Black hole instabilities and weak cosmic censorship in higher dimensions

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Numerical Relativity and Holography
Santiago de Compostela

UNIVERSITY OF CAMBRIDGE Department of Applied Mathematics and Theoretical Physics (DAMTP)

Science & Technology Facilities Council

This is a PDF version of the slides without the videos. Some of the videos are available online at the **GRChombo** youtube channel or at:

http://grchombo.github.io/movies.html.

Outline

- Background
  - Weak cosmic censorship
  - Black strings and the Gregory-Laflamme instability
  - Black rings
- Numerical methods
- The endpoint of black ring instabilities
- Summary
- Work in progress

Why are we interested?

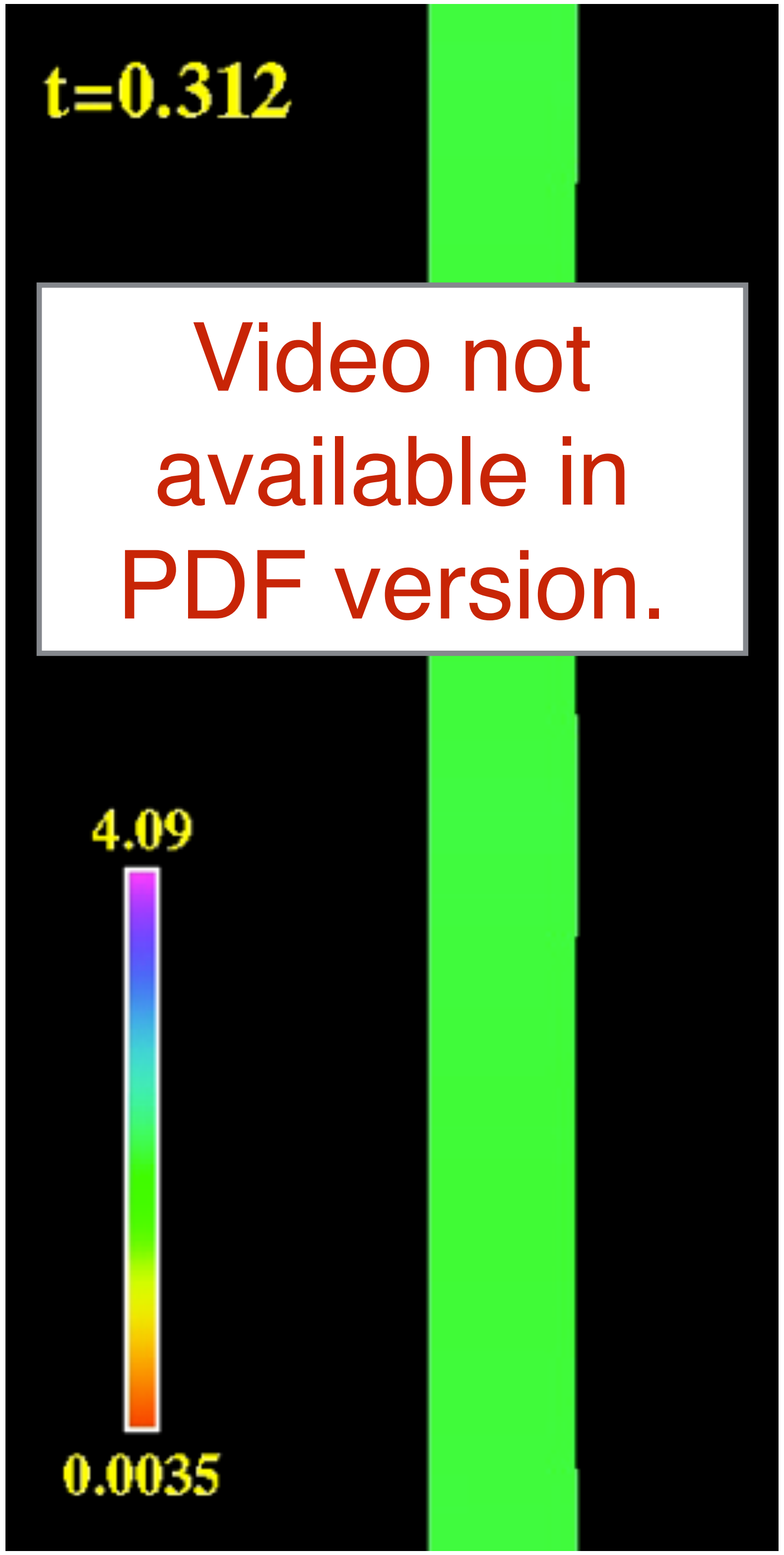
- Theories with more than four dimensions
  - String theory/M-theory
  - AdS/CFT
- Mathematical interest
  - weak cosmic censorship
  - rich mathematical structure
- Advance numerical relativity
  - Techniques for AdS evolution
  - More robust gauge choices, algorithms,…

Weak cosmic censorship

- Context: initial value problem
- Mathematically:
  - Related to *global existence* of solutions
  - No naked singularities
- Physically:
  - *"GR cannot evolve to a regime where quantum gravity is important (outside of event horizon)"*
  - "*CFT side: in large N_c strong coupling limit $1/N_c$ corrections do not become important*"
- Counterexamples: black strings in 5D, …

Black strings

- Gregory-Laflamme instability



[Lehner&Pretorius,2010]

Black strings

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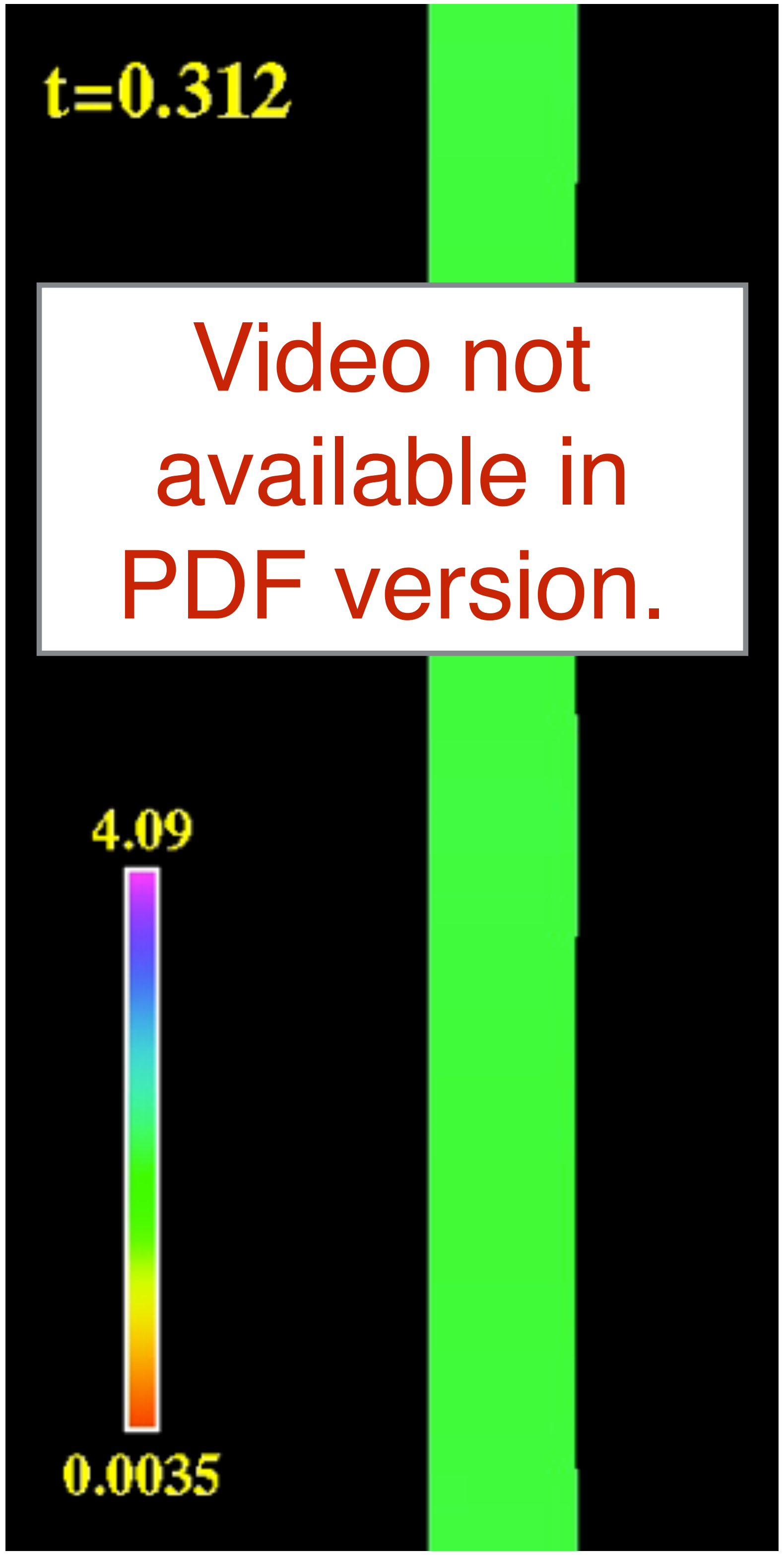
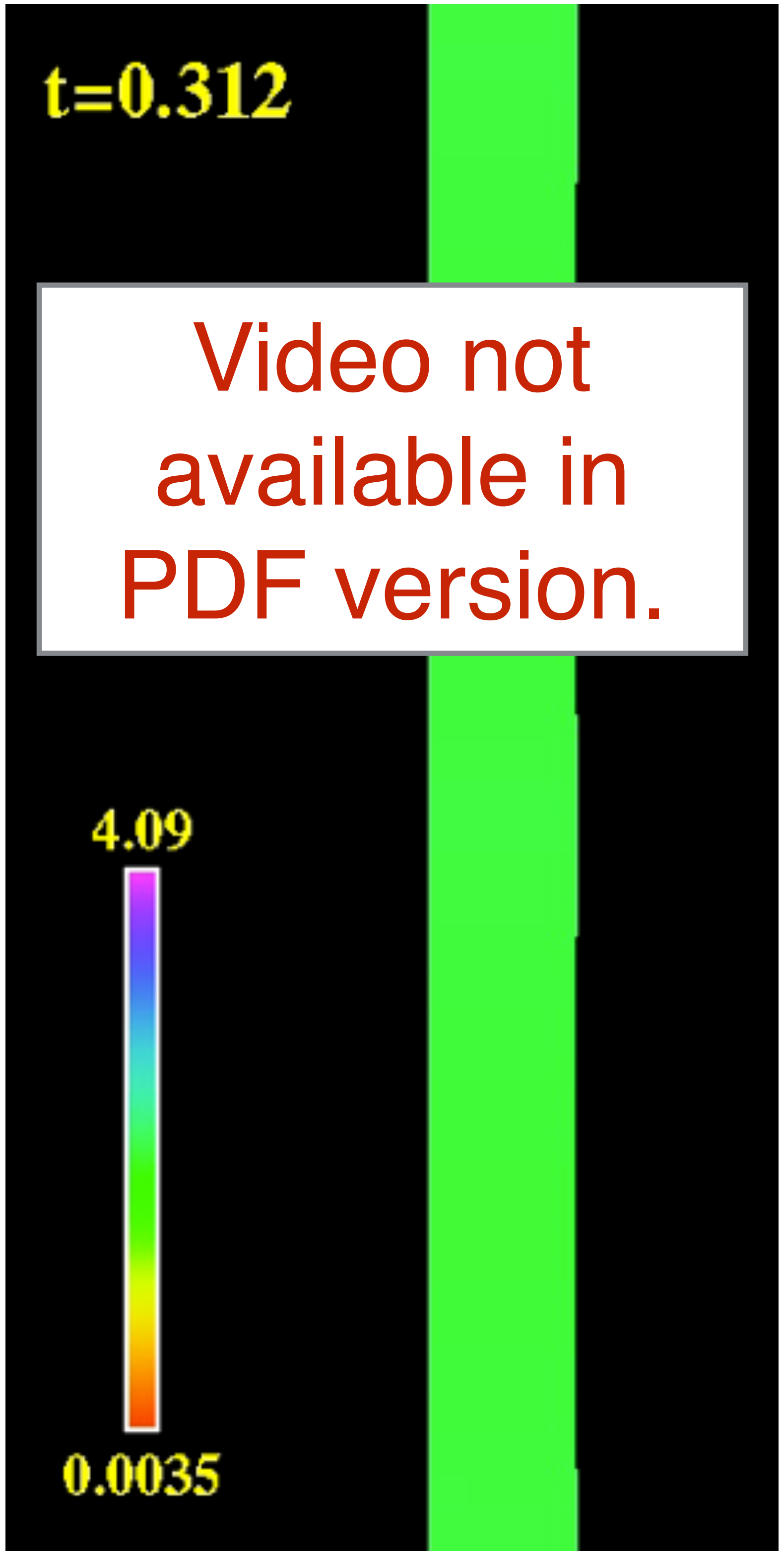


[Lehner&Pretorius,2010]

Black strings

- Gregory-Laflamme instability
- Fractal structure of bulges and necks.
- Pinch-off in finite asymptotic time.
- Weak Cosmic Censorship may be violated in spacetimes with compact extra dimensions.

[Lehner&Pretorius,2010]

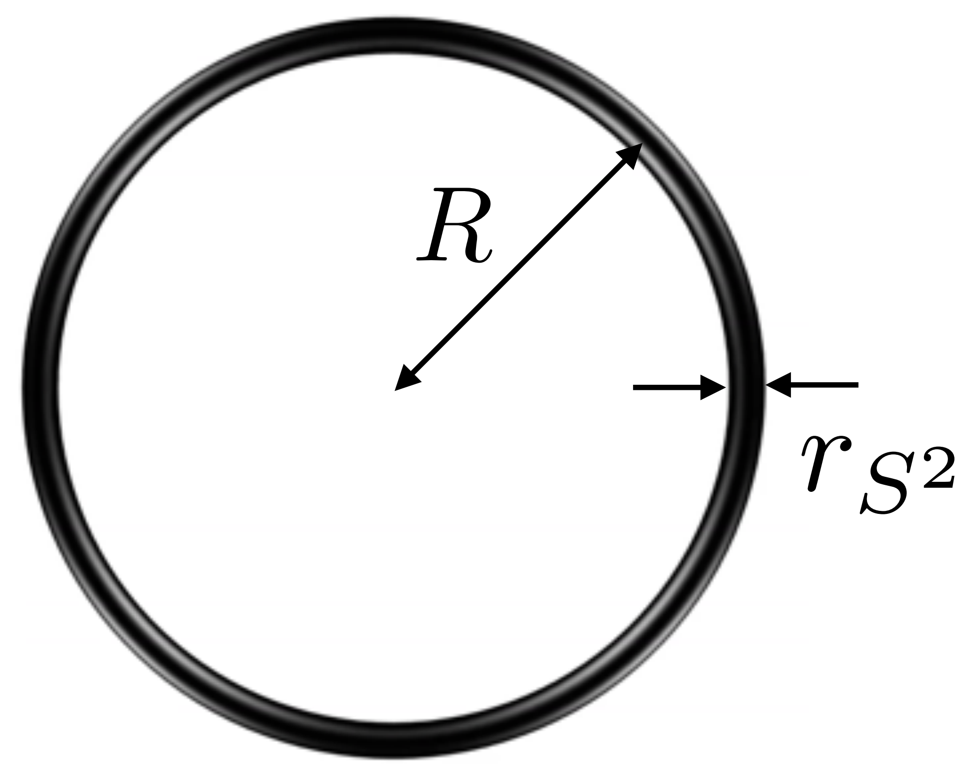


[Lehner&Pretorius,2010]

Black rings

Black rings

- Explicit 5D black hole solution with horizon topology $S^1 \times S^2$.
- Asymptotically flat or asymptotically AdS
  [Emparan&Reall] [Figueras&Tunyasuvunakool]
- Free parameters: $0 < R < \infty$ $0 < \nu < 1$



