NumHol2016

Numerical Relativity and Holography

27 June - 1 July 2016
Santiago de Compostela, Spain

Homes' law in holographic superconductors

2016.06.28

Keun-Young Kim
GIST Korea

Gwangju Institute of Science and Technology

Motivation:

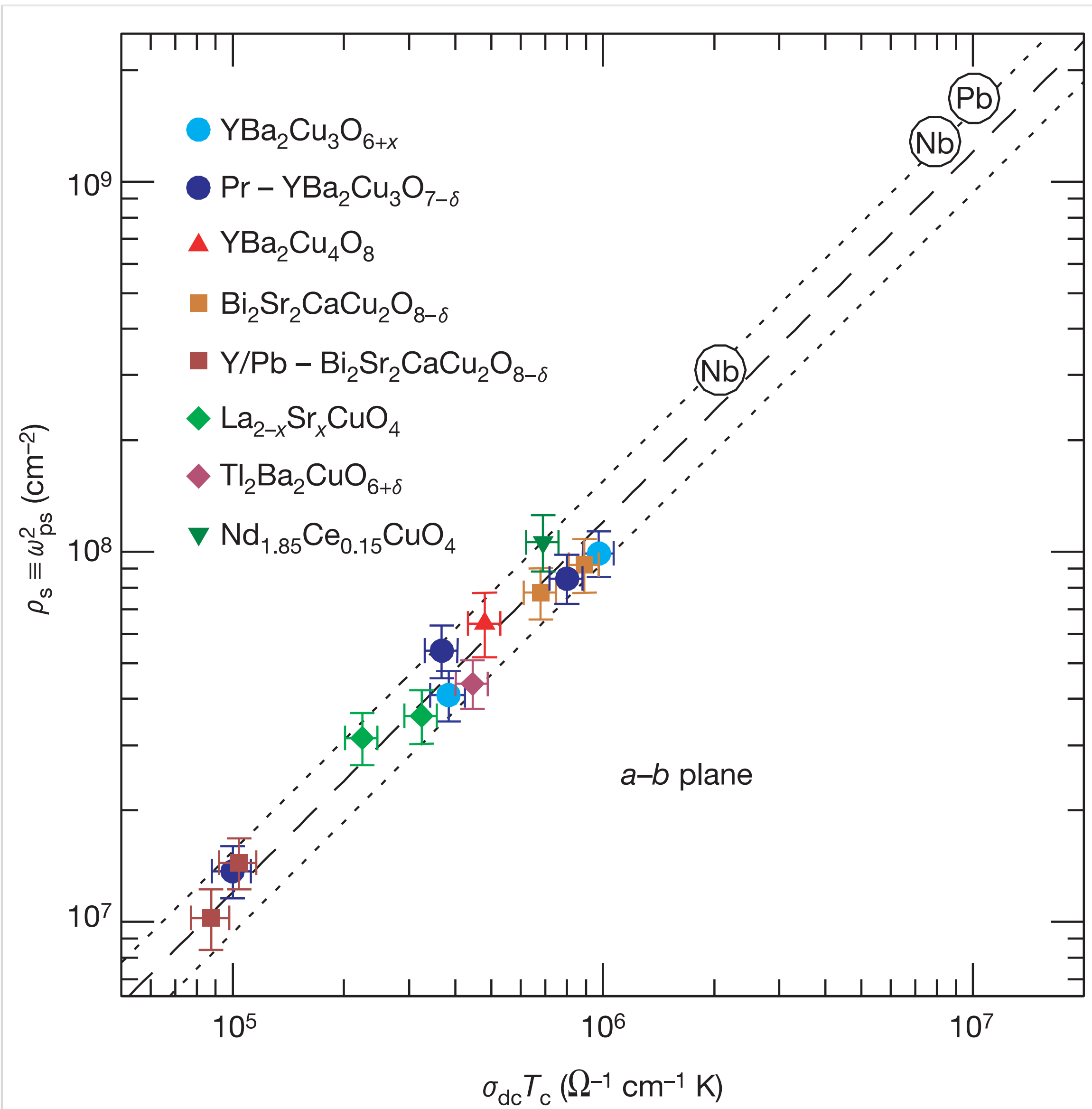
What is Homes' law?

Why is Homes' law interesting?

[Homes, et al. 2004 Nature]

A universal scaling relation in high-temperature superconductors

C. C. Homes[1], S. V. Dordevic[1], M. Strongin[1], D. A. Bonn[2], Ruixing Liang[2], W. N. Hardy[2], Seiki Komiya[3], Yoichi Ando[3], G. Yu[4], N. Kaneko[5*], X. Zhao[5], M. Greven[5,6], D. N. Basov[7] & T. Timusk[8]



Superconducting transition temperature

Electric DC conductivity

Superfluid density

- Homes' law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

C is constant regardless of doping level, nature of dopant, crystal structure and type of disorder.

C=4.4: a-b plane high-Tc superconductor, clean BCS superconductor

C=8.1: c-axis high-Tc superconductor, dirty BCS superconductor

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

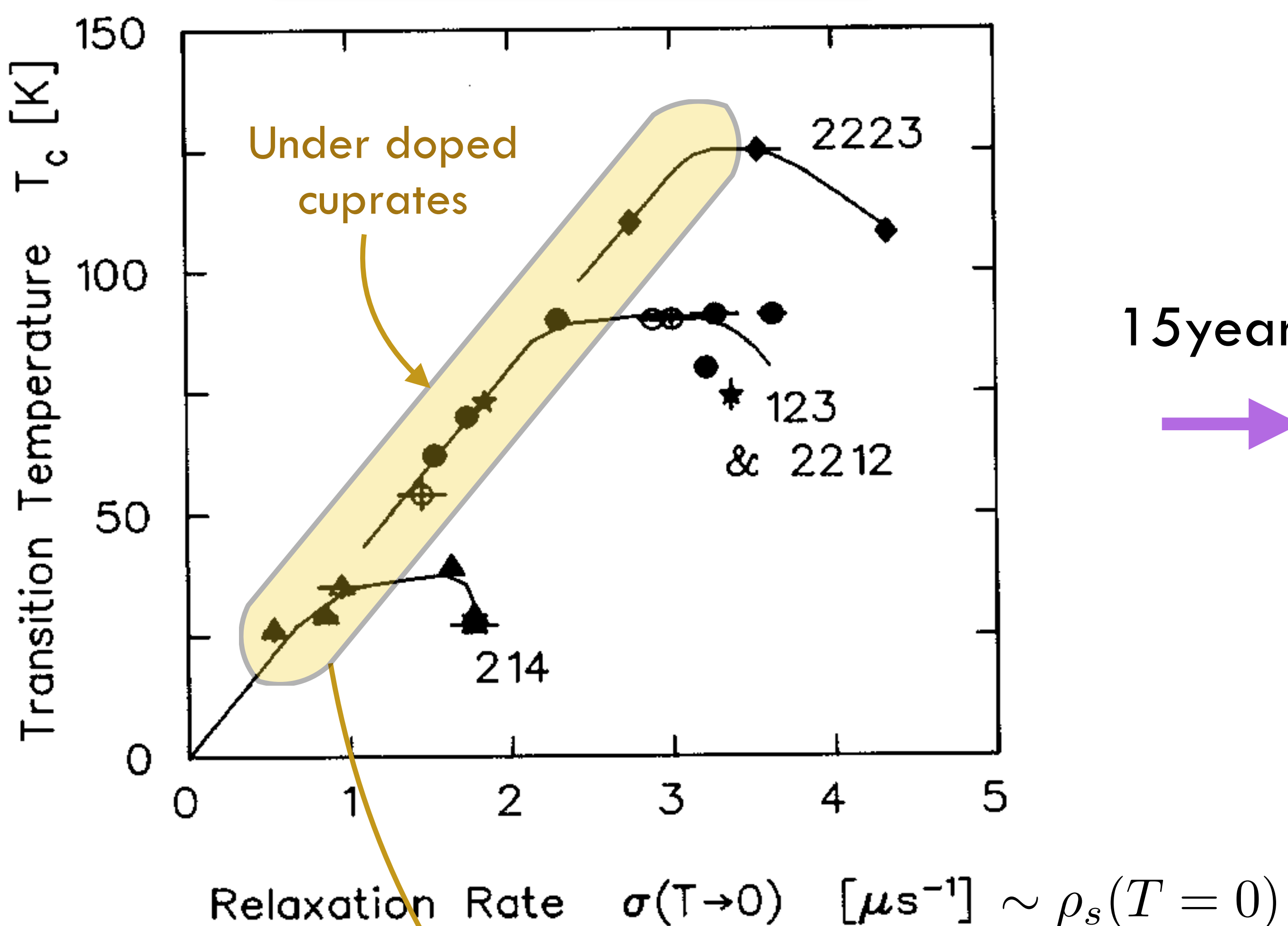
- Understanding high Tc superconductivity?
- Universal property of the hairy black holes?

History for finding universality: Uemura's law

1986: Discovery of cuprate, Bednorz, et al. (Z. Phys. B)

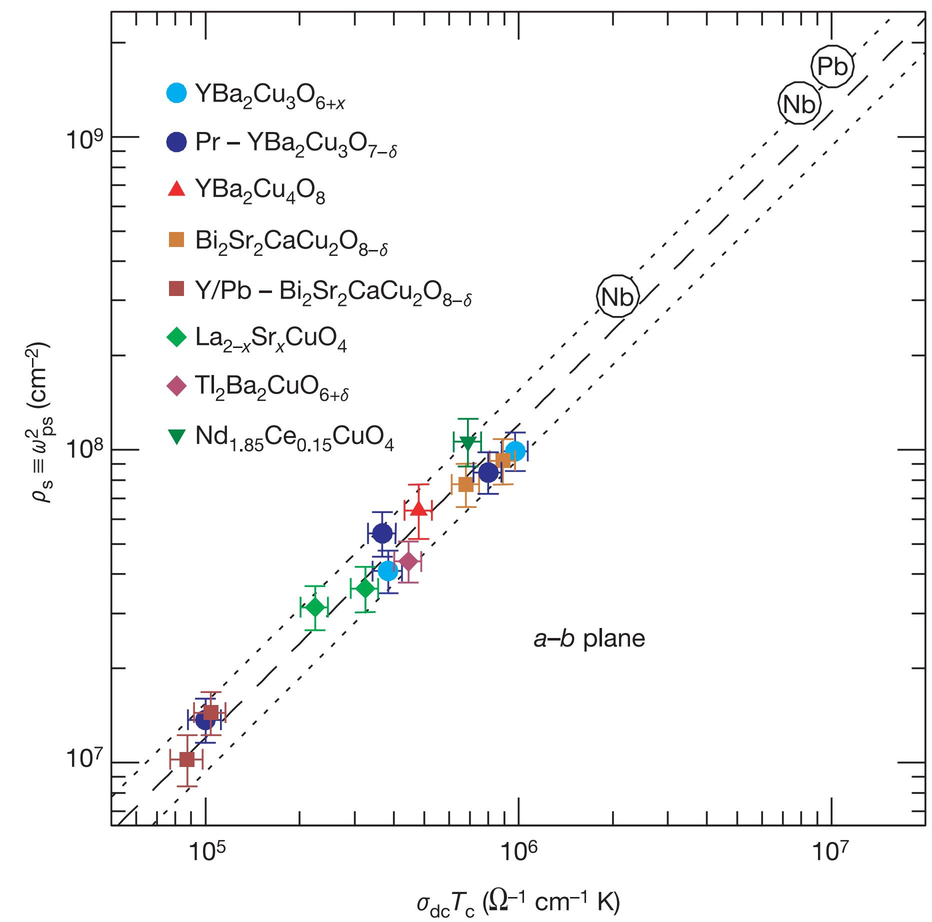
3years

1989: Uemura et al (PRL)



15years

2004: Homes et al (Nature)



Uemura's law

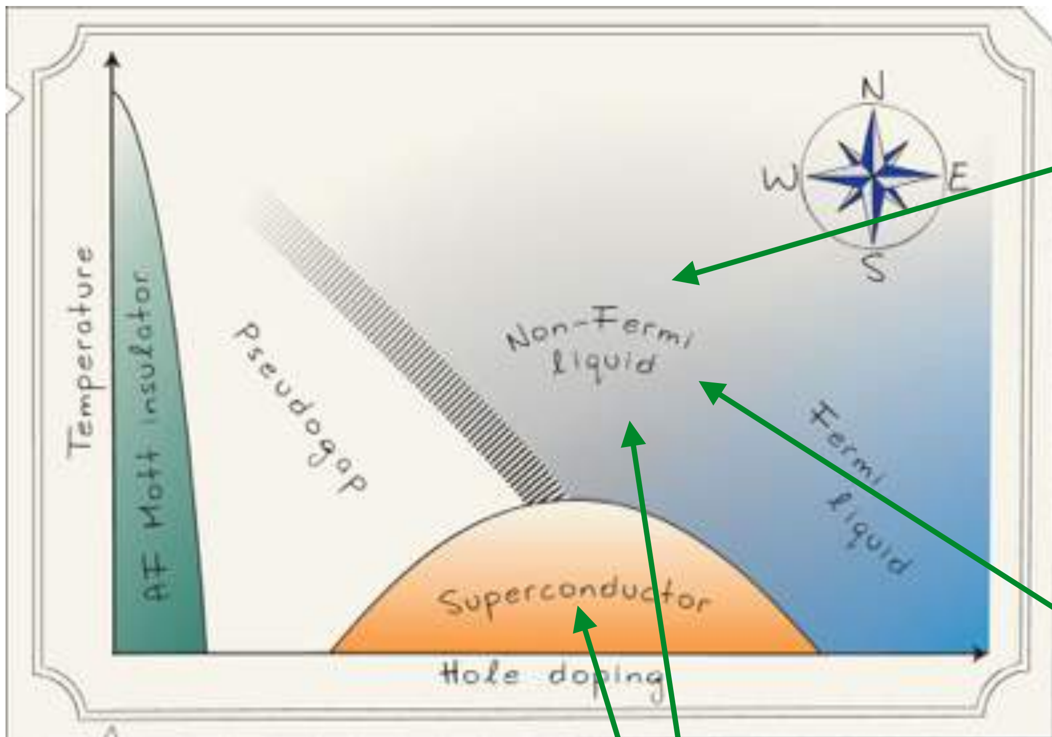
$$\rho_s(T=0) = BT_c$$

Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

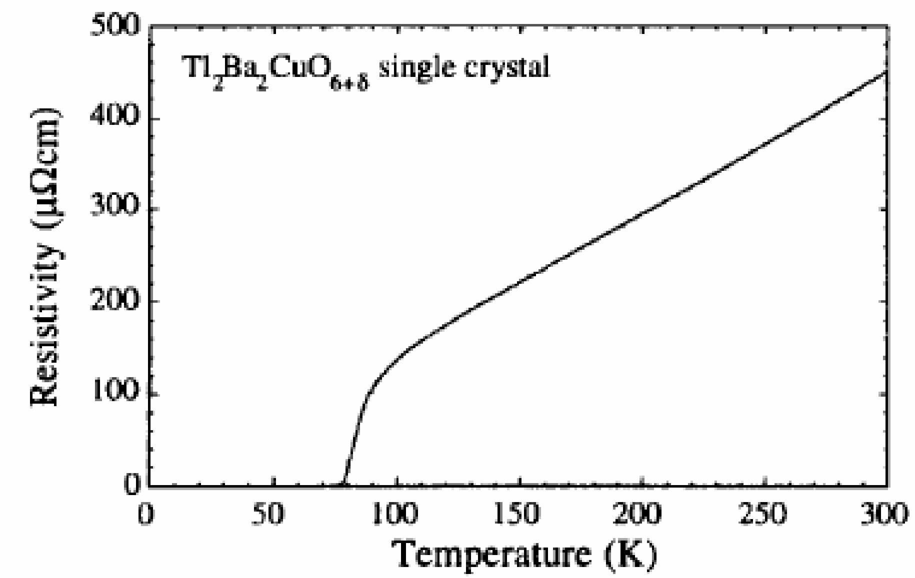
Universal properties in cuprates

Cuprate phase diagram

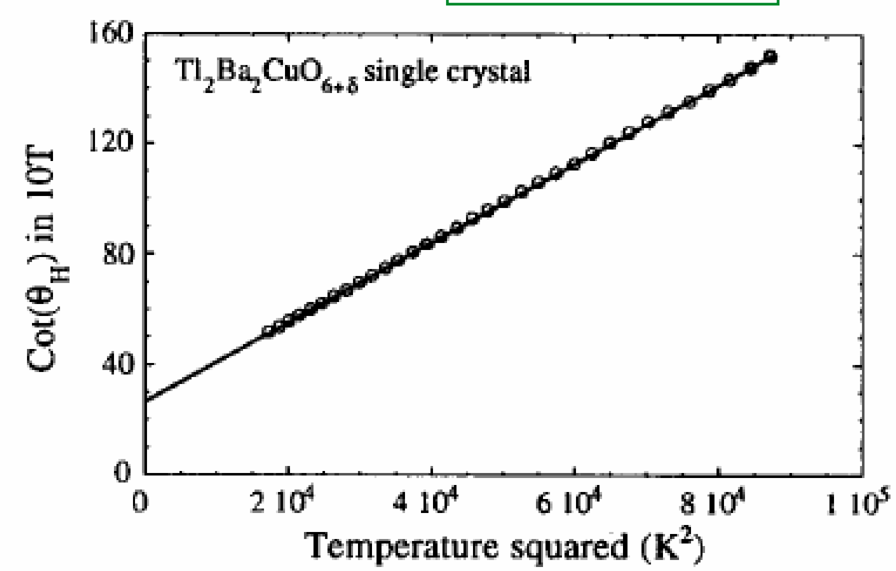


[Peter Wahl: 2012, Nature

- DC resistivity $\rho \sim T$

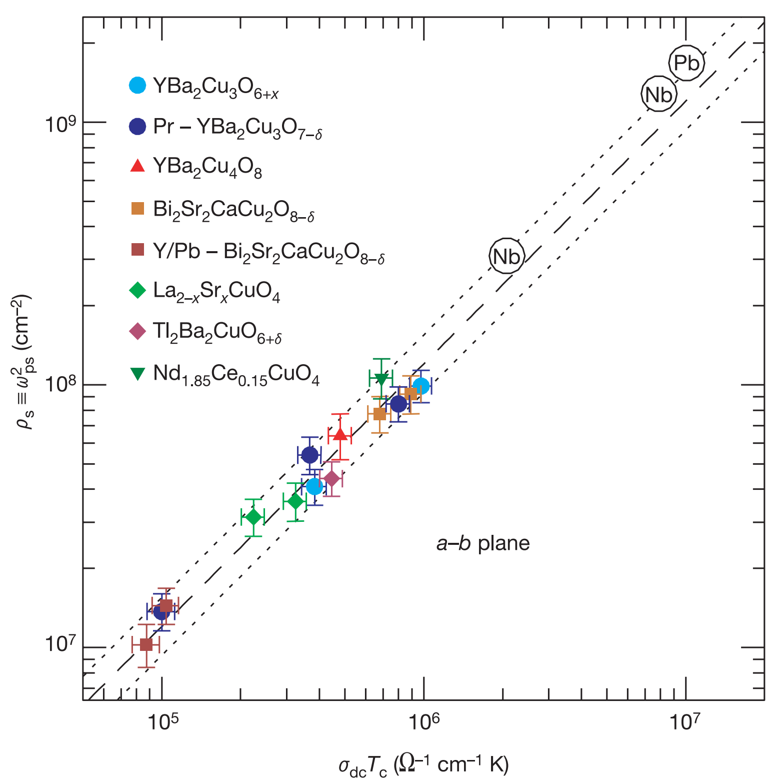


- Hall angle $\sigma_{xx}/\sigma_{xy} \sim T^2$



Mackenzie, 1997

- Homes law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$



Contents

Homes' law in Holographic context

arXiv.org > hep-th > arXiv:1002.1722

High Energy Physics – Theory

Introduction to Holographic Superconductors

Gary T. Horowitz

8.1 Open problems

We close with a list of open problems[15]. They are roughly ordered in difficulty with the easier problems listed first. (Of course, this is my subjective impression. With the right approach, an apparently difficult problem may become easy!)

1. In the probe limit below the critical temperature, there is an infinite discrete set

.
.
.

10. The high temperature cuprate superconductors satisfy a simple scaling law relating the superfluid density, the normal state (DC) conductivity and the critical temperature [36]. Can this be given a dual gravitational interpretation?

