Quenching an electric field in metallic holography

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based on 1606.03457

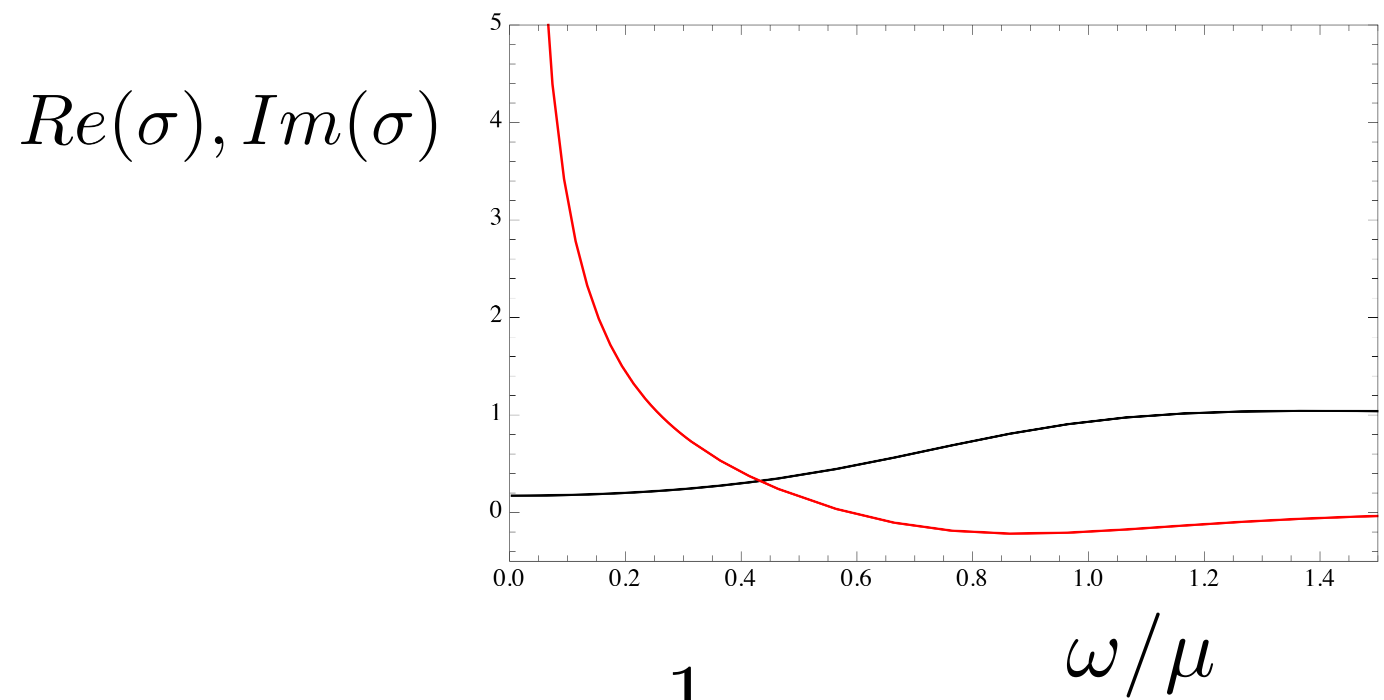
NumHol2016, Santiago de Compostela

- Transport properties of strongly coupled systems of interest both experimentally and theoretically.
- Holography furnishes us with tractable examples of theories dual to classical gravity
- Transport properties can be straightforwardly extracted by perturbing black branes describing equilibrium states
- Here, electric (σ) thermoelectric ($\beta, \bar{\beta}$) and thermal ($\bar{\kappa}$)
- Turn on sources $E,\ \nabla T$
- Measure electric (J^i) and heat ($Q^i = T^{ti} - \mu J^i$) currents

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \beta T \\ \bar{\beta} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

- e.g. RN-AdS4

  [Hartnoll]



- In DC limit,

$$Re(\sigma) \sim \delta(\omega) \qquad Im(\sigma) \sim \frac{1}{\omega}$$

- Translationally invariant, momentum conserved, $\omega = 0$ vector mode in the bulk

- Desirable to explicitly break translations, introduces a momentum relaxation timescale

$$\omega = \frac{-i}{\tau_{\rm rel}}$$

- For long $\tau_{\rm rel}$ transport governed by `modified' hydro modes due to only approximately conserved momentum, coherent

- For short $\tau_{\rm rel}$ no long lived momentum and no hydro, incoherent

$$\omega = \frac{-i}{\tau_{\rm rel}}$$

- A variety of stationary black branes which realise momentum relaxation
  - Holographic lattices [Horowitz, Santos, Tong][Chesler, Lucas, Sachdev][Balasubramanian, Herzog][Rangamani, Rozali, Smyth][…]
  - Holographic disorder [Hartnoll, Santos][…]
  - Helical black branes [Donos, Hartnoll][Donos, Gouteraux, Kiritsis][…]
  - Q-lattices, Axion models [Donos, Gauntlett] [Andrade, B.W.][…]
  - …

This talk: electric field quench

- Here we take an inhomogeneous model and do a time dependent quench of an applied electric field
- From boundary perspective we'll see:
    - bounded momentum due to inhomogeneities
    - unbounded energy response due to Joule heating
    - relaxation of currents
- From bulk point of view expect qualitatively different dynamics (momentum not conserved), e.g. in approach to equilibrium.
- QNM spectrum governing current relaxation can depend qualitatively on inhomogeneity, we will see this imprint in the dynamics

This talk: electric field quench

- Time dependent, need numerics (characteristic formulation)
- Usually a 2+1 problem, but with an axion-like model can reduce to 1+1
- Exception for no net charge density: current but no momentum, generalises results of [Horowitz, Iqbal, Santos] — nonlinear response encoded by a Vaidya spacetime
- Here we don't find any nonlinear steady states, due to Joule heating. Contrast with probe limit e.g. [Karch][Sonner, Green][Baggioli, Pujolas], where one effectively has a heat bath.

Outline

- Model
- Exact solutions at $\rho = 0$
- Numerics at $\rho \neq 0$
  - Holding E constant
    - linear E and steady states,
    - nonlinear E and Joule heating
  - Returning to equilibrium
    - event horizon,
    - QNMs
    - relaxation of currents

Model and equilibrium

Bulk model

- Model of choice: Einstein-Maxwell + axions ϕ_I

$$S = \frac{1}{2\kappa^2}\int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_{I=1}^{2}(\partial\phi_I)^2 - \frac{1}{4}F^2\right] d^4x$$

- Ward identities

$$\partial_\mu \langle T^\mu{}_\nu \rangle = \partial_\nu \phi_I^{(0)} \langle O_I \rangle + \left(F^{(0)}\right)_\nu{}^\mu \langle J_\mu \rangle$$

$$\partial_\mu \langle J^\mu \rangle = 0$$

- Giving the axions linear sources $\phi_I^{(0)} \propto x^I$ turns on the RHS and relaxes momentum
- In linear response get a steady state with balance on RHS

[Andrade, B.W.]

Bulk model - equilibrium

- For isotropic configurations of axions, $\phi_I^{(0)} = \alpha x^I$
  there are analytic stationary black brane solutions

$$ds^2 = \frac{1}{r^2}(-f(r)dv^2 - 2dvdr + dx^2 + dy^2)$$

$$A = (\mu - \rho\, r)dv, \quad \phi_I = \alpha x^I$$

$$f(r) = 1 - \frac{1}{2}\alpha^2 r^2 - mr^3 + \frac{1}{4}\rho^2 r^4$$

- $\mathrm{AdS}_2 \times \mathrm{R}^2$ at low temperatures, even for $\mu = \rho = 0$
- For some parameters have negative energy density
- Curious special point: $\epsilon = \rho = 0 \quad T \neq 0$
- Finite DC electric and thermoelectric conductivities

$$\sigma = 1 + \frac{\mu^2}{\alpha^2} \qquad \bar{\beta} = \frac{4\pi\rho}{\alpha^2}$$

Bulk model - driving with sources

- Throughout this talk sources are:

$$\phi_I^{(0)} = \alpha x^I \qquad\qquad F^{(0)} = E(t) dx \wedge dt$$

- Insert in Ward identities:

$$\begin{aligned} \partial_t \epsilon &= EJ \\ \partial_t J_E &= \alpha \langle O_1 \rangle + E\rho \end{aligned}$$

- Joule heating unavoidable — time dependent black branes
- In general there will also be a time dependent energy current, but cancellations can occur

Exact results at $\rho = 0$

- For no net charge there is electric current but no momentum.
- Vaidya solution in 4D Einstein-Maxwell [Horowitz, Iqbal, Santos]
- With axions:

$$
\begin{aligned}
ds^2 &= \frac{1}{r^2}(-f(v,r)dv^2 - 2dvdr + dx^2 + dy^2) \\
A &= E(v)x\,dv, \quad \phi_I = \alpha x^I
\end{aligned}
$$

$$f(v,r) = 1 - \frac{1}{2}\alpha^2 r^2 - m(v)r^3.$$

- Joule heating $\partial_v(2m(v)) = E(v)^2$
- Instantaneous response $J = E$
- Electric current but no energy currents: $\sigma = 1, \bar{\beta} = 0$
- Here α is surplus to requirements

$$\rho \neq 0$$

Numerical evolution

- Characteristic evolution:

$$
\begin{aligned}
ds^2 &= \frac{1}{r^2}\big(-F(v,r)dv^2 - 2dvdr + 2e^{B(v,r)}F_x(v,r)dvdx \\
&\qquad +S(v,r)(e^{2B(v,r)}dx^2 + e^{-2B(v,r)}dy^2)\big) \\
A &= (E(v)x + a_v(v,r))dv + a_x(v,r)dx \\
\phi_1 &= \alpha\, x + \Phi(v,r) \\
\phi_2 &= \alpha\, y
\end{aligned}
$$

- x,y inhomogeneity doesn't appear in the equations of motion
- Crank-Nicolson for stepping in v
- Chebyschev collocation in r
- For large injections of energy we adapt the radial coordinate so AH sits at r=1

$$\rho \neq 0$$

Holding E constant

- Initial temperature T_i, charge density ρ, relaxation param. α

- Quench an applied electric field to a constant value E_f

$$E(t) = \frac{1}{2}\left(\tanh\left(\frac{t}{w}\right) + 1\right) E_f$$

- Read off electric and thermoelectric conductivities

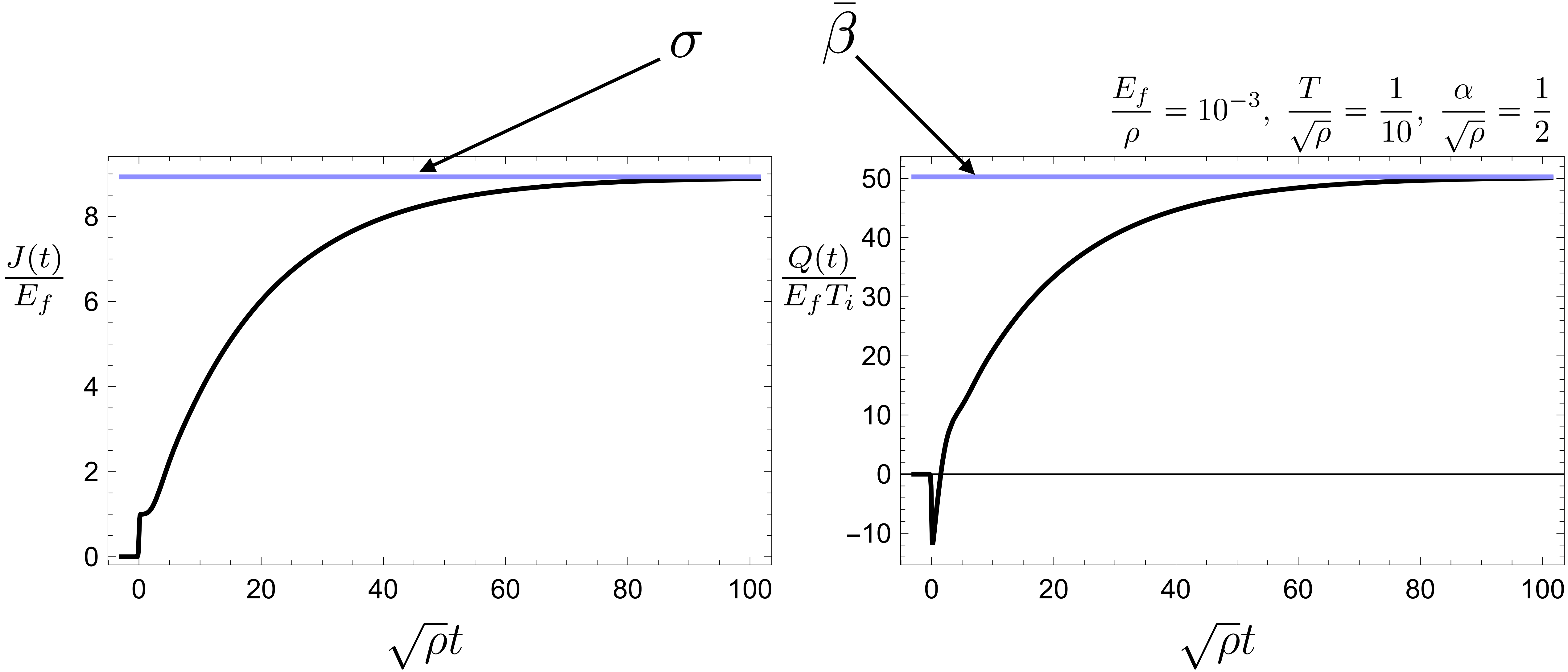
$$\begin{aligned} J &= \sigma E_f, \\ Q \equiv J_E - \mu J &= \bar{\beta} T \, E_f \end{aligned}$$

- Consider two cases: **Small electric fields**

  Large electric fields

Small electric fields

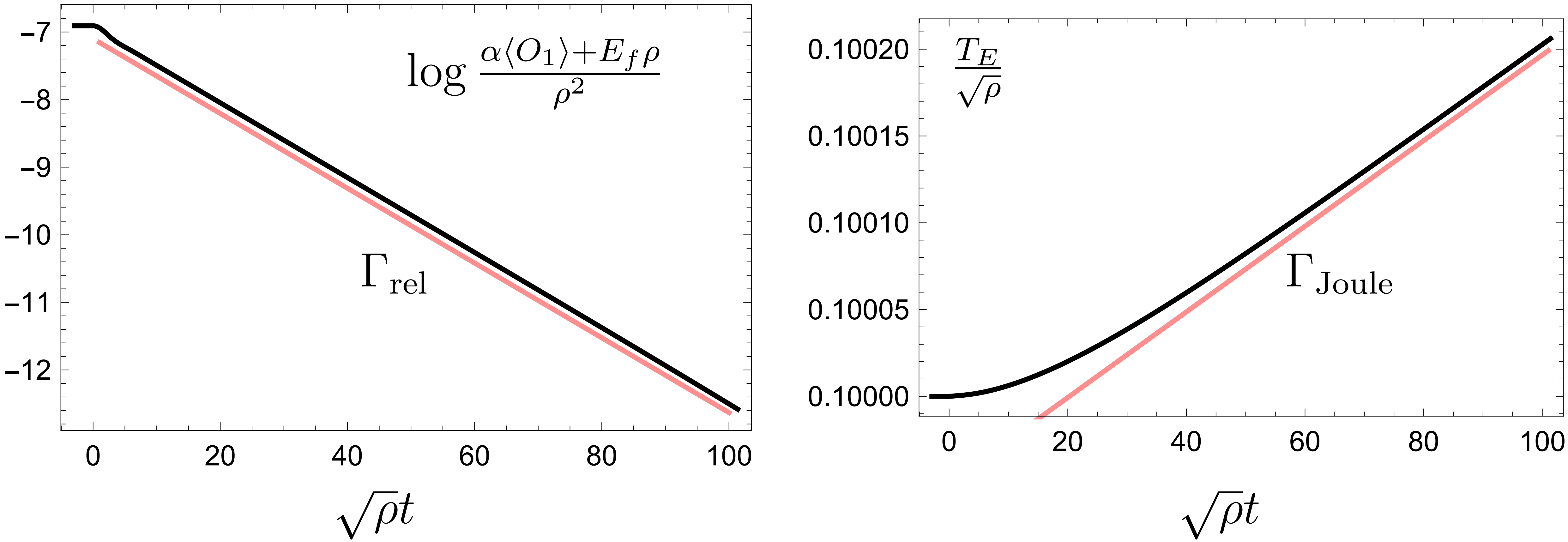
DC linear response



- Instantaneous response (cf. neutral case)
- Approaches linear steady state

Small electric fields

$$\partial_t \epsilon = EJ$$
$$\partial_t J_E = \alpha \langle O_1 \rangle + E\rho$$



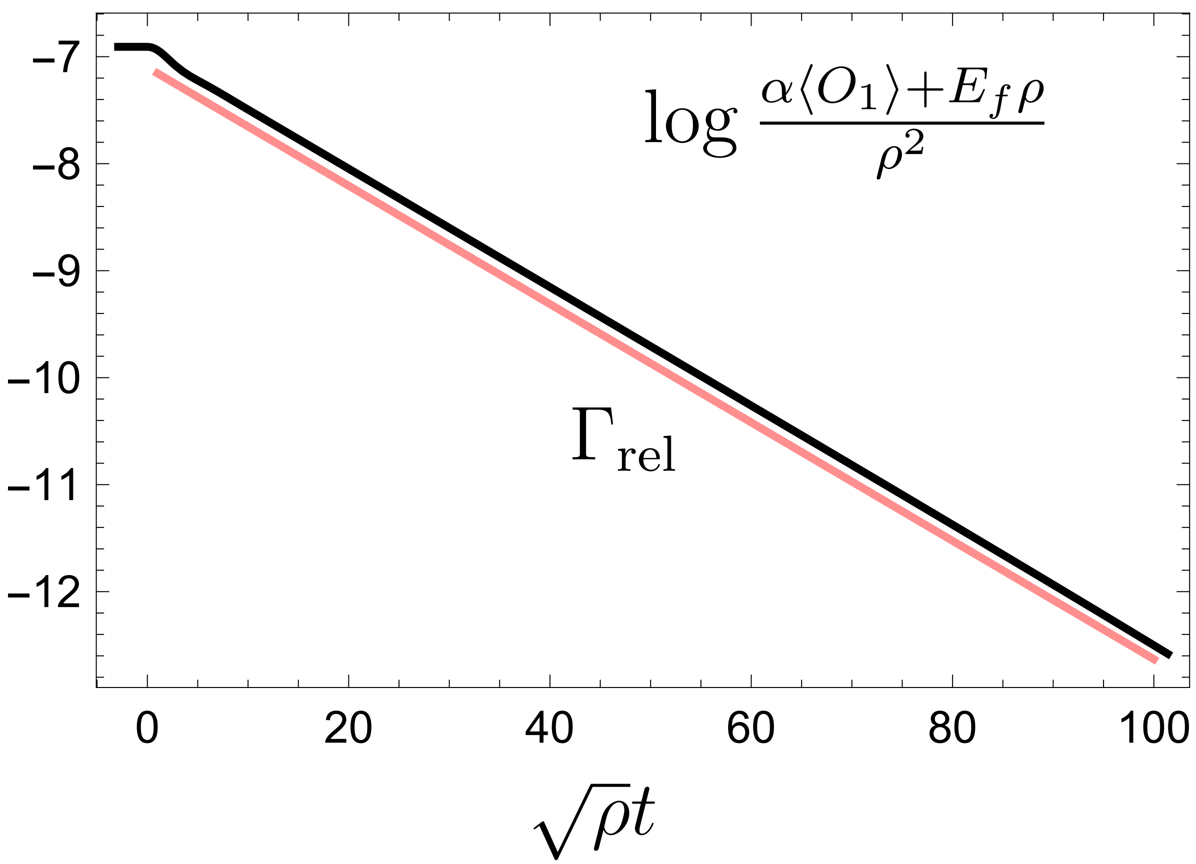
- Longest lived vector QNM governs approach to linear steady state (set by momentum relaxation parameter α - more later)
- Heating rate determined by specific heat & Ward identity at equilibrium

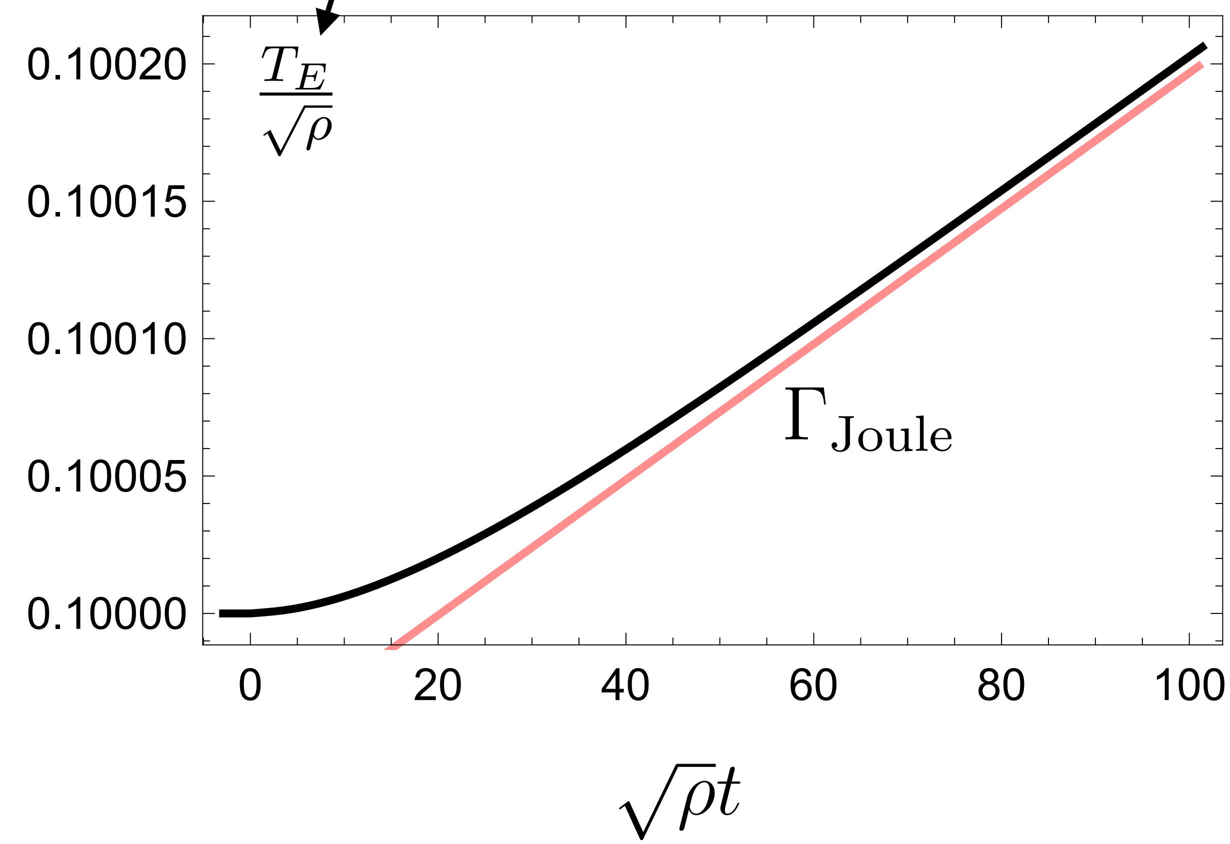
$$\Gamma_{\text{Joule}} = \frac{1}{\sqrt{\rho}} \frac{\partial T}{\partial t}\bigg|_\rho = \frac{1}{\sqrt{\rho}} \frac{1}{c_\rho} \sigma E^2$$