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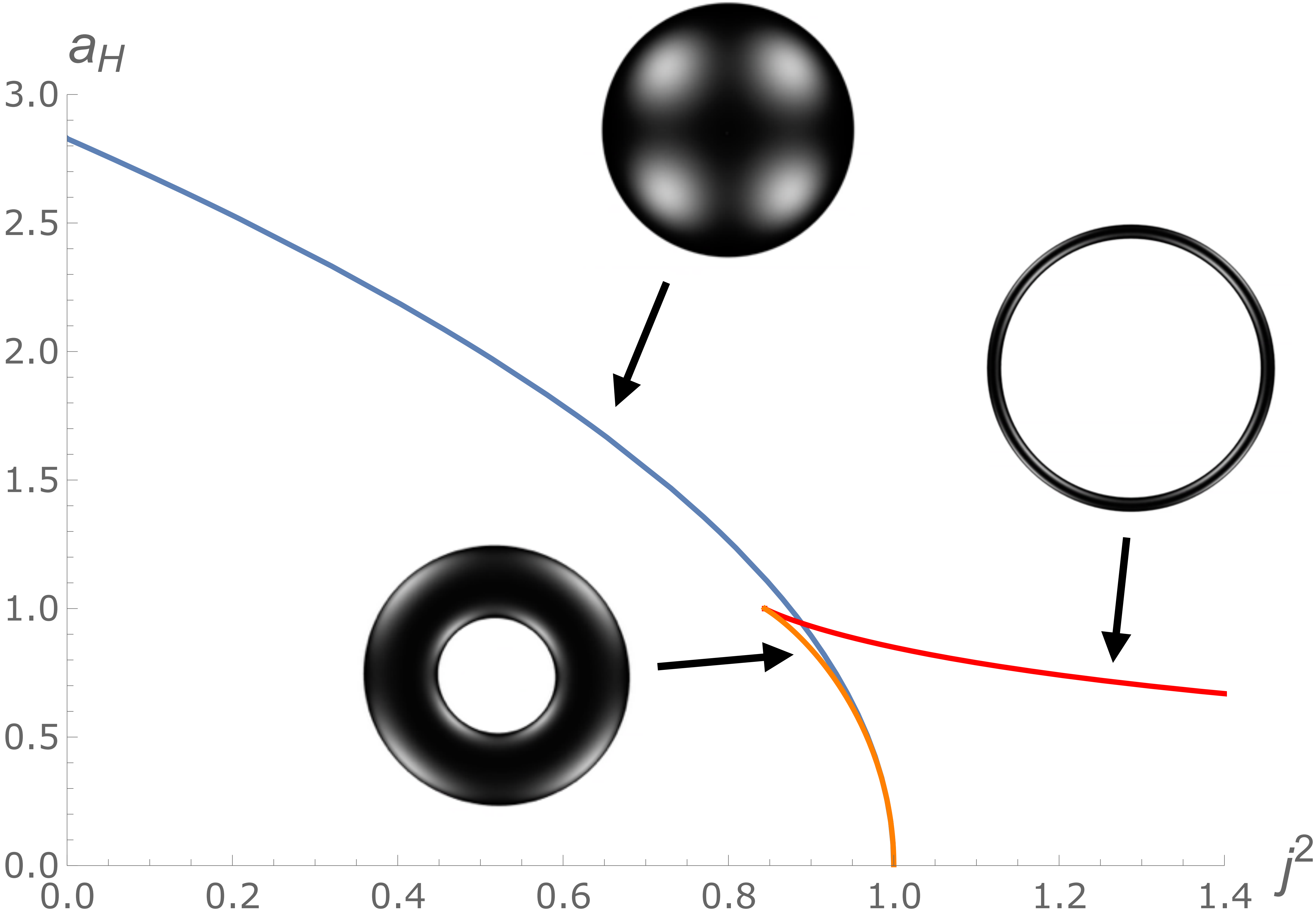

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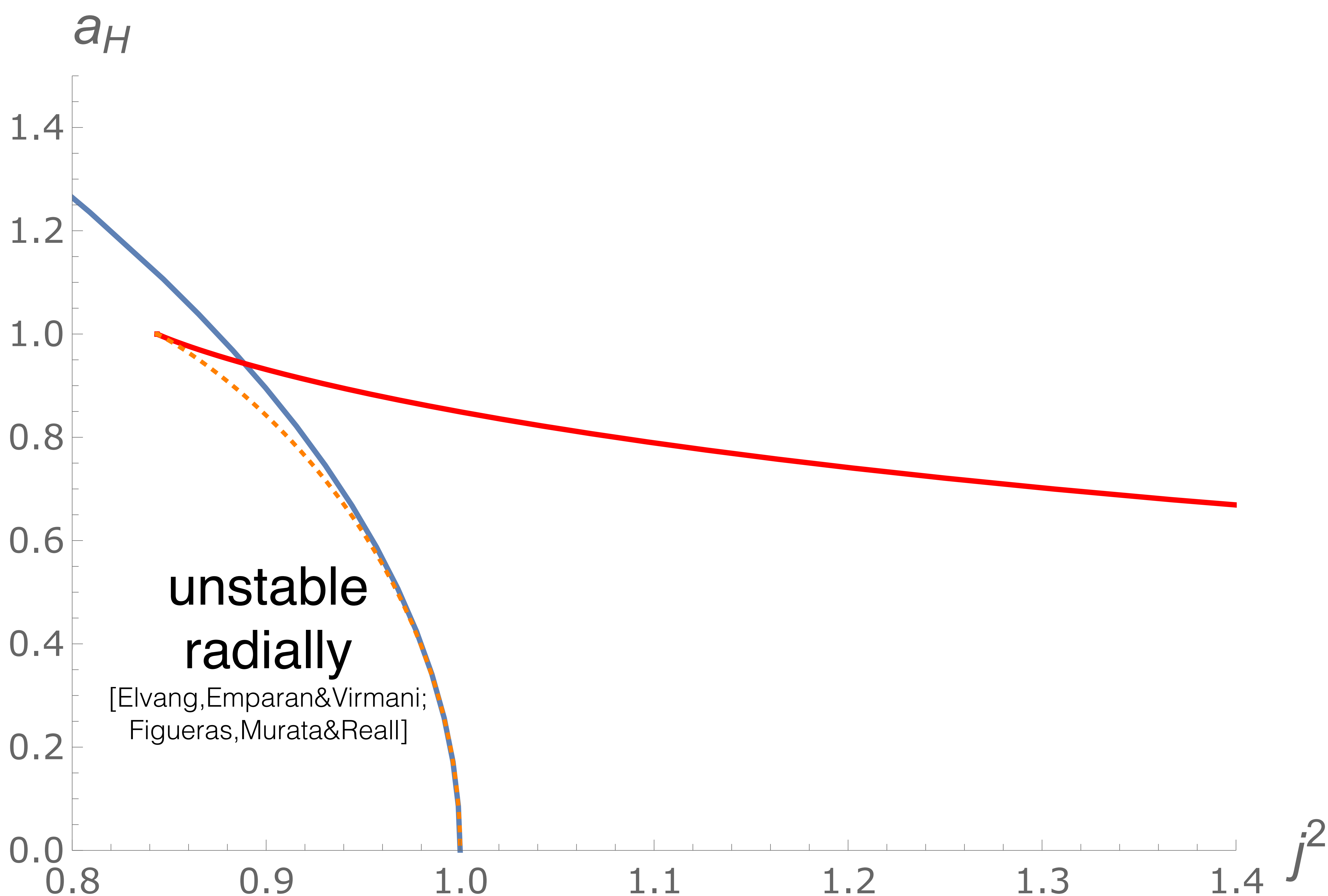

a_H
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.8
0.9
1.0
1.1
1.2
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1.4
j^2
unstable
radially
[Elvang,Emparan&Virmani;
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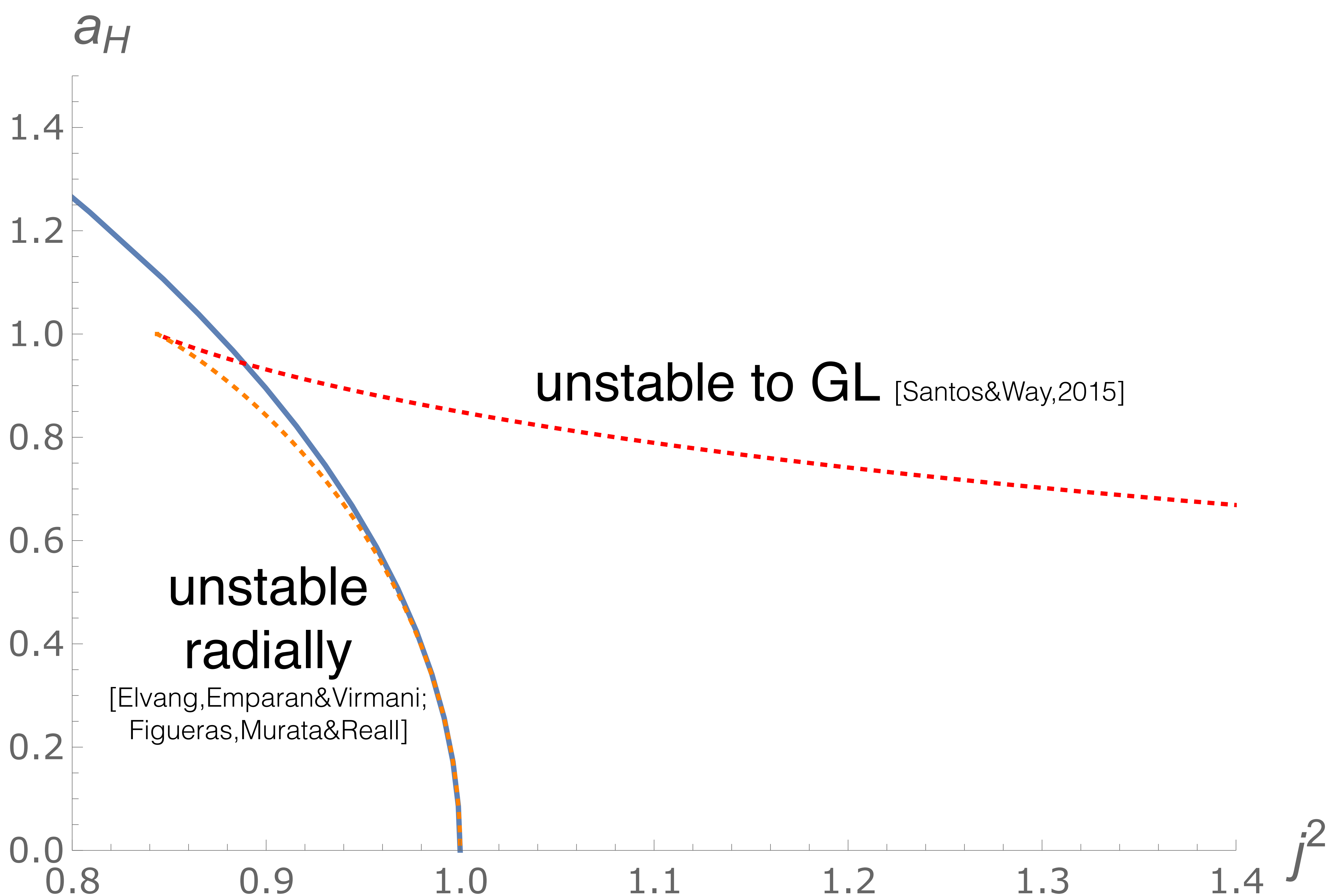

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unstable to GL [Santos&Way,2015]
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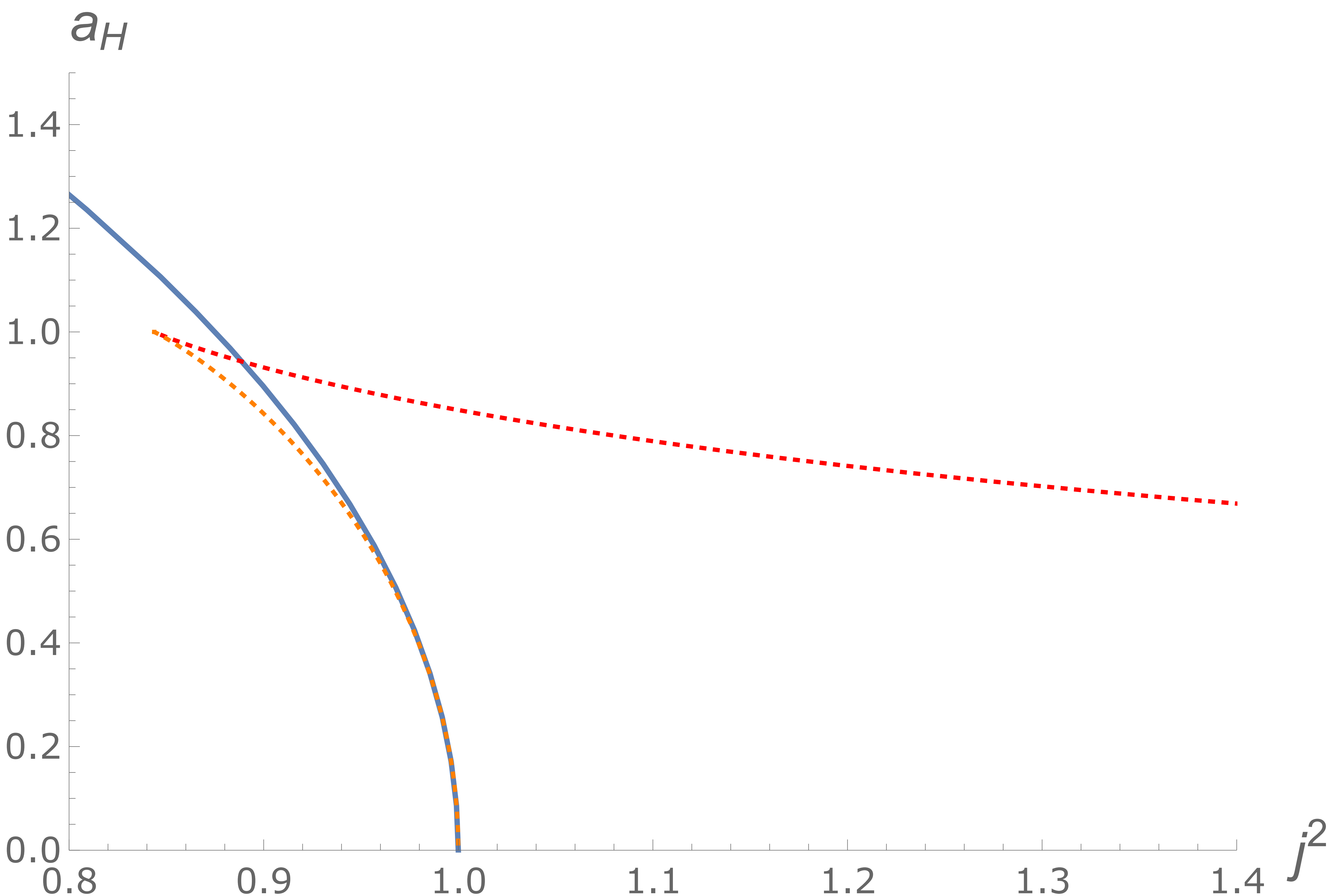

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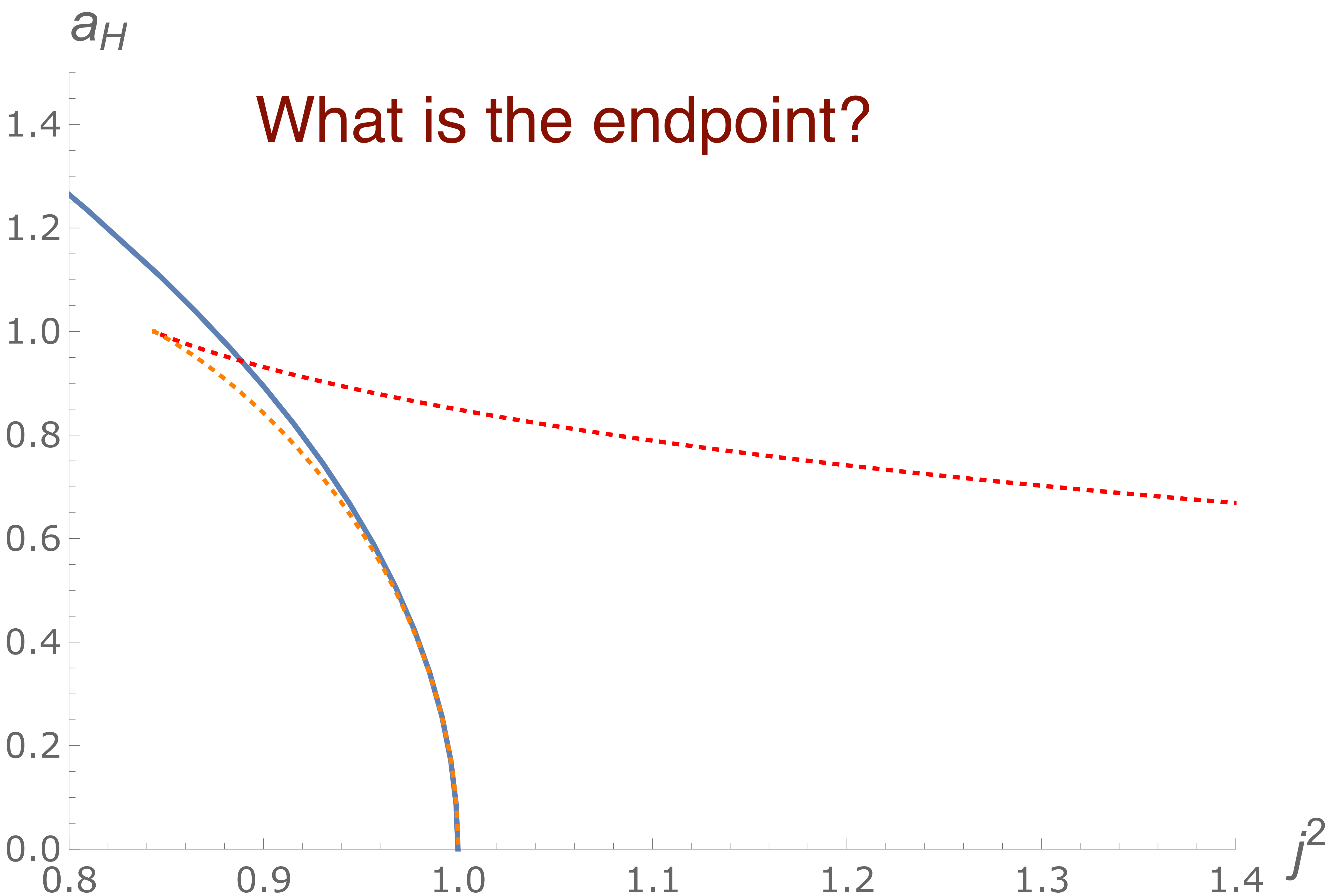