Homes' law in Holographic context

- Homes' law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

arXiv.org > hep-th > arXiv:1206.5305

High Energy Physics - Theory

Towards a Holographic Realization of Homes' Law

Johanna Erdmenger, Patrick Kerner, Steffen Muller

arXiv.org > hep-th > arXiv:1501.07615

High Energy Physics - Theory

S-Wave Superconductivity in Anisotropic Holographic Insulators

Johanna Erdmenger, Benedikt Herwerth, Steffen Klug, Rene Meyer, Koenraad Schalm

arXiv.org > hep-th > arXiv:1604.06205

High Energy Physics - Theory

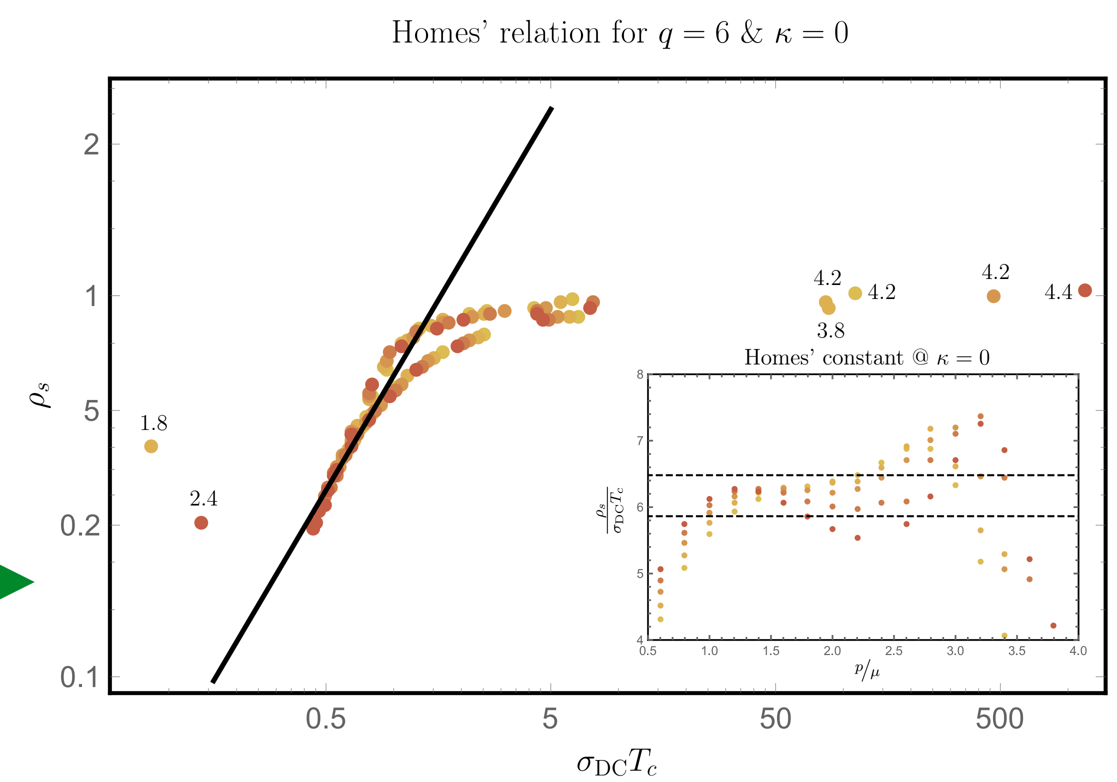
Ward Identity and Homes' Law in a Holographic Superconductor with Momentum Relaxation

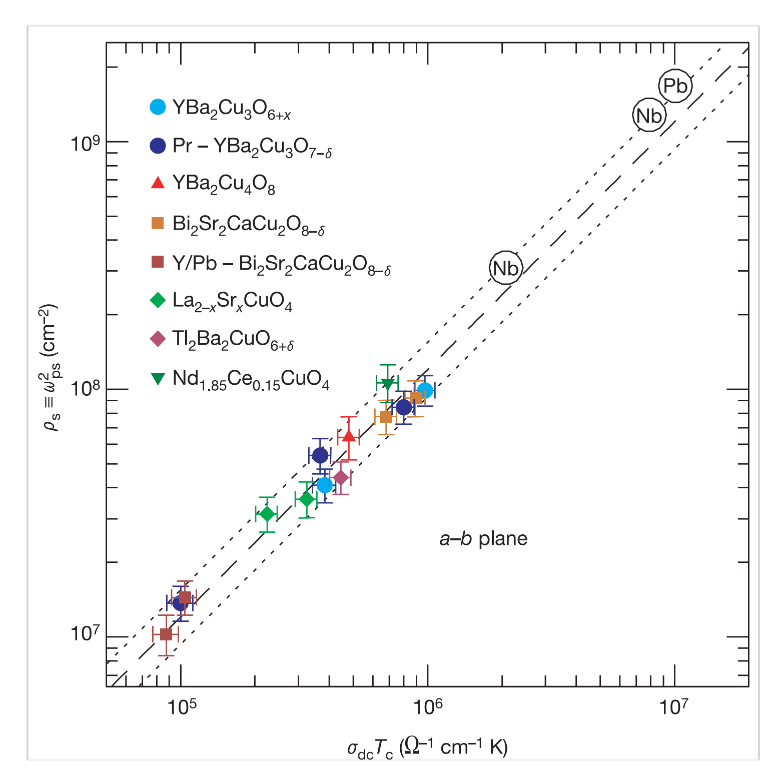
Keun-Young Kim, Kyung Kiu Kim, Miok Park

arXiv:1607.XXXXX

Homes' law in Holographic Superconductor with Q-lattices

Keun-Young Kim and Chao Niu



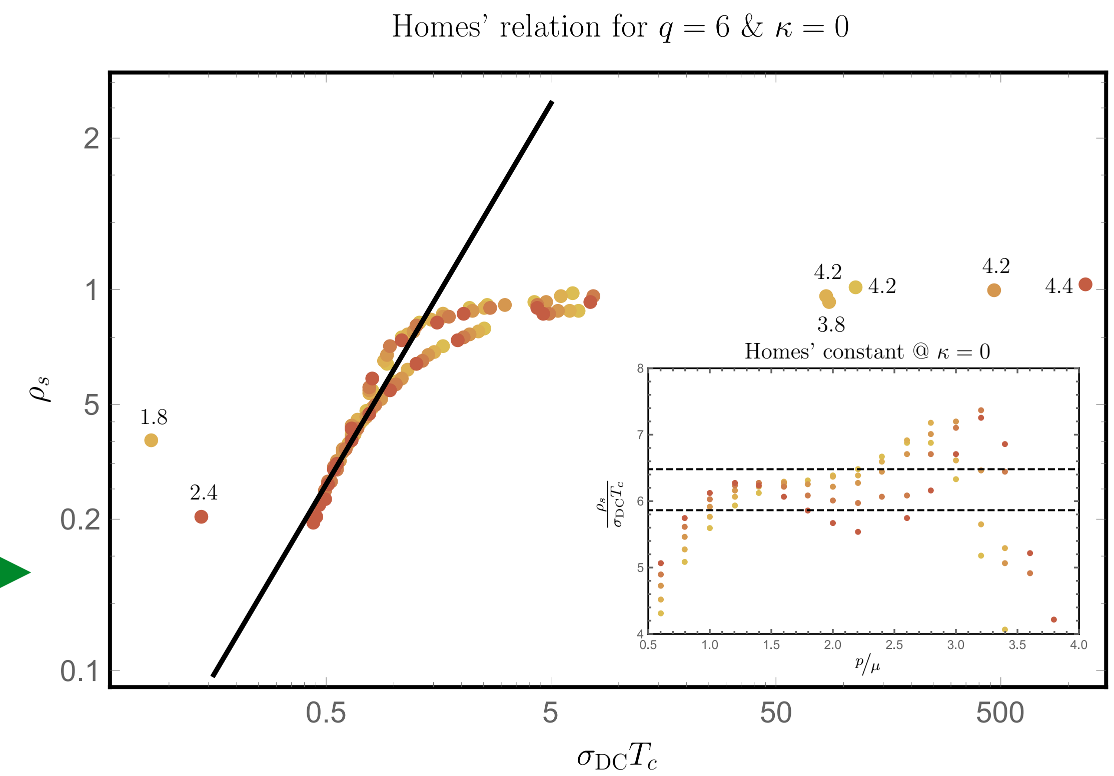


Homes' law in Holographic context

- Homes' law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

arXiv.org > hep-th > arXiv:1206.5305
High Energy Physics – Theory
Towards a Holographic Realization of Homes' Law
Johanna Erdmenger, Patrick Kerner, Steffen Muller

arXiv.org > hep-th > arXiv:1501.07615
High Energy Physics – Theory
S-Wave Superconductivity in Anisotropic Holographic Insulators
Johanna Erdmenger, Benedikt Herwerth, Steffen Klug, Rene Meyer, Koenraad Schalm



arXiv.org > hep-th > arXiv:1604.06205
High Energy Physics – Theory
Ward Identity and Homes' Law in a Holographic Superconductor with Momentum Relaxation
Keun-Young Kim, Kyung Kiu Kim, Miok Park

arXiv:1607.XXXXX
Homes' law in Holographic Superconductor with Q-lattices
Keun-Young Kim and Chao Niu

This talk
- Physical understanding?
- How much model dependent?

arXiv.org > hep-th > arXiv:1409.8346
High Energy Physics – Theory
Coherent/incoherent metal transition in a holographic model
Keun-Young Kim, Kyung Kiu Kim, Yunseok Seo, Sang-Jin Sin

arXiv.org > hep-th > arXiv:1501.00446
High Energy Physics – Theory
A Simple Holographic Superconductor with Momentum Relaxation
Keun-Young Kim, Kyung Kiu Kim, Miok Park

Goals and method

Goals

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

Holographer's tool box

1. Need a holographic superconductor ~ hairy black hole (0803.3295: Hartnoll, Herzog, Horowitz)
2. Conductivity?

Linear response theory

$$\sigma(\omega) = \frac{G^R_{JJ}(\omega)}{i\omega}$$

Holography

$$G^R$$

Son and Starinets, hep-th/0205051
Herzog and Son, hep-th/0212072
Skenderis and van Rees, 0805.0150

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

The model and method are well established.
Why is the progress slow?

Momentum relaxation matters

Original holographic superconductor: HHH

[Hartnoll, Herzog, Horowitz: 0803.3295]

The first holographic superconductor

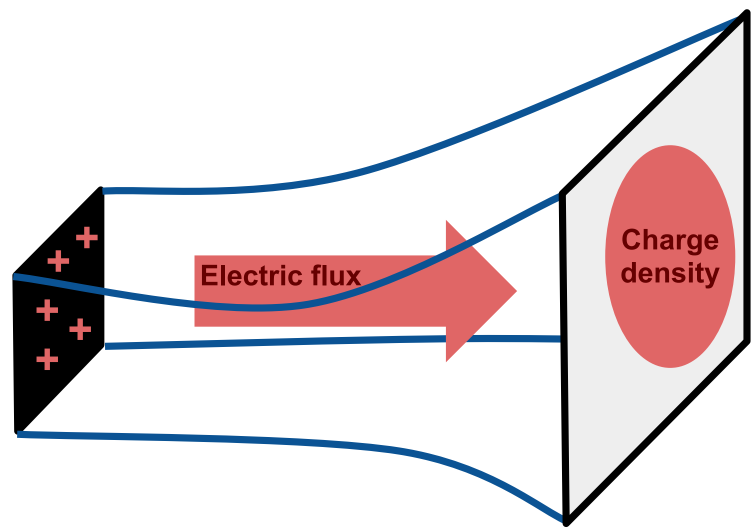
$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

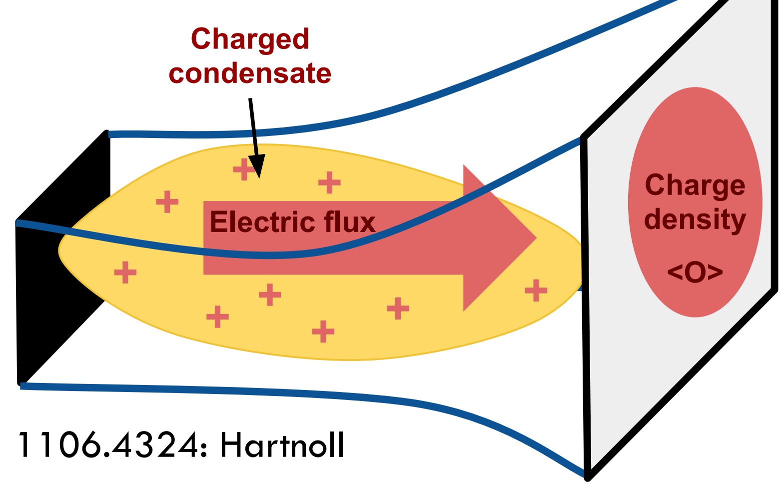
$\Phi = 0$

AdS-RN-black brane



$\Phi \neq 0$

Holographic superconductor



1106.4324: Hartnoll

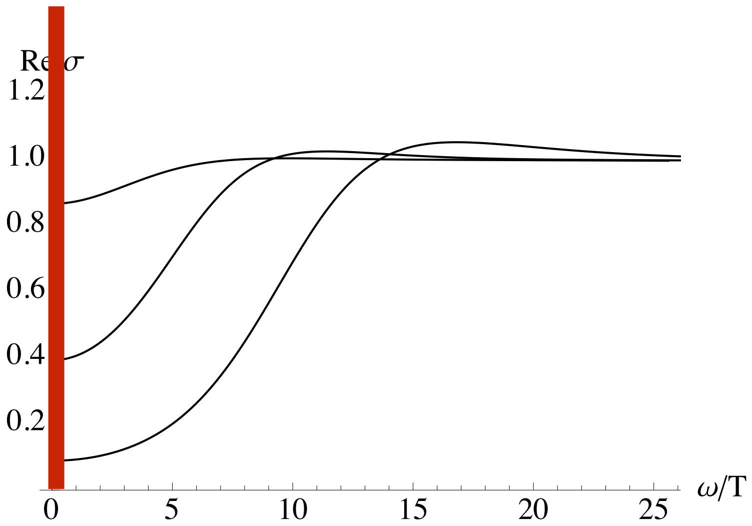
Optical conductivity

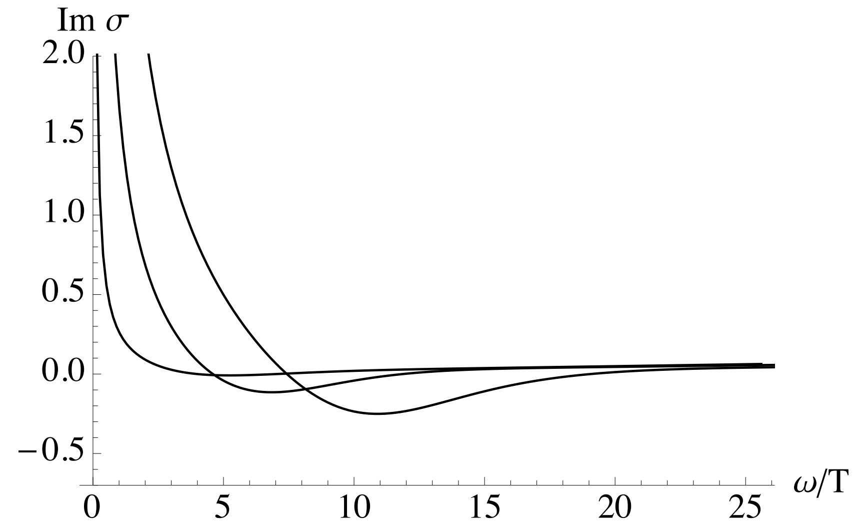
Conductivity: normal phase

[Hartnoll: 0903.3234]

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$





$$\mathrm{Im}\,\sigma \sim 1/\omega \quad \Leftrightarrow \quad \mathrm{Re}\ \sigma(\omega) \sim \delta(\omega)$$

Kramers-Kronig relation

$$\chi(\omega) = \chi_R(\omega) + i\chi_I(\omega)$$

$$\chi_R(\omega) = \frac{1}{\pi}\mathcal{P}\int \frac{\chi_I(\omega')}{\omega'-\omega}d\omega'\,, \qquad \chi_I(\omega) = -\frac{1}{\pi}\mathcal{P}\int \frac{\chi_R(\omega')}{\omega'-\omega}d\omega'$$

Translation invariance + finite density

Holographic superconductor with momentum relaxation

[Horowitz and Santos: 1302.6586]

The first holographic superconductor + momentum relaxation

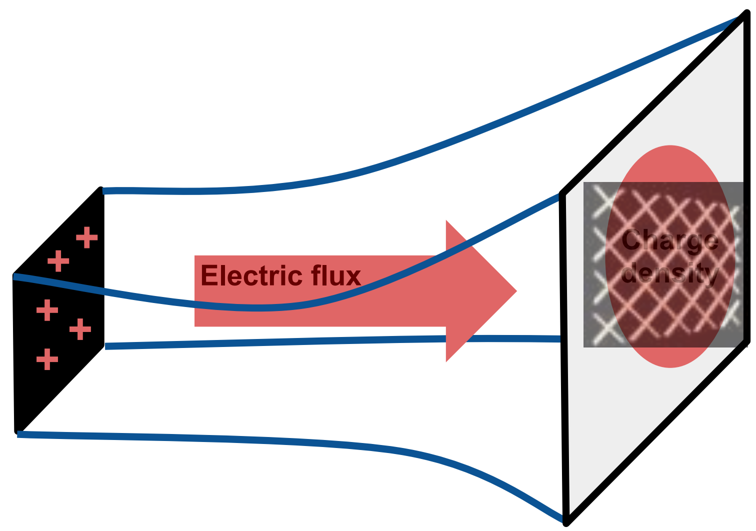
$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$\Phi = 0$

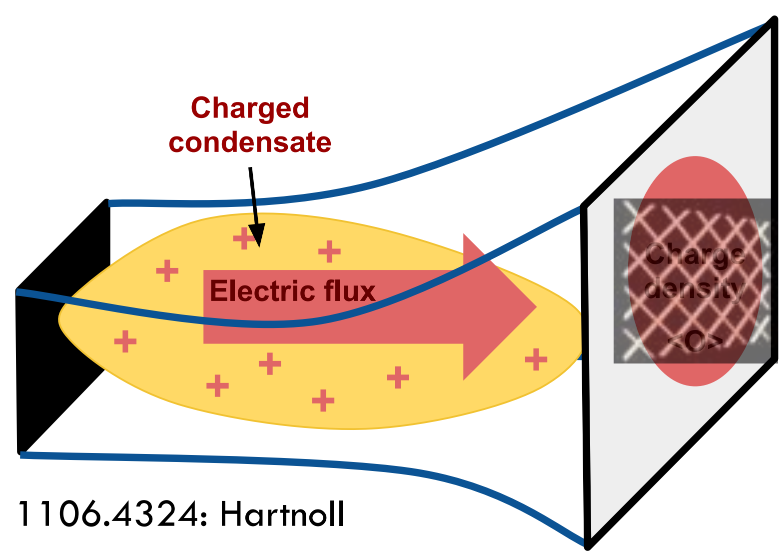
AdS-RN-black brane



$$A_t \sim 1 + A_0\cos(k_0 x)$$

$\Phi \neq 0$

Holographic superconductor



1106.4324: Hartnoll

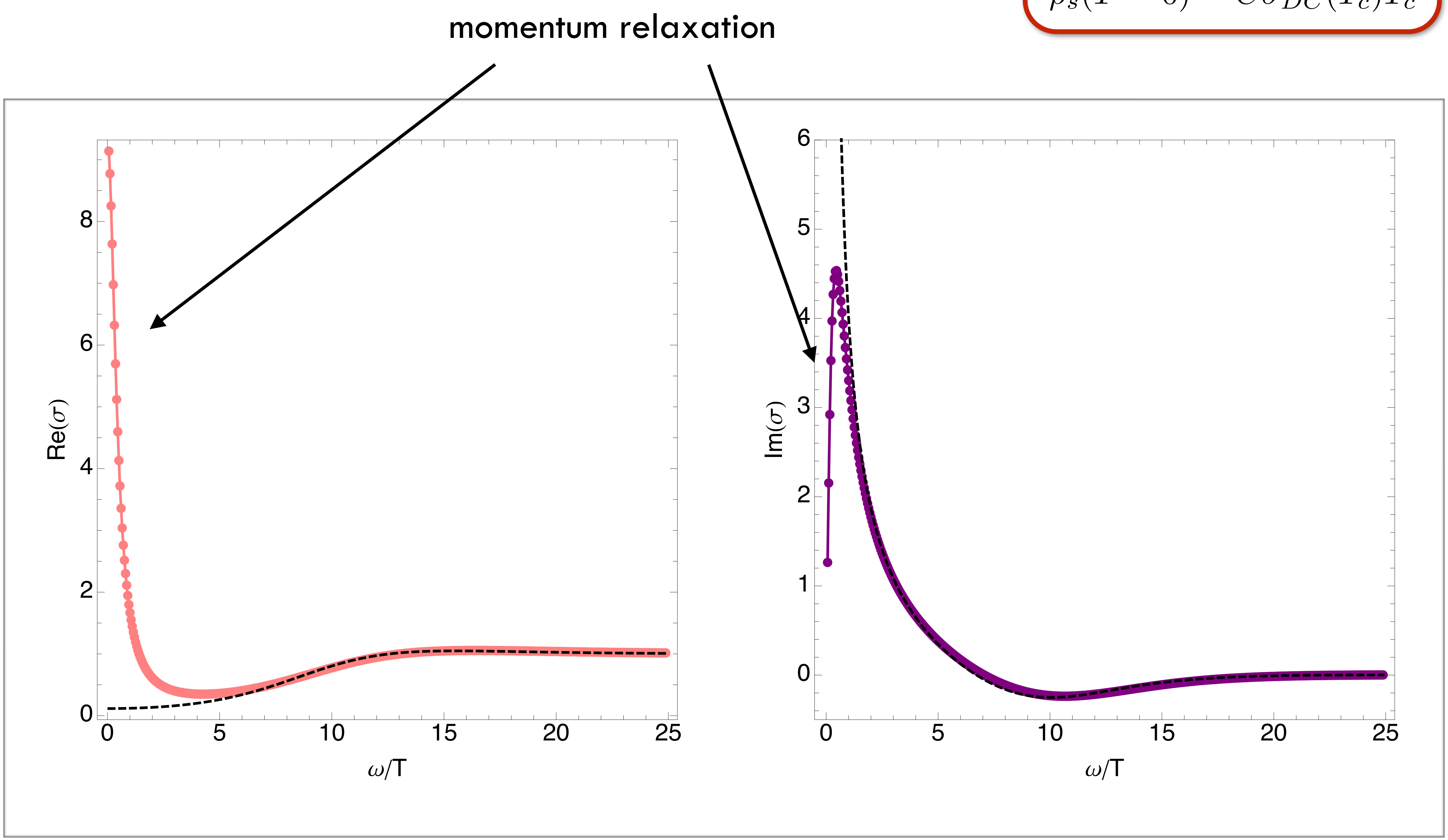
Optical conductivity

[Horowitz, Santos, Tong: 1204.0512, 1209.1098]