Small electric fields

$$\partial_t \epsilon = EJ$$
$$\partial_t J_E = \alpha \langle O_1 \rangle + E\rho$$

define $T_E(t)$ as the equilibrium temperature reached if we stop driving at time t





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$$\Gamma_{\rm Joule} = \frac{1}{\sqrt{\rho}} \frac{\partial T}{\partial t}\bigg|_\rho = \frac{1}{\sqrt{\rho}} \frac{1}{c_\rho} \sigma E^2$$

Large electric fields

- How can we model the current response when there is a large electric field?
- Assume some transient regime governed by α (just as in linear response)
- Then describe time dependence by

$$\sigma(t) = \sigma\left(\frac{\alpha}{\sqrt{\rho}}, \frac{T_E(t)}{\sqrt{\rho}}, \frac{E_f}{\rho}\right)$$

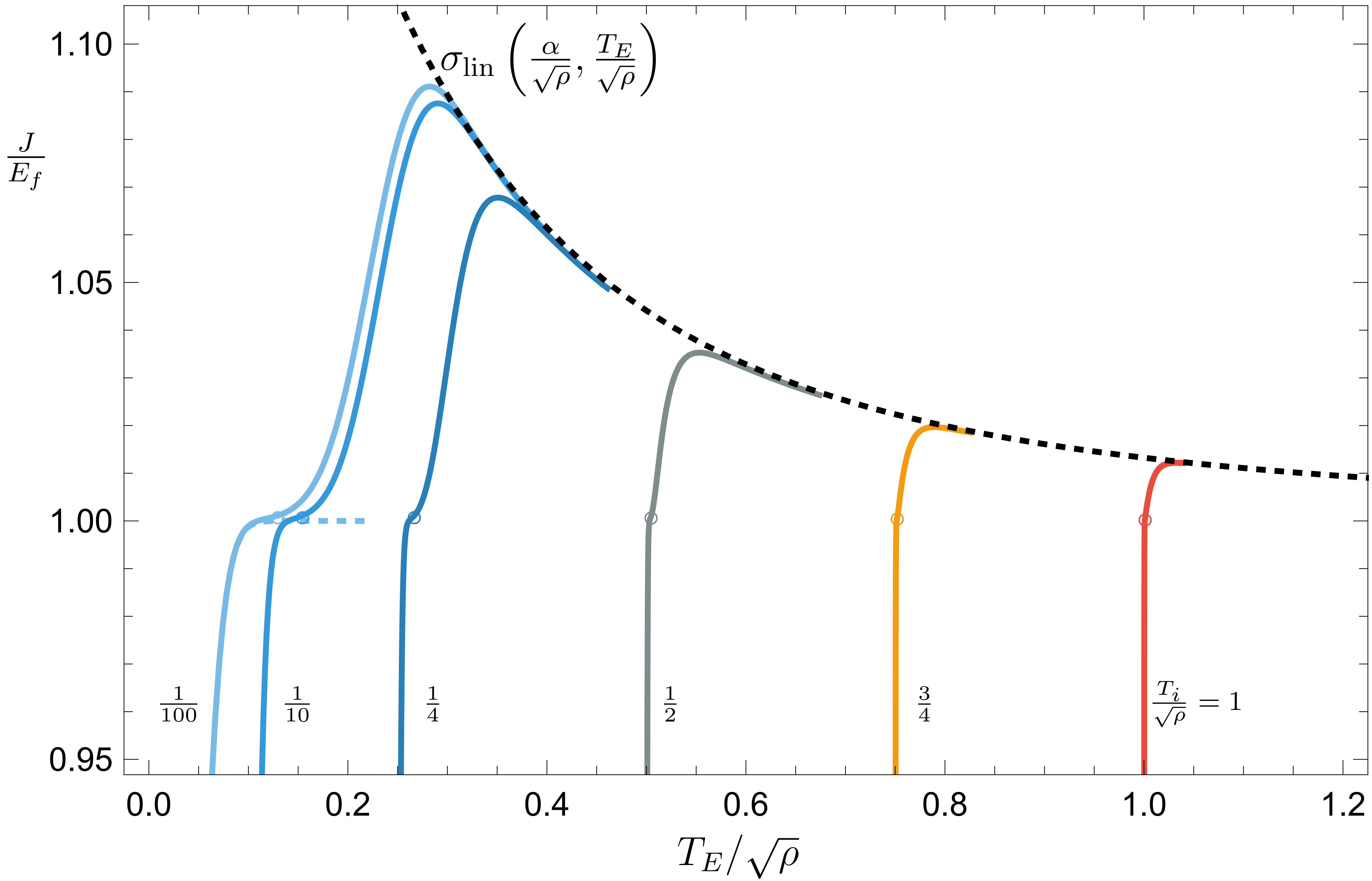
- Recall in linear response at equilibrium:

$$\sigma = \sigma\left(\frac{\alpha}{\sqrt{\rho}}, \frac{T}{\sqrt{\rho}}\right)$$

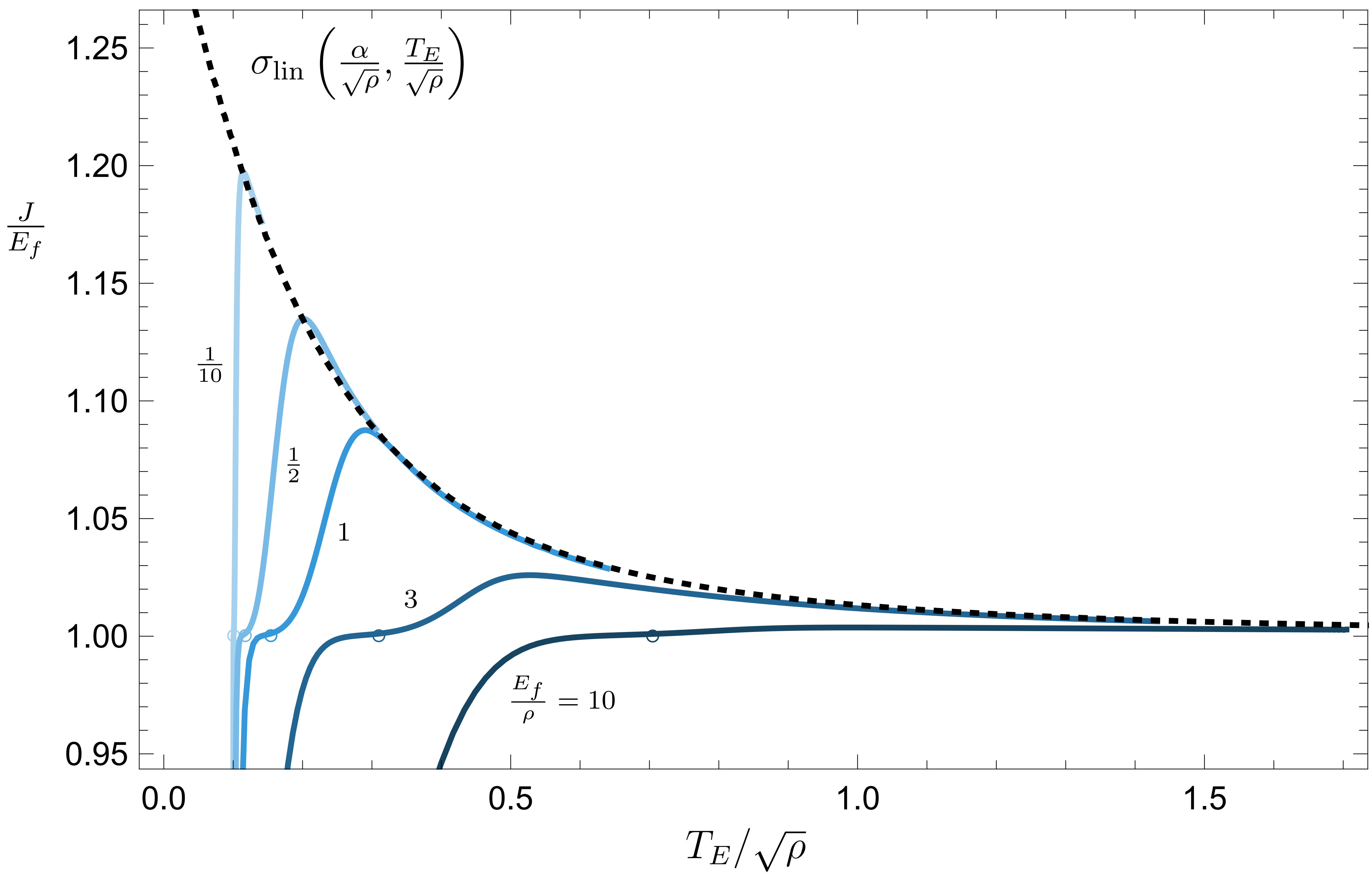
- Remarkably, a good approximation is achieved by using the linear response result and promoting