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What is the endpoint?
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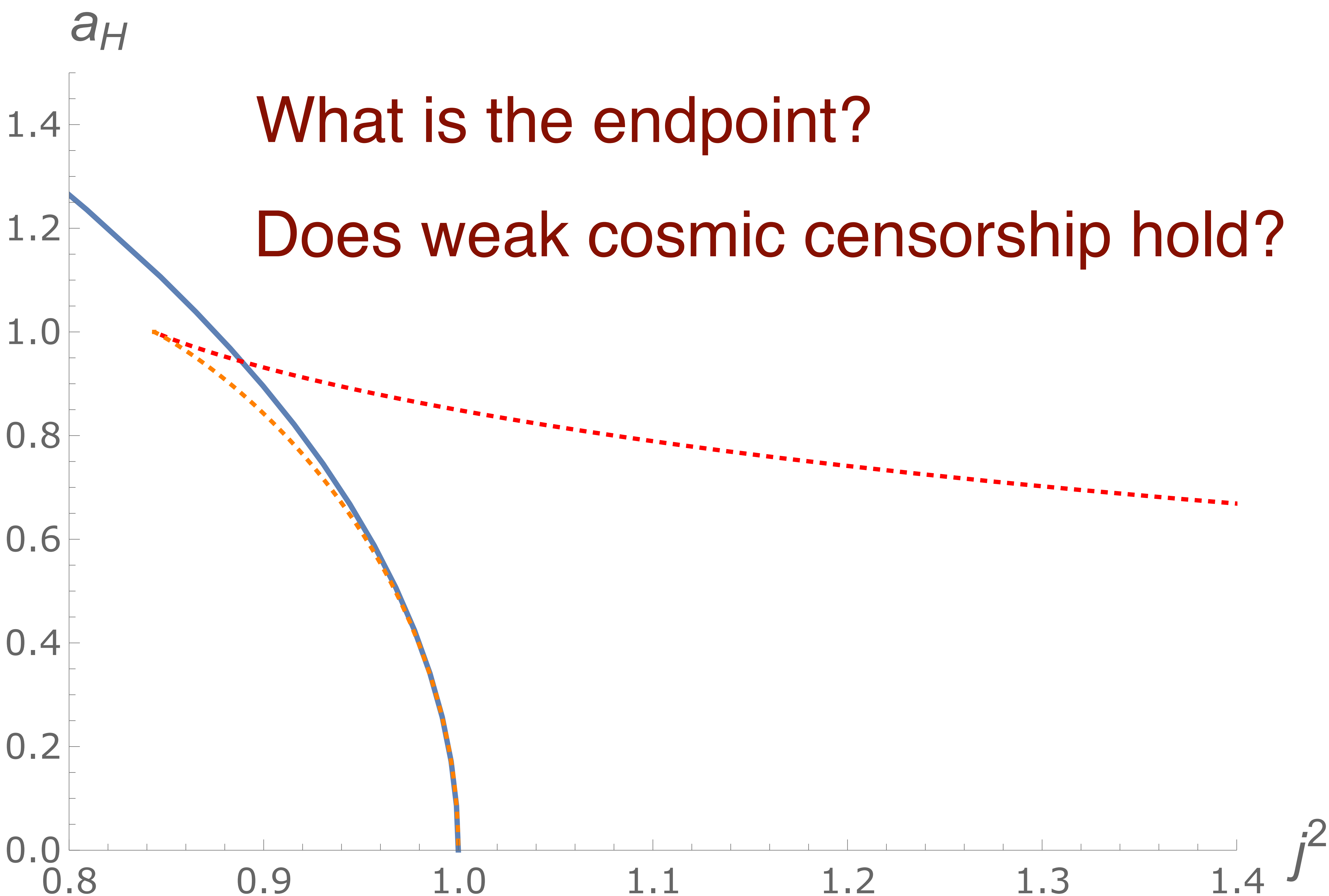


What is the endpoint?

Does weak cosmic censorship hold?

Numerical methods

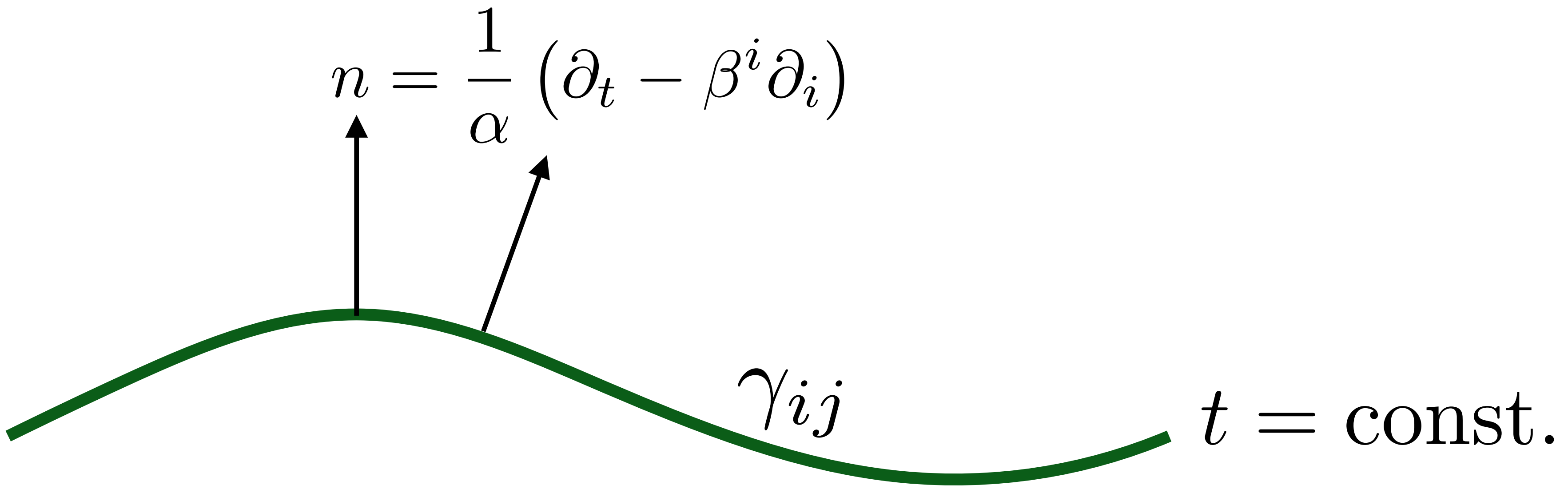
Numerical methods

d+1 numerical relativity

- d+1 split:

$$ds^2 = -\alpha^2\, dt^2 + \gamma_{ij}(dx^i + \beta^i\, dt)(dx^j + \beta^j\, dt)$$

- Evolve γ_{ij} and $K_{ij} = -\dfrac{1}{2}\mathcal{L}_n \gamma_{ij}$.
- Specify gauge conditions for α and β^i.



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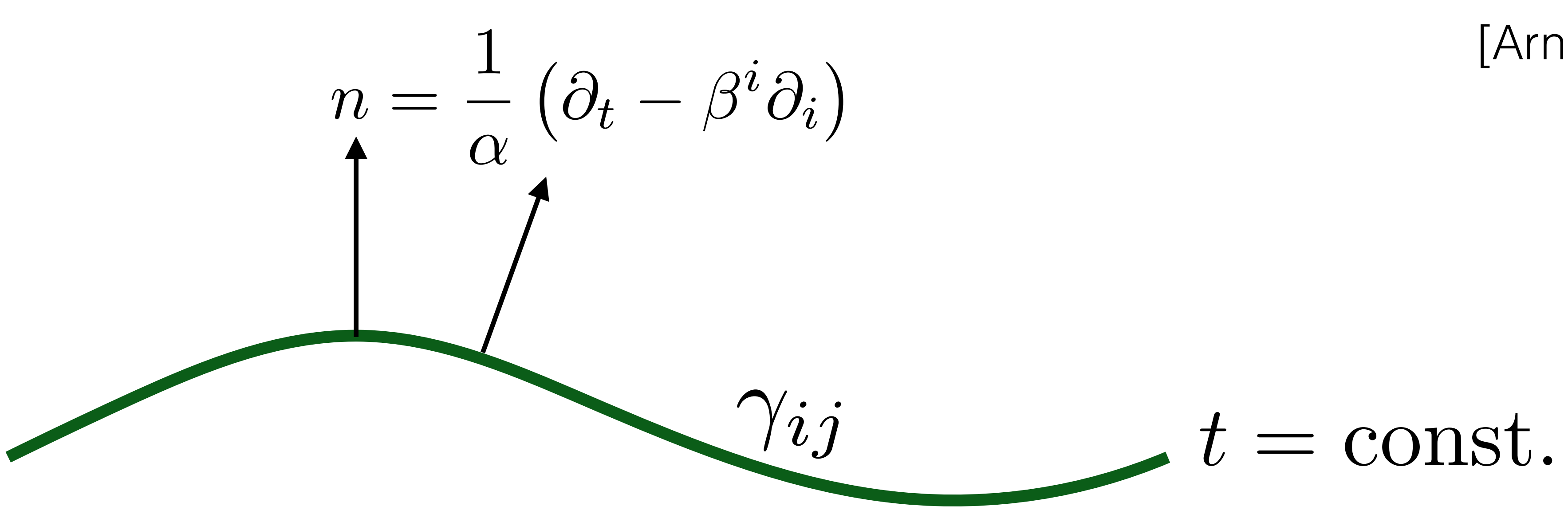
ADM formalism

[Arnowitt,Deser&Misner]



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ADM formalism
(only weakly hyperbolic)

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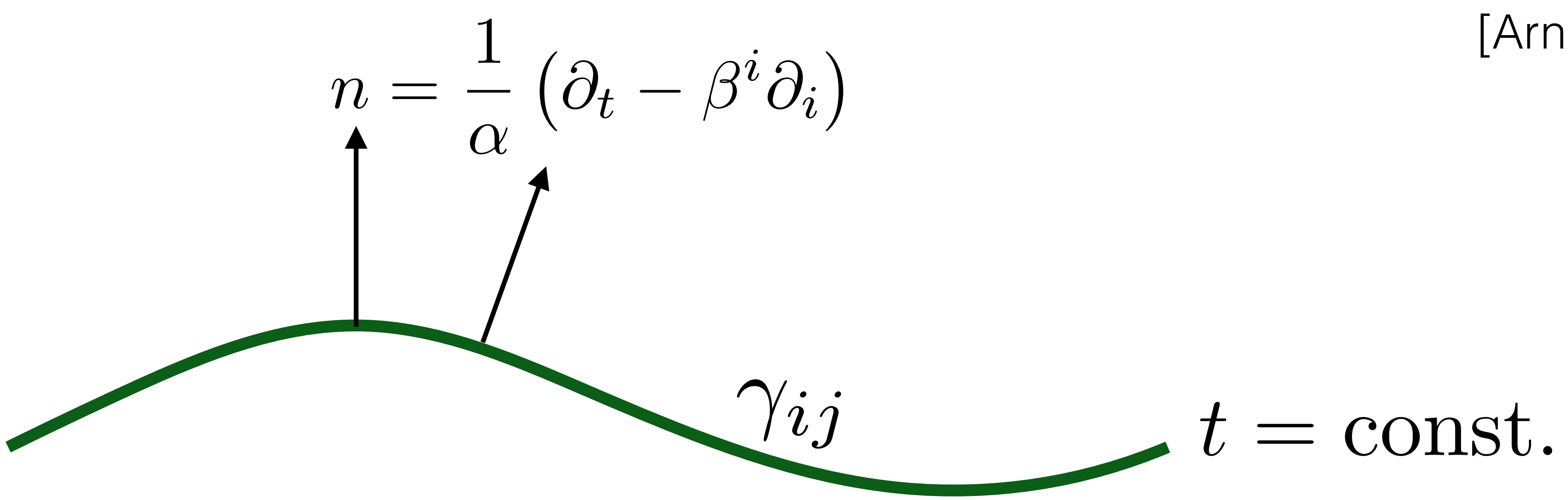

CCZ4 [Baumgarte&Shapiro;Shibata&Nakamura; Bona et al.; Hilditch et al.; Alic et al.; Gundlach et al.]

- Reorganise evolution variables
  - Separate out conformal factor and trace.
  - Evolve contracted connection separately.

$$(\gamma_{ij}, K_{ij}) \rightarrow (\chi, \tilde{\gamma}_{ij}, \tilde{A}_{ij}, K, \tilde{\Gamma}^i)$$

$$\chi = \gamma^{-1/d} \quad \tilde{\gamma}_{ij} = \chi\gamma_{ij} \quad \tilde{A}_{ij} = \chi K_{ij}^{\mathrm{TF}} \quad \tilde{\Gamma}^i = \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk}$$

- Constraint damping terms:

$$R_{ab} + 2\nabla_{(a}Z_{b)} - \kappa_1\left[2n_{(a}Z_{b)} - (1+\kappa_2)g_{ab}\, n^c Z_c\right] = 0$$

Constraints propagate

Constraints are damped [Gundlach et al.]

Why simulating rings is hard

- Separation of scales
- Problems with singularity:
  - Extended, dynamic
  - Far from conformally flat
- Finding distorted non-spherical AH
- Very expensive

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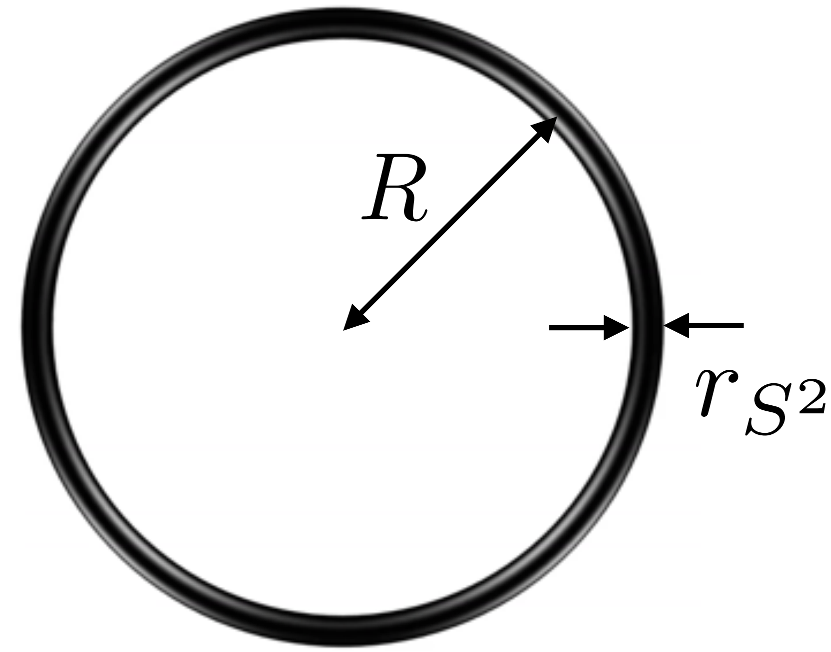

