(e^{\frac{2}{d-2}\phi}dr + Fdz)\,.$$

Schematic evolution equations:

$$\partial_r S = H_S(S, B)$$
$$\partial_t\partial_r B = H_B(B, S, \partial_t B)$$

Advantages of characteristic evolution

- Initial data is free (no elliptic constraints on the data);
- No second time derivatives (therefore smaller number of basic variables);
- Equations have convenient hierarchical structure in which variables are integrated in turn in terms of characteristic data from prior members of the hierarchy.

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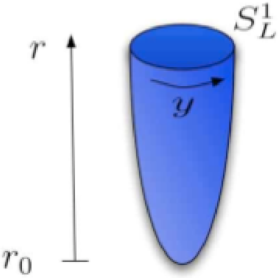
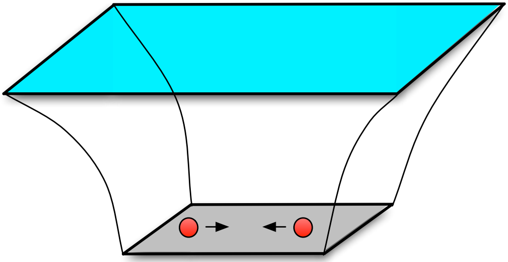
Localised Plasma Balls

[Figueras, Tunyasuvunakool 2014]





- Finite size black holes sitting at IR bottom of AdS-soliton geometry.
- Dual to bubbles of deconfined plasma within the confining vacuum.

Localised Plasma Balls

Einstein-DeTurck equations

$$R_{\mu\nu} + \frac{d}{L^2} g_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} = 0$$

solved for a global reference metric

$$d\bar{s}^2 = \left[(1 - I(r))\bar{g}_{\mu\nu}^{\text{near}-\text{horizon}} + I(r)\bar{g}_{\mu\nu}^{\text{soliton}}\right] dx^\mu dx^\nu$$

interpolating between the near-horizon geometry of a topologically spherical black hole and the AdS-soliton metric [Figueras, Tunyasuvunakool 2014]

Outline

Final Remarks

- Collisions in AdS spaces provide convenient framework to study heavy-ion collisions.
- How does the presence of a confinement scale affect the relaxation process?
- We use an AdS-soliton model to model confinement and study its effect in the evolution of the system:
  - Estimates for the energy spectrum through the ZFL;
  - Formalism for non-linear evolutions;
  - Localised Plasma Balls as initial data.