Conductivity: normal phase

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$



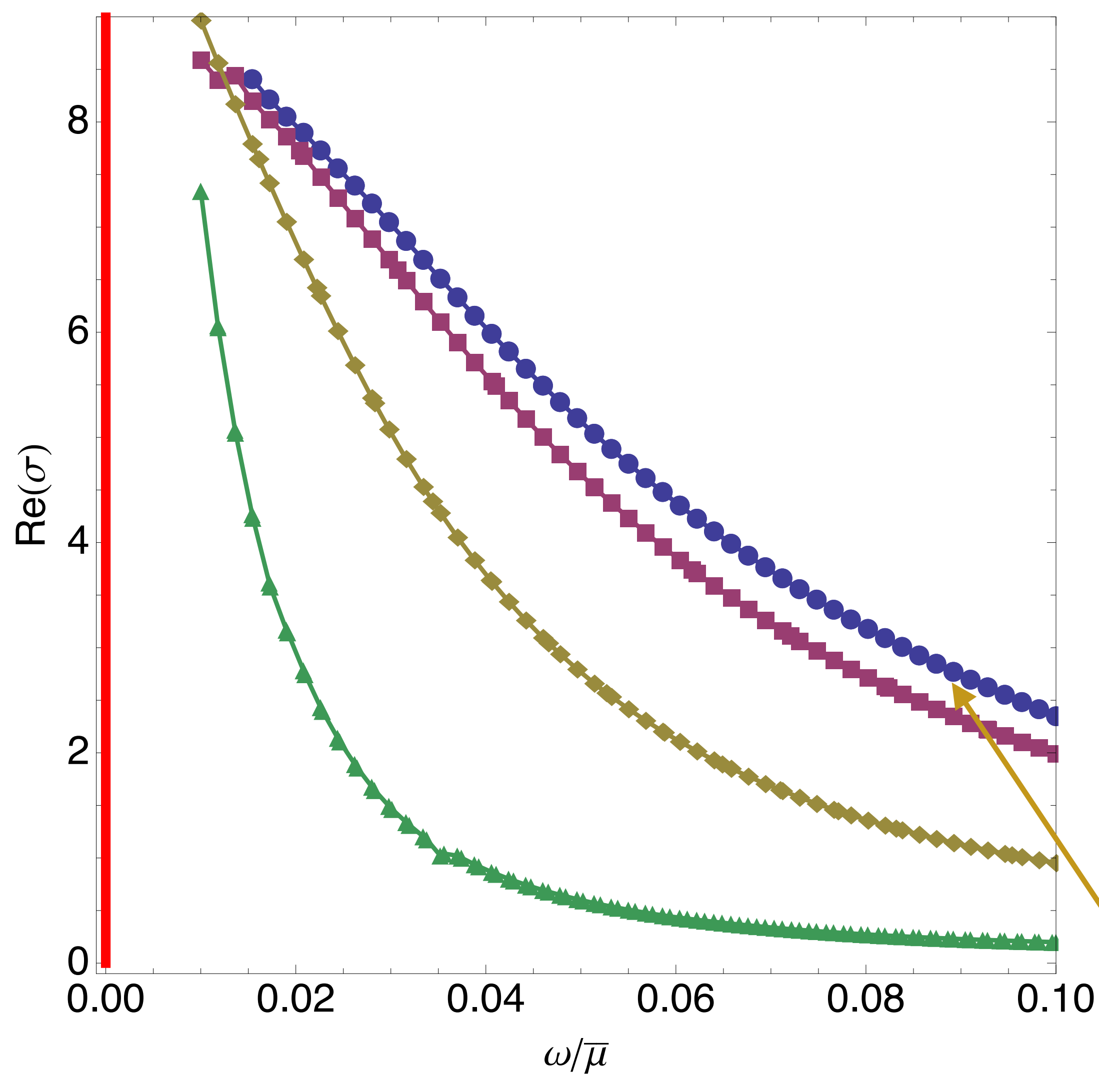
Optical conductivity

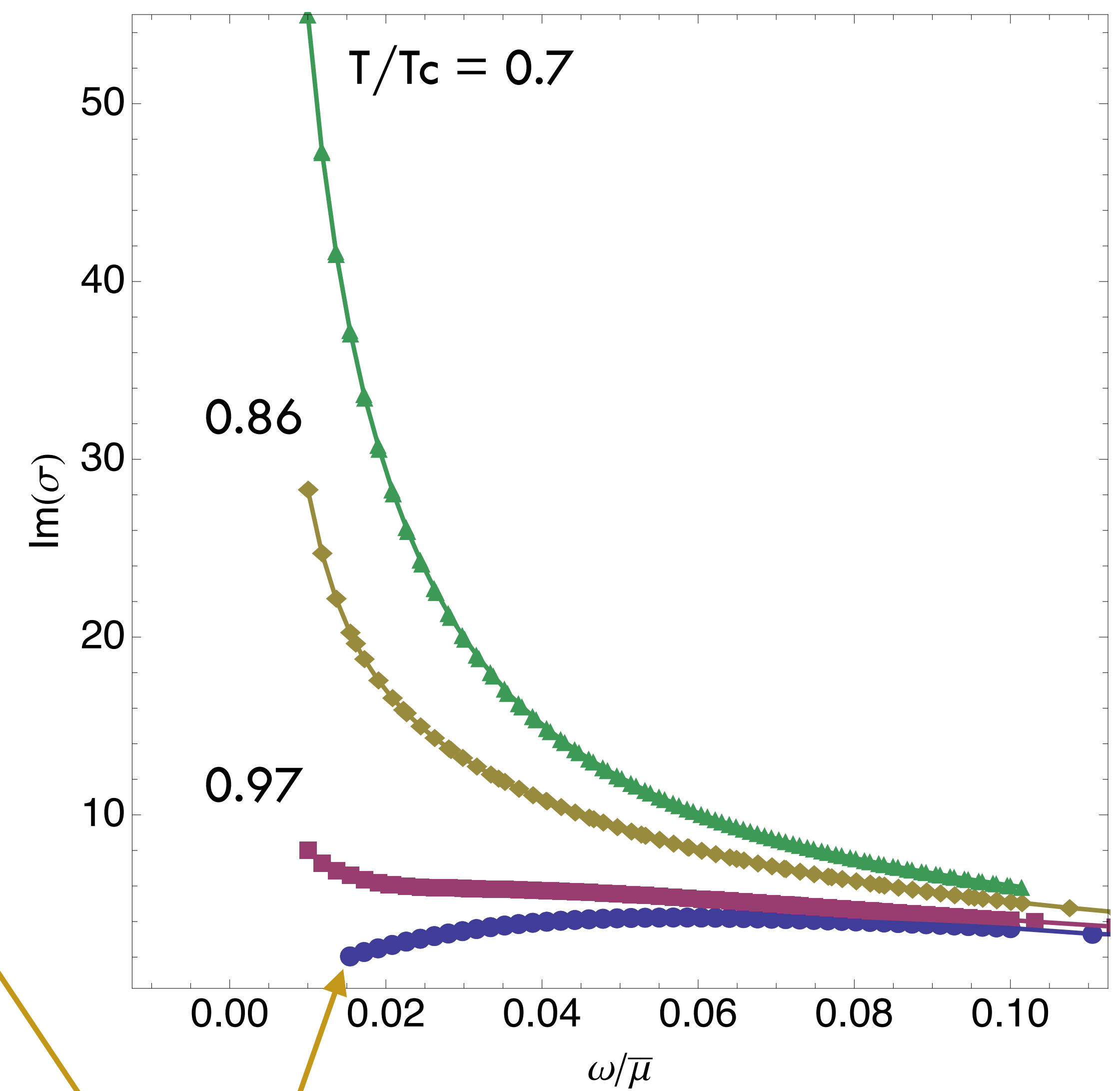
Horowitz and Santos (1302.6586)

Conductivity: normal and superconducting phase

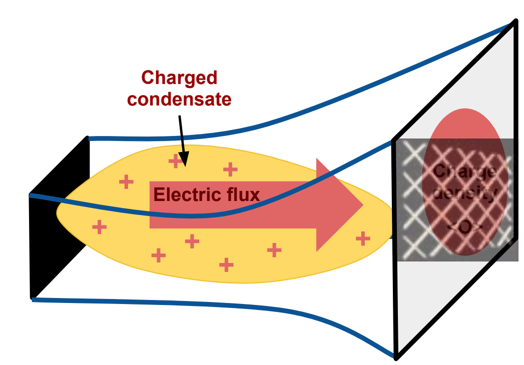
- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$





normal phase



Holographic superconductor with momentum relaxation

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

Massless scalar [Andrade, Withers: 1311.5157] → [Andrade, Gentle: 1412.6521] [KYK, Kim, Park: 1501.00446]

$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

$\psi_I = (\beta x, \beta y)$

Q-lattice [Donos, Gauntlett: 1311.3292] → [Ling, Liu, Niu, Wu, Xian: 1410.6761] [Andrade, Gentle: 1412.6521]

$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$$

$\Psi = e^{ikx}z\psi(z)$

$\psi(0) = \lambda$

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

$$B = \frac{\rho_s(T=0)}{(T_c)T_c}$$

We want to check if C or B is universal
(independent of momentum relaxation parameters)

Holographic superconductor with momentum relaxation

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

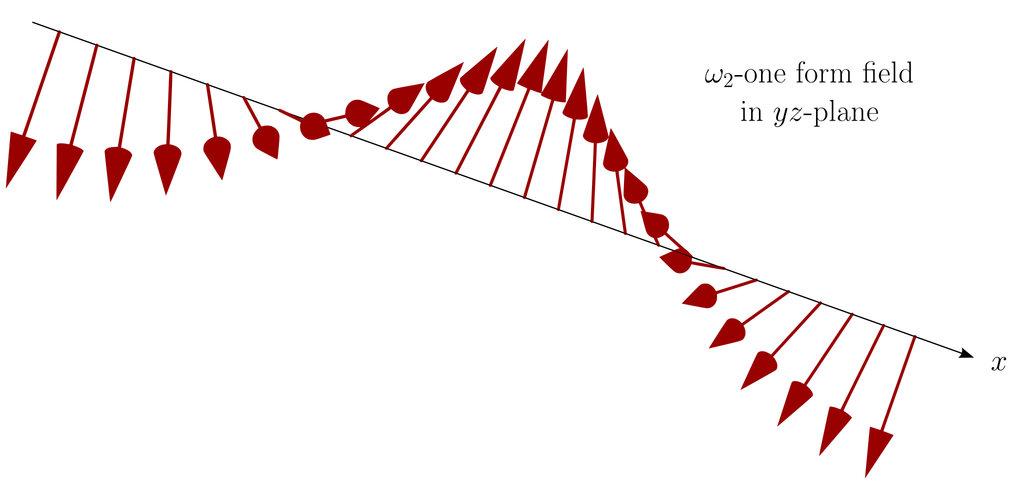
Helical lattice model

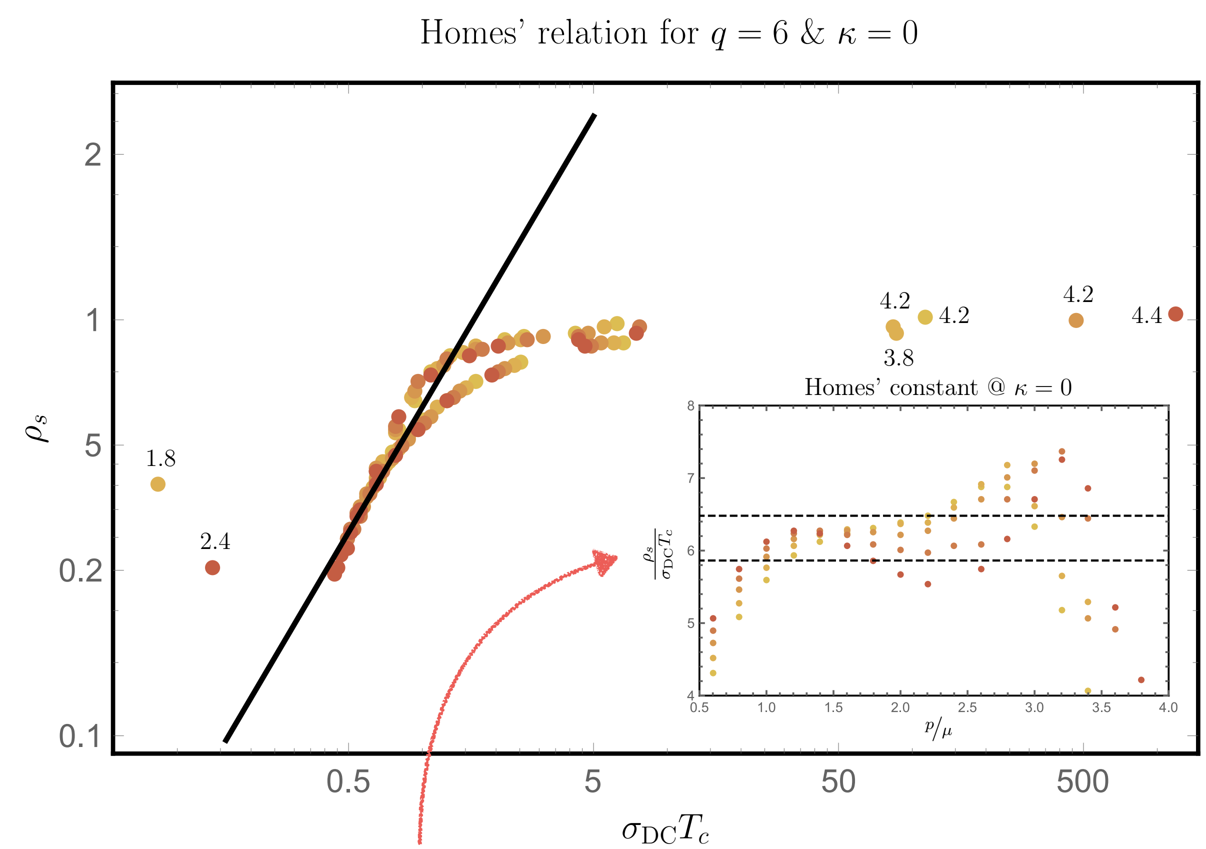
$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\,\sqrt{-g}\Big[-|\partial\rho - \mathrm{i}qA\rho|^2 - m_\rho^2|\rho|^2\Big]$$

$$S_{\text{helix}} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\Big[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2 B_\mu B^\mu\Big]$$

$$B = w(r)\omega_2, \qquad w(\infty) = \lambda,$$

$$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$$





$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

Contents

[KYK, Kim, Park: 1501.00446]

Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - \frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

- Homes' law

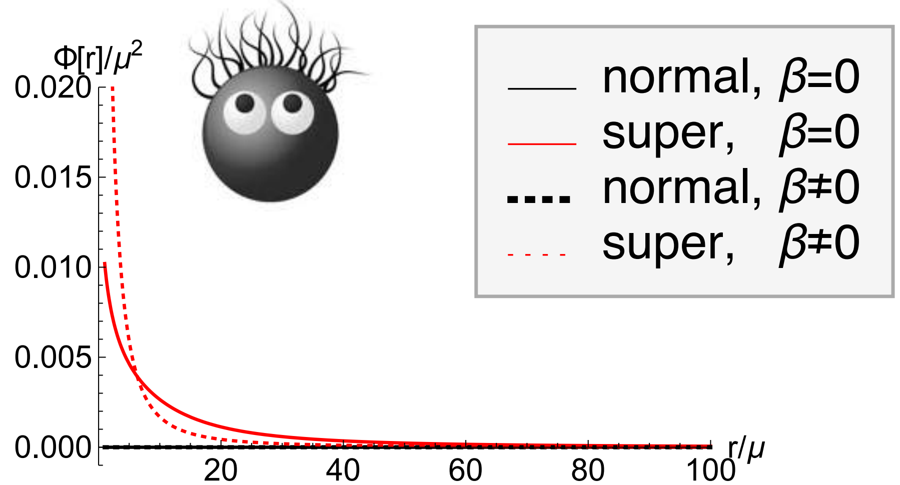
$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

Ansatz

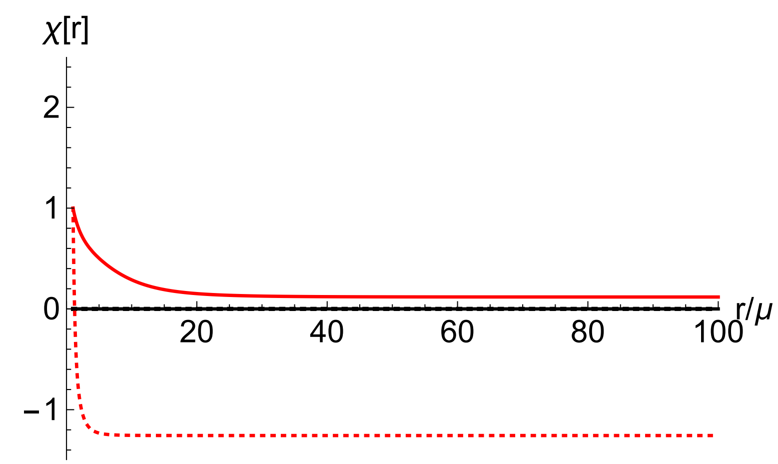
$$A = A_t(r)\mathrm{d}t \qquad \Phi = \Phi(r) \qquad \psi_I = (\beta x, \beta y)$$

$$\mathrm{d}s^2 = -U(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{U(r)} + r^2(\mathrm{d}x^2 + \mathrm{d}y^2)$$

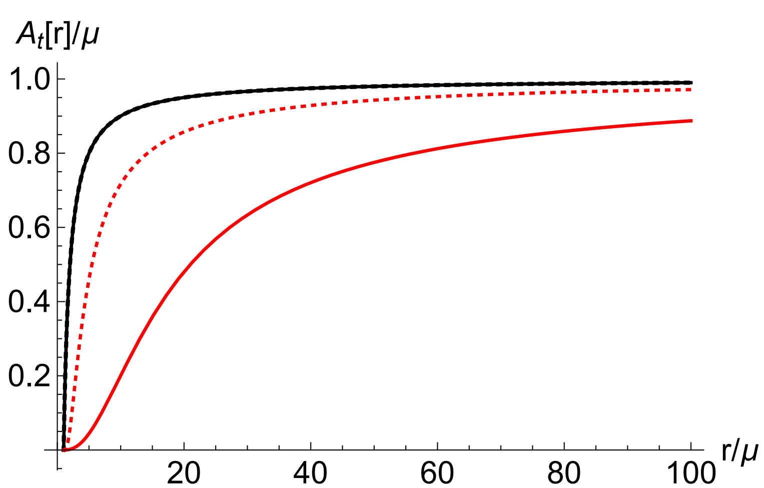
Solutions



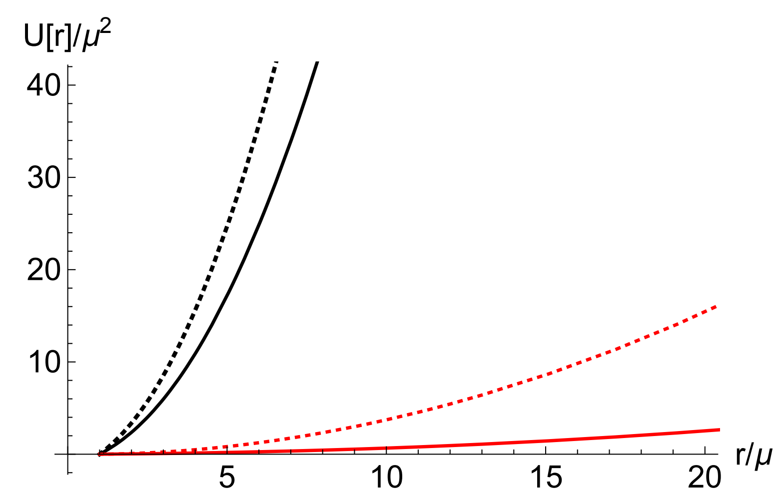
(a) $\Phi(r)/\mu^2$



(b) $\chi(r)$



(c) $A_t(r)/\mu$



(d) $U(r)/\mu^2$

[KYK, Kim, Park: 1501.00446]

Action

$$S = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - \frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

Background

$$A = A_t(r)\mathrm{d}t \qquad \Phi = \Phi(r) \qquad \psi_I = (\beta x, \beta y)$$

$$\mathrm{d}s^2 = -U(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{U(r)} + r^2(\mathrm{d}x^2 + \mathrm{d}y^2)$$

+

Fluctuations

$$\delta A_x(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)$$

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}r^2 h_{tx}(\omega,r)$$

$$\delta\psi_1(t,r) = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\xi(\omega,r)$$

$$h_{tx} = h_{tx}^{(0)} + \frac{1}{r^2}h_{tx}^{(2)} + \frac{1}{r^3}h_{tx}^{(3)} + \cdots,$$

$$a_x = a_x^{(0)} + \frac{1}{r}a_x^{(1)} + \cdots,$$

$$\xi = \xi^{(0)} + \frac{1}{r^2}\xi^{(2)} + \frac{1}{r^3}\xi^{(3)} + \cdots,$$

$$S_{\mathrm{ren}}^{(2)} = \frac{V_2}{2}\int_0^{\infty}\frac{\mathrm{d}\omega}{2\pi}\left(-\rho\bar{a}_x^{(0)}h_{tx}^{(0)} - \epsilon\bar{h}_{tx}^{(0)}h_{tx}^{(0)} + \bar{a}_x^{(0)}a_x^{(1)} - 3\bar{h}_{tx}^{(0)}h_{tx}^{(3)} + 3\bar{\xi}^{(0)}\xi^{(3)}\right)$$

$$\frac{1}{2}\int_0^{\infty}\frac{\mathrm{d}\omega}{2\pi}\left[J_{-\omega}^a G_{ab}J_\omega^b\right]$$

$$\begin{pmatrix} a_x^{(1)} \\ h_{tx}^{(3)} \\ \xi^{(3)} \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}\begin{pmatrix} a_x^{(0)} \\ h_{tx}^{(0)} \\ \xi^{(0)} \end{pmatrix},$$

$$R^a = \mathbb{M}_b^a J^b$$

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

Numerical method for multi fields

[Kaminski, Landsteiner, Mas, Shock, Tarrio: 2009]
[KYK, Kim, Sin, Seo: 2014)]

How to compute $\mathbb{M}^a_b$

$$\Phi^a_i(r) \to \mathbb{S}^a_i + \cdots + \frac{\mathbb{O}^a_i}{r^{\delta_a}} + \cdots$$

$$h_{tx} = h^{(0)}_{tx} + \frac{1}{r^2} h^{(2)}_{tx} + \frac{1}{r^3} h^{(3)}_{tx} + \cdots ,$$

$$a_x = a^{(0)}_x + \frac{1}{r} a^{(1)}_x + \cdots ,$$

$$\xi = \xi^{(0)} + \frac{1}{r^2}\xi^{(2)} + \frac{1}{r^3}\xi^{(3)} + \cdots ,$$

$$\Phi^a(r) = \Phi^a_i(r) c^i \to \mathbb{S}^a_i c^i + \cdots + \frac{\mathbb{O}^a_i c^i}{r^{\delta_a}} + \cdots$$

$$\equiv J^a + \cdots + \frac{R^a}{r^{\delta_a}} + \cdots ,$$

$$c^i = (\mathbb{S}^{-1})^i_a J^a \qquad R^a = \mathbb{O}^a_i c^i = \mathbb{O}^a_i (\mathbb{S}^{-1})^i_b J^b$$

ex) one field case: $\frac{a^{(1)}_x}{a^{(0)}_x}$

$$\frac{1}{2}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left[J^a_{-\omega} G_{ab} J^b_\omega\right]$$

$$\begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} = \begin{pmatrix} -\frac{iG_{11}}{\omega} & \frac{i(G_{11}\mu - G_{12})}{\omega} \\ \frac{i(G_{11}\mu - G_{21})}{\omega} & -\frac{i(G_{22} + \mu(-G_{12} - G_{21} + G_{11}\mu))}{\omega} \end{pmatrix}$$

[Hartnoll: 0903.3234]

Optical electric conductivities: superconducting phase

[KYK, Kim, Park: 1501.00446]

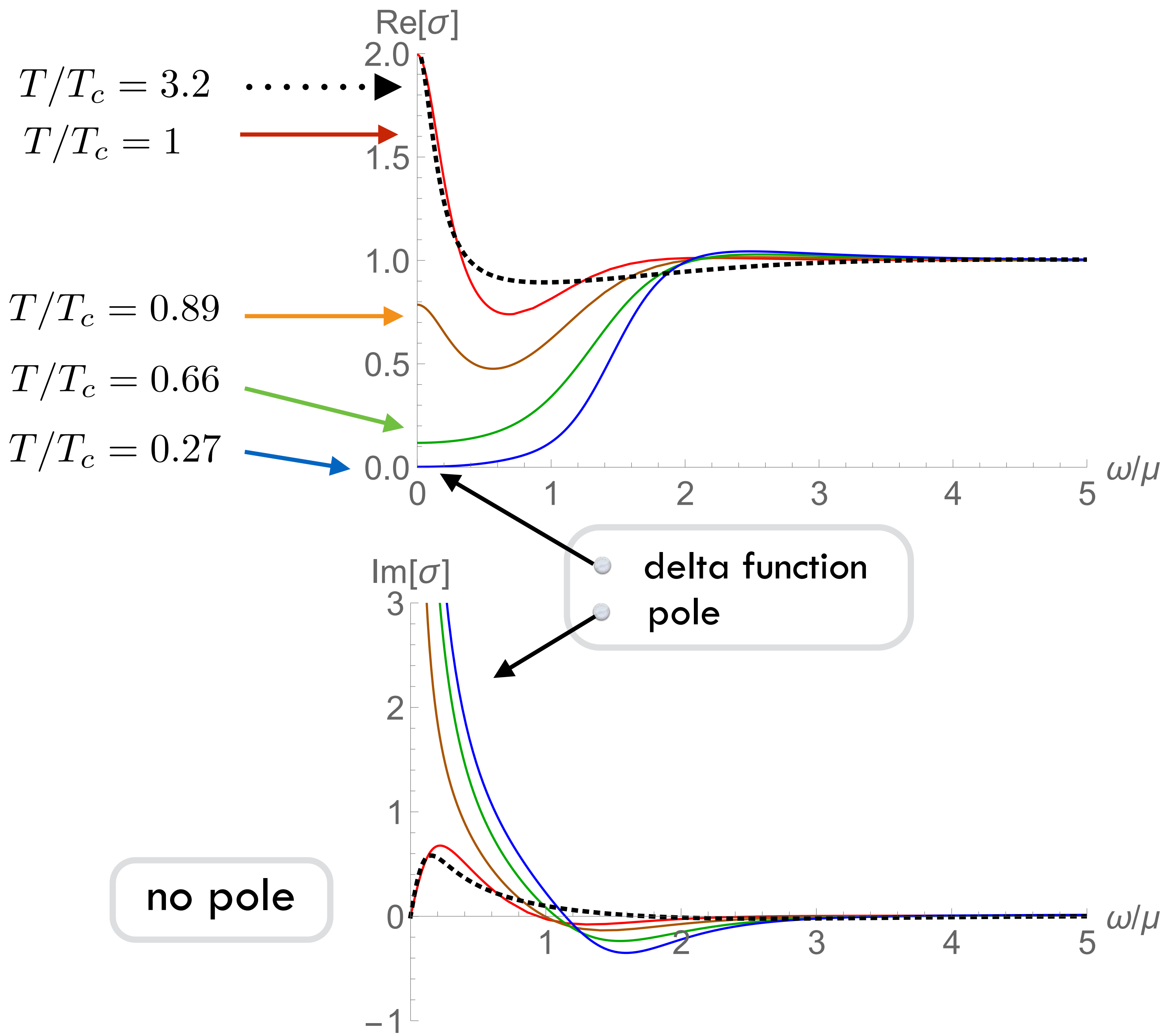
$\beta/\mu = 1$

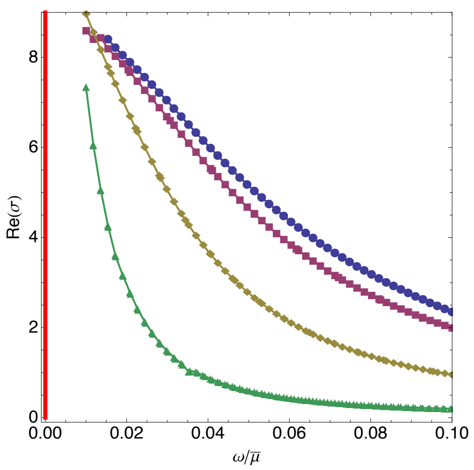
- Homes' law

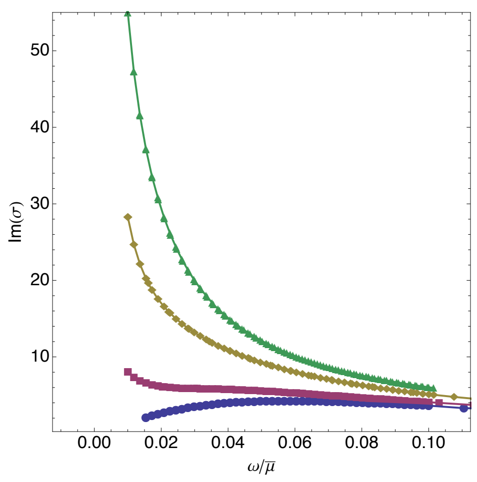
$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$







Digression: Thermoelectric conductivities and DC limits

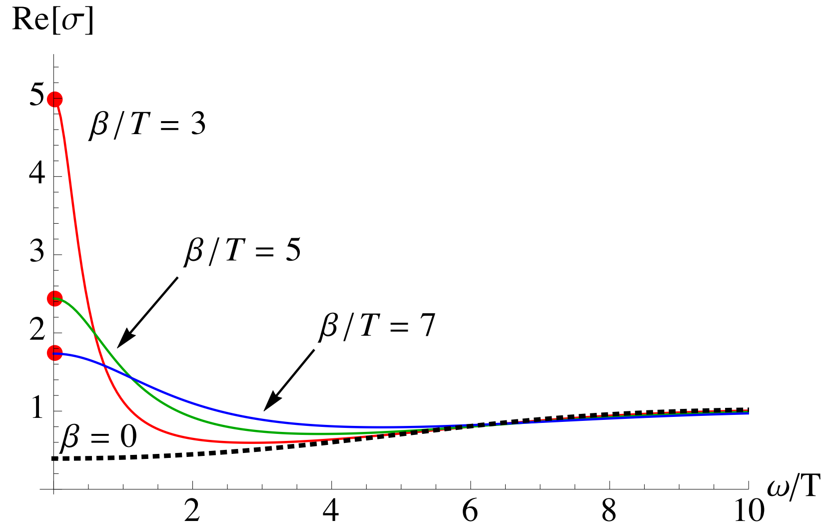
[KYK, Kim, Sin, Seo: 1409.8346]

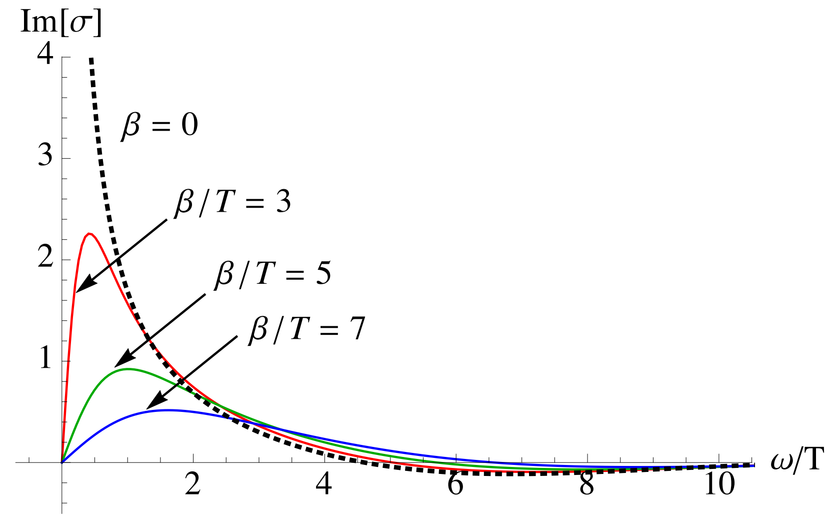
Electric conductivity

$\mu/T = 6$

DC result:
Andrade, Withers
1311.5157

$$\sigma = 1 + \frac{\mu^2}{\beta^2}$$



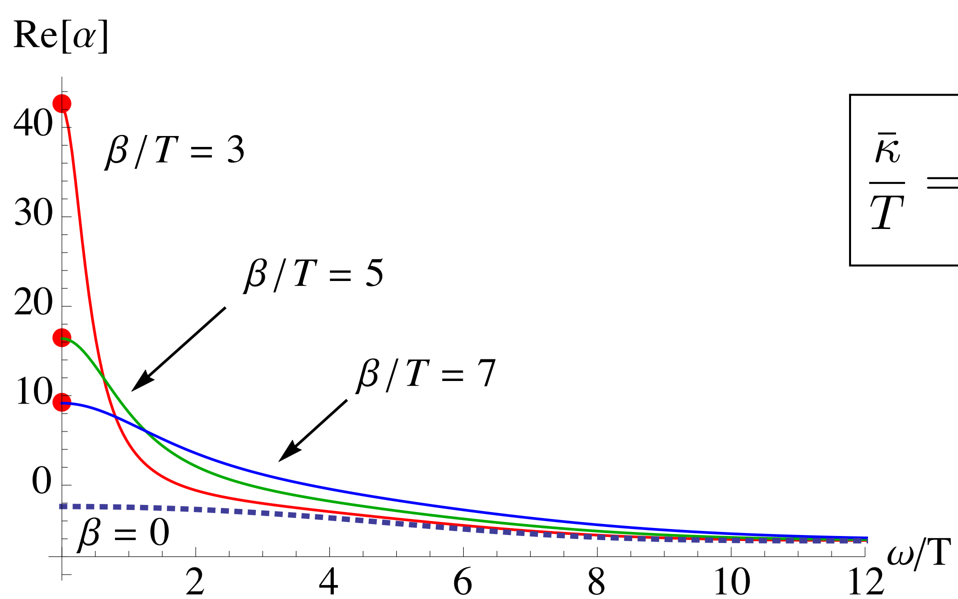


Thermoelectric conductivity

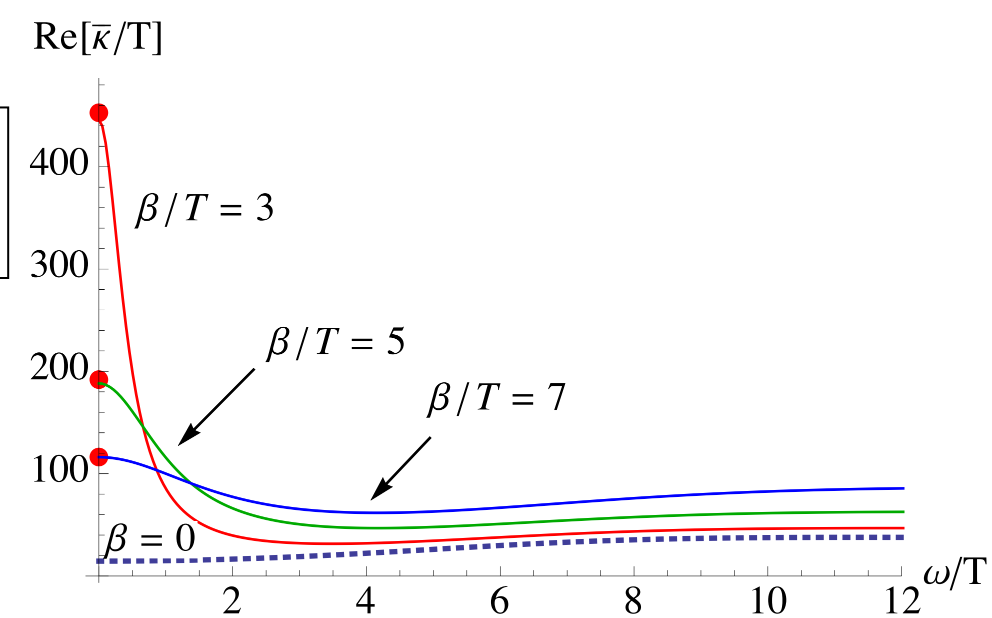
DC results:
Donos and Gauntlett
1406.4742

Thermal conductivity

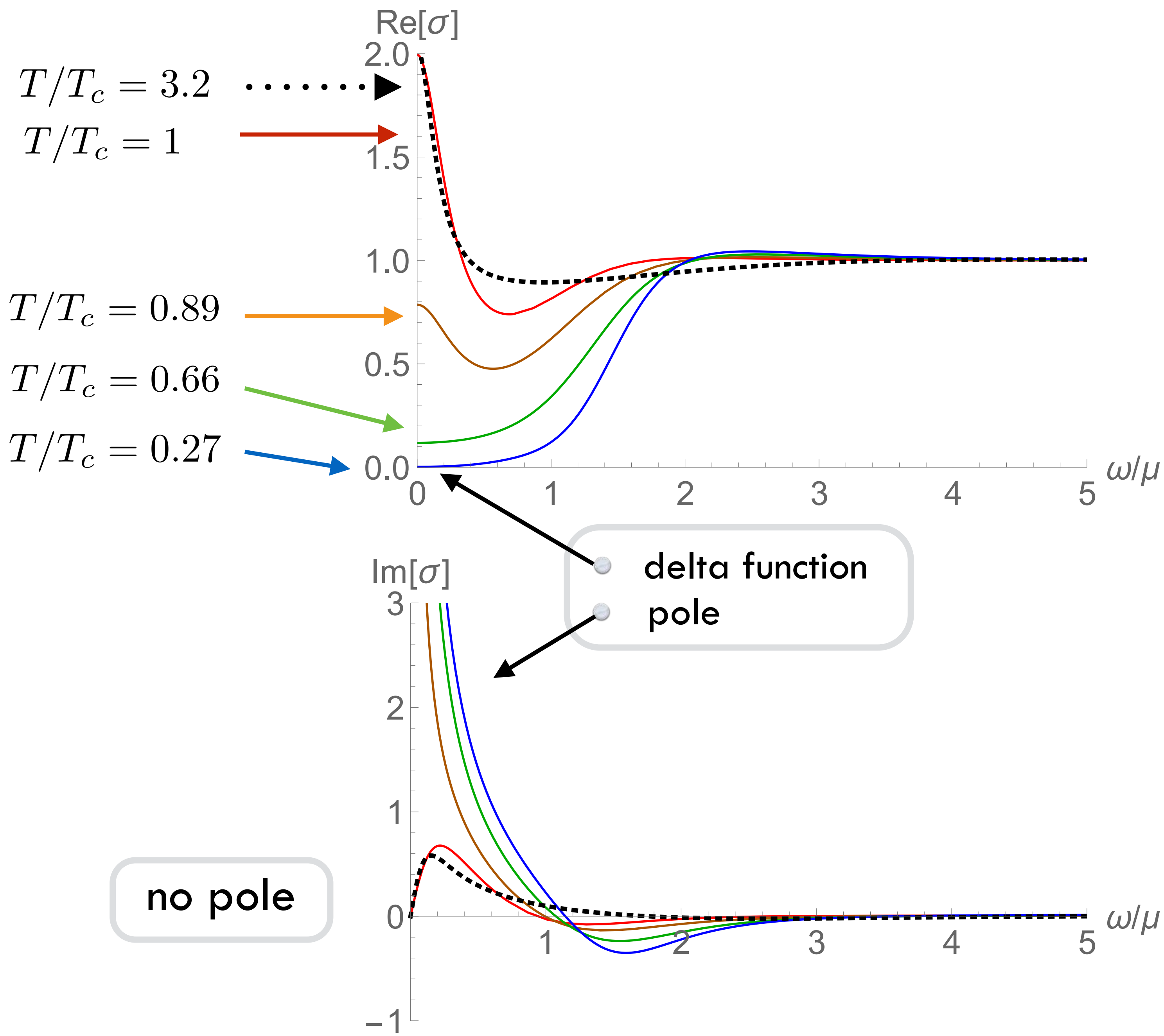
$$\alpha = \frac{4\pi\mu}{\beta^2} r_0$$



$$\frac{\bar{\kappa}}{T} = \frac{(4\pi)^2}{\beta^2} r_0^2$$



$\beta/\mu = 1$



Ferrell-Glover-Tinkham(FGT) Sum rule works: conservation of charged degrees of freedom

$$\int_{0+}^{\infty} d\omega \mathrm{Re}[\sigma_n(\omega) - \sigma_s(\omega)] = \rho_s = \frac{\pi}{2} K_s$$

$$\mathrm{Re}\sigma(\omega) = \rho_s \delta(\omega) + \cdots$$

$$\mathrm{Im}\sigma(\omega) = \frac{K_s}{\omega} + \cdots$$

Kramers-Kronig relation

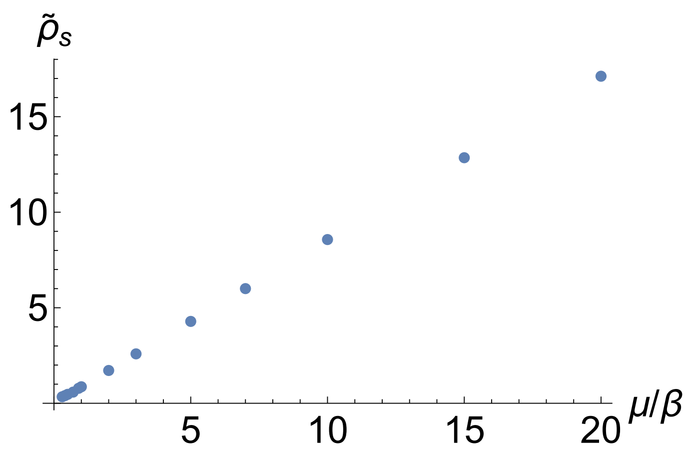
$$\mathrm{Im}\,\sigma(\omega) = -\frac{2\omega}{\pi}\mathcal{P}\int_0^{\infty} d\tilde{\omega}\, \frac{\mathrm{Re}\,\sigma(\tilde{\omega})}{\tilde{\omega}^2 - \omega^2}.$$

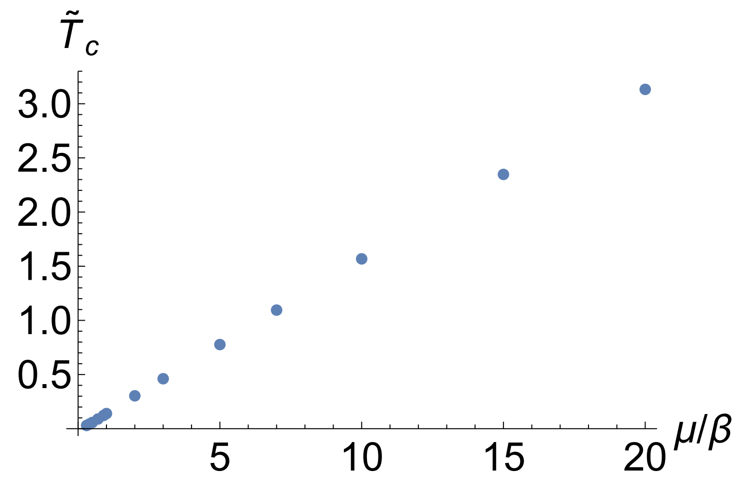
$$\mathrm{Im}\sigma(\omega) = \frac{K_s}{\omega} \leftrightarrow \mathrm{Re}\sigma(\omega) = \rho_s \delta(\omega)$$

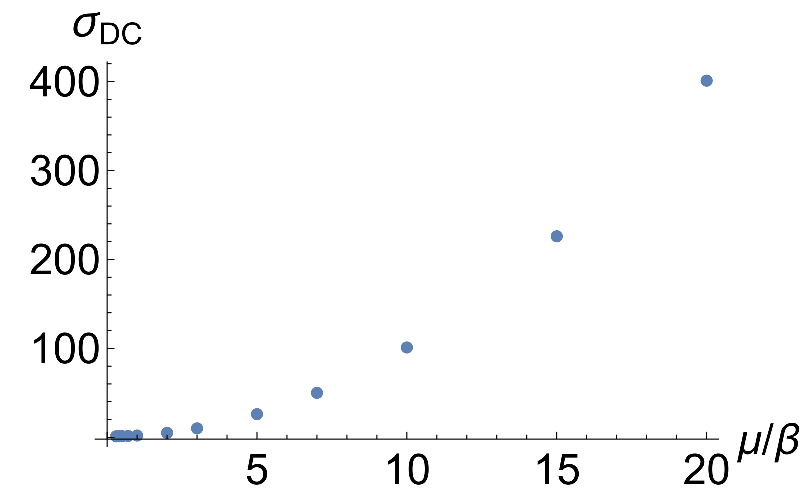
$$\rho_s = \frac{\pi}{2} K_s$$

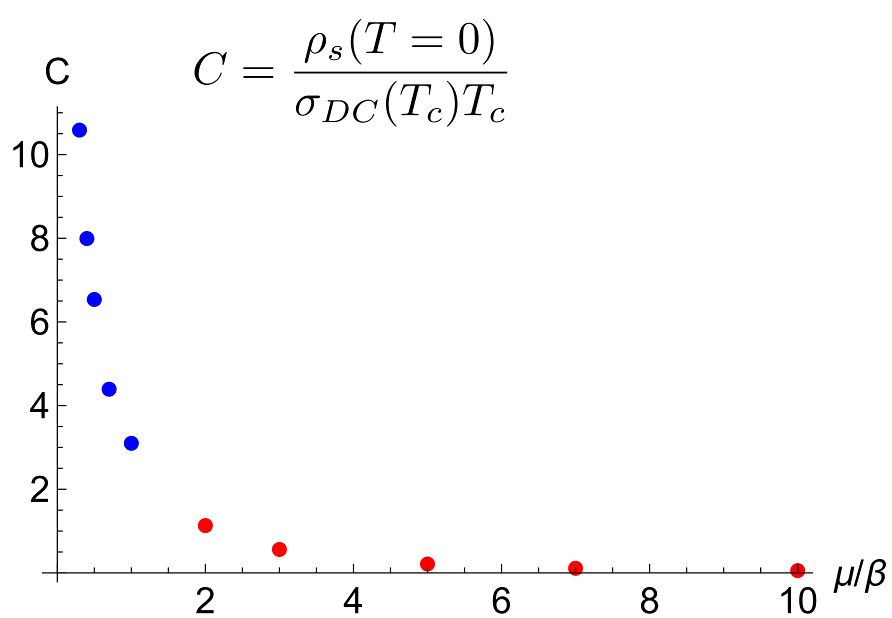
Homes' law and Uemura's law

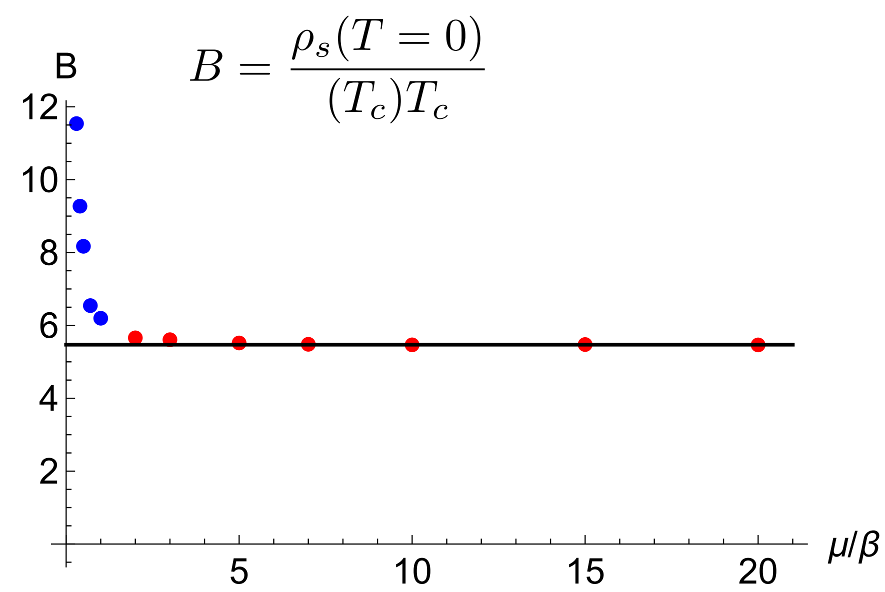
[KYK, Kim, Park: 1604.06205]











- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

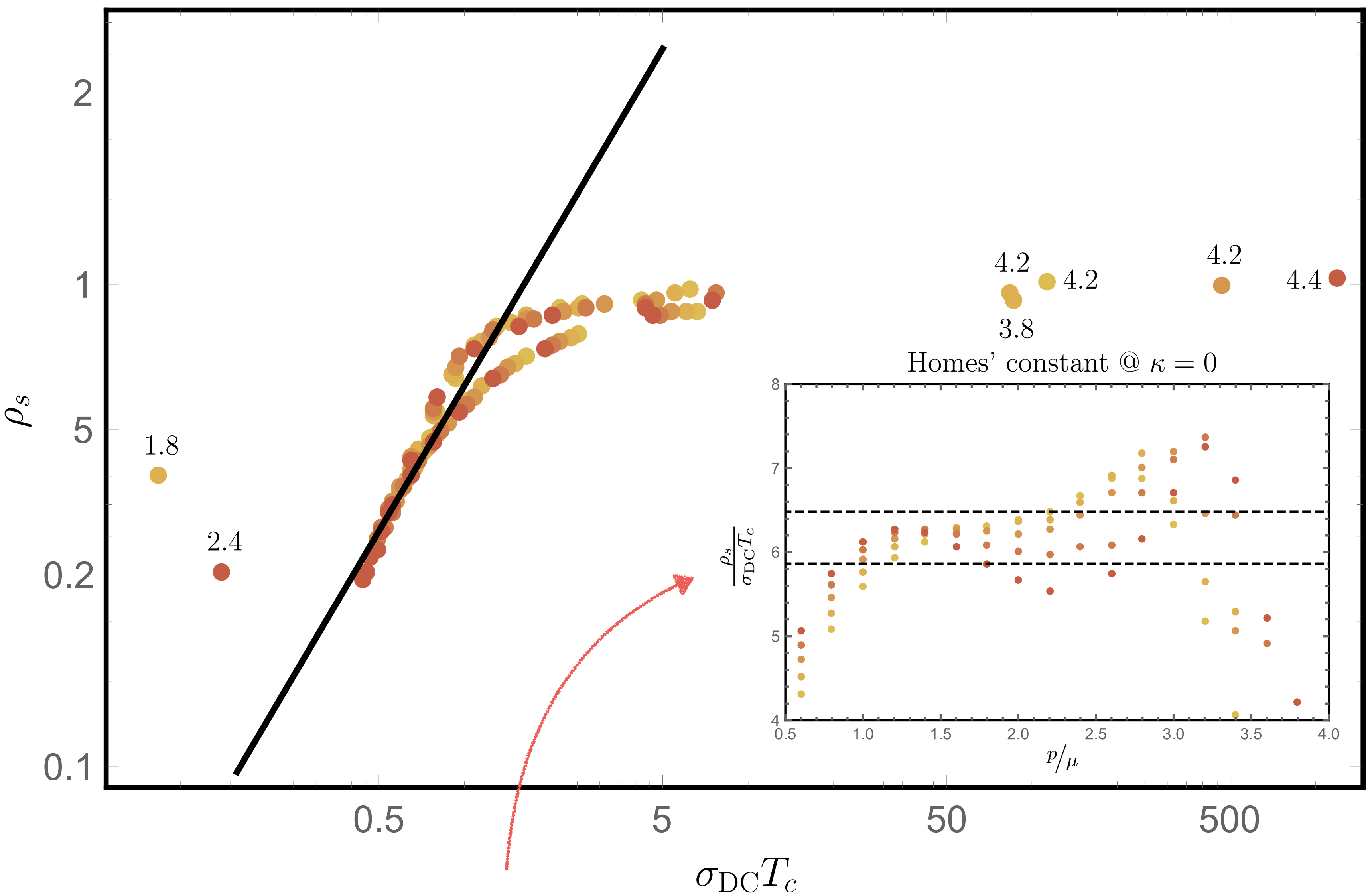
~~Homes' law~~

Helical lattice case

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

Homes' relation for $q = 6$ & $\kappa = 0$

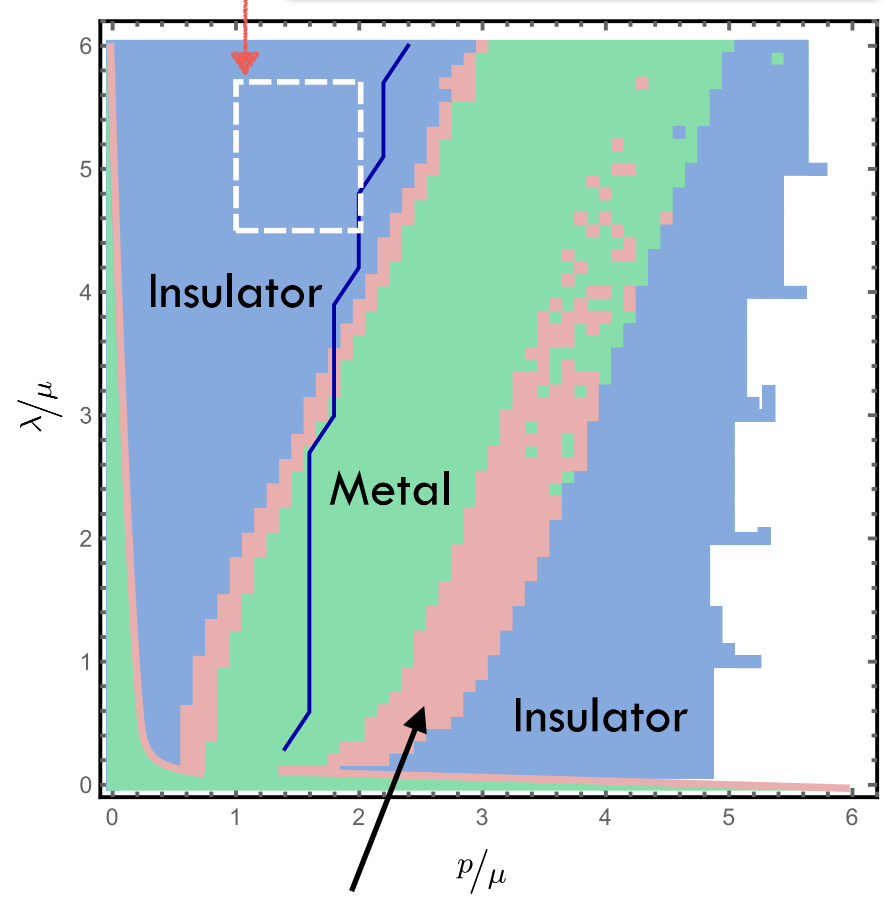


$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

λ/μ = 4.5, 4.8, 5.1, 5.4, 5.7

$p/\mu: \; 1 \sim 2$
$\lambda/\mu: 4.5 \sim 5.7$

Phase diagram ($\Phi = 0$)



undetermined

Contents

- Motivations
- Holographic superconductor with momentum relaxation
- Massless scalar model
- Q-lattice model
- Summary and outlook

Q-lattice model

[KYK, Niu: 1607.XXXXX]

[Donos, Gauntlett: 1311.3292]

[Ling, Liu, Niu, Wu, Xian: 1410.6761]
[Andrade, Gentle: 1412.6521]

Action $S = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - |\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$

Ansatz $ds^2 = \frac{1}{z^2}\left[-(1-z)U(z)dt^2 + \frac{dz^2}{(1-z)U(z)} + V_1(z)dx^2 + V_2(z)dy^2\right]$

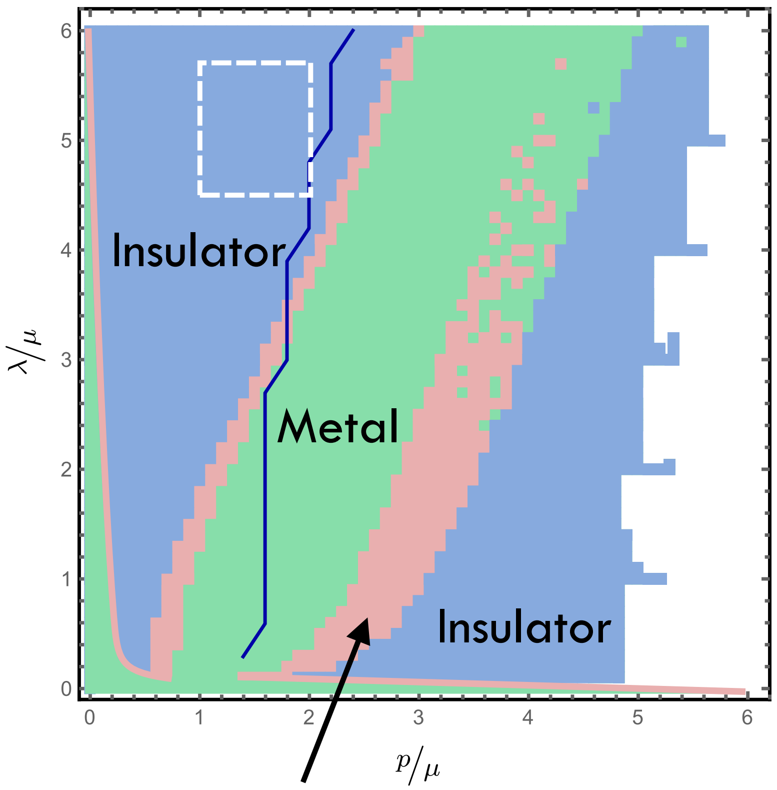
$A = \mu(1-z)a(z)dt \qquad \Phi = z\phi(z) \qquad \Psi = e^{ikx}z\psi(z) \quad (\psi(0) = \lambda)$

Two parameters k, λ

with $m_\Psi^2 = m_\Phi^2 = -2, \; q = 6$

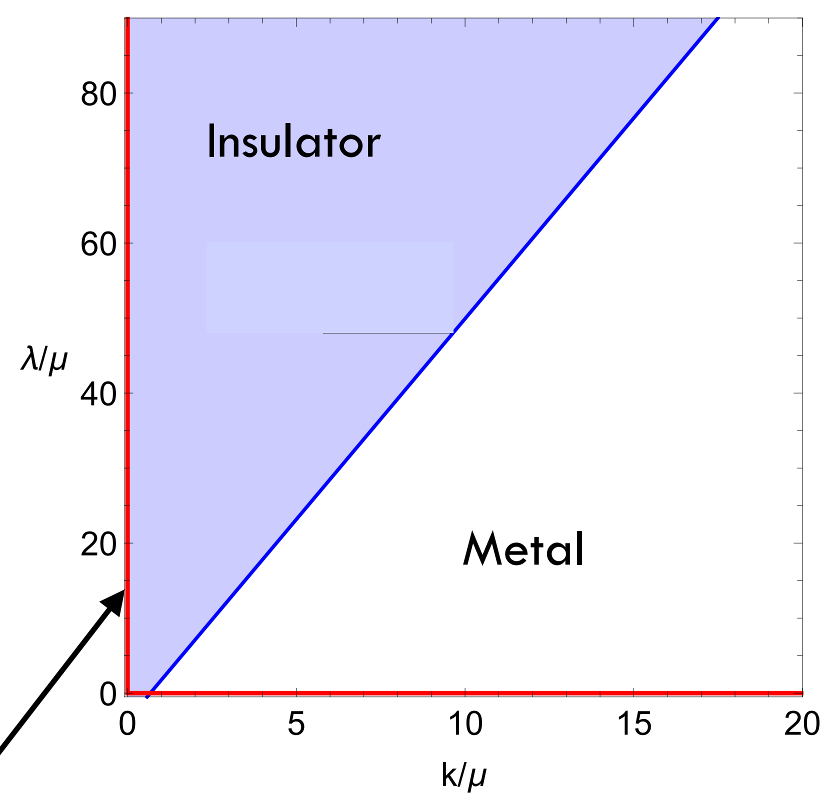
Phase diagram $(\Phi = 0)$

Helical lattice



undetermined

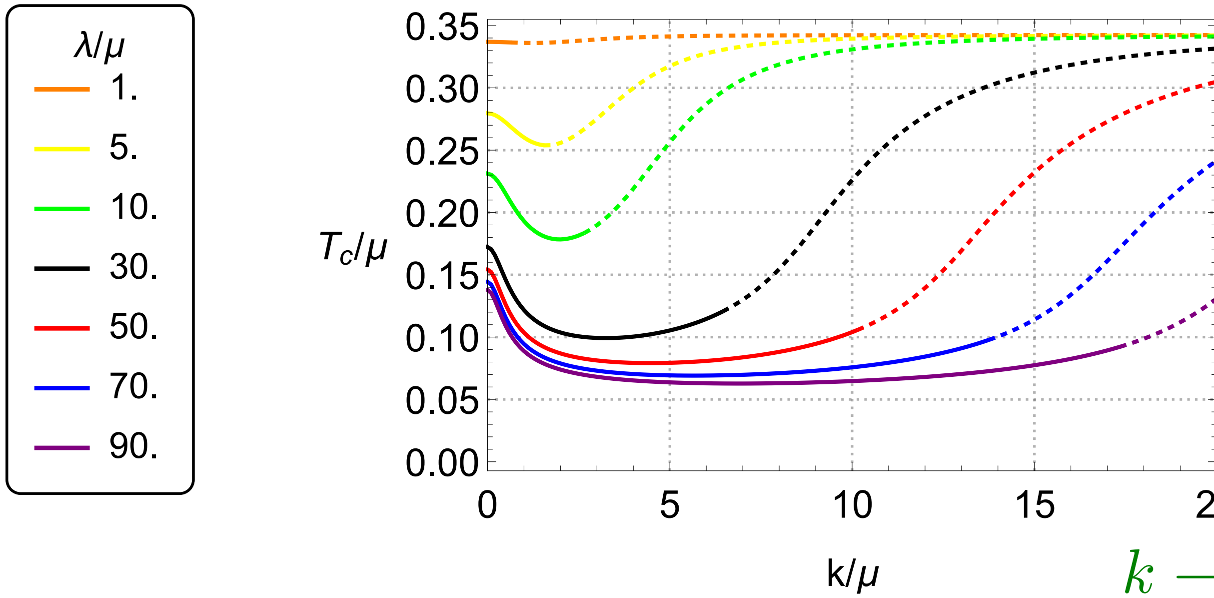
Q lattice



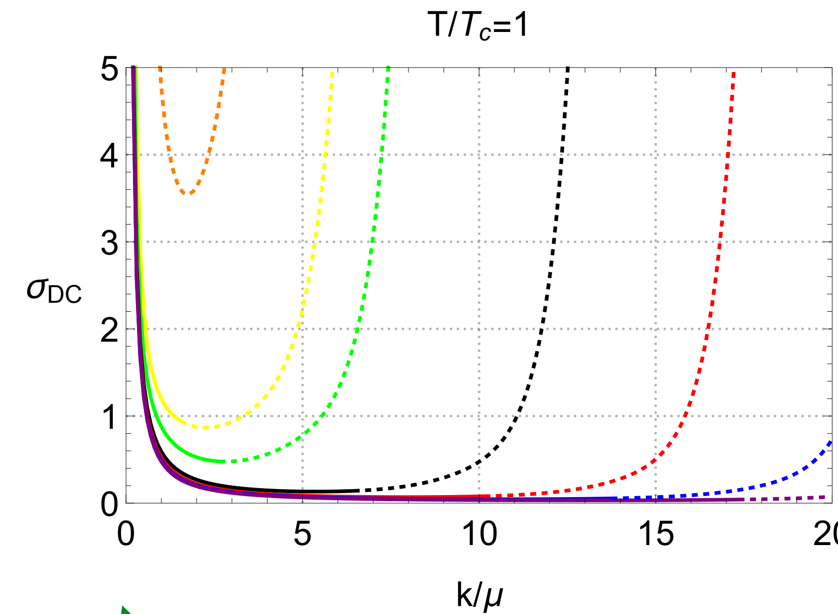
There may be metal regime near k=0: numerical issue

Transition temperature and DC conductivity

[KYK, Niu: 1607.XXXXX]



$k \to \infty$
AdS RN limit

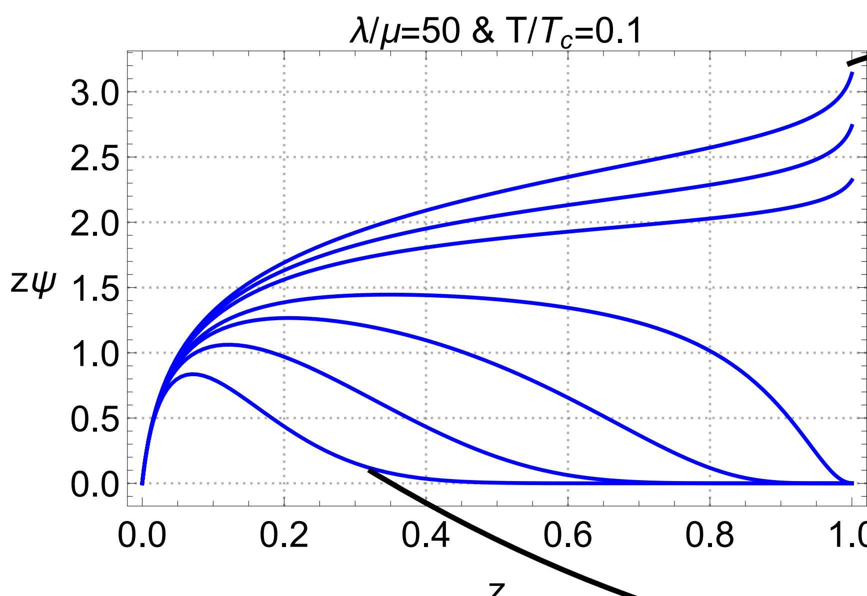


$k \to \infty$
AdS RN limit

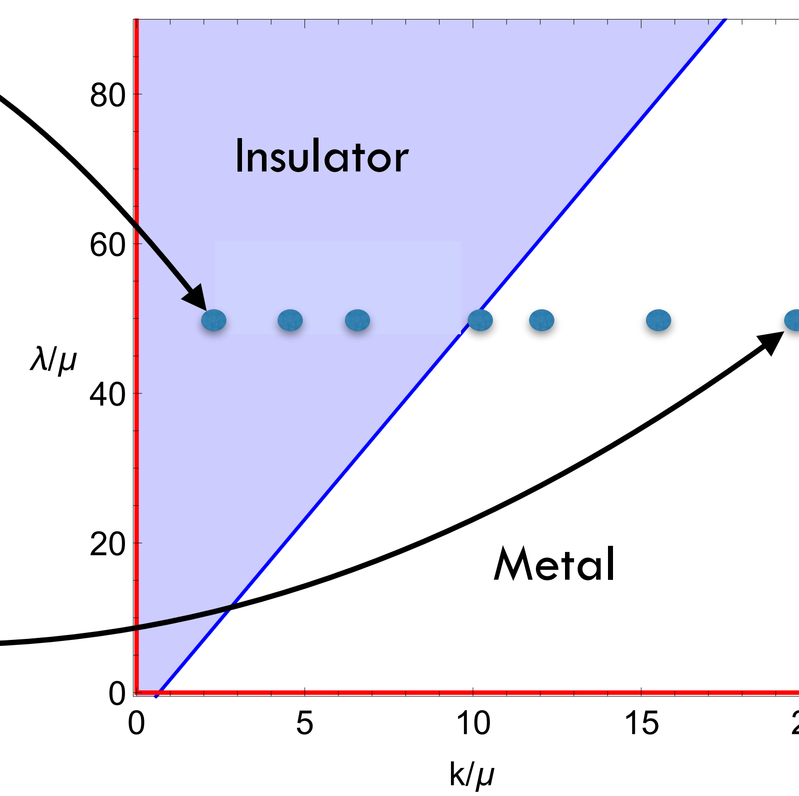
No momentum relaxation

$$\Psi = e^{ikx} z\psi(z)$$

$$\psi(0) = \lambda$$



$k/\mu = 2, 4, 6, 10.1, 12, 15, 20.$



Superfluid density

[KYK, Niu: 1607.XXXXX]

London equation

$$J_i(\omega,p) = -K_s A_i(\omega,p)$$

bulk gauge field

$$a_i(z,\omega,p) = a_i^{(0)}(\omega,p) + z a_i^{(1)}(\omega,p) + \cdots$$

$$K_s = -\left.\frac{a_x^{(1)}(\omega,p)}{a_x^{(0)}(\omega,p)}\right|_{\omega,p\to 0}$$

A. In the limit $p=0$ and $\omega \to 0$

$$K_s = -\left.\frac{a_x^{(1)}(\omega,0)}{a_x^{(0)}(\omega,0)}\right|_{\omega\to 0}$$

$$J_i(\omega,0) = \frac{iK_s}{\omega}E_i(\omega,0) \equiv \sigma(\omega)E_i(\omega,0)$$

$$\mathrm{Im}[\sigma(\omega)] = \frac{K_s}{\omega} + \cdots$$

$$\mathrm{Re}[\sigma(\omega)] = \frac{\pi}{2}K_s\delta(\omega)$$

Infinite DC conductivity

B. In the limit $\omega = 0$ and $p \to 0$

$$\tilde{K}_s = -\left.\frac{a_x^{(1)}(0,p)}{a_x^{(0)}(0,p)}\right|_{p\to 0}$$

$$\nabla \times \vec{J} = -K_s\vec{B}$$

$$-\nabla^2\vec{B} = \nabla\times(\nabla\times\vec{B})$$
$$= 4\pi\nabla\times\vec{J} = -4\pi K_s\vec{B} \equiv -\frac{1}{\lambda^2}\vec{B}$$

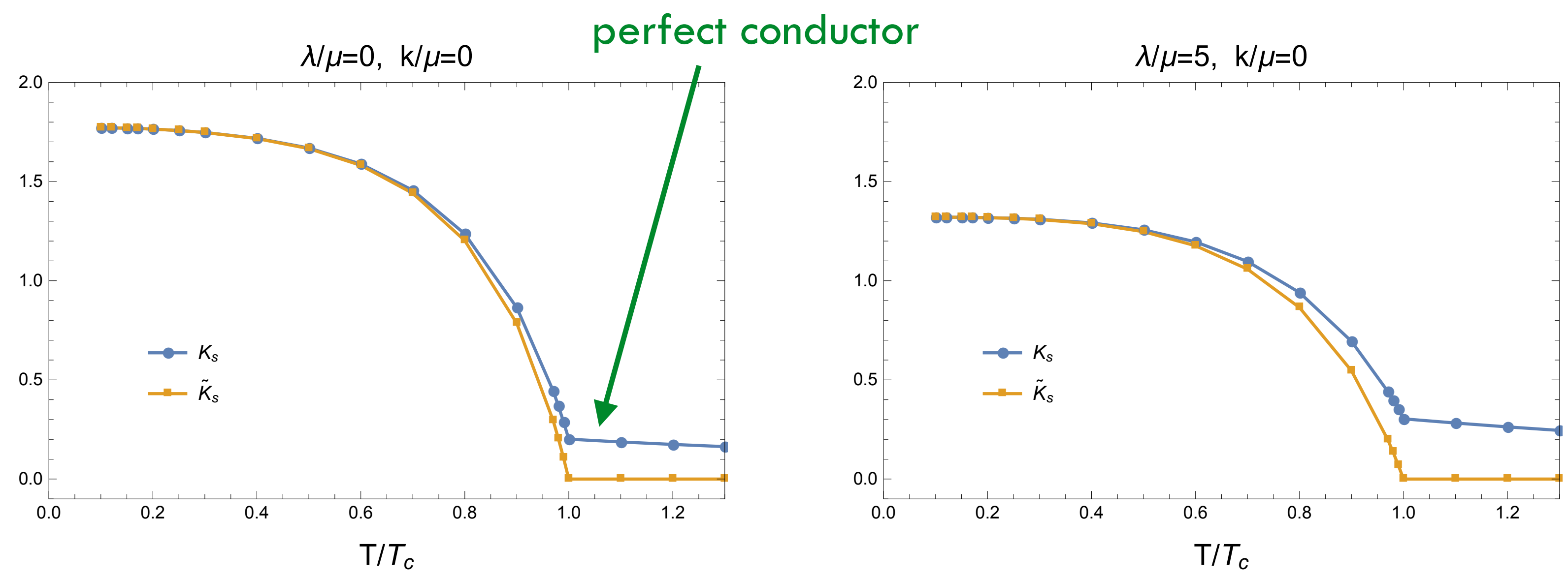
Meissner effect:
Magnetic penetration depth

Superfluid density

[KYK, Niu: 1607.XXXXX]

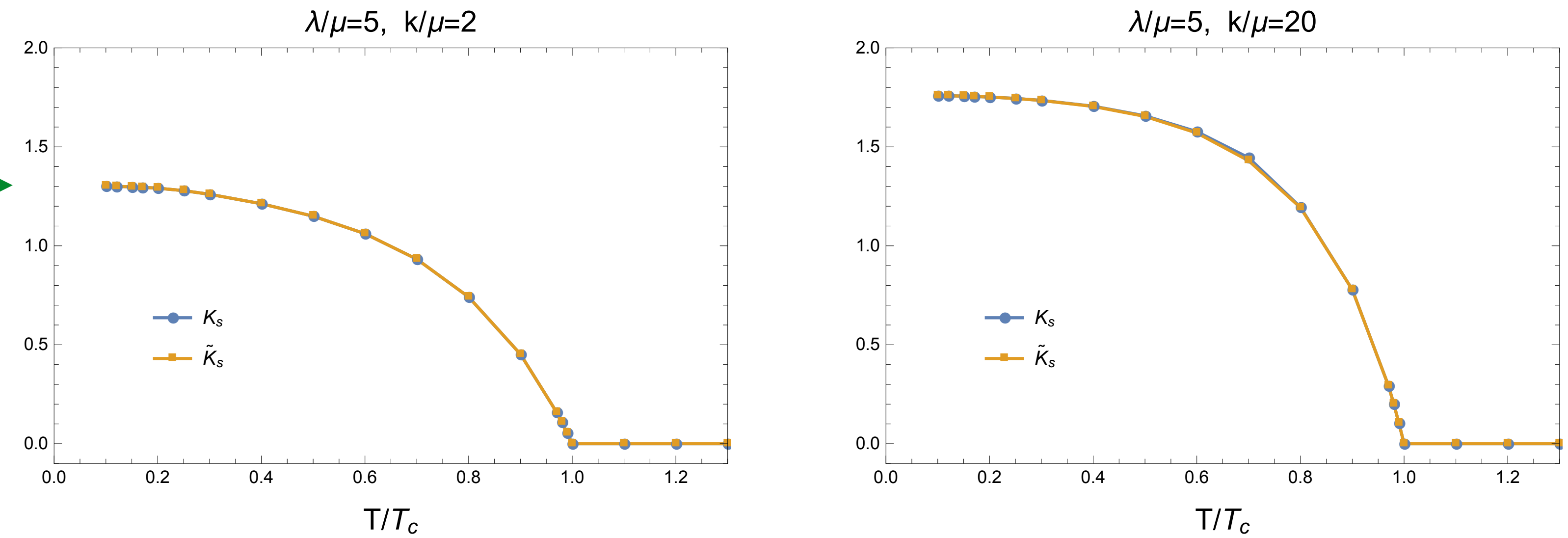
$$K_s = -\frac{a_x^{(1)}(\omega,0)}{a_x^{(0)}(\omega,0)}\bigg|_{\omega\to 0}$$

$$\tilde{K}_s = -\frac{a_x^{(1)}(0,p)}{a_x^{(0)}(0,p)}\bigg|_{p\to 0}$$

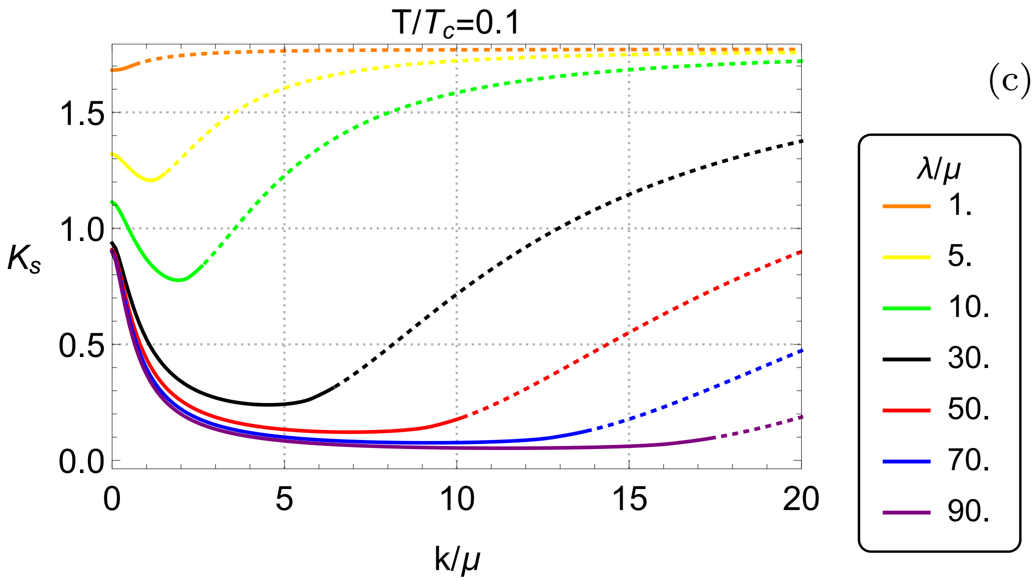


(a) no momentum relaxation (b) no momentum relaxation

extrapolate →



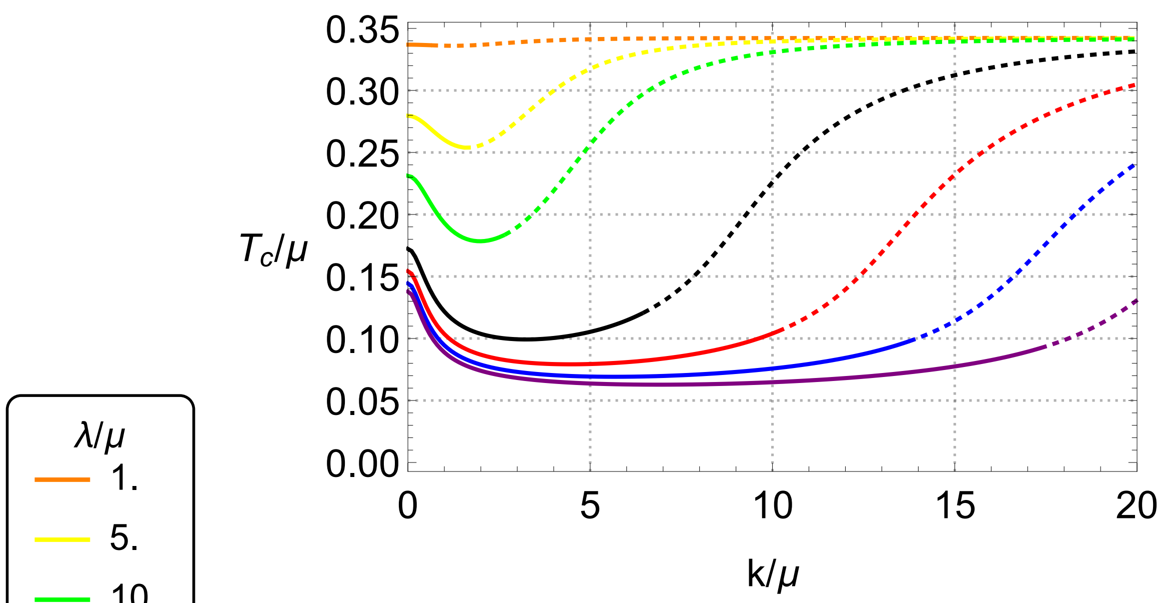
(c) large momentum relaxation (d) small momentum relaxation

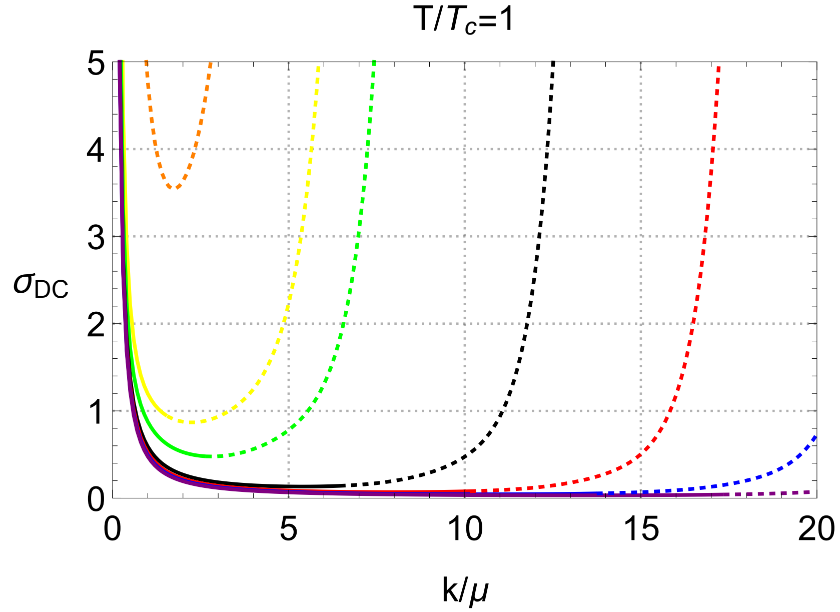


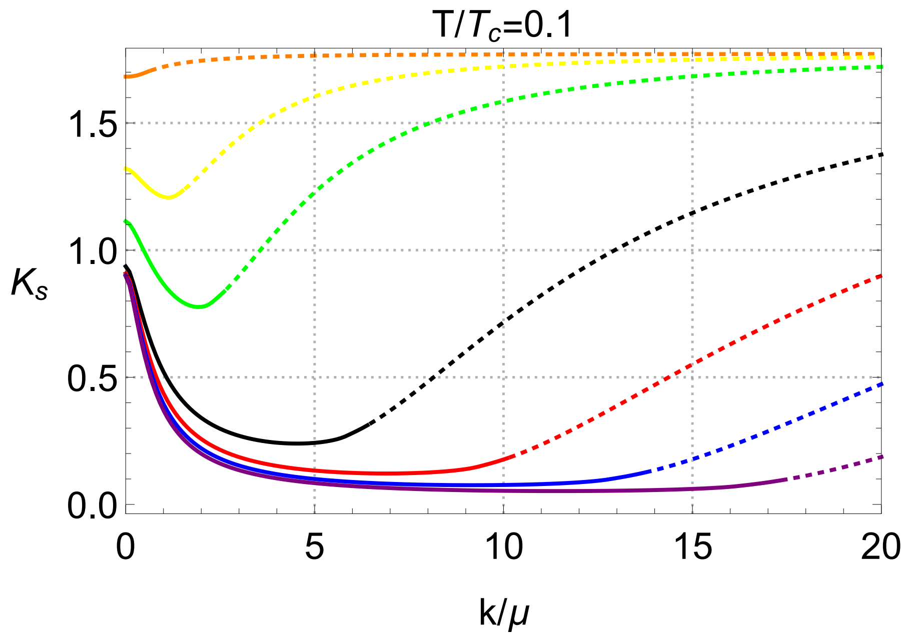
Homes' law?

[KYK, Niu: 1607.XXXXX]

Homes' law $K_s(T=0) = C\sigma_{DC}(T_c)T_c$

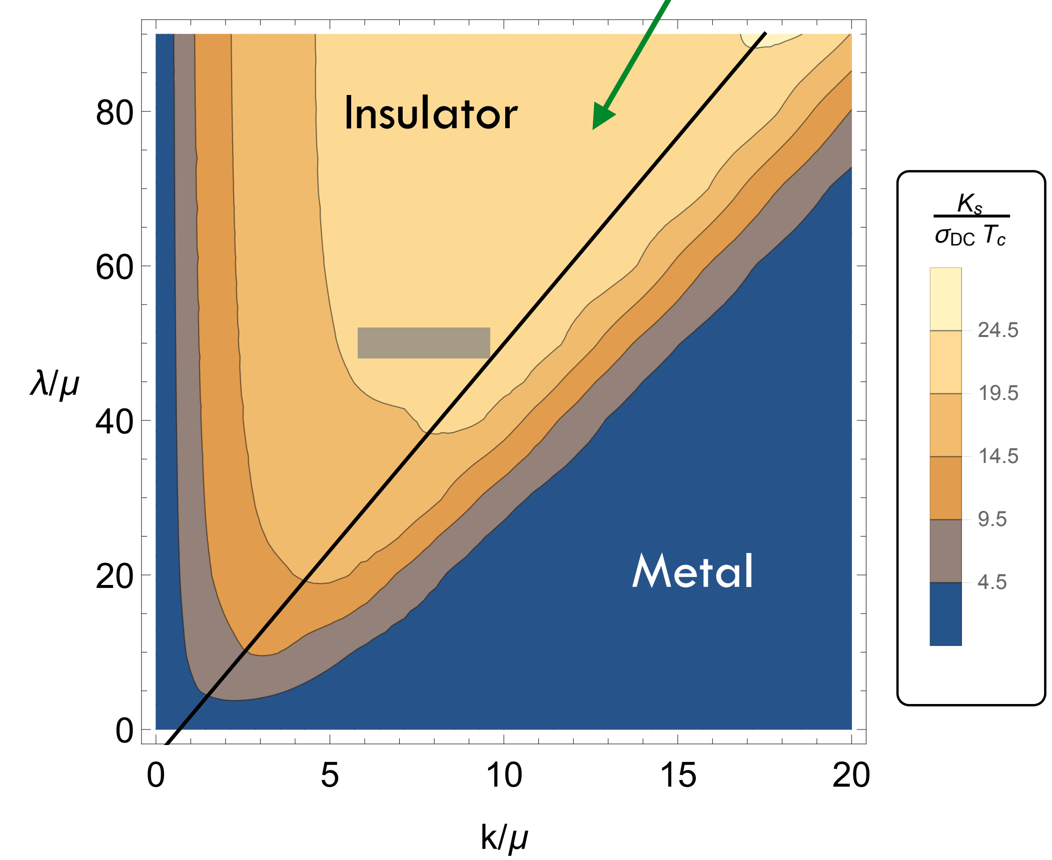






At fixed k, plateau for large λ

Small variation of C



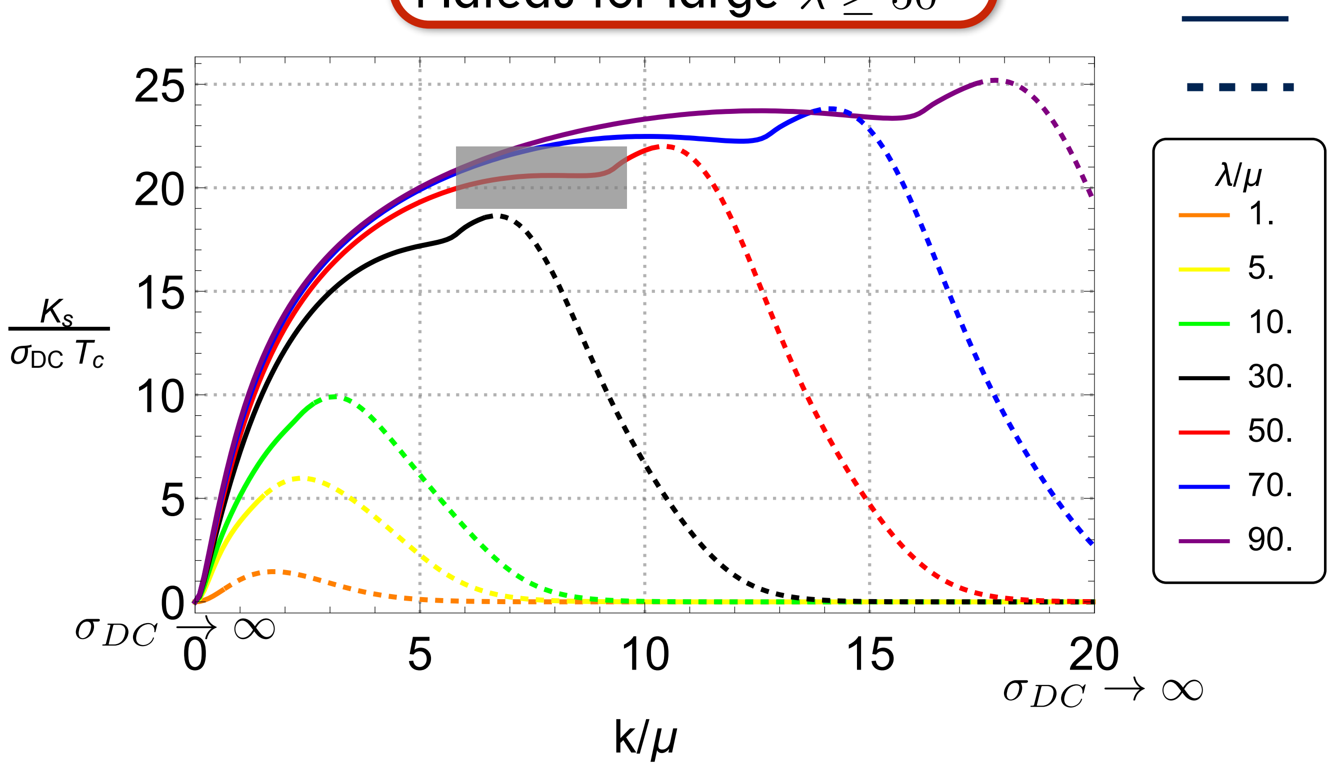
$$C = \frac{\rho_s(T=0)}{\sigma_{DC}(T_c)T_c}$$

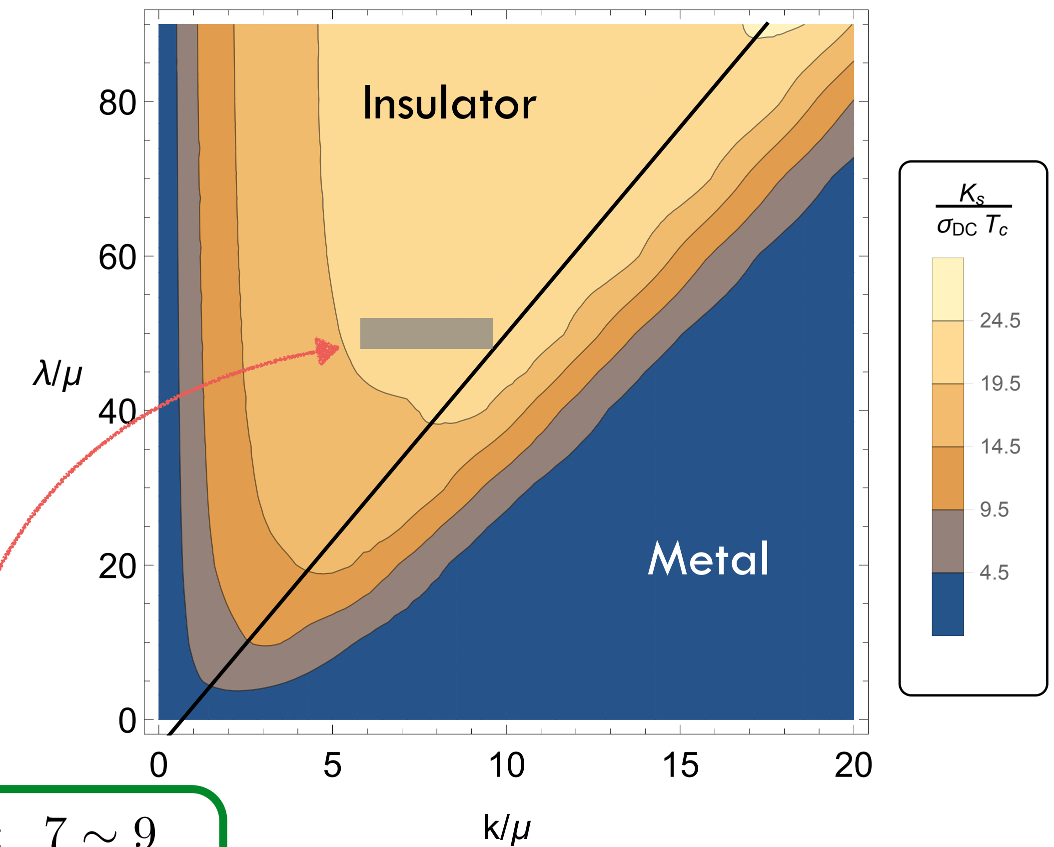
Homes' law: Insulating phase near transition ?

[KYK, Niu: 1607.XXXXX]

Homes' law

Plateau for large $\lambda \geq 50$



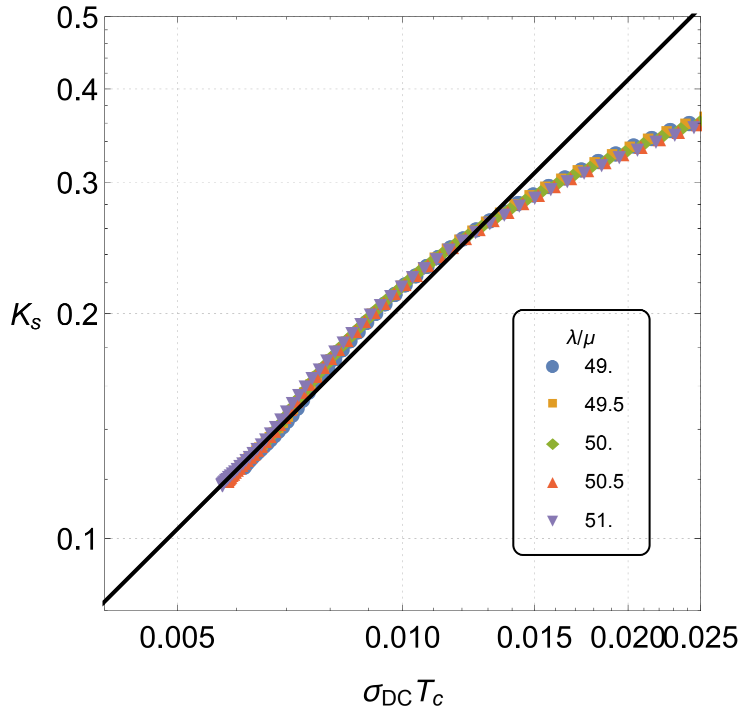


$k/\mu : \ 7 \sim 9$

$\lambda/\mu : 49 \sim 51$

Comparison: Helical lattice and Q-lattice

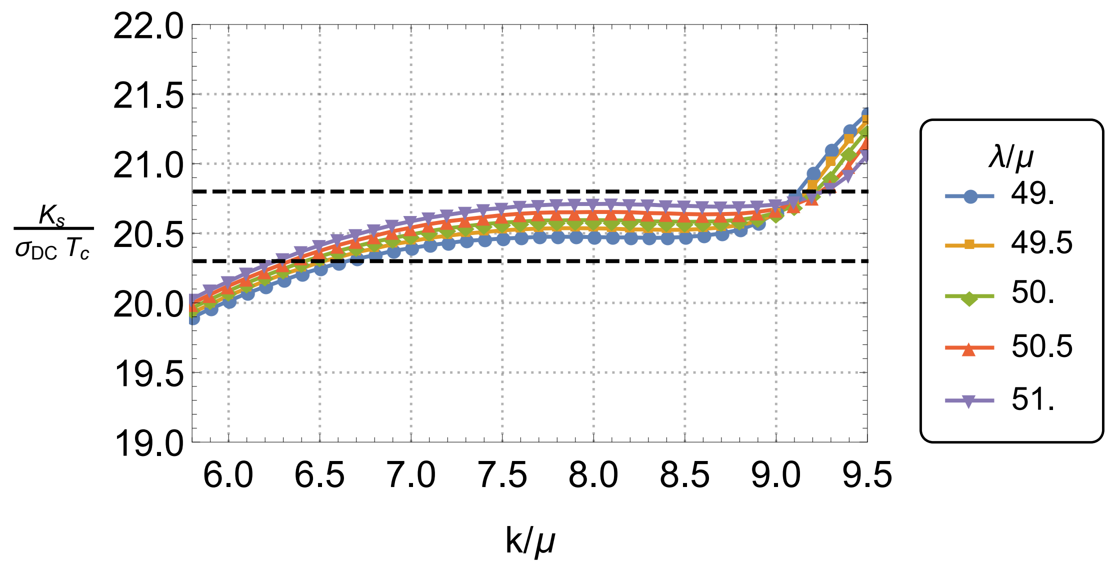
[KYK, Niu: 1607.XXXXX]