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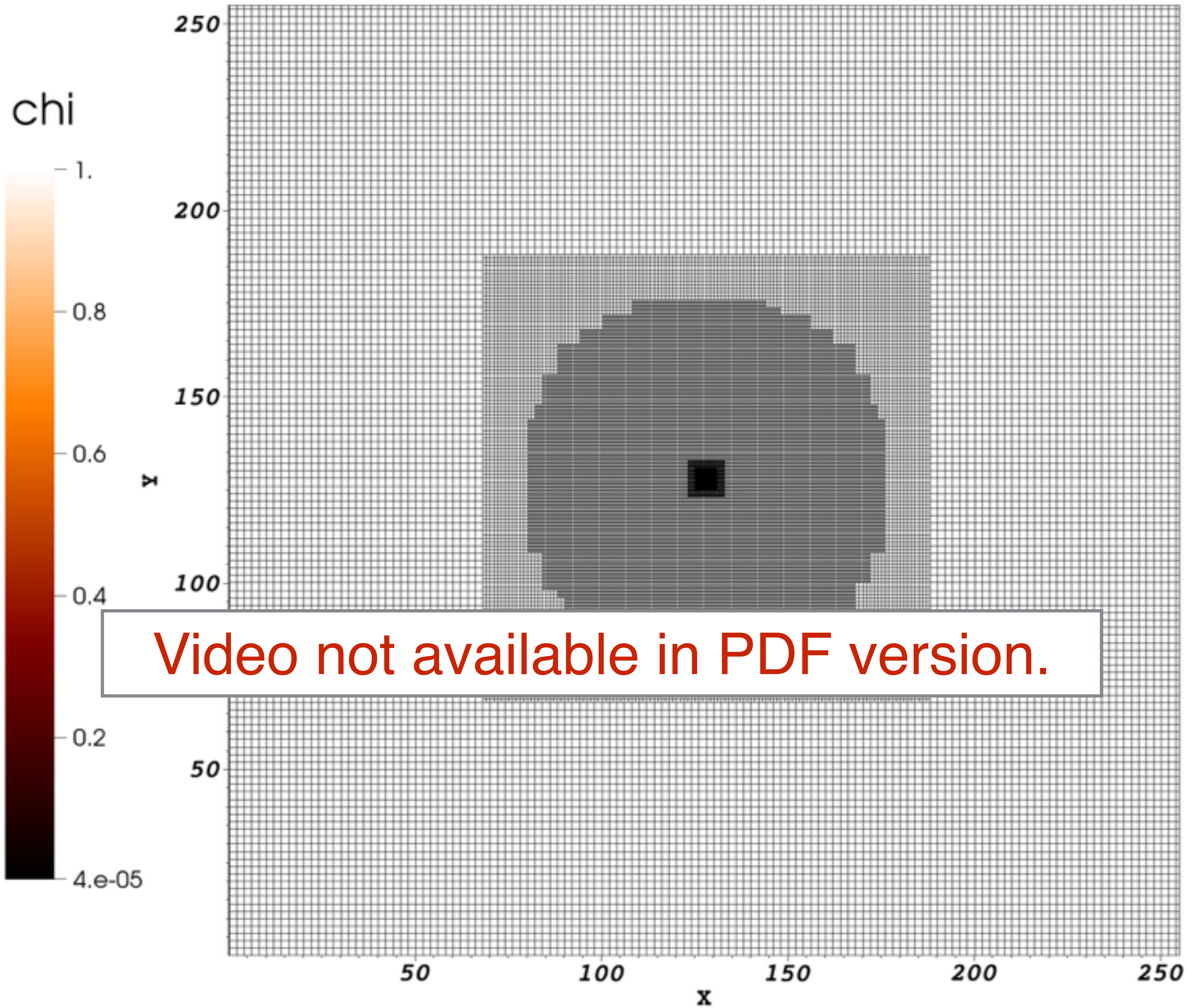
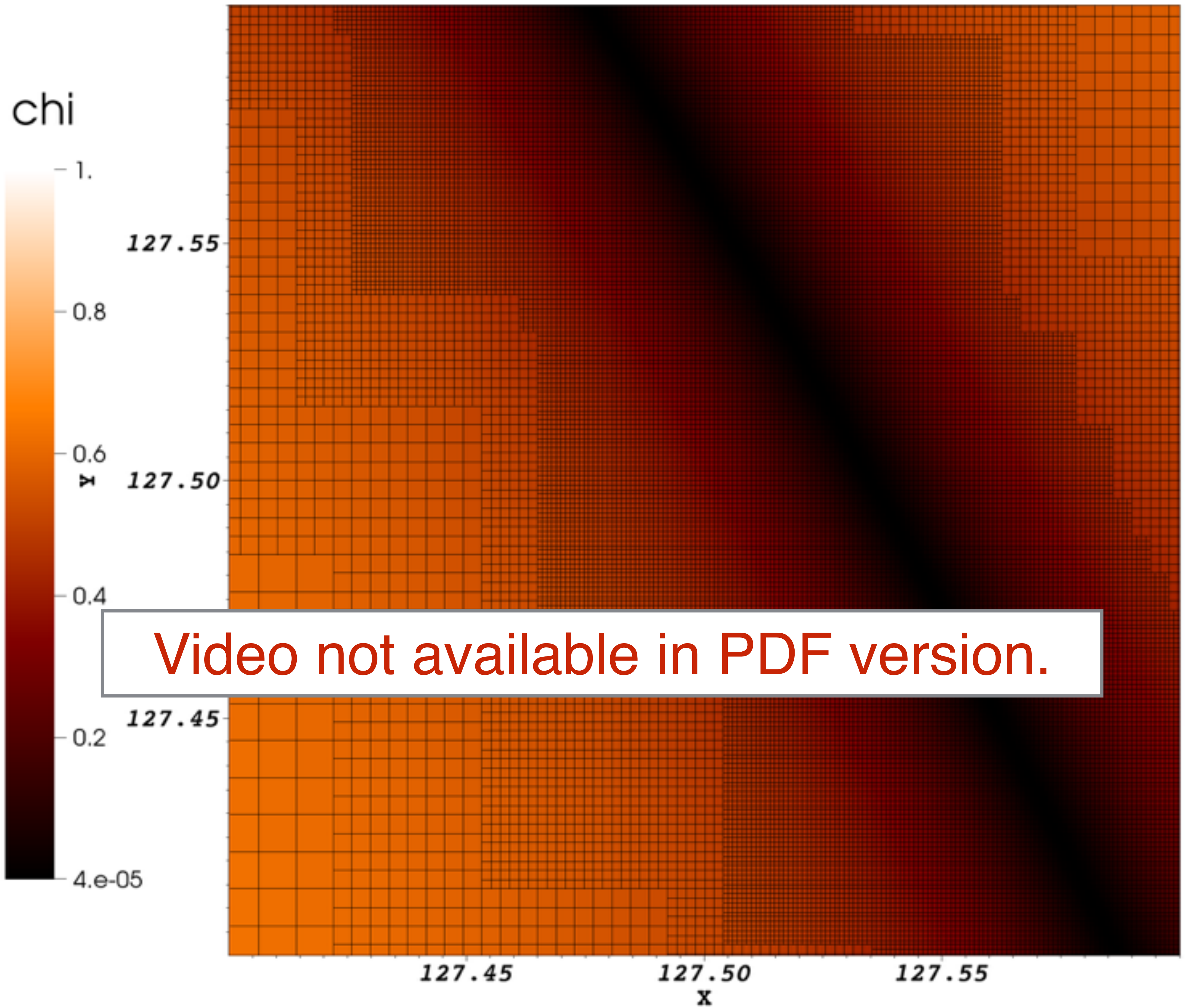
Adaptive mesh refinement



Adaptive mesh refinement



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- Very expensive ➡ **Supercomputers**

The results

Results

Very fat rings ($\nu \gtrsim 0.7$)

- Know that fat rings are unstable radially.
  [Elvang,Emparan&Virmani; Figueras,Murata&Reall]
- Endpoint?

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- Endpoint? ➡ **Myers-Perry black hole (change of topology)**

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Intermediate rings ($0.35 \lesssim \nu \lesssim 0.7$)

- Unstable radially for $\nu \geq 0.5$.
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- GL mode starts to kick in. [Santos&Way]

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- Gravitational wave emission very efficient
- End-state: Myers-Perry black hole

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$\nu = 0.3$

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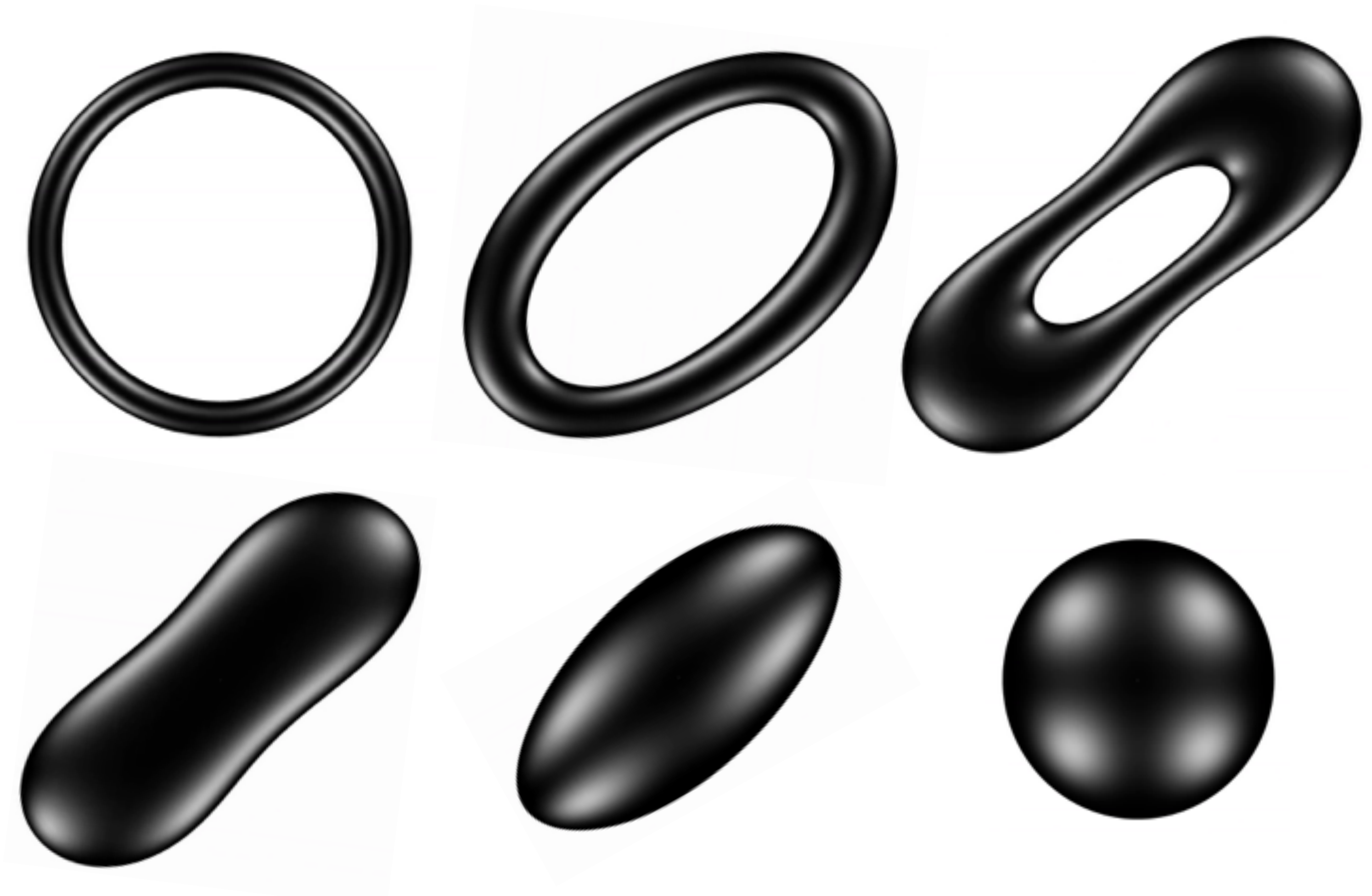
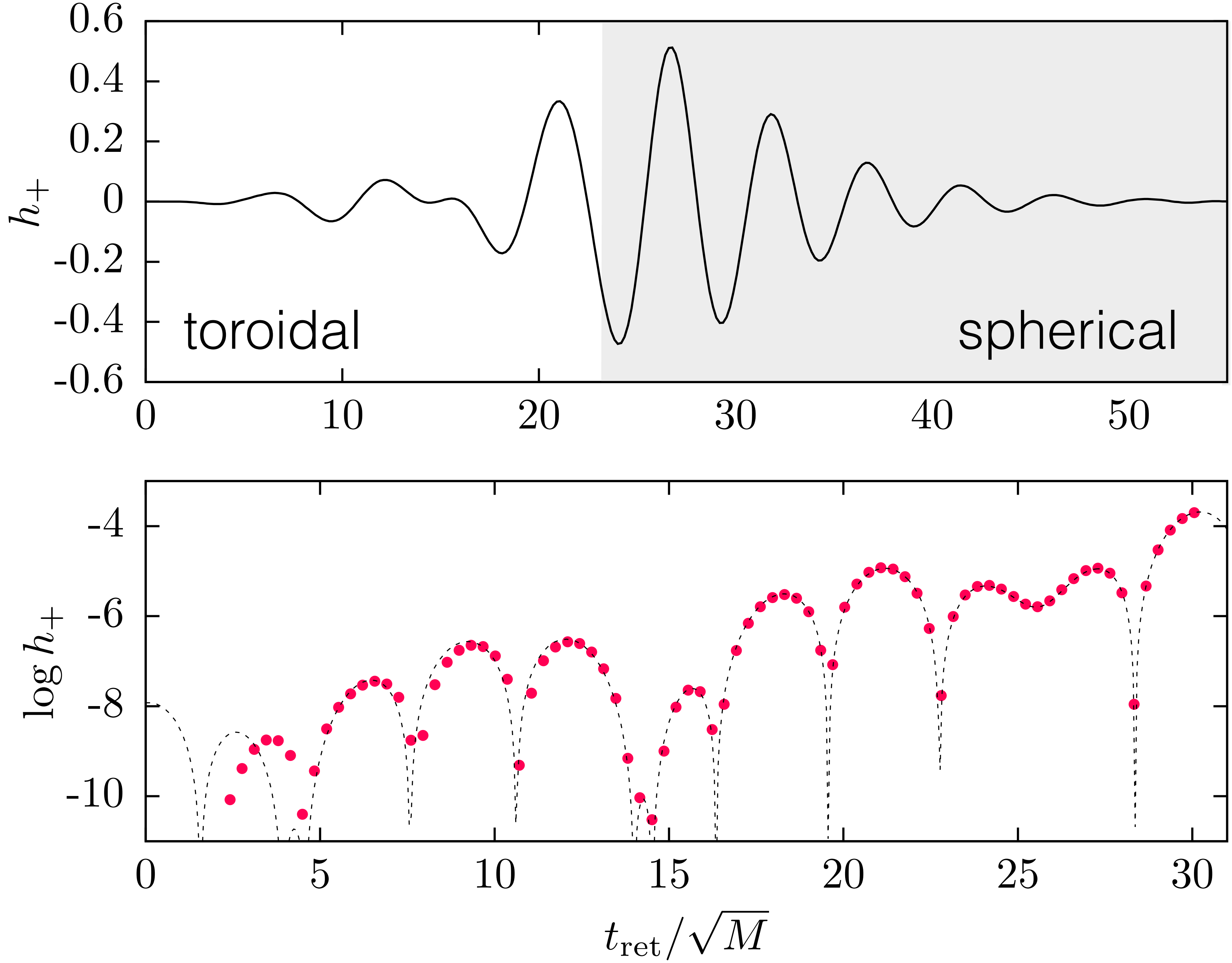
- Competition of GL and "elastic" mode.
- Endpoint: Myers-Perry black hole



Wave signal

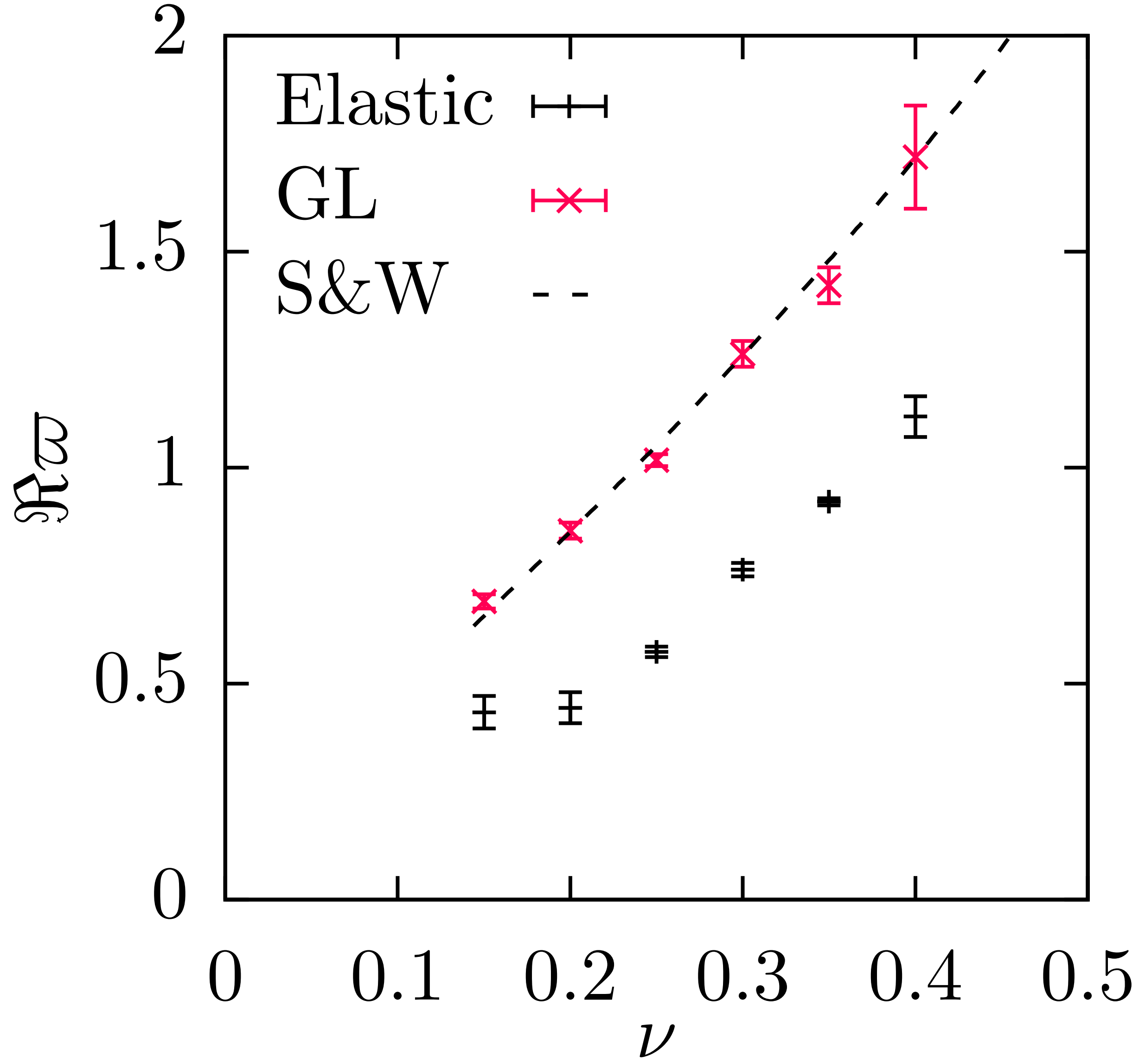


● Numerical results

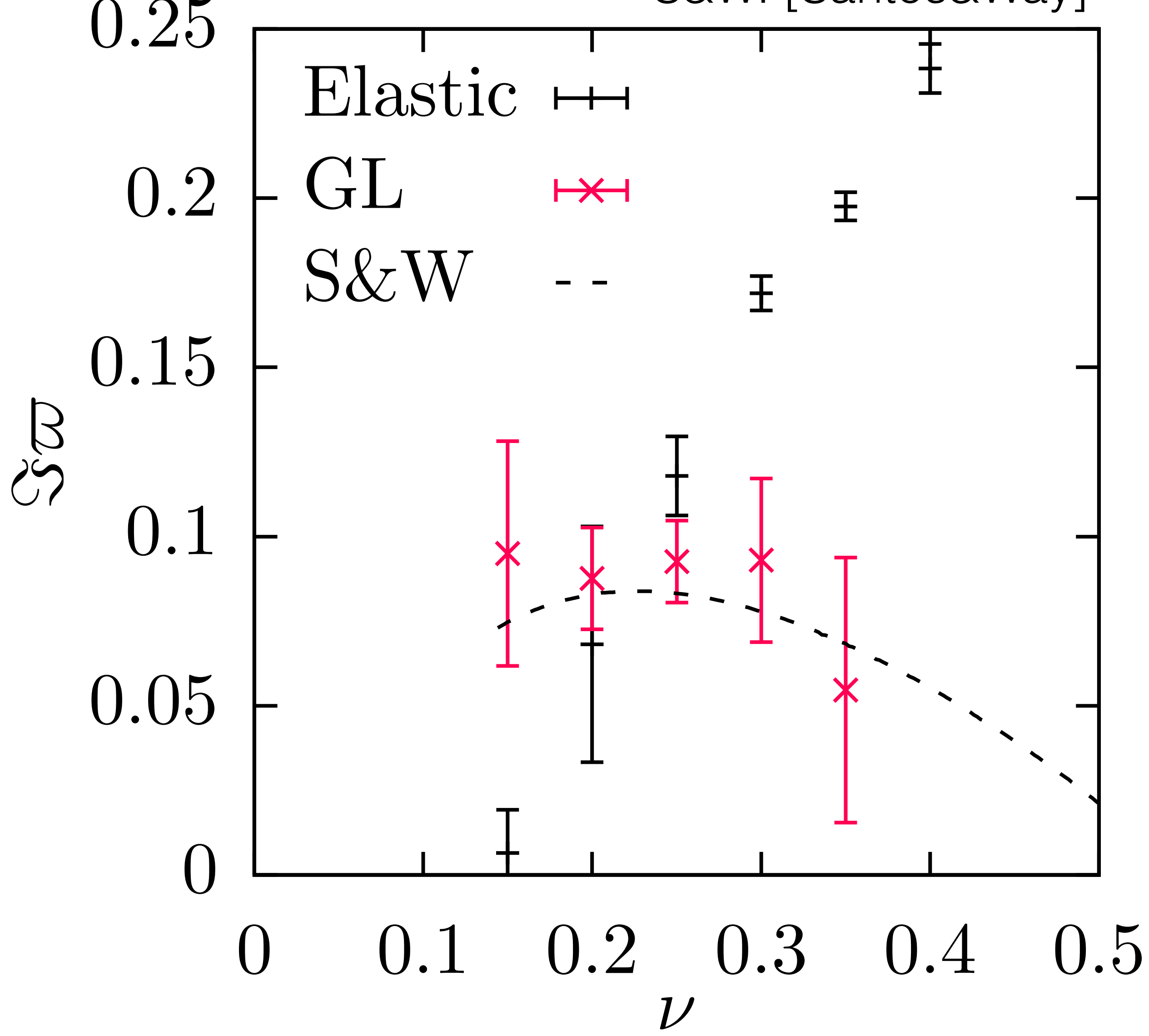
- - - $A_1 \sin(\Re\varpi_1 t + \varphi_1) e^{\Im\varpi_1 t} + A_2 \sin(\Re\varpi_2 t + \varphi_2) e^{\Im\varpi_2 t}$

Linear analysis - results

S&W: [Santos&Way]



frequency



growth rate

Very thin rings ($\nu \lesssim 0.15$)

- Growth rates suggest: GL should dominate
- Have to consider $m = 4$ mode as we cannot suppress its perturbation sufficiently.

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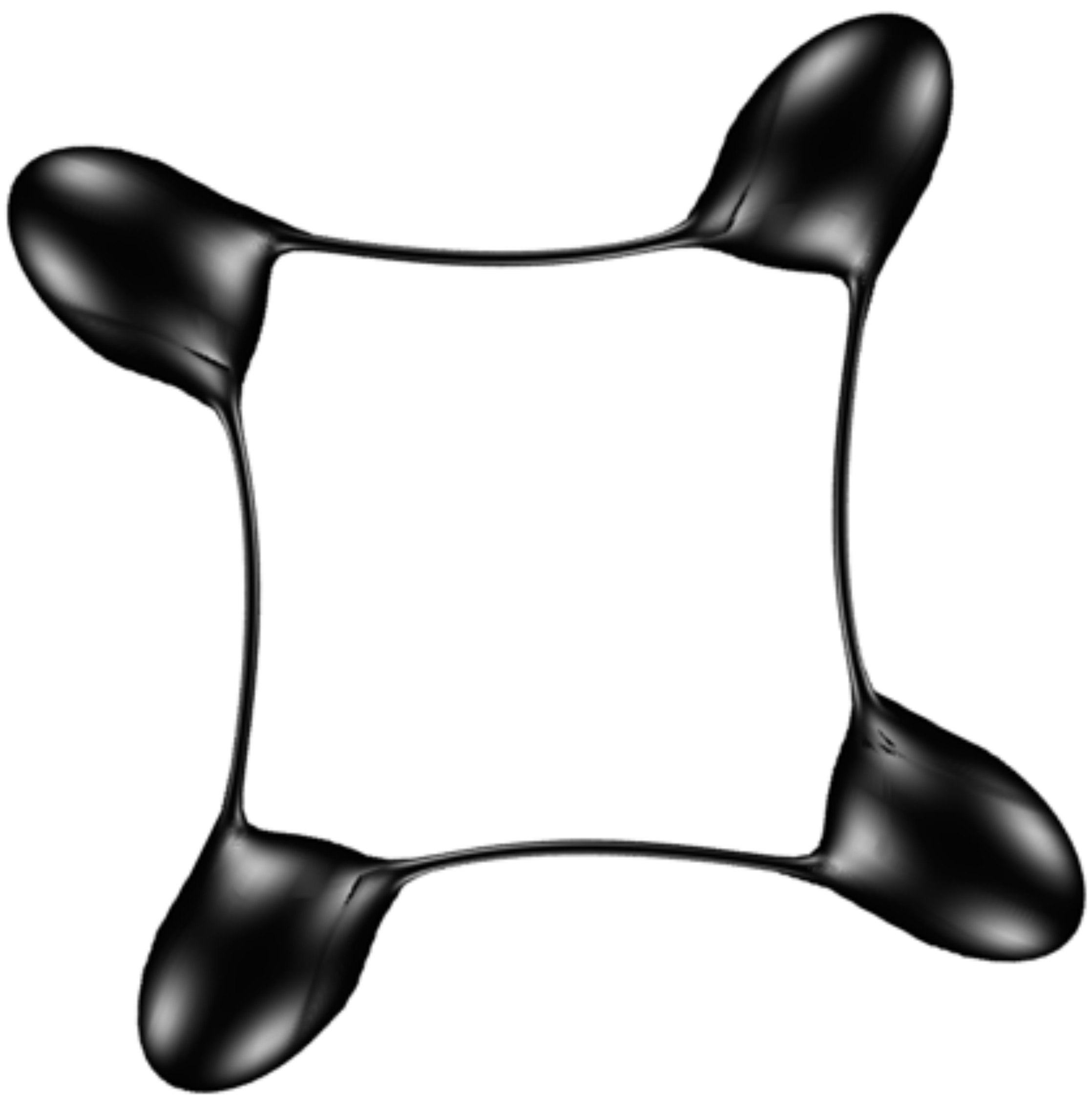

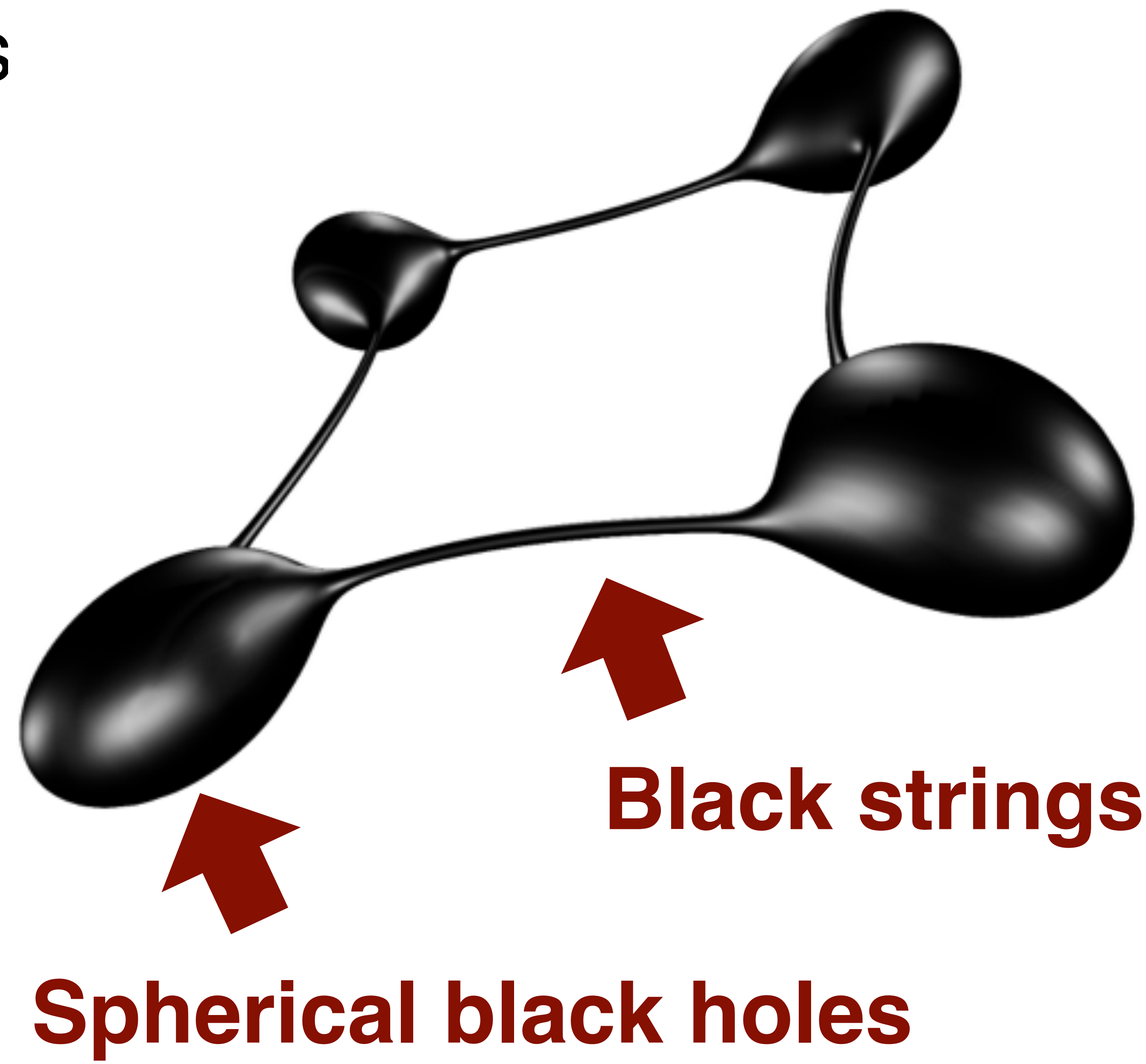

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$\chi = 0.2$

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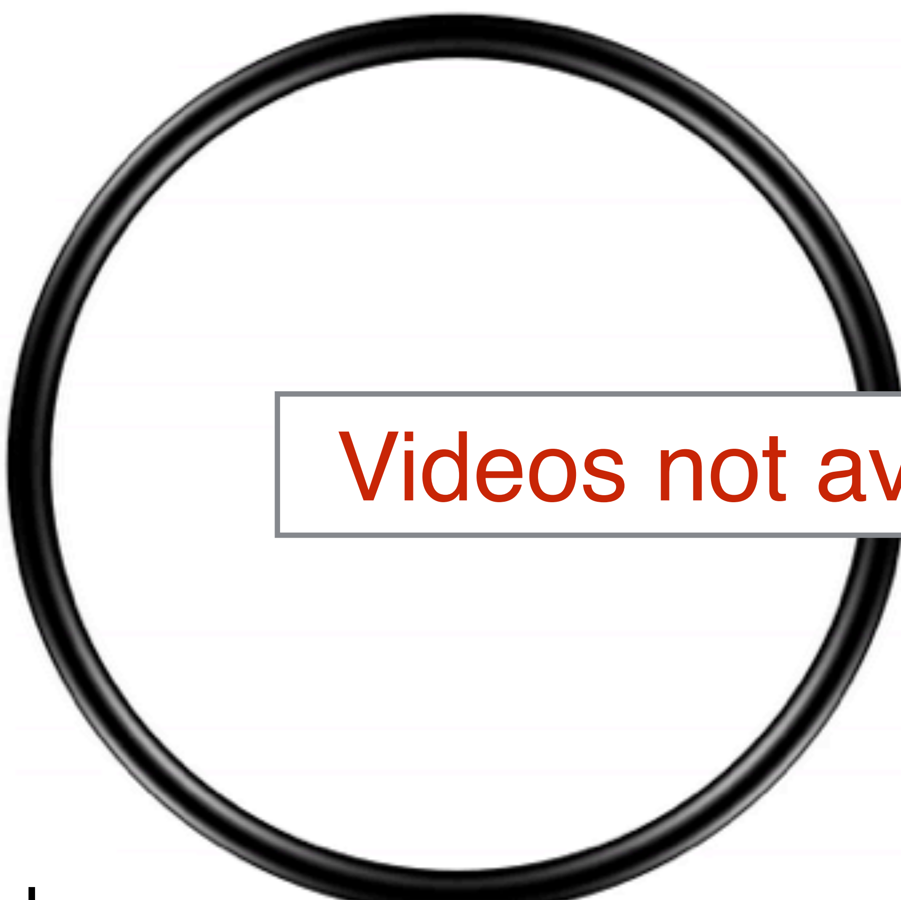

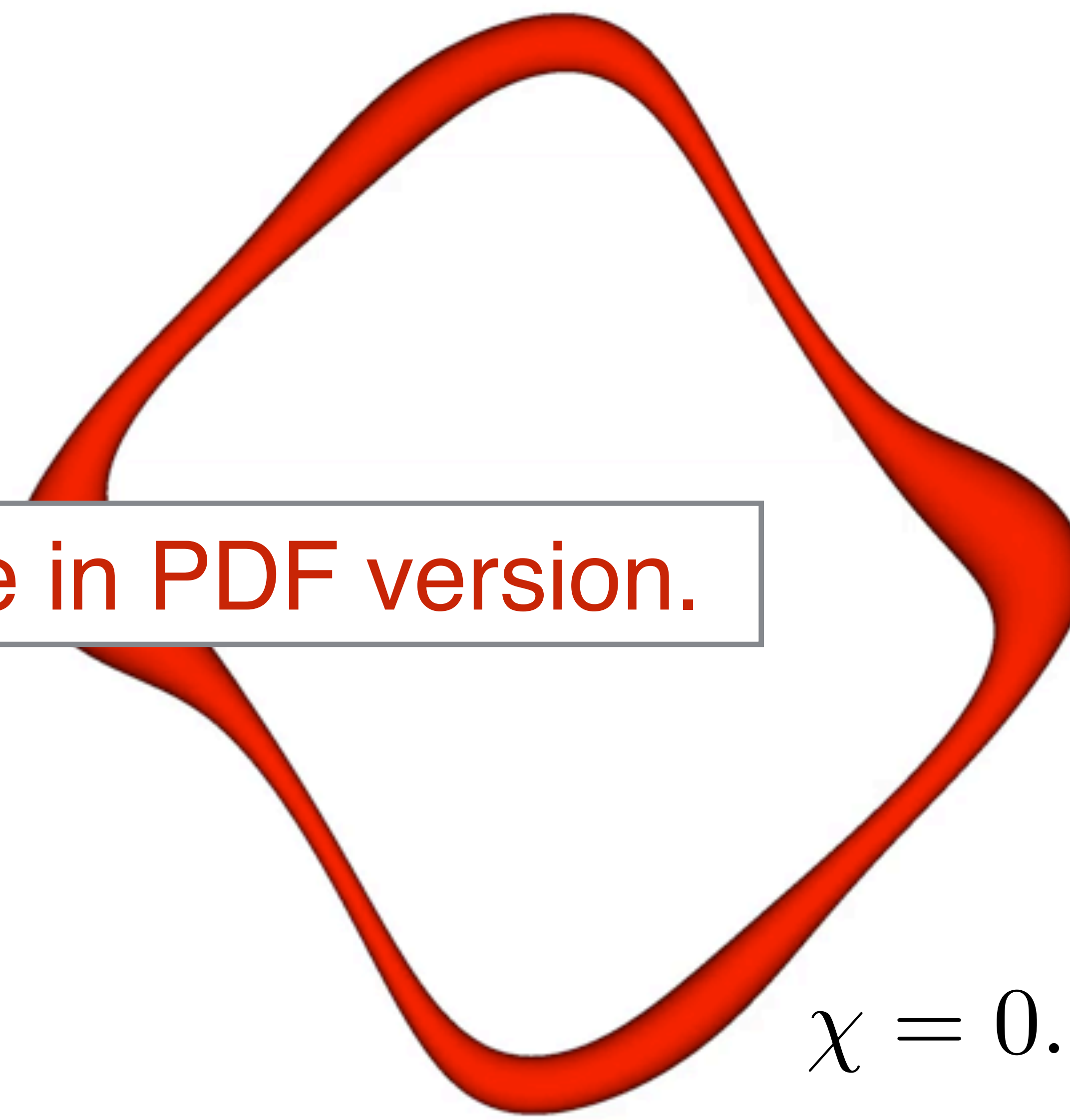

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AH

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Summary

- Fat rings collapse to Myers-Perry black holes
  - Change of horizon topology
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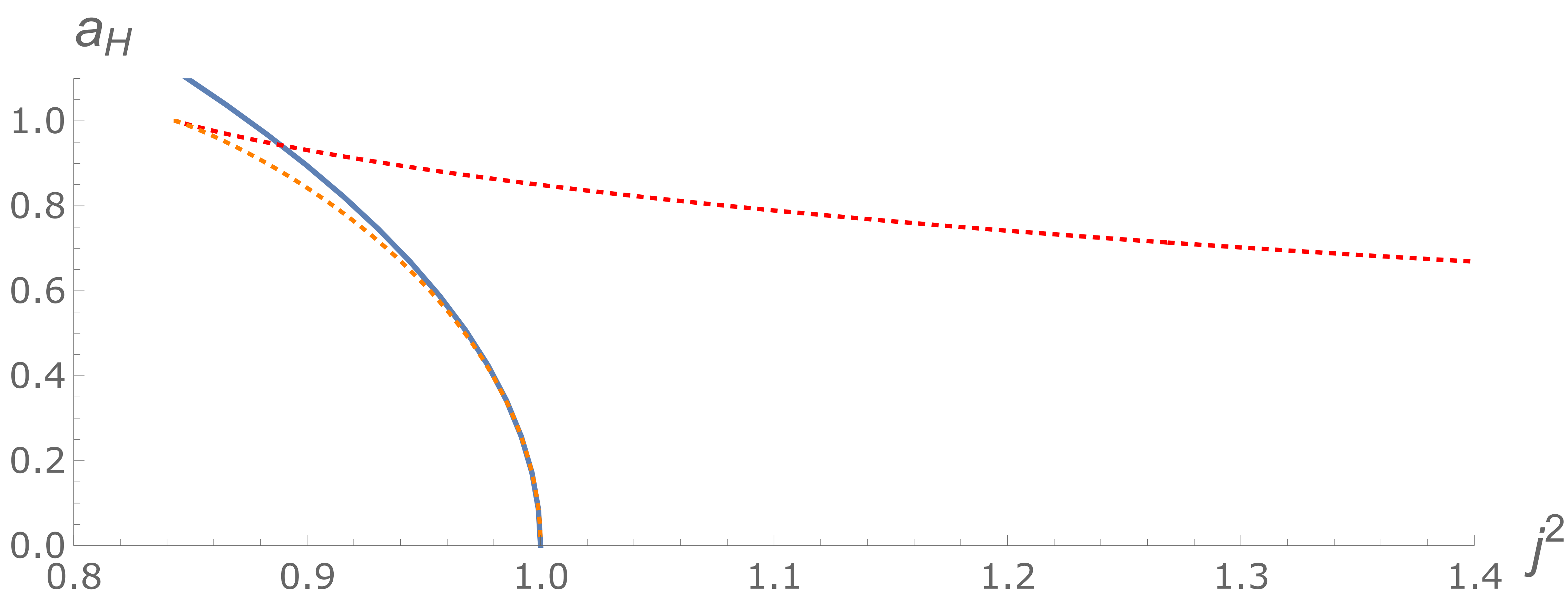

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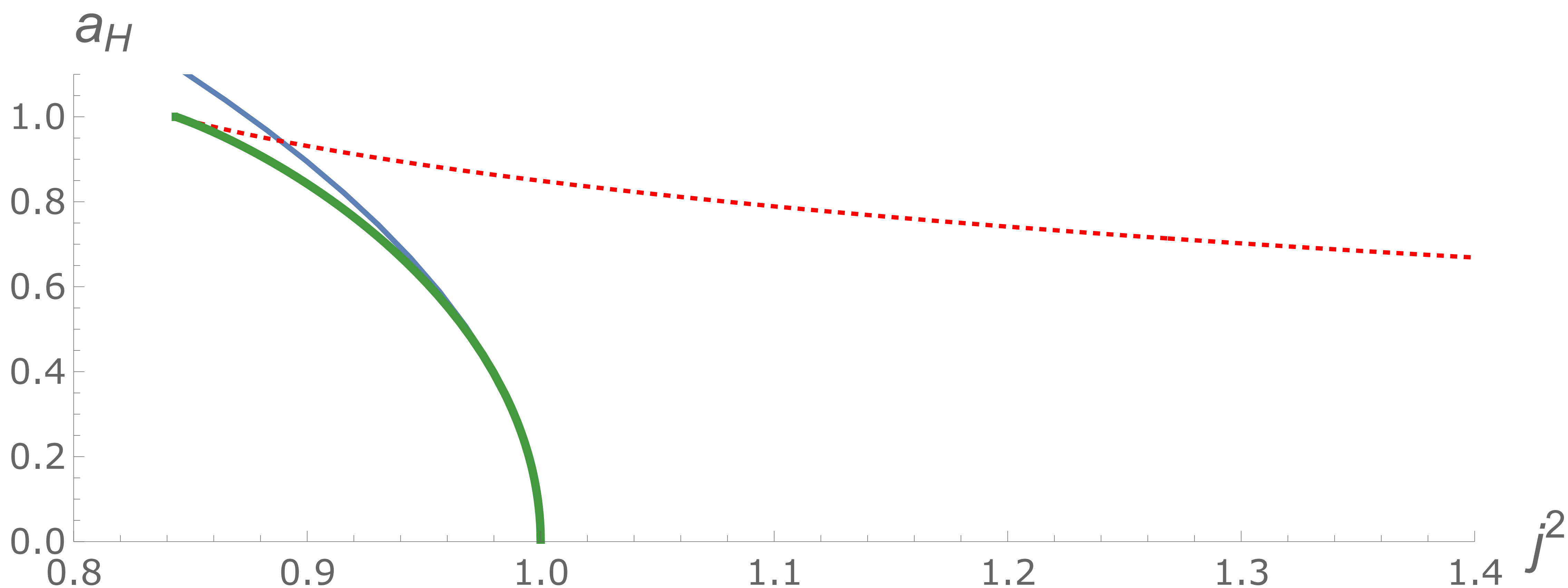

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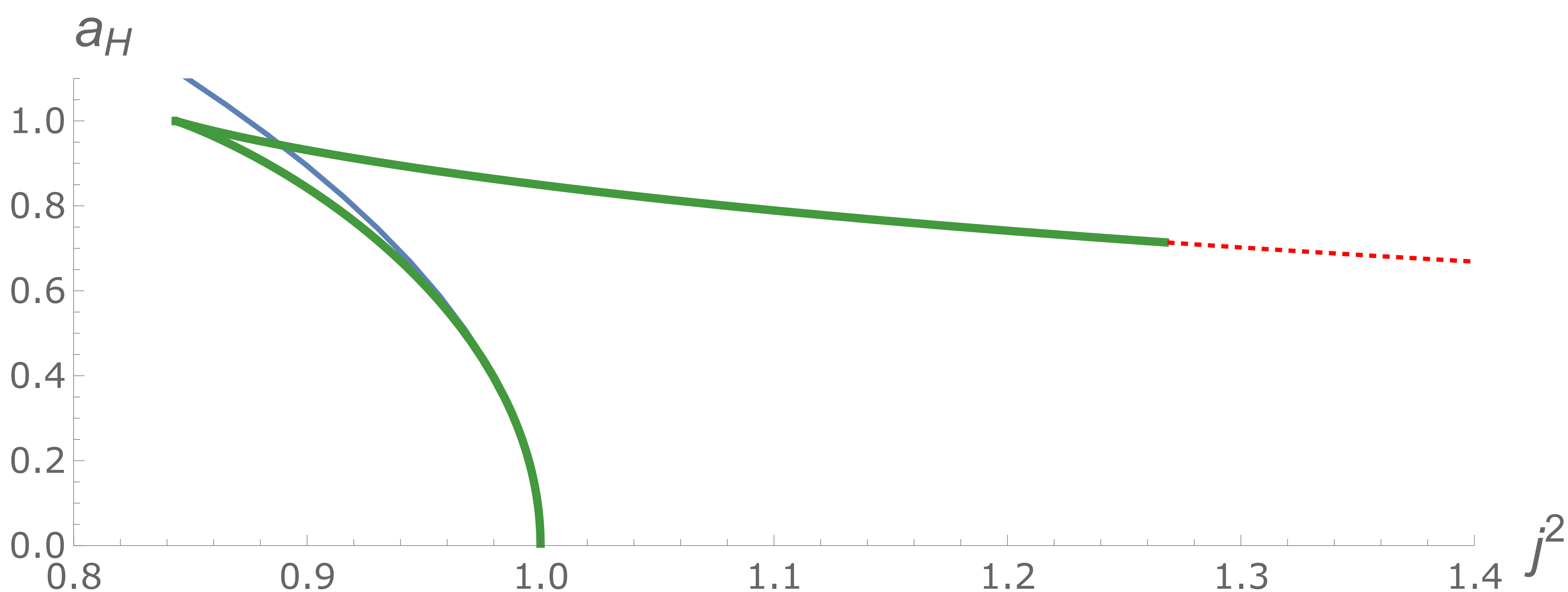

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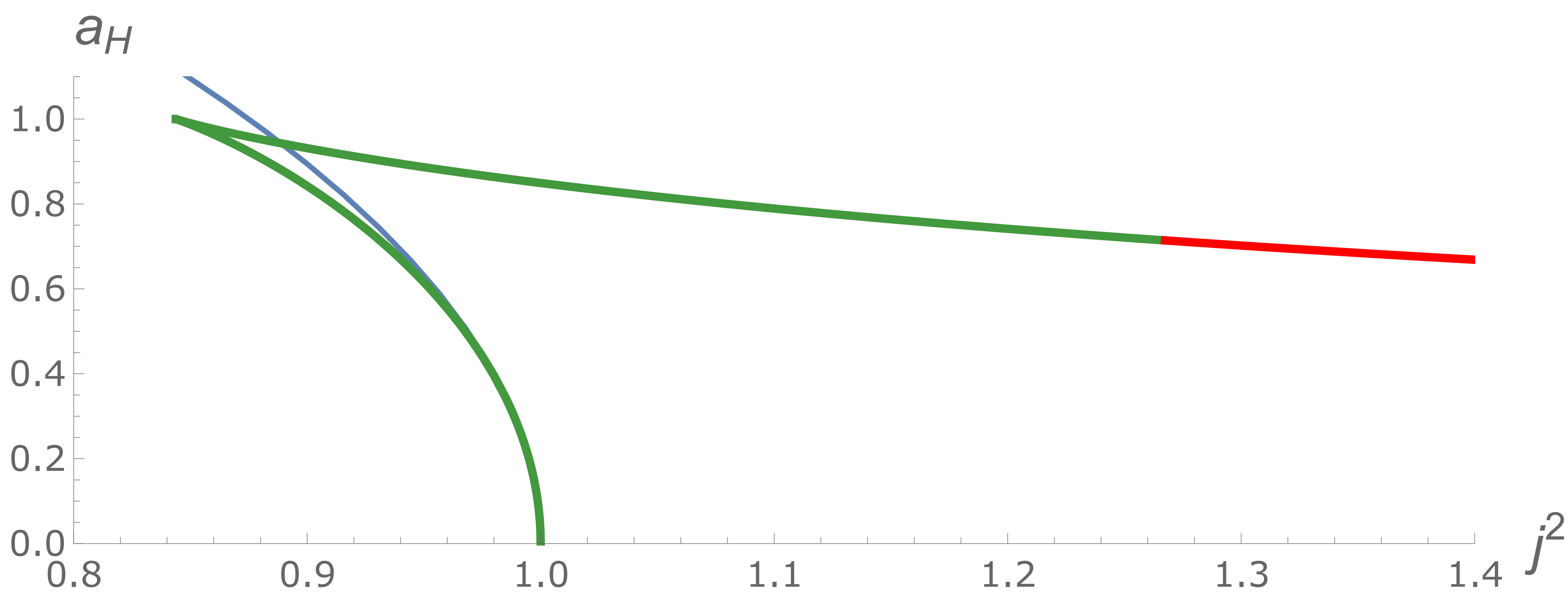

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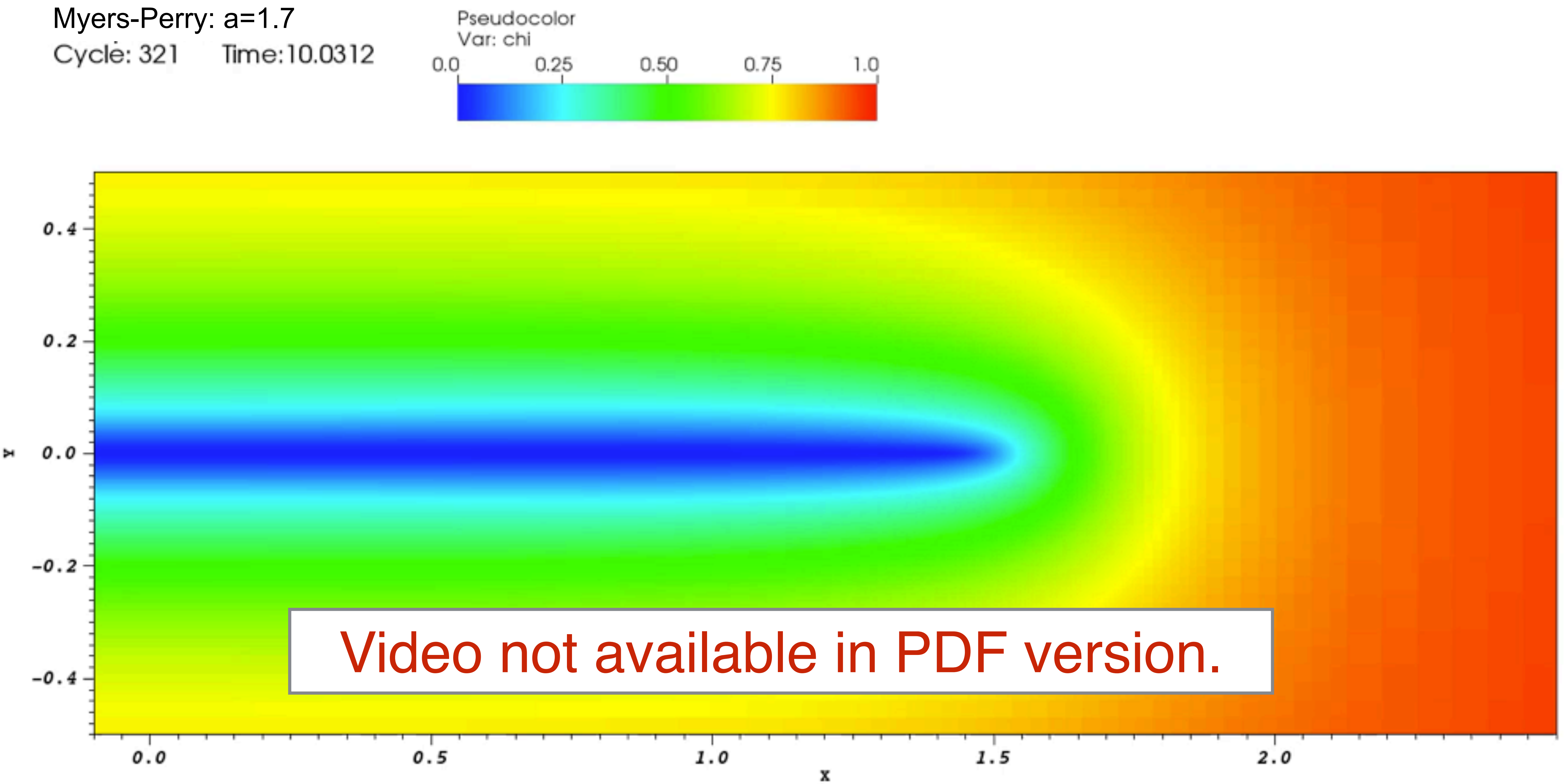
Numerical evidence that weak cosmic censorship around very thin black rings is false.