$$T \to T_E(t)$$

Varying T_i $\qquad E_f/\rho = 1$



Varying E_f $T_i/\sqrt{\rho} = 1/10$



$$\rho \neq 0$$

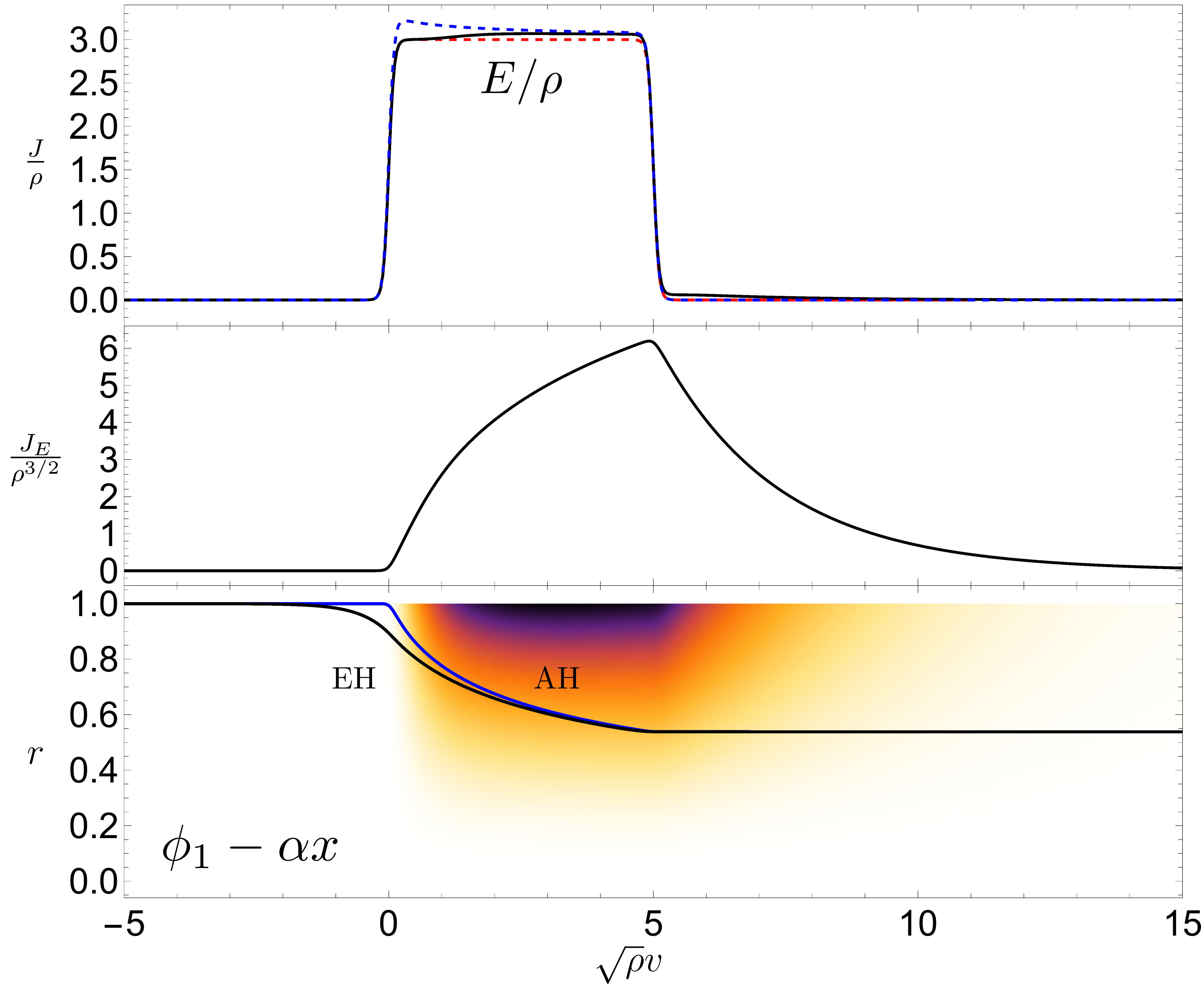
Returning to equilibrium

- Apply top hat

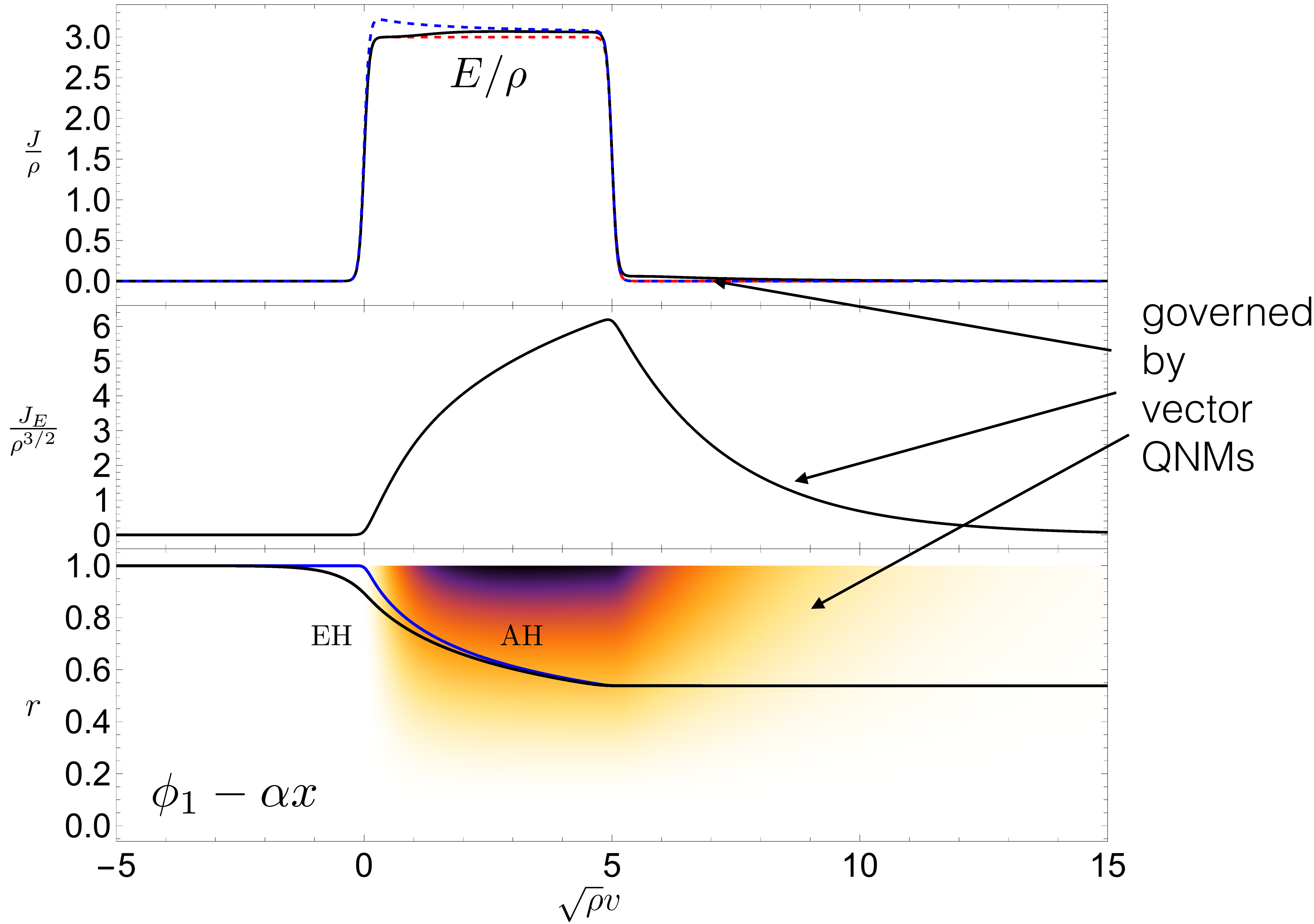
$$E(t) = (\Theta(t) - \Theta(t - t_*))\, E_c$$

$$\Theta(t) \equiv \frac{1}{2}\left(\tanh\left(\frac{t}{w}\right) + 1\right)$$

- What happens to the black hole?
- Return to equilibrium



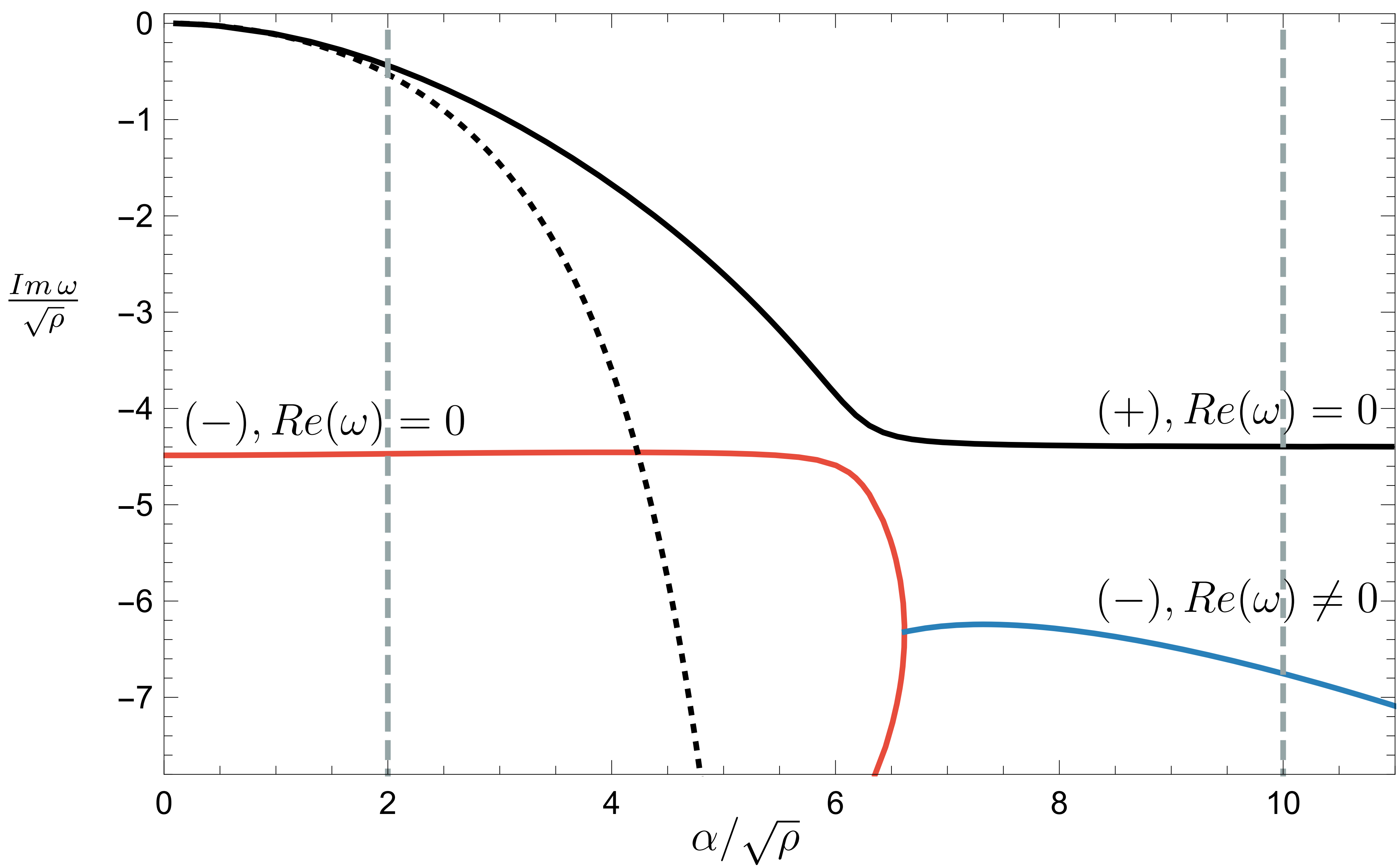
E/ρ
$\frac{J}{\rho}$
$\frac{J_E}{\rho^{3/2}}$
EH
AH
r
$\phi_1 - \alpha x$
$\sqrt{\rho}v$



E/ρ
$\frac{J}{\rho}$
$\frac{J_E}{\rho^{3/2}}$
r
EH
AH
$\phi_1 - \alpha x$
$\sqrt{\rho}v$
governed by vector QNMs

Vector quasi-normal modes

- Two sectors of vector perturbations (+) / (-)
- Small α use matching calculation $\Gamma_{\rm rel} = \tau_{\rm rel}^{-1} = \frac{s\alpha^2}{6\pi\epsilon}$ [Davison]
- Otherwise construct numerically:



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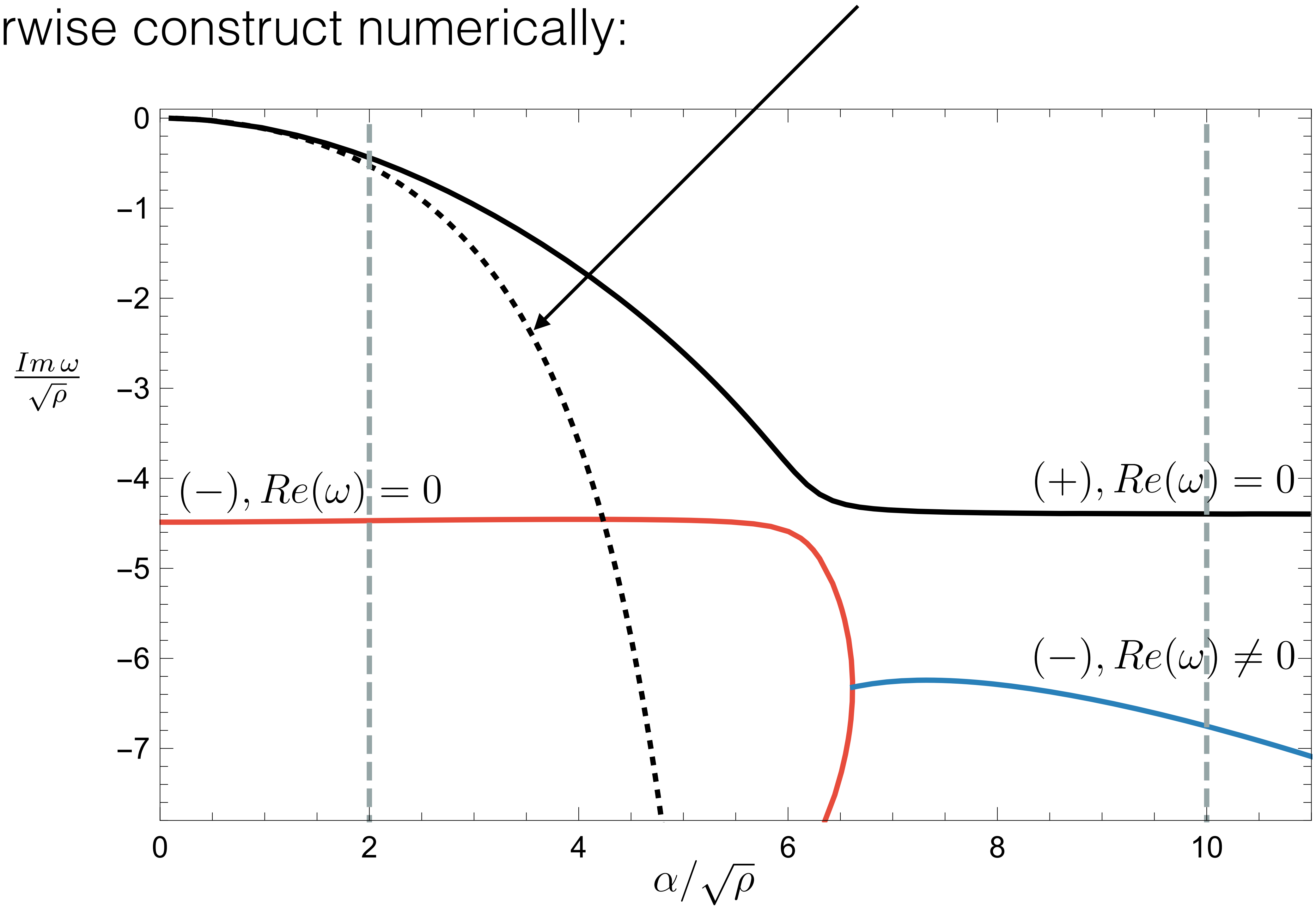
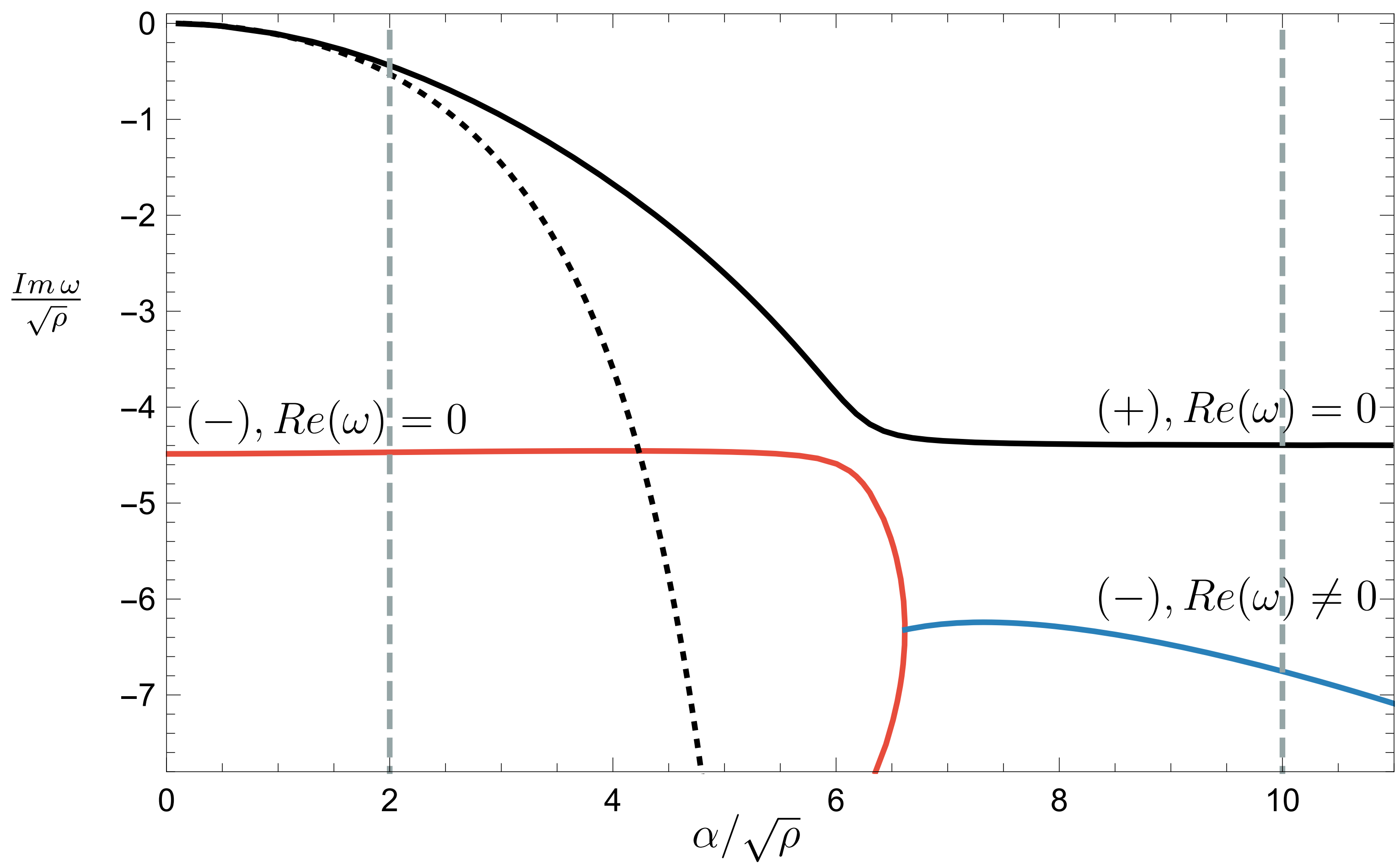
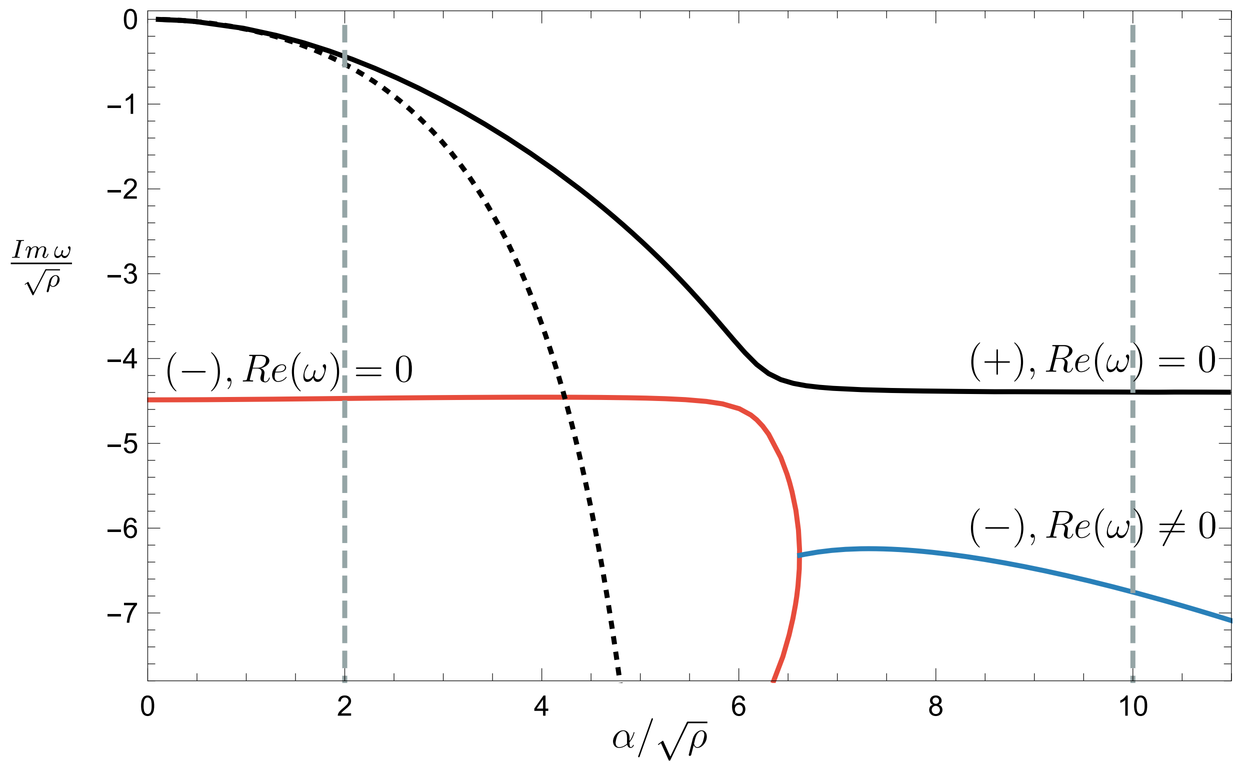




- In general Q, J relaxation involve a mixture of both sectors
- But, the qualitative changes appear in shorter lived modes… (not the case at $\rho = 0$)
- So why are these features relevant?
- As $\alpha \rightarrow \infty$, Q only involves (-)-sector modes
- At large α there is a parametric enhancement of amplitude

relaxation of currents



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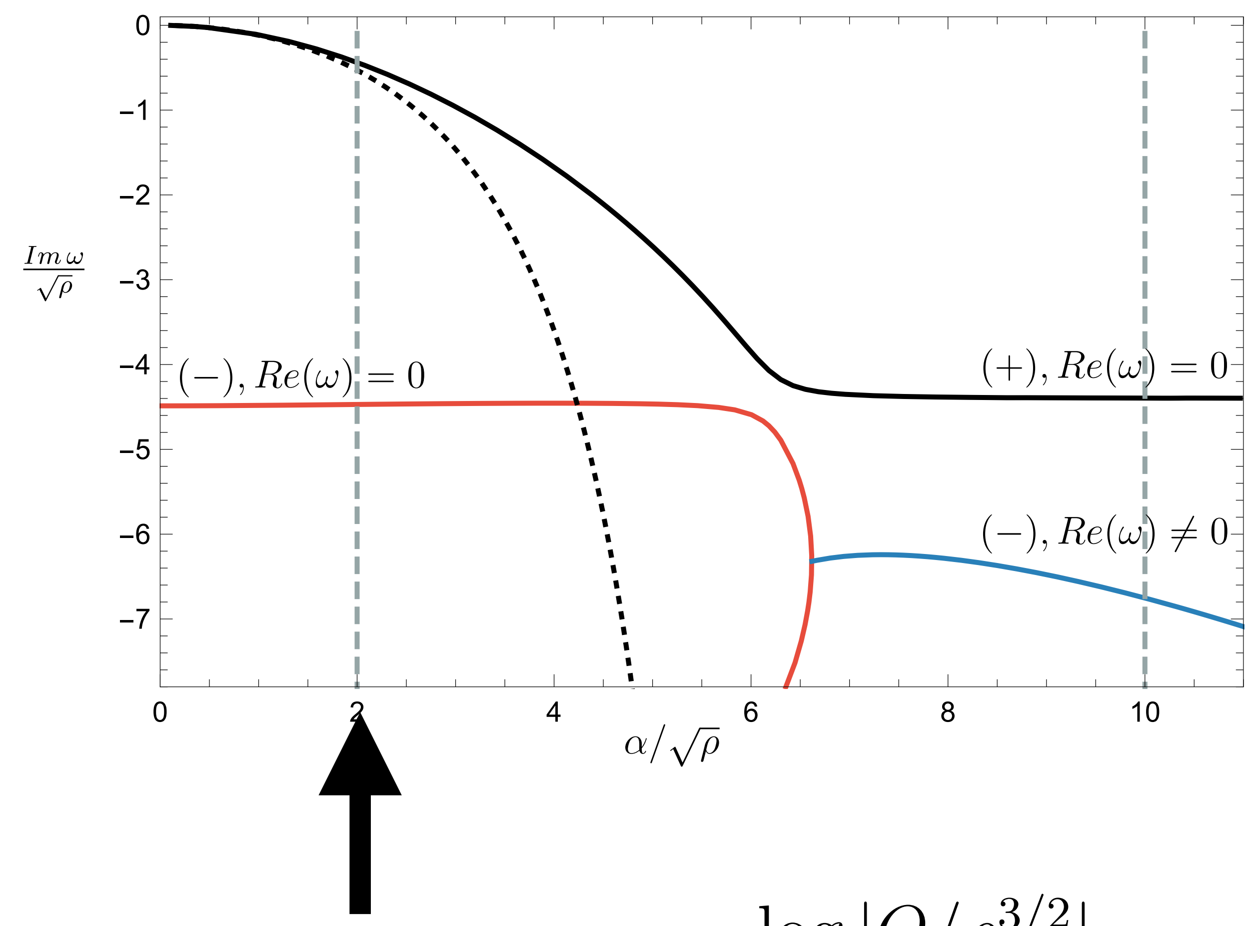

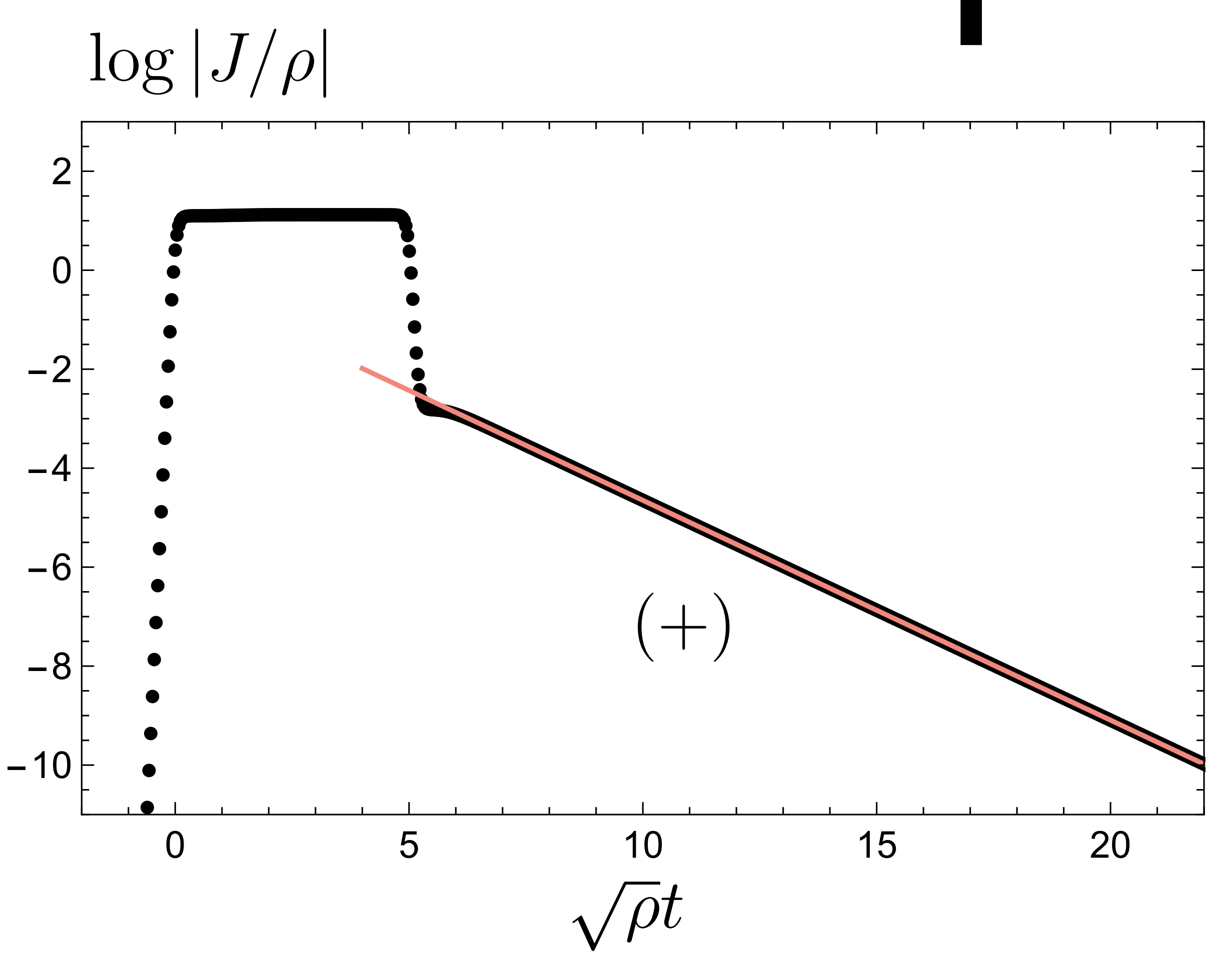

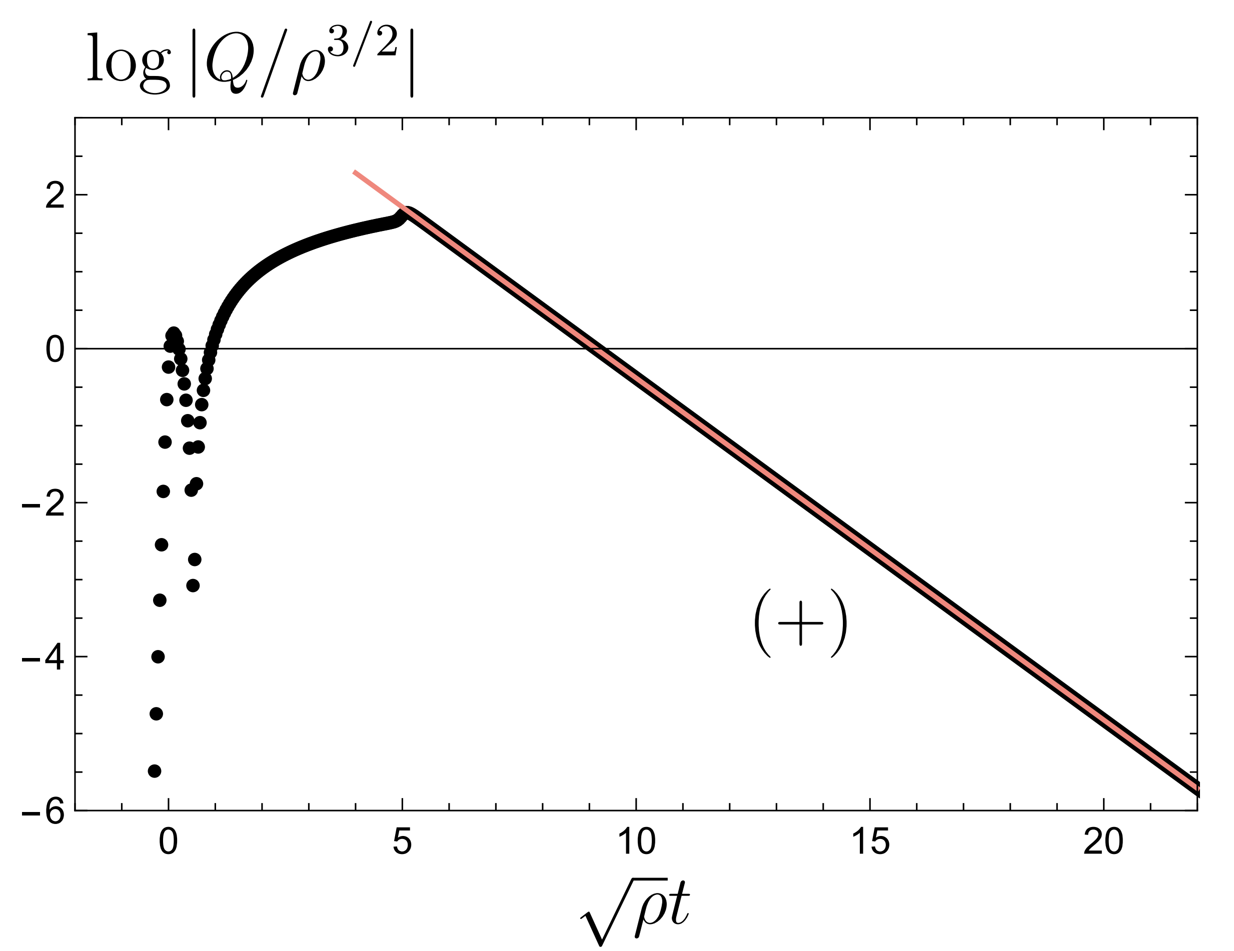

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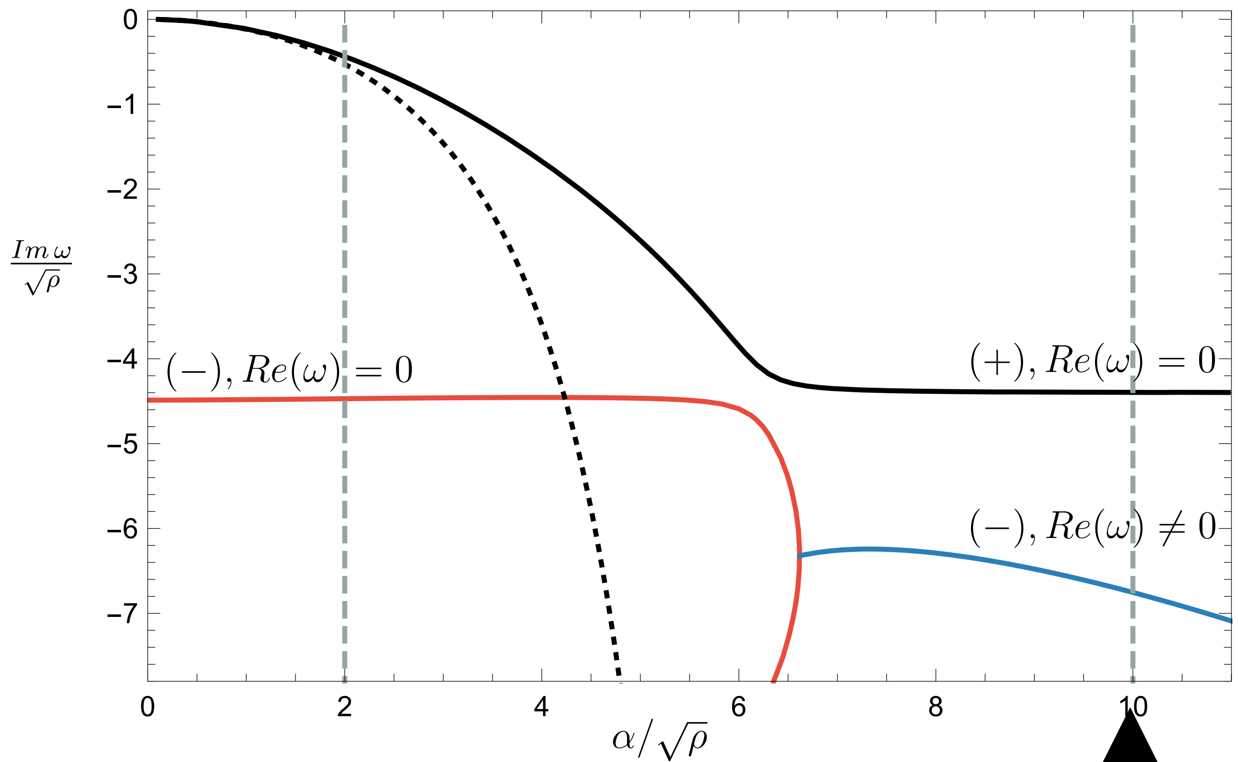

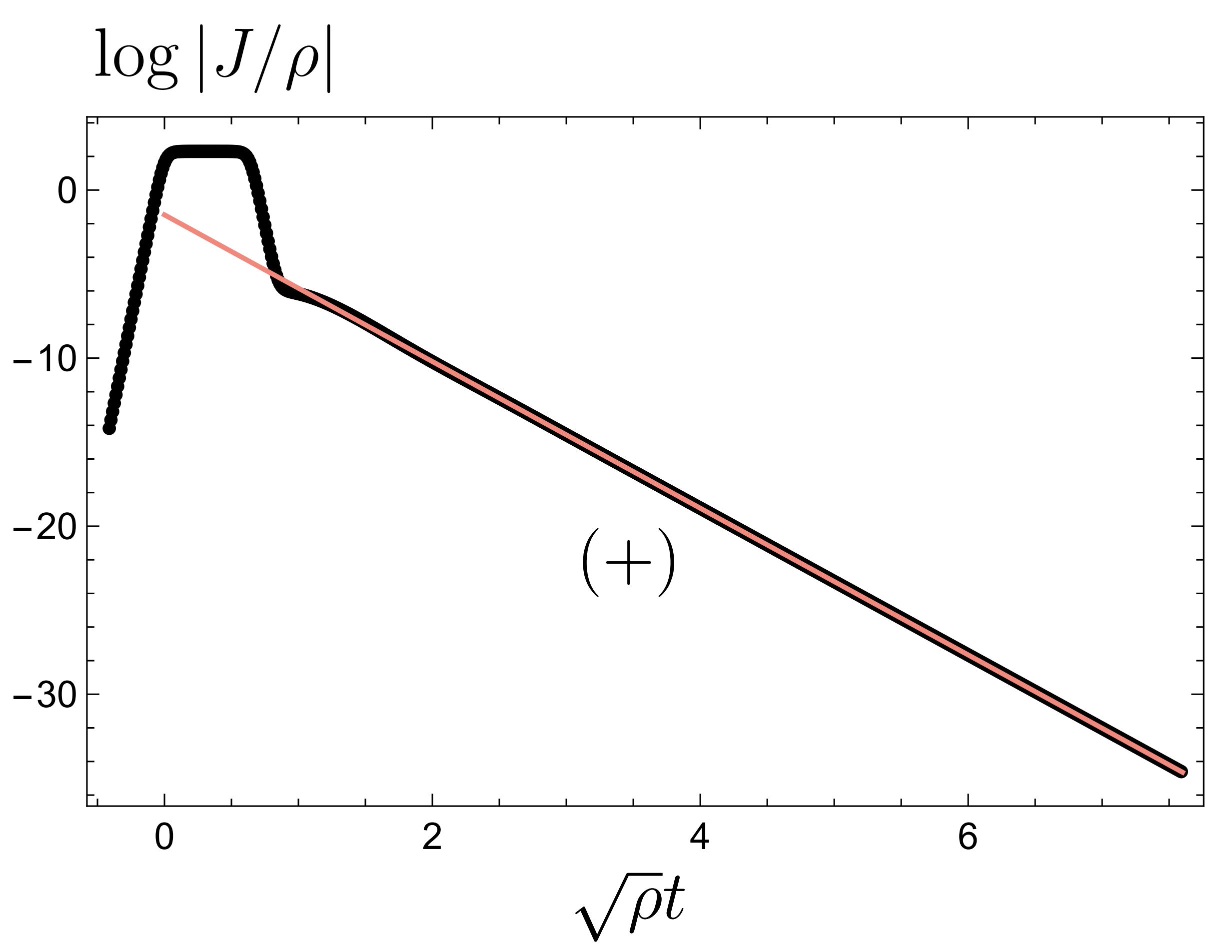

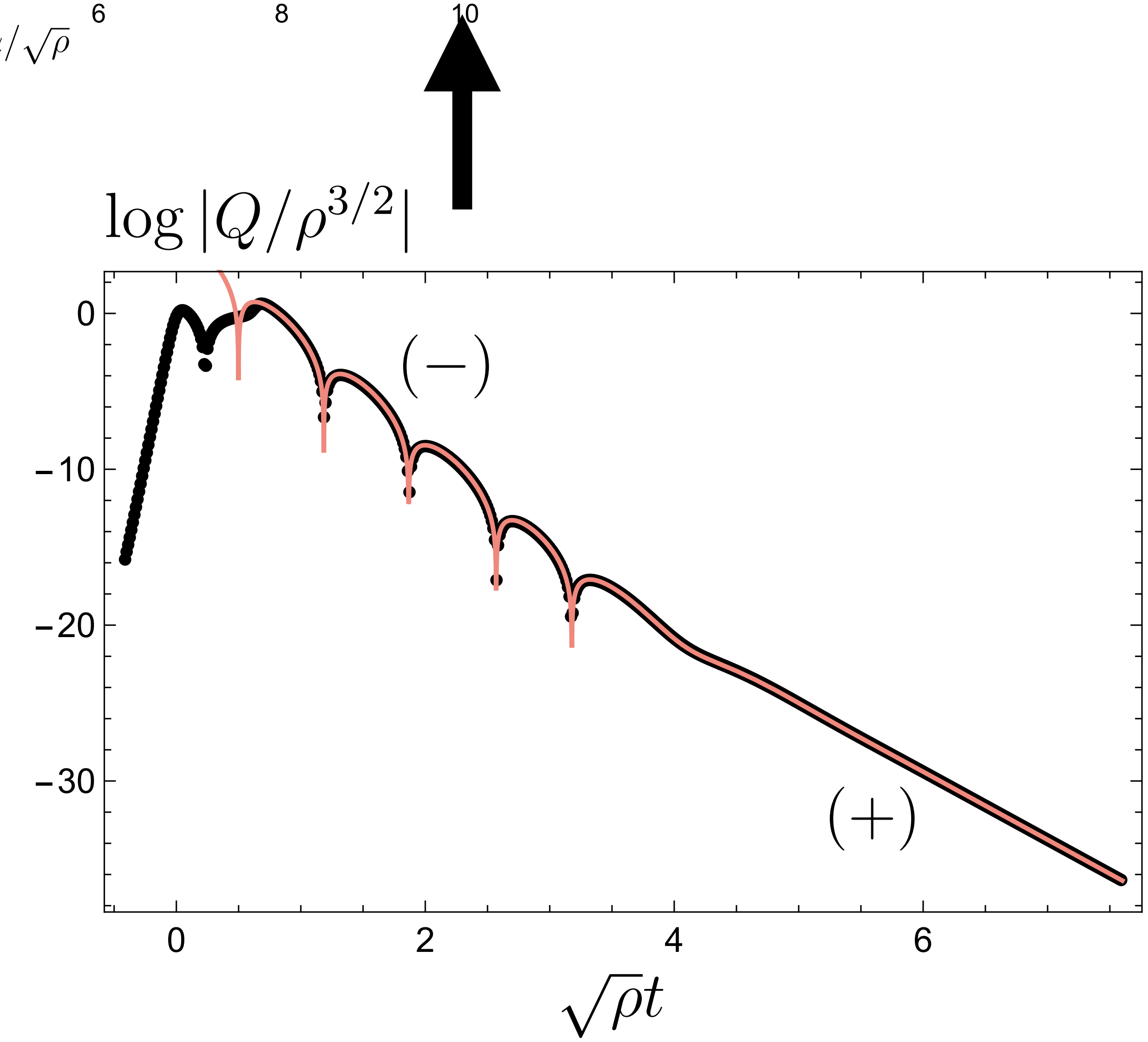

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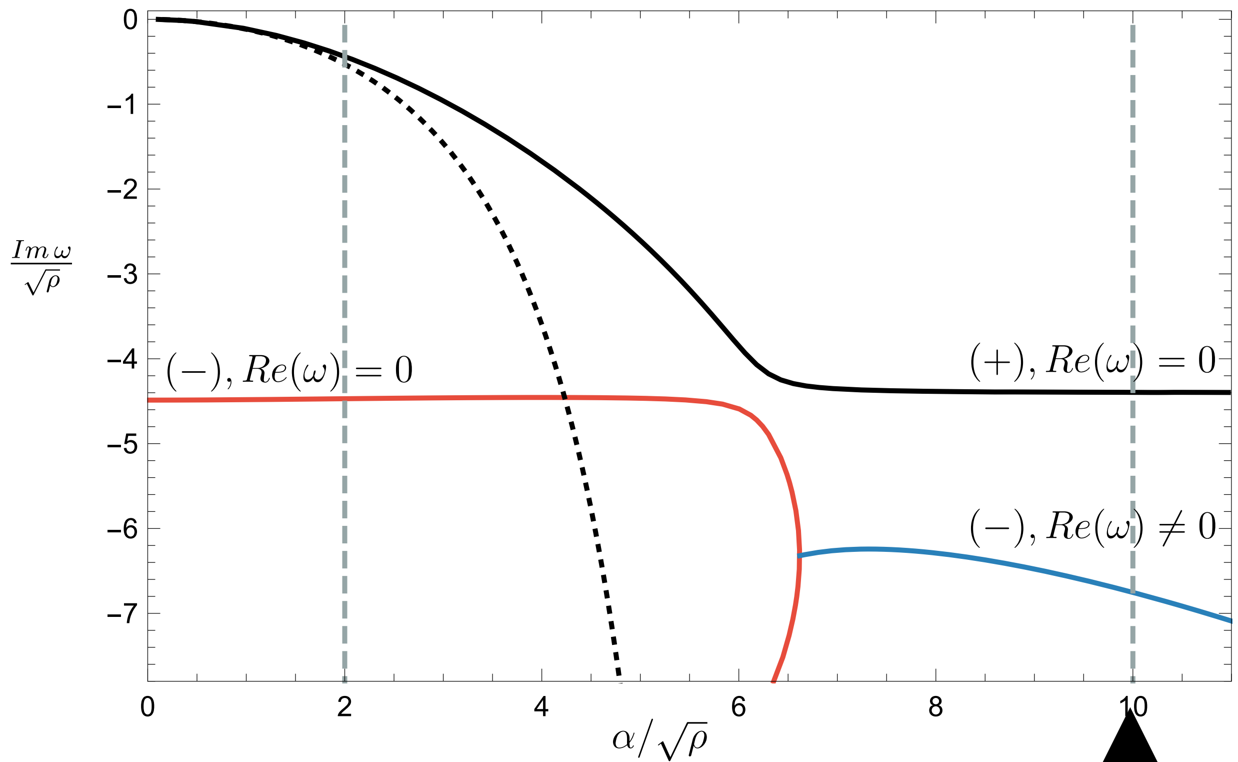

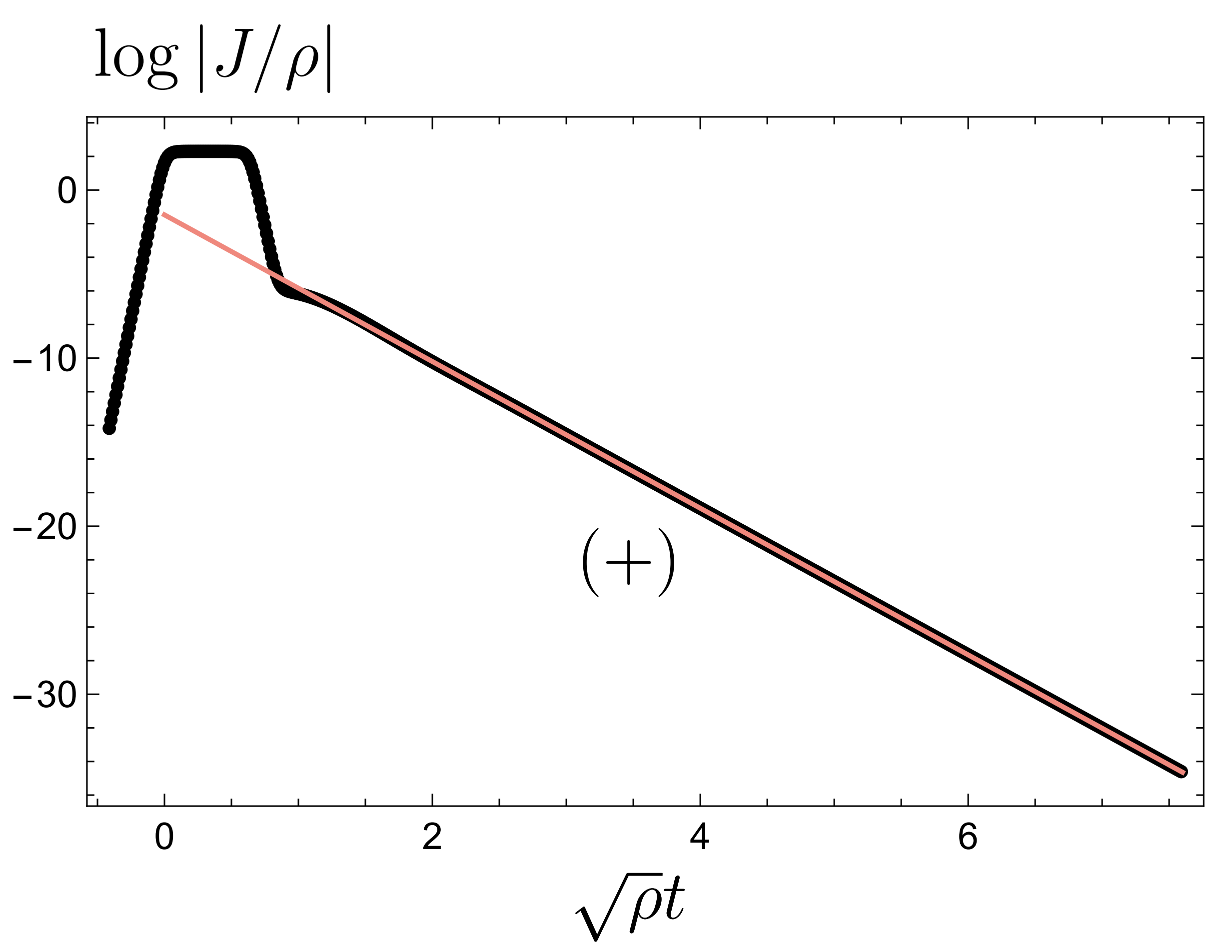

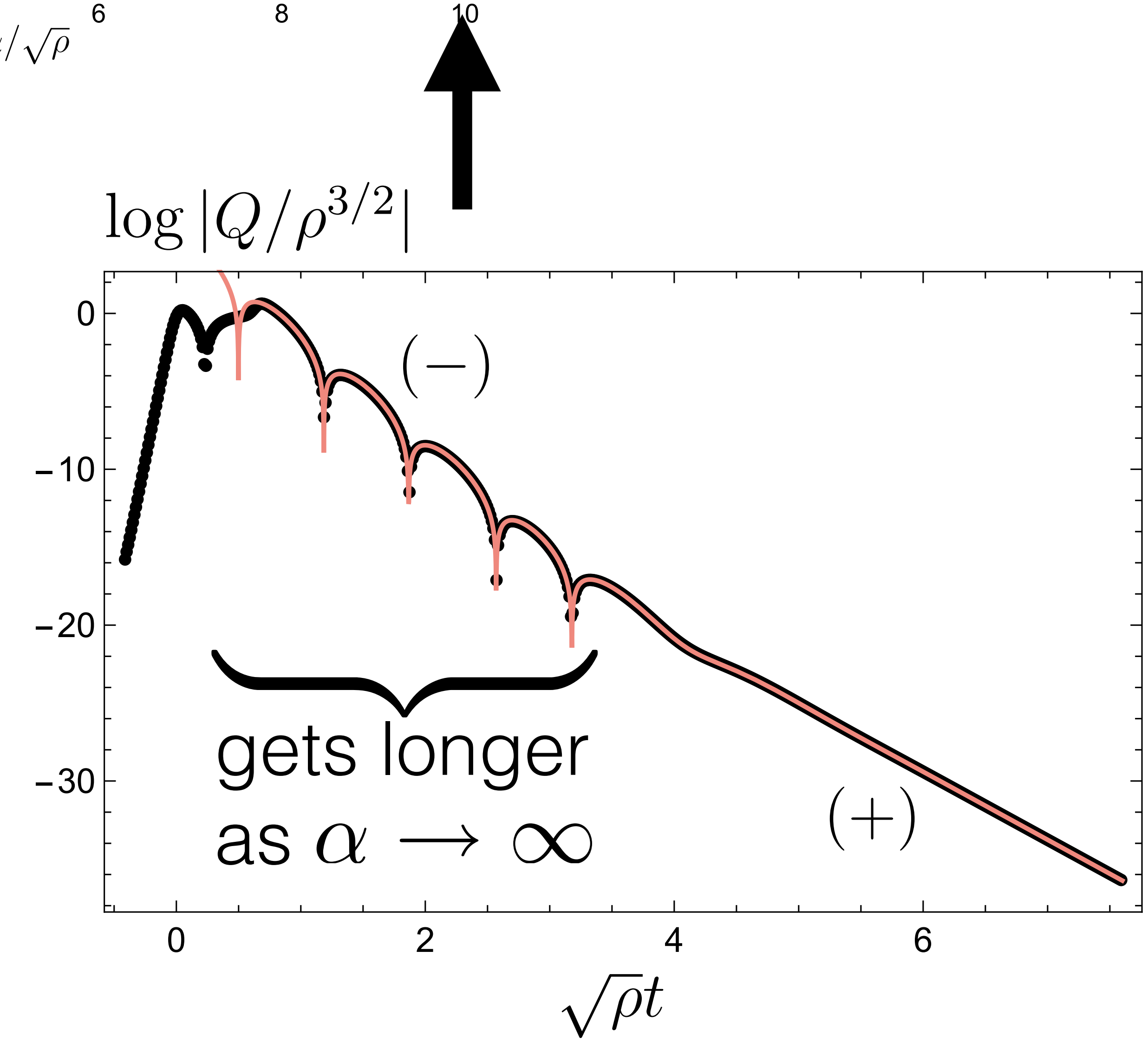

Summary of key points and *outlook*

- Studied far-from-equilibrium holographic metals, using an electric field quench

1. Joule heating

- At $\rho = 0$ Vaidya-like solutions capture Joule heating together with an instantaneous electrical current response
- also, govern the initial response for a rapid quench when $\rho \neq 0$
- Linear steady states (DC linear response)
- Otherwise, $\rho \neq 0$ studied numerically
- *How can we get nonlinear steady states? Heat bath? Insulators?*

2. Nonlinear conductivity

- Electric conductivity well approximated by DC linear response result after promoting $T \to T_E(t)$
- Includes cases where $E_f \gtrsim T_E^2, \quad E_f \gtrsim \rho, \quad E_f \gtrsim \alpha^2$
- Generalises E_f -independence of $\rho = 0$ result, $\sigma = 1$
- *Why is σ E_f - independent?*

3. Current relaxation

- QNMs display qualitative changes as a function of inhomogeneity
- Naively subdominant QNM gets parametrically enhanced
- Oscillatory decay of heat current as a signature of sufficiently incoherent metals
- *Is it a generic phenomenon? compute QNM spectrum for other inhomogeneous models*

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— Thank you —