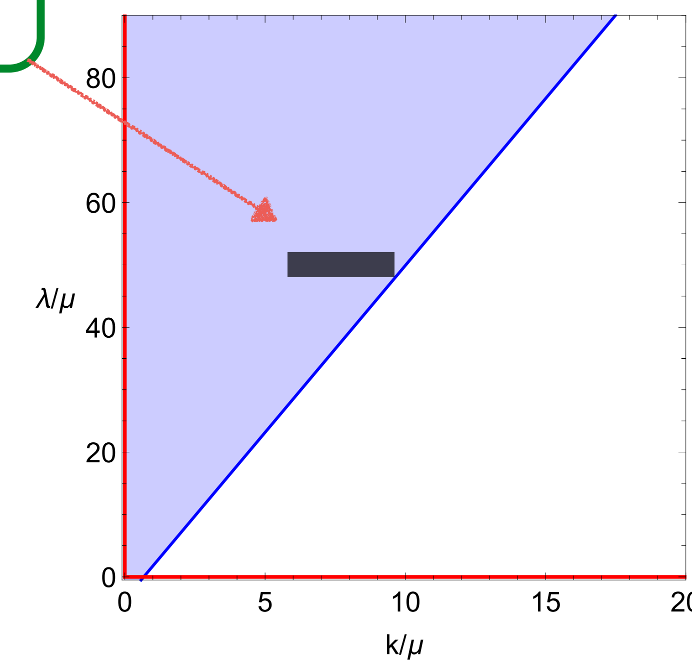


$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$k/\mu: \ 7 \sim 9$

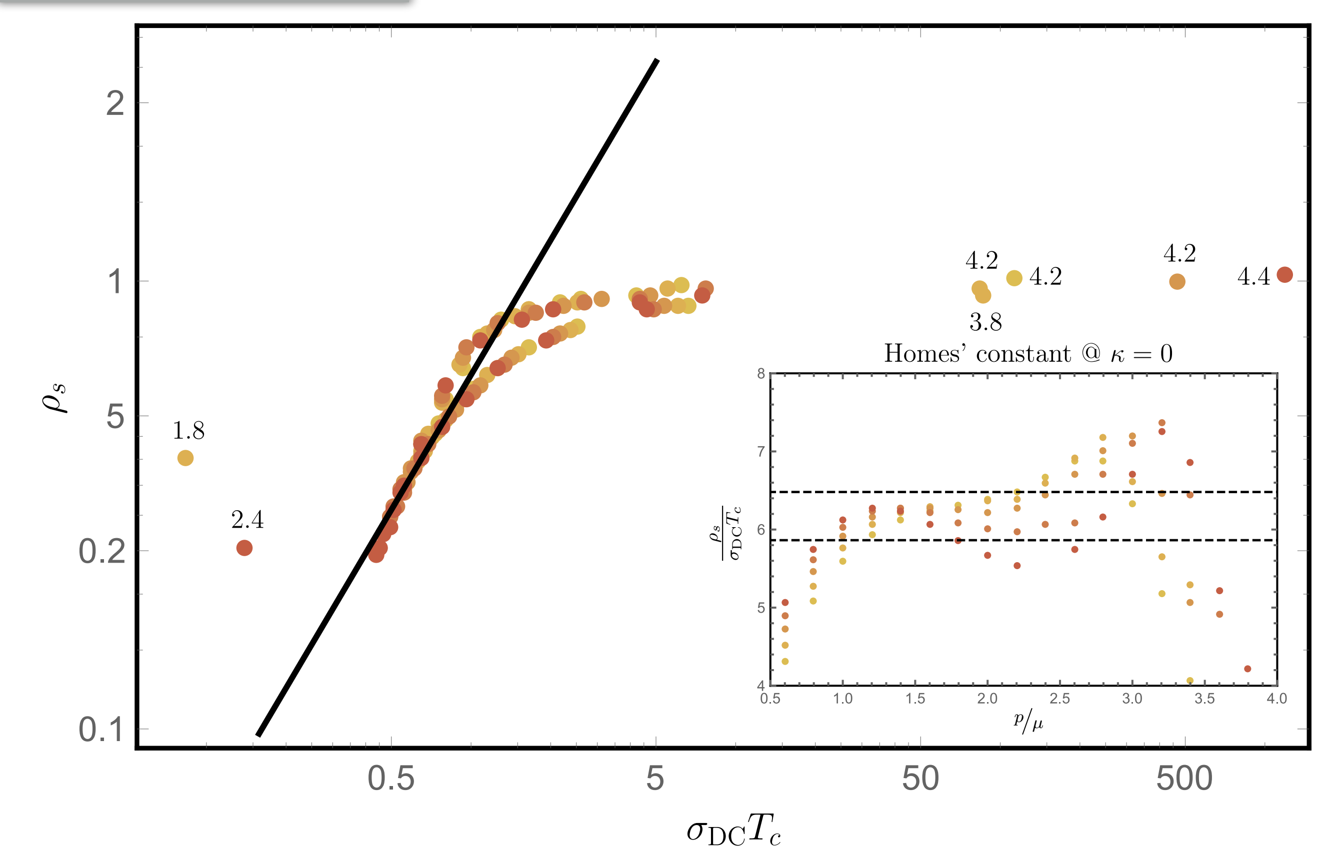
$\lambda/\mu: 49 \sim 51$





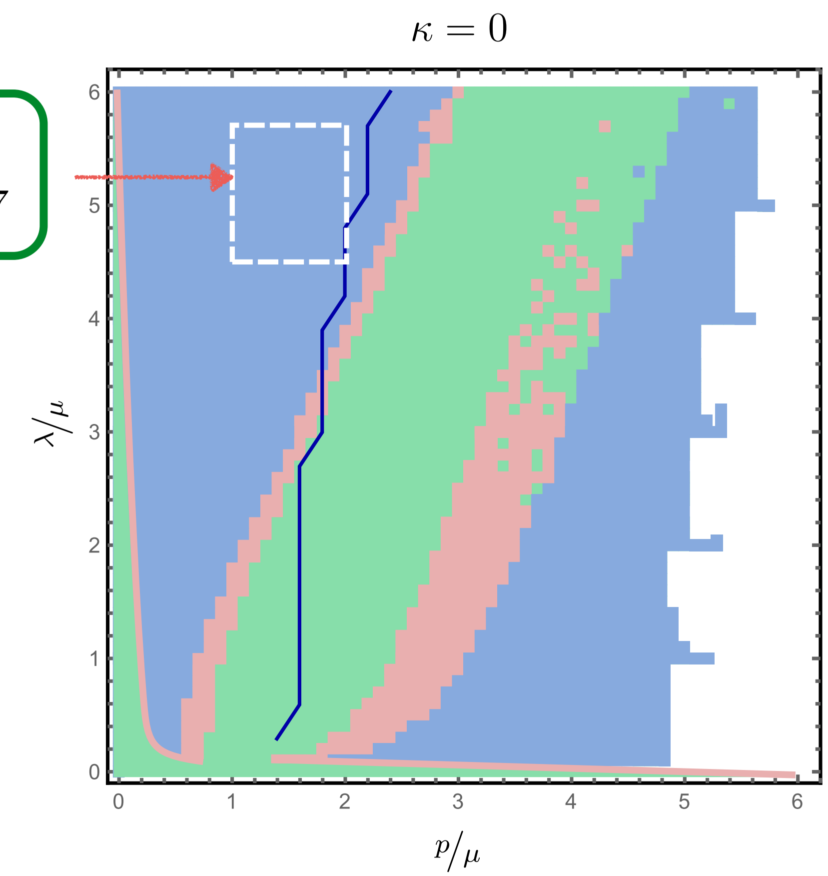
Helical lattice

Homes' relation for $q = 6$ & $\kappa = 0$



$p/\mu: \ 1 \sim 2$

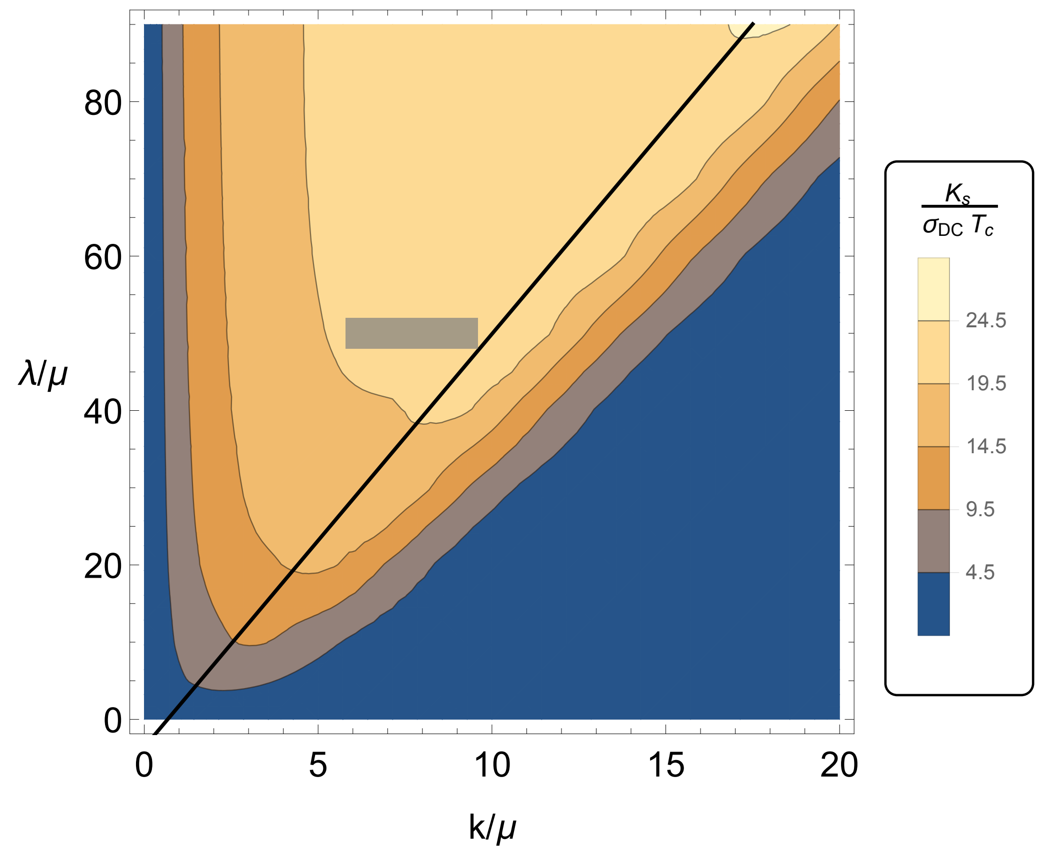
$\lambda/\mu: 4.5 \sim 5.7$

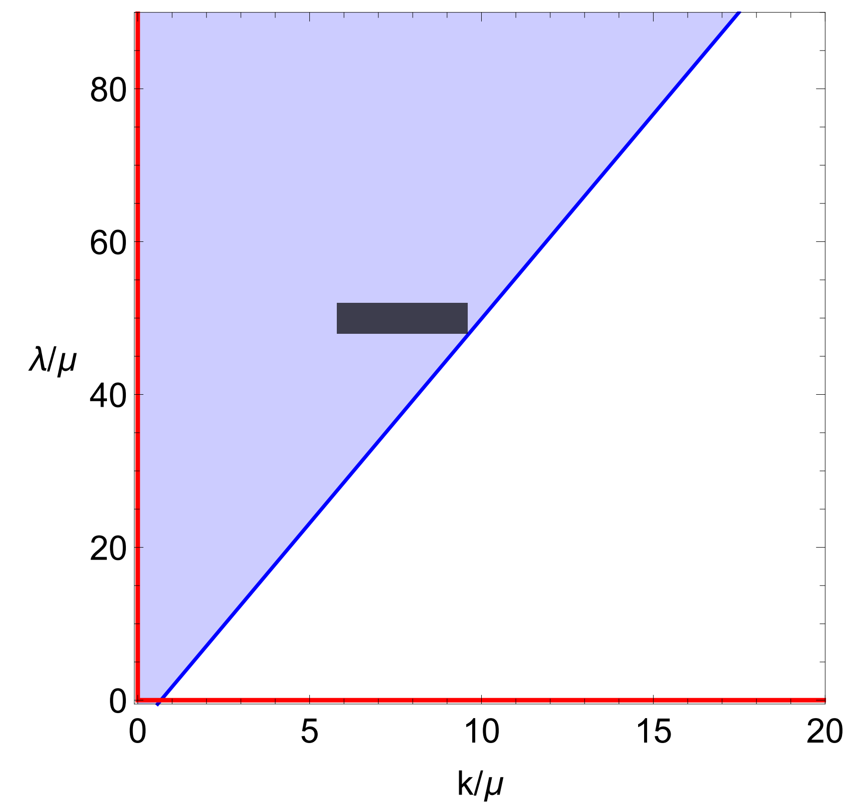


Comparison: Helical lattice and Q-lattice

[KYK, Niu: 1607.XXXXX]

Homes' law

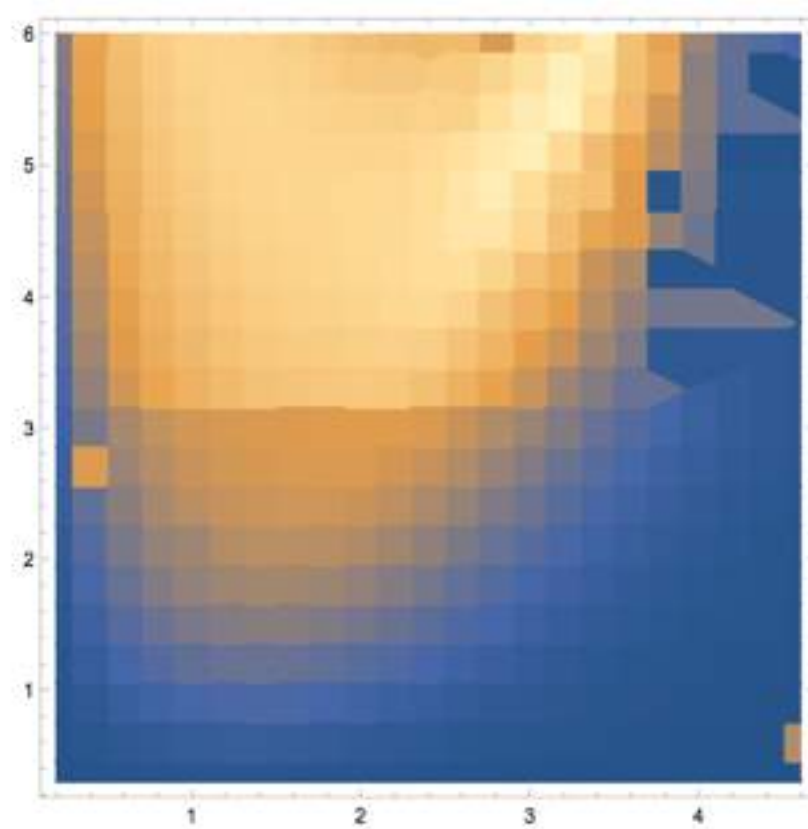


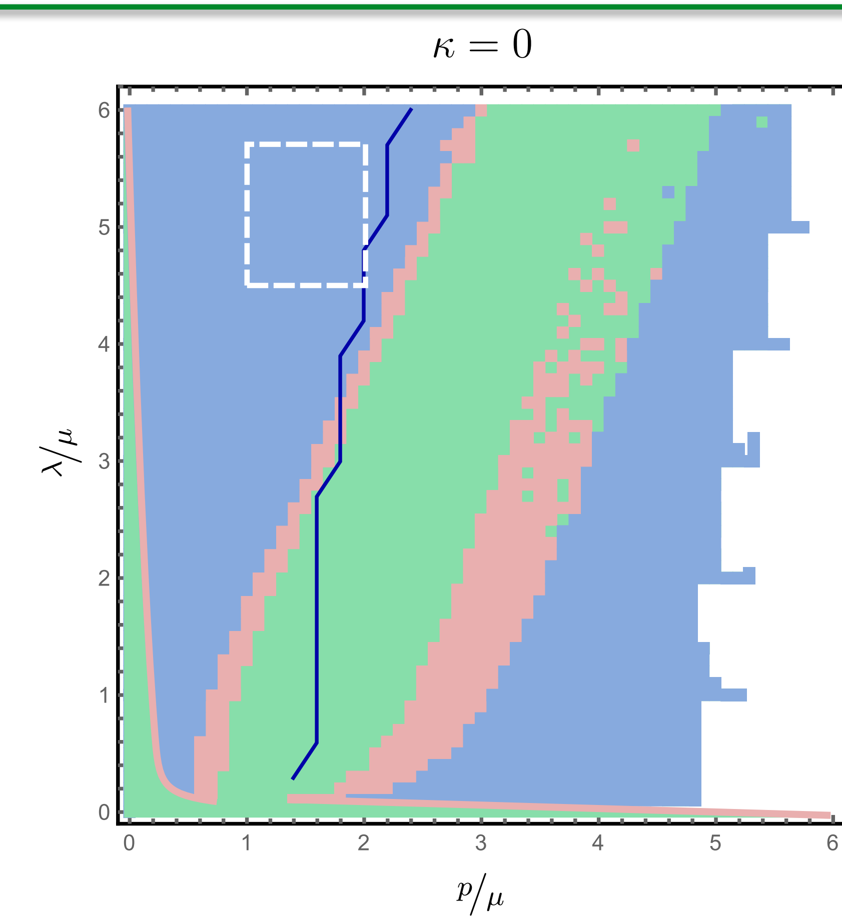


Helical lattice

Preliminary data

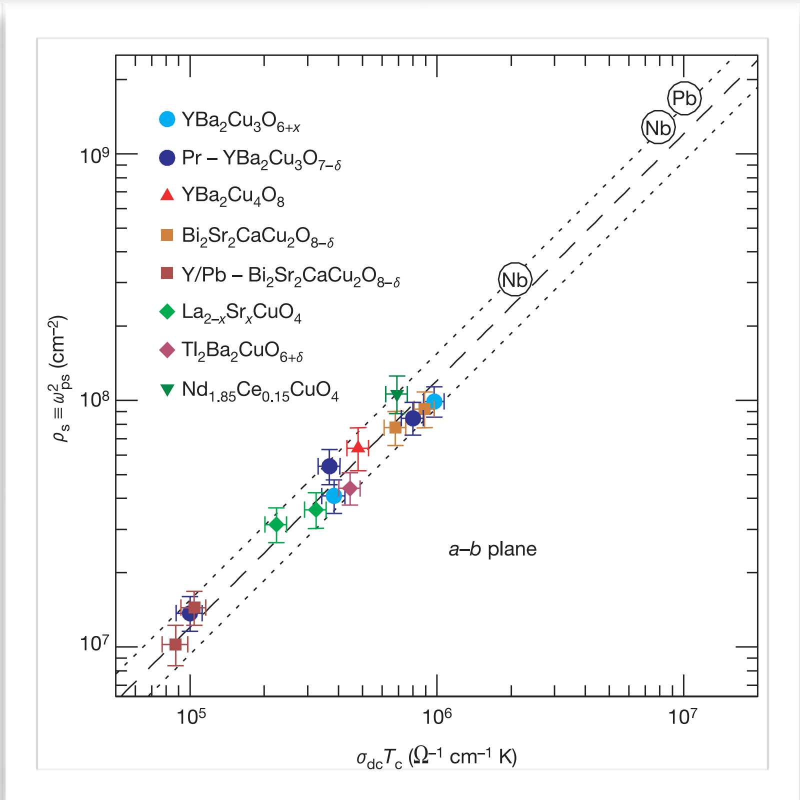
[Erdmenger, Meyer, Schalm, Shock: in progress]





Contents

Summary and outlook



- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

$\psi_I = (\beta x, \beta y)$

$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$$

$\Psi = e^{ikx}z\psi(z) \quad \psi(0) = \lambda$

$$S_{\rm helix} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2B_\mu B^\mu\right]$$

$B = w(r)\omega_2, \qquad w(\infty) = \lambda,$

$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$

~~Homes' law~~ Uemura's law (△)

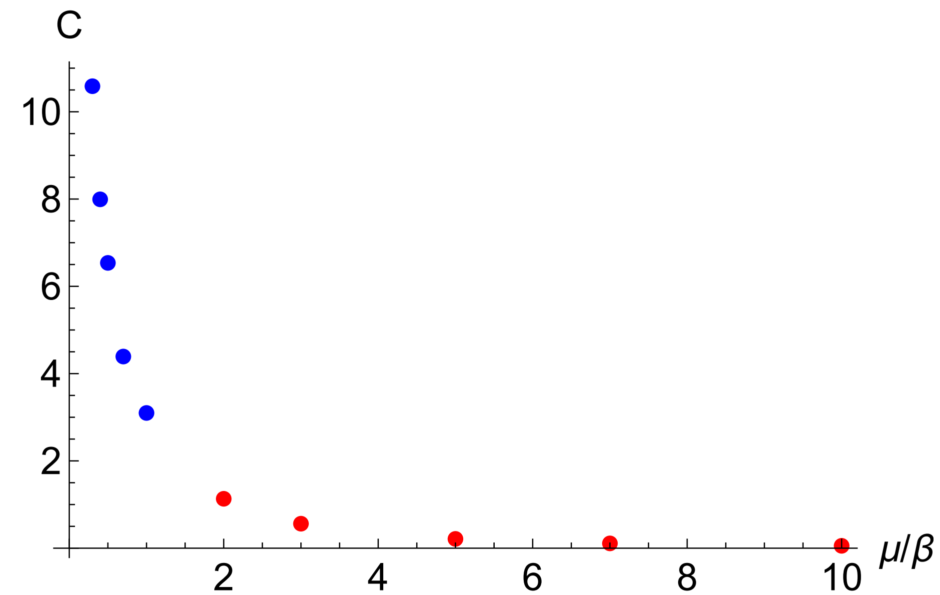
Homes' law (△) ~~Uemura's law~~

Homes' law (△) ~~Uemura's law~~

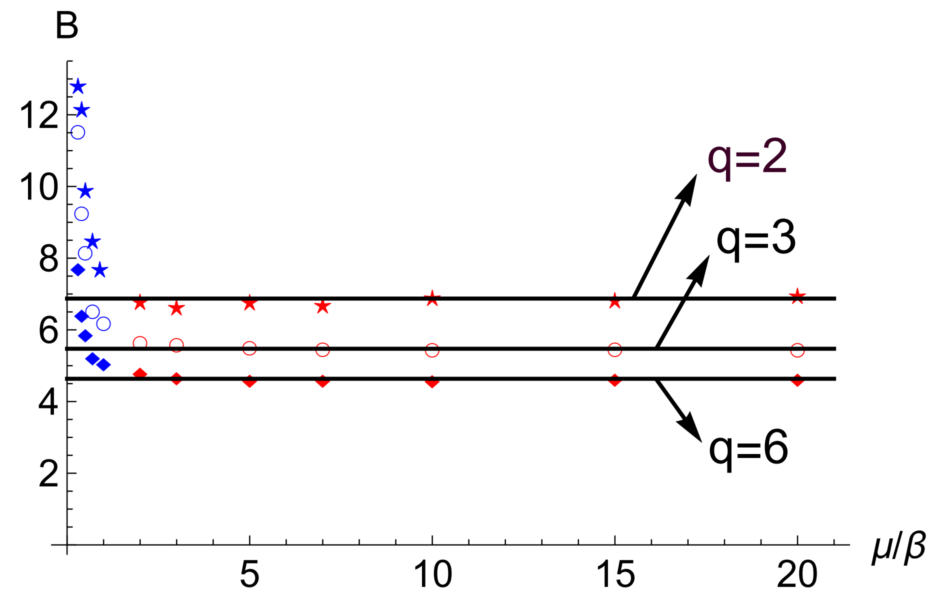
Summary and outlook

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

Massless scalar model



(a) $C(=\tilde{\rho}_s/(\sigma_{DC}\tilde{T}_c))$, $q=3$

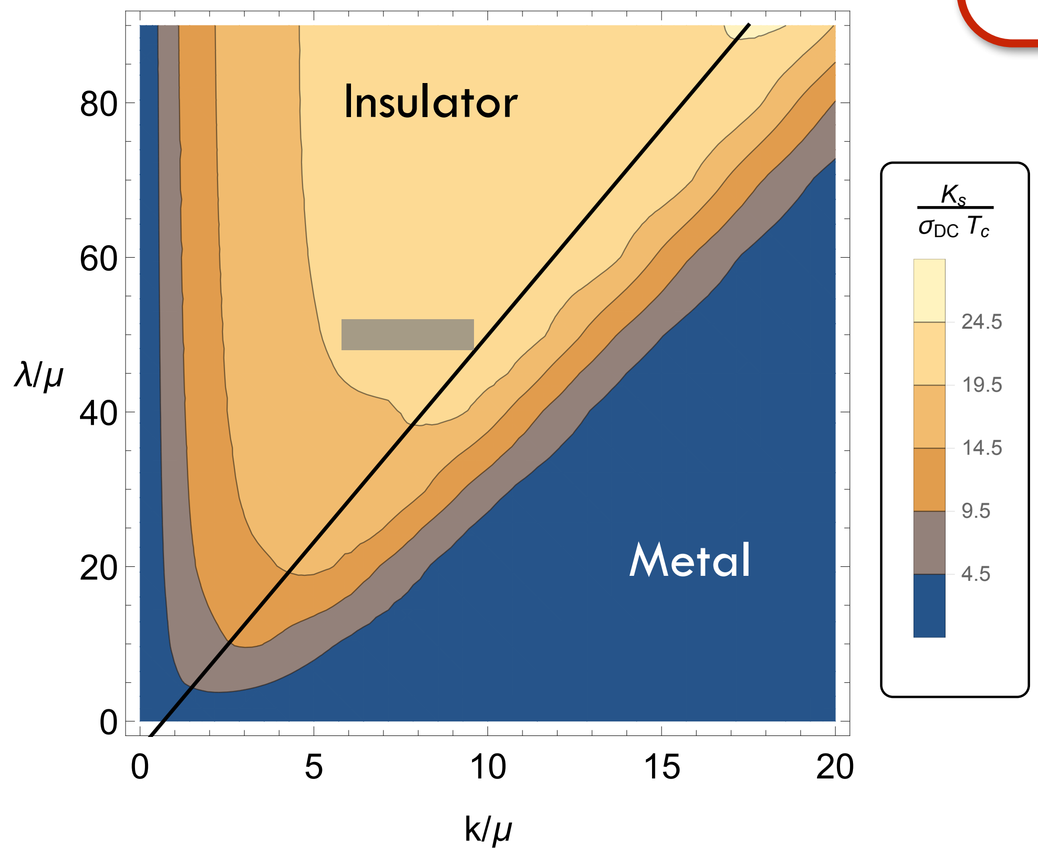


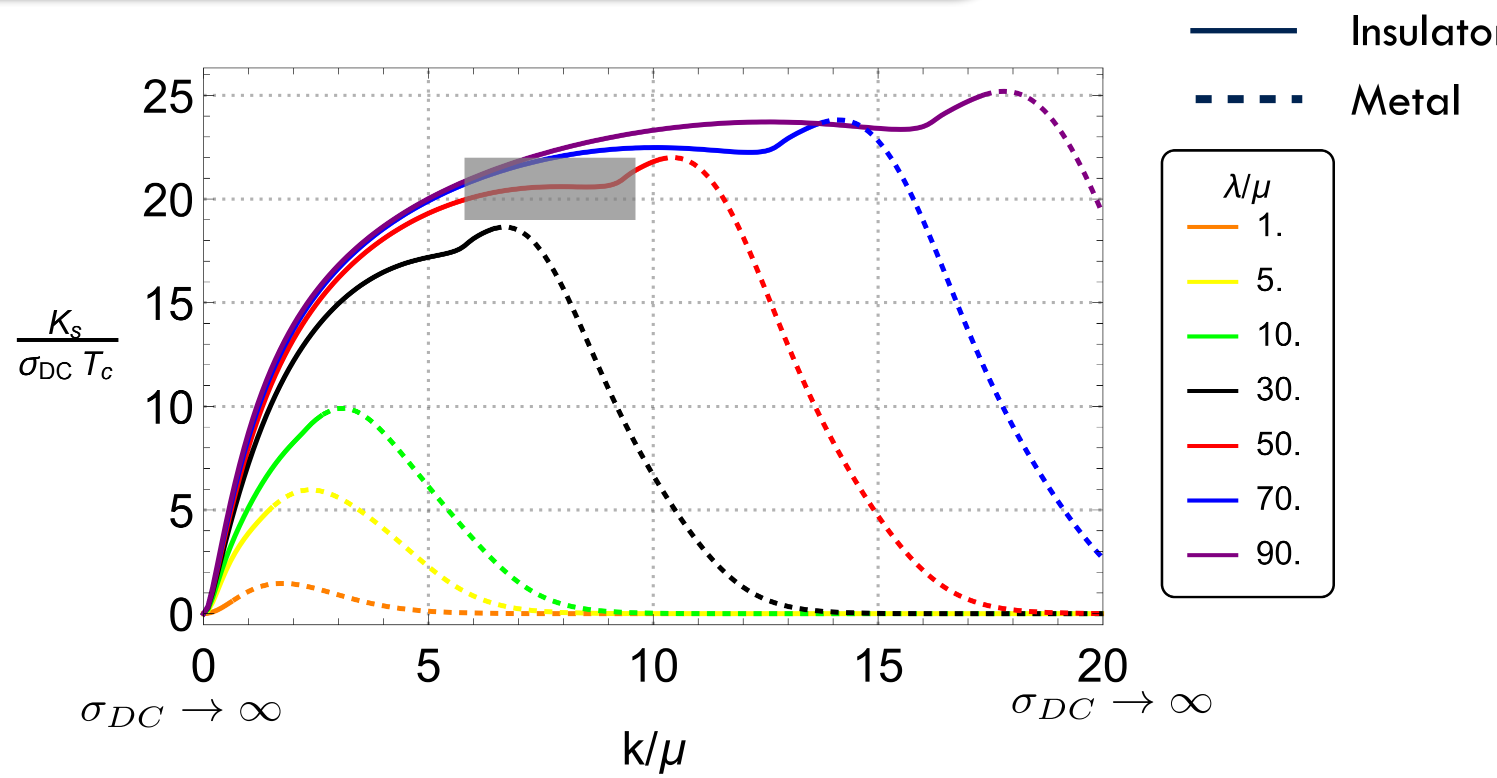
(b) $B(=\tilde{\rho}_s/\tilde{T}_c)$ for $q=2,3,6$

Summary and outlook

Q-lattice model

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$





At given k , plateau for large λ

Plateau for large $\lambda \geq 50$

- Similar to helical lattice model: wider range of parameters analysed
- Metal/insulator property without condensate seems to affect the properties of superconductor and Homes' law
- Uemura's law does not hold: DC conductivity plays a role
- Superfluid density cross-checked by two methods: missing spectral weight transferred to higher frequencies
- Other models with linear T resistivity will be more interesting

Thank you