Work in progress



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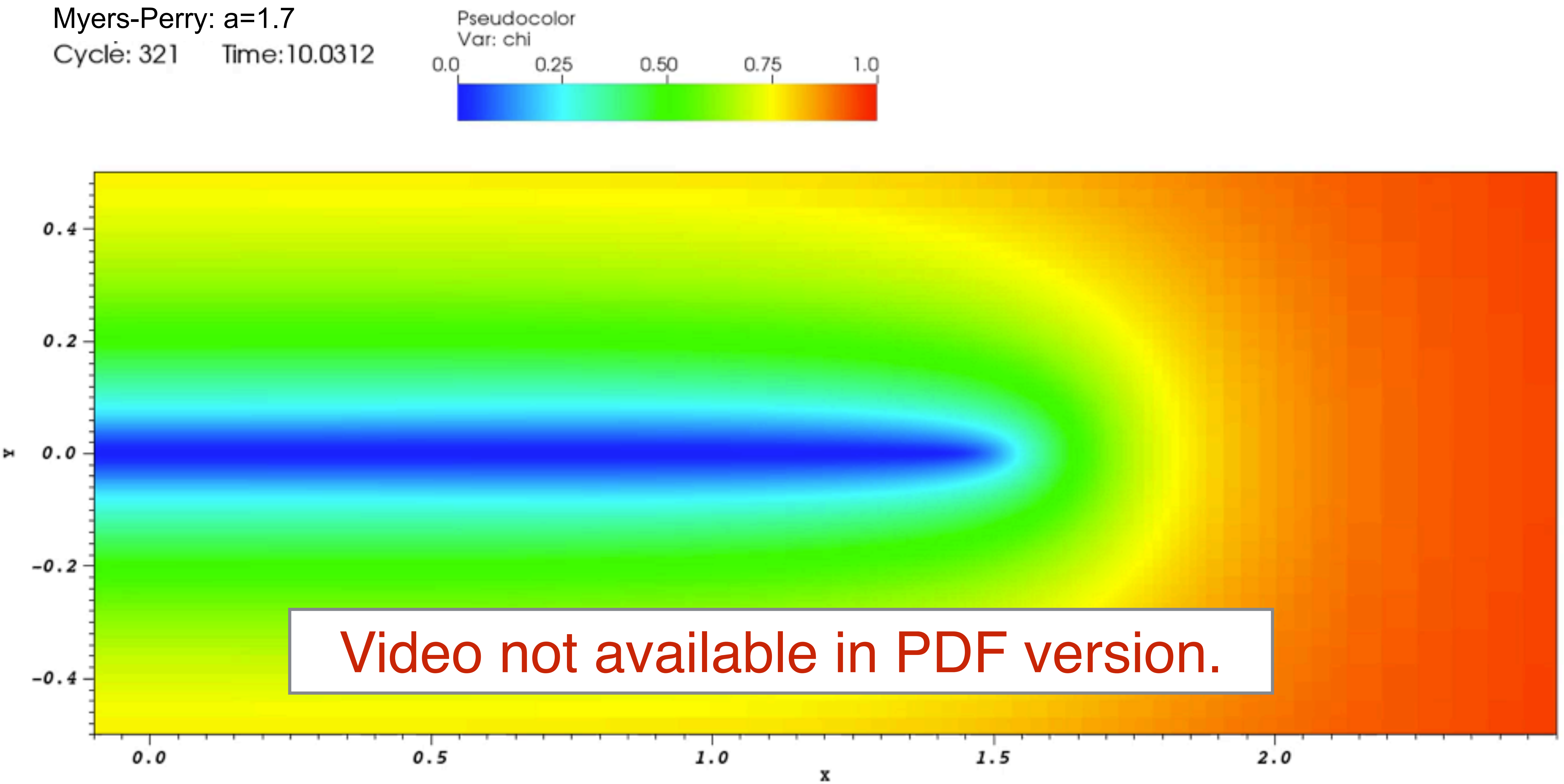

Work in progress

Thank you very much
for your attention!

Extra slides

Question: artificial viscosity

- Features that cannot be resolved
- Automatically triggered artificial viscosity

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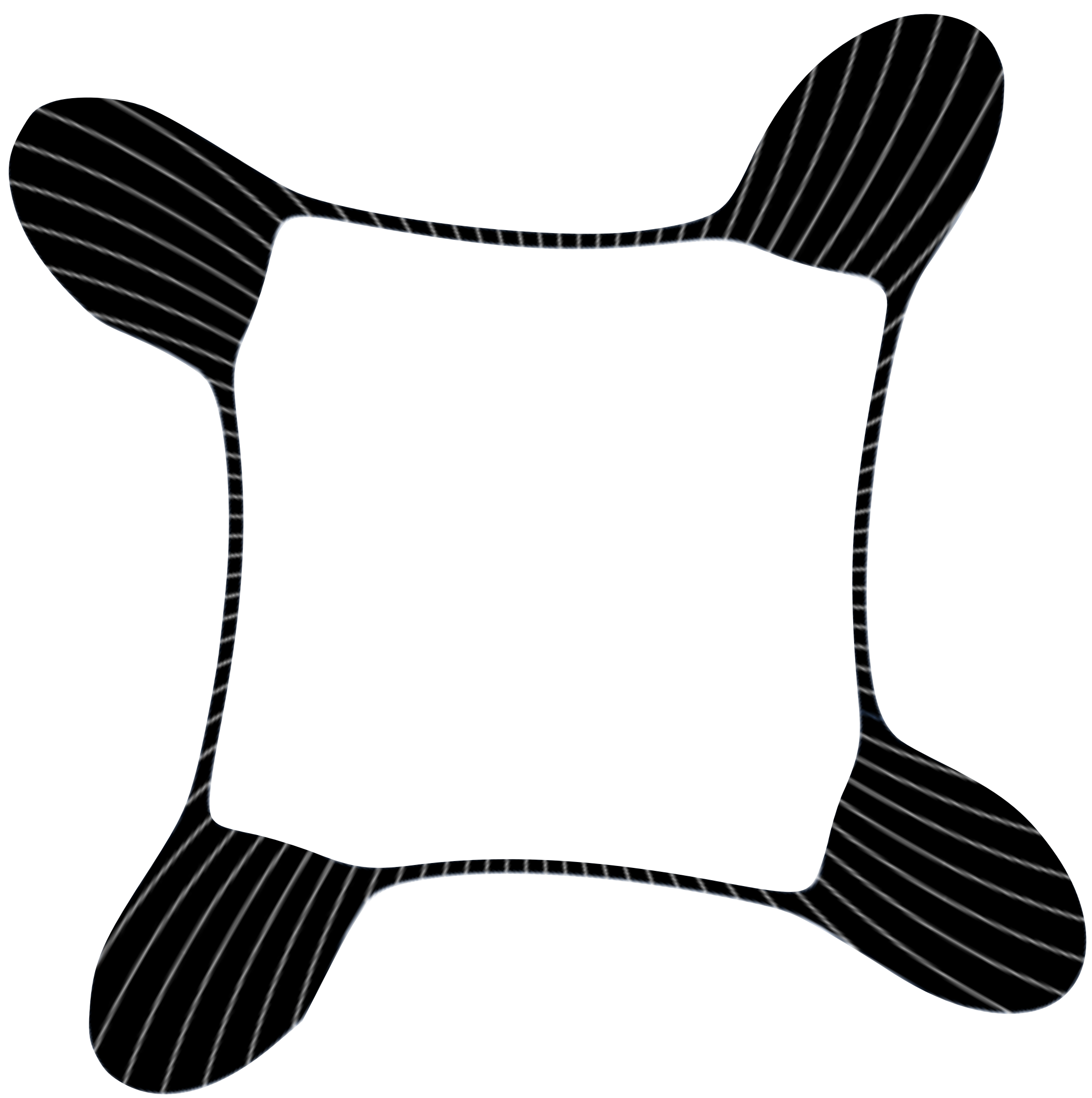

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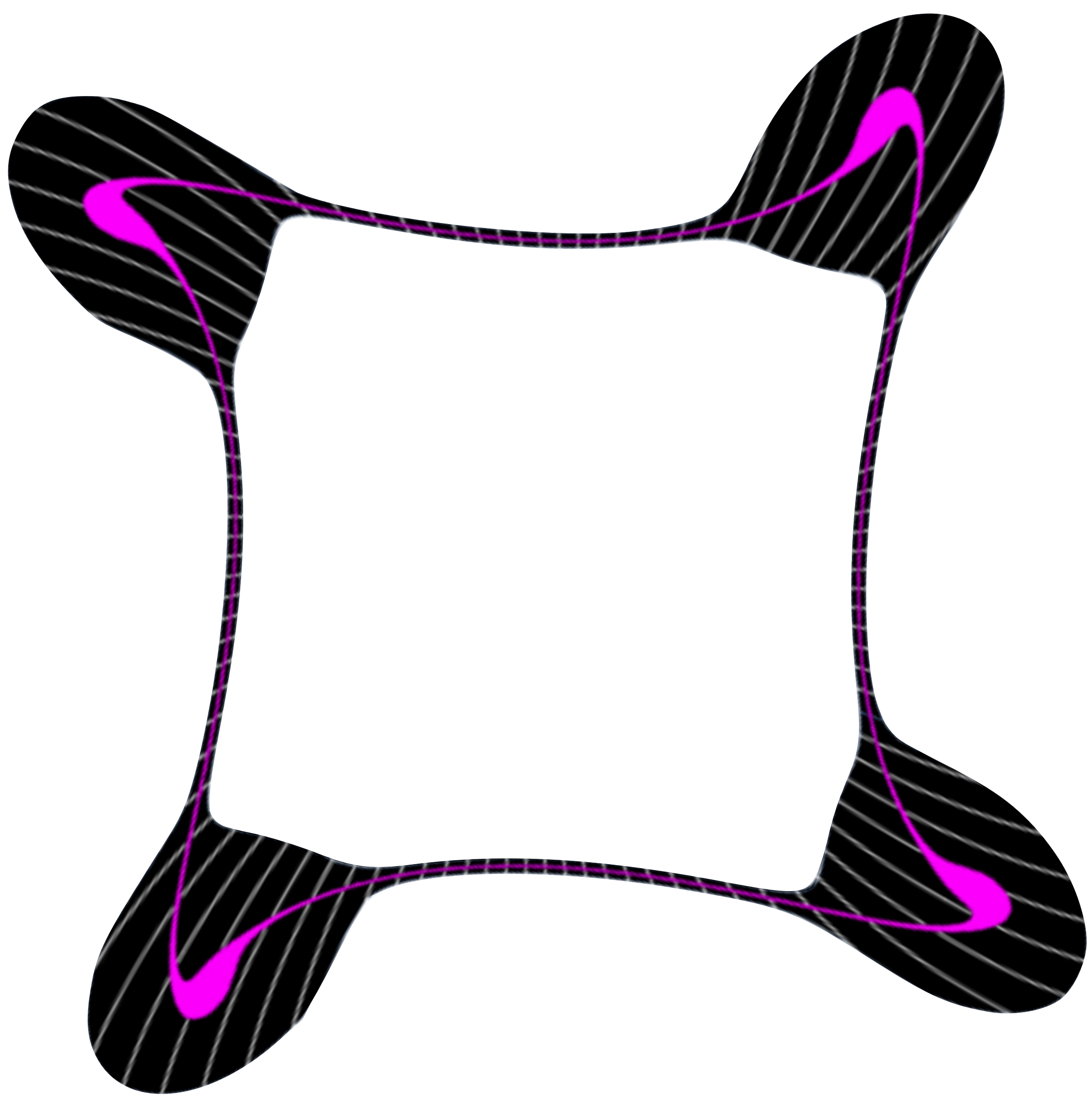

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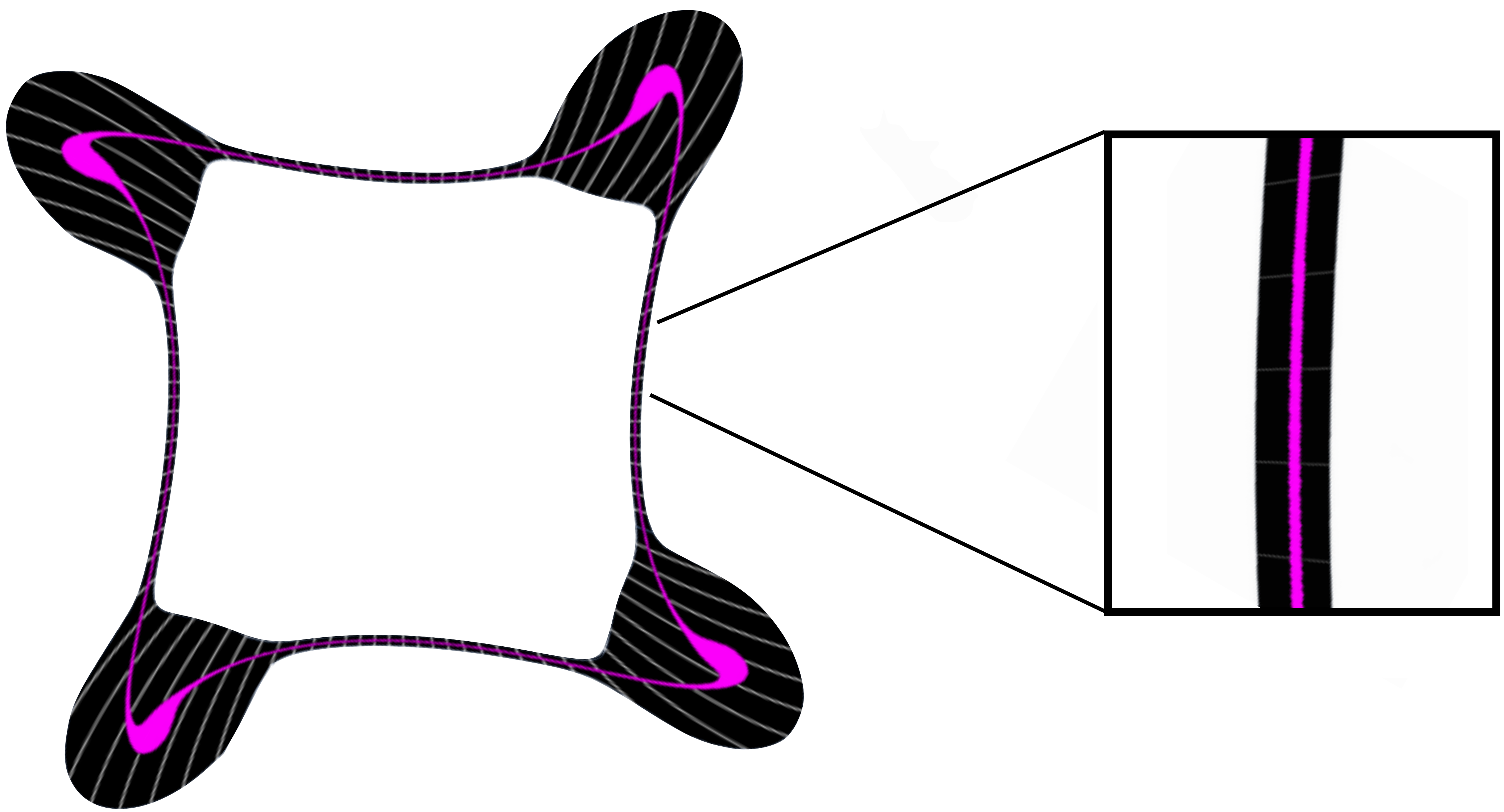
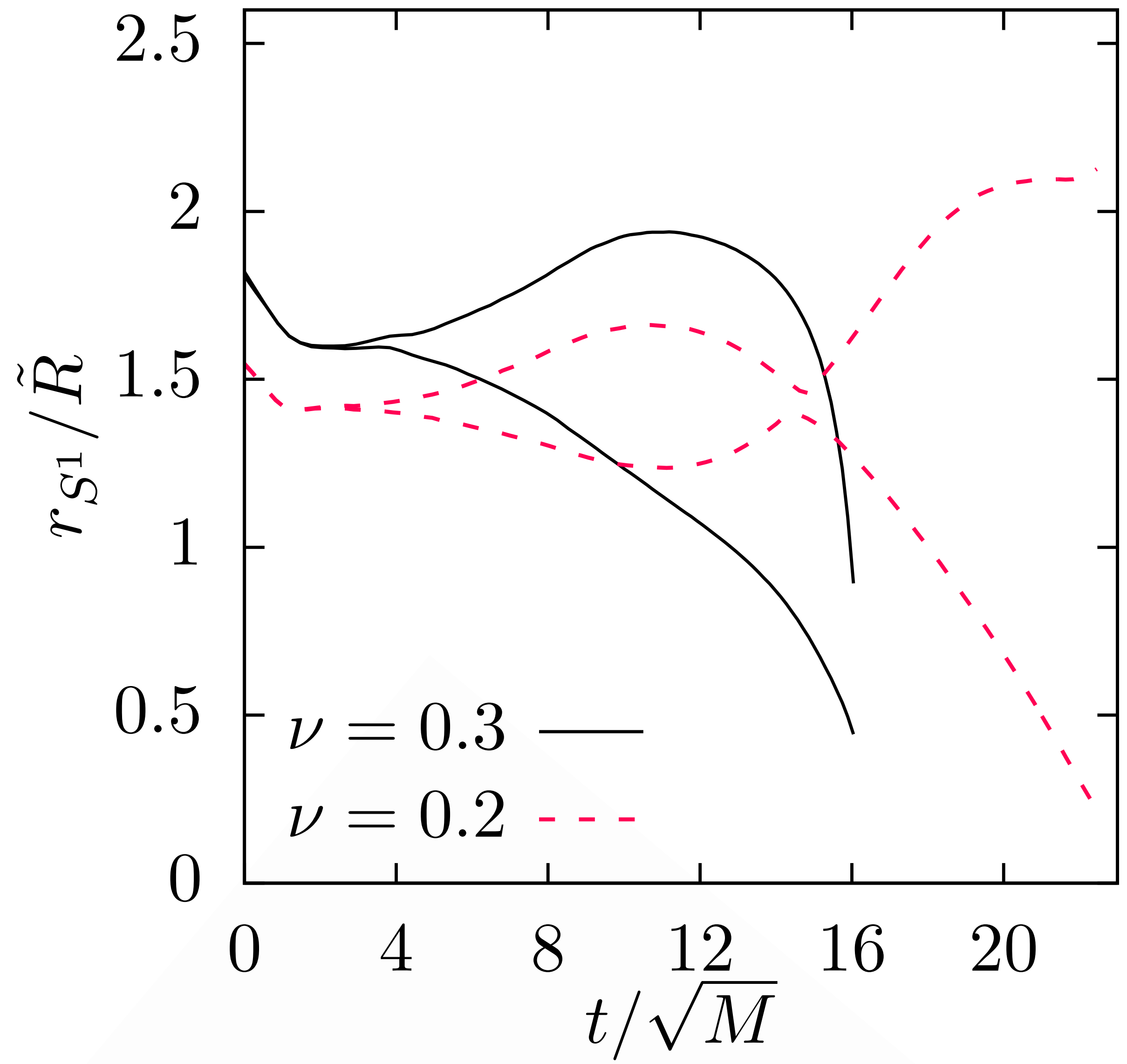
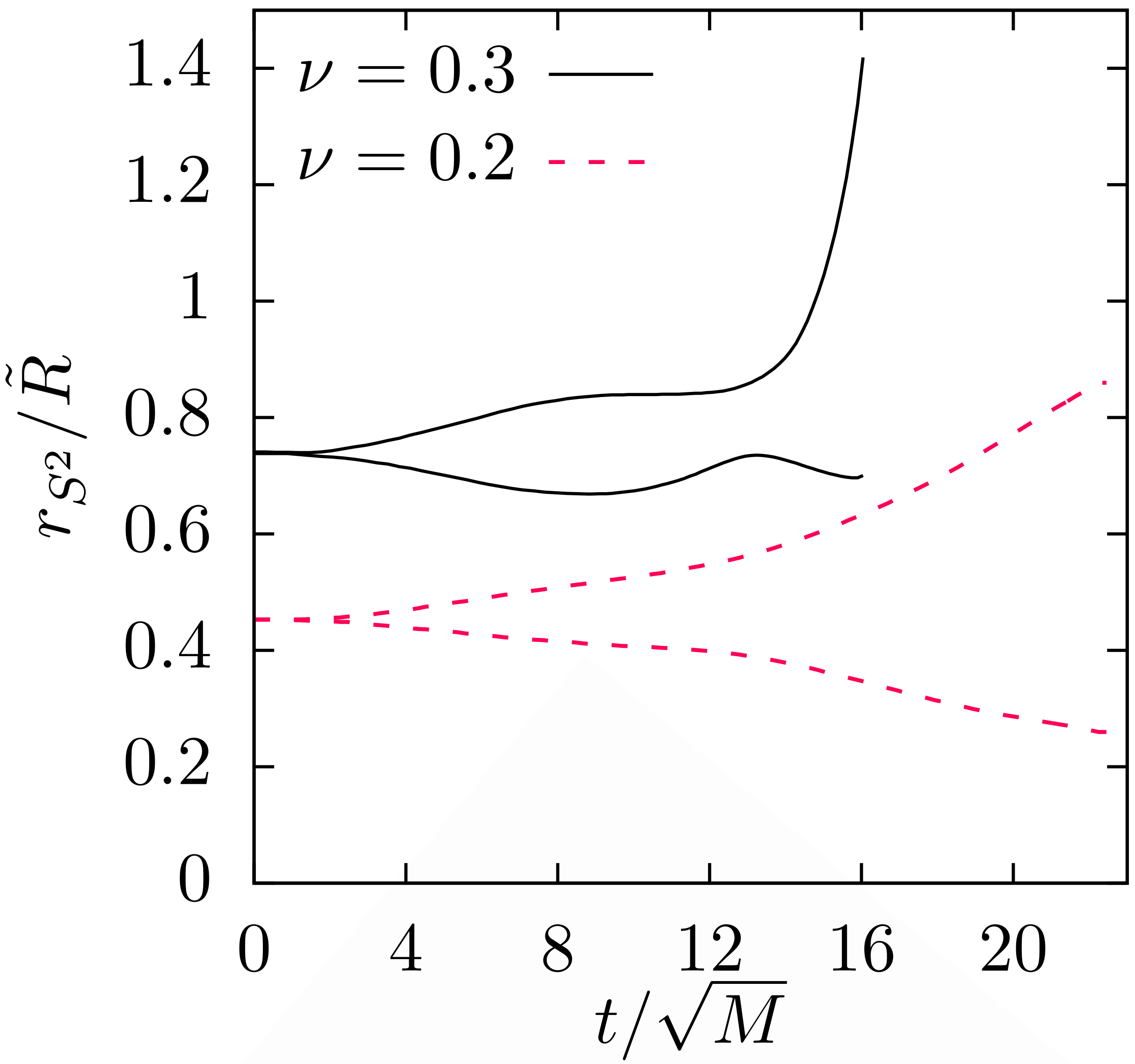
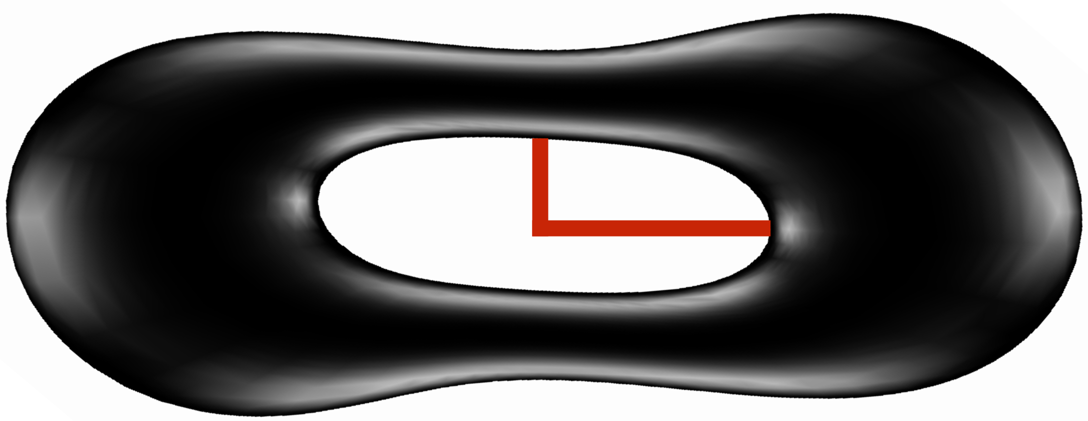
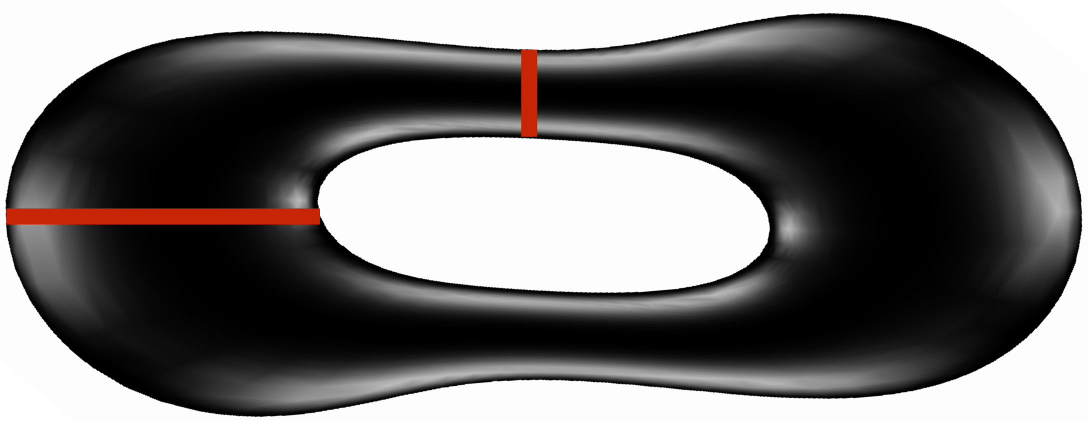


Question: physical distances



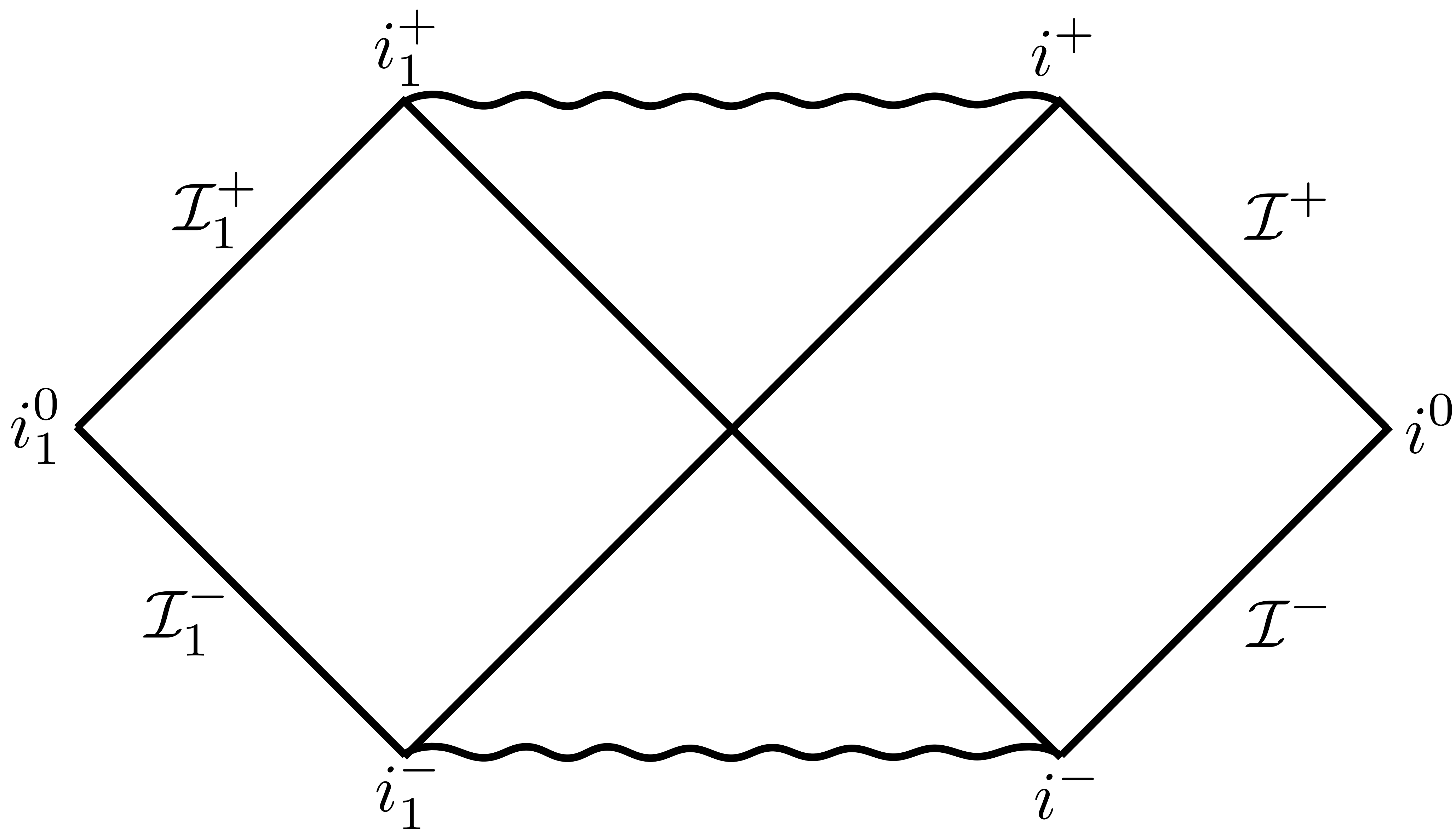






Question: puncture method

- Absorb (some) coordinate singularities in $\chi = \gamma^{-1/d}$
- Must not have physical singularities



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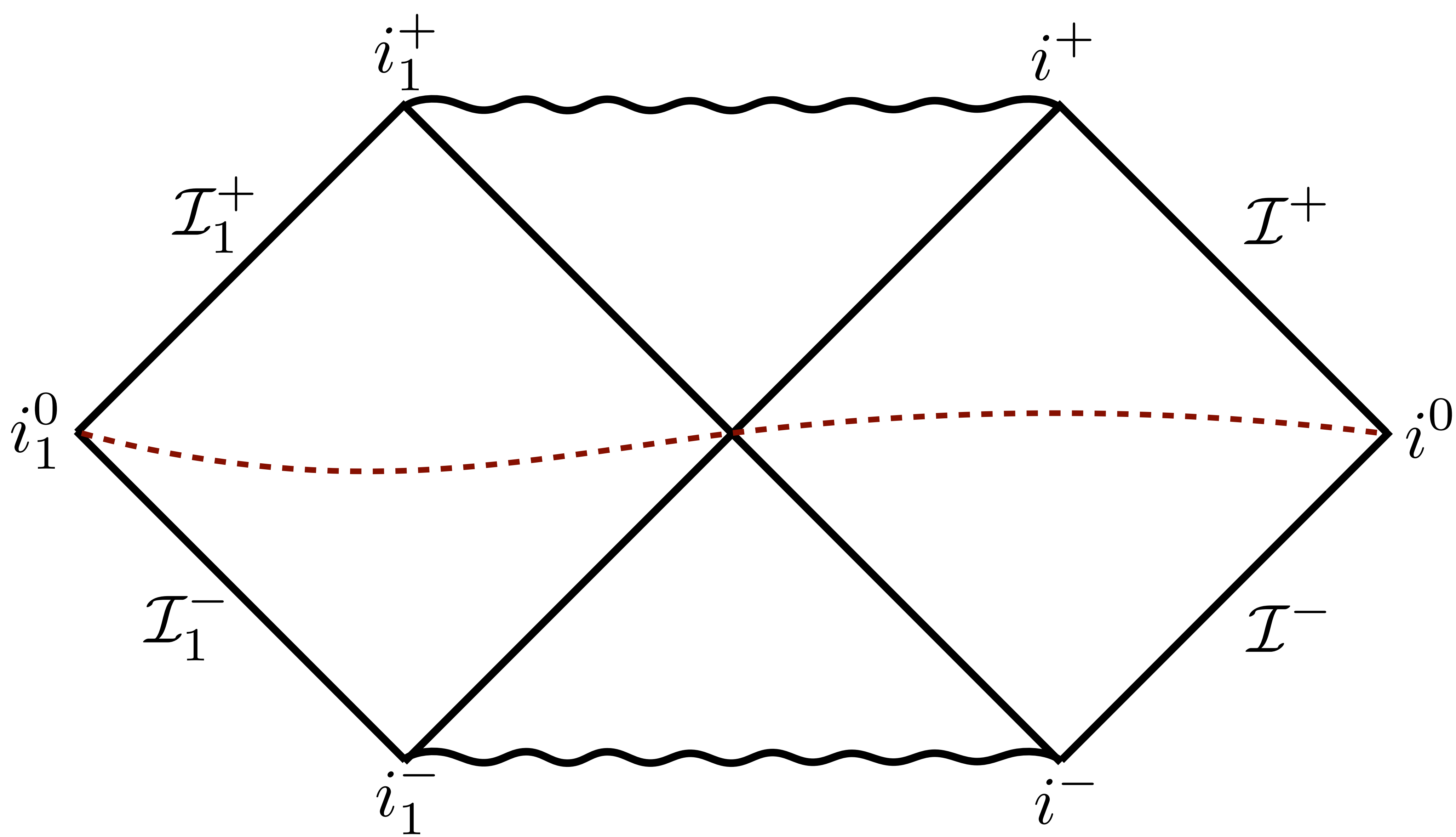

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