Cold Holographic matter in top-down models

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with

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Motivation

We want to explore new phases of matter at finite density by using the holographic gauge/gravity duality

Holography allows to study strongly interacting systems with no quasiparticle description

Physical examples:

Quark-gluon plasma

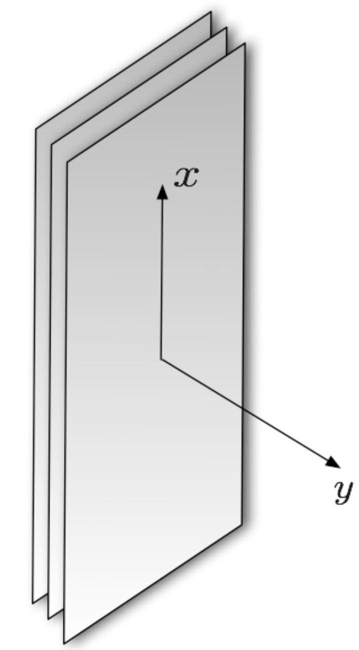
Strange metals

Heavy electron systems

They are non-Fermi liquids

(Top-down) Holographic model

SYM theory at strong coupling and large N → Stack of color Dp-branes



There is a 10d metric associated
Fields in the adjoint rep.

Charge carriers → Flavor Dq-branes

They add fields in the fundamental rep. (quarks) living in the intersection

mass of the quarks → distance between the Dp and Dq branes

Brane setup

Dp-Dq brane intersection of the type $(n\,|\,p \perp q)$

	x^1	$\cdots$	x^n	x^{n+1}	$\cdots$	x^p	y^1	$\cdots$	y^{q-n}	y^{q-n+1}	$\cdots$	y^{9-p}
$Dp:$	$\times$	$\cdots$	$\times$	$\times$	$\cdots$	$\times$	$-$	$\cdots$	$-$	$-$	$\cdots$	$-$
$Dq:$	$\times$	$\cdots$	$\times$	$-$	$\cdots$	$-$	$\times$	$\cdots$	$\times$	$-$	$\cdots$	$-$

Dp $\rightarrow$ N_c color branes ($p+1$-dimensional gauge theory on the bulk)

Dq $\rightarrow$ N_f flavor branes (fundamental hypermultiplets)

Probe approximation ($N_f << N_c$)

Dp $\rightarrow$ represented by a gravity solution

Dq $\rightarrow$ a probe in the Dp-brane background

Coordinates transverse to both branes

$\vec{z} = (z^1, \cdots, z^{9+n-p-q}) \rightarrow$ embedding functions $\quad (z = 0 \rightarrow$ massless quarks$)$

Probe action

$$S = T_{Dq} \int d^{q+1}\xi \, e^{-\phi} \sqrt{-\det(g+F)}$$

No WZ term

$g \longrightarrow$ induced metric $\qquad \phi \longrightarrow$ dilaton

$F \longrightarrow$ worldvolume gauge field

For the Dp-brane background (massless quarks)

$$ds^2_{q+1} = \rho^{\frac{7-p}{2}} \left[-f_p(\rho) dt^2 + (dx^1)^2 + \cdots (dx^n)^2 \right] + \rho^{\frac{p-7}{2}} \left[\frac{d\rho^2}{f_p(\rho)} + \rho^2 \, d\Omega^2_{q-n-1} \right]$$

$$f_p = 1 - \left(\frac{r_h}{\rho} \right)^{7-p} \qquad e^{-2\phi} = \left(\frac{R}{\rho} \right)^{\frac{(7-p)(p-3)}{2}}$$

r_h is related to the temperature $\longrightarrow$ $T = \frac{7-p}{4\pi} \, r_h^{\frac{5-p}{2}}$

Baryonic charge density $\rightarrow$ dual the DBI gauge field A^t

$$\langle J^t \rangle = \frac{\delta S}{\delta A_t'} \qquad\qquad A_t' \equiv \partial_\rho A_t$$

The dynamics of the probe depends on p and λ

$$\lambda = 2n + \frac{1}{2}(p-3)(p+q-2n-8)$$

Ansatz for F $\longrightarrow$ $F = A_t'\, d\rho \wedge dt + B\, dx^1 \wedge dx^2$

Action $\longrightarrow$ $S_{Dq} = -\mathcal{N}\, V_{\mathbb{R}^{(n,1)}} \int d\rho \sqrt{\rho^\lambda + B^2 \rho^{\lambda+p-7}} \sqrt{1 - A_t'^2}$

A_t is a cyclic variable

$$\frac{\sqrt{\rho^\lambda + B^2 \rho^{\lambda+p-7}}}{\sqrt{1 - A_t'^2}} A_t' = d \longrightarrow A_t' = \frac{d}{\sqrt{\rho^\lambda + B^2 \rho^{\lambda+p-7} + d^2}} \qquad \langle J^t \rangle = \mathcal{N}\, d$$

Thermodynamics at zero T

Chemical potential (for $B = 0$)

$$\mu = A_t(\infty) = \int_0^\infty d\rho\, A_t' = \gamma\, d^{\frac{2}{\lambda}}$$

$$\gamma = \frac{1}{\sqrt{\pi}}\,\Gamma\Big(\frac{1}{2} - \frac{1}{\lambda}\Big)\,\Gamma\Big(1 + \frac{1}{\lambda}\Big)$$

Grand Canonical potential

$$\Omega = -S^{reg}_{on-shell} = \mathcal{N} \int_0^\infty \rho^{\frac{\lambda}{2}} \left[\frac{\rho^{\frac{\lambda}{2}}}{\sqrt{\rho^\lambda + d^2}} - 1\right] d\rho$$

$$\Omega = -\frac{2}{\lambda+2}\,\mathcal{N}\,\gamma\, d^{1+\frac{2}{\lambda}} = -\frac{2}{\lambda+2}\,\mathcal{N}\,\gamma^{-\frac{\lambda}{2}}\,\mu^{1+\frac{\lambda}{2}}$$

$\Longrightarrow$

$$\rho = -\frac{\partial \Omega}{\partial \mu} = \mathcal{N} d$$

Energy density

$$\epsilon = \Omega + \mu\rho = \frac{\lambda}{\lambda+2}\,\mathcal{N}\,\gamma\, d^{1+\frac{2}{\lambda}}$$

Pressure

$$\boxed{p = -\Omega = \frac{2}{\lambda}\,\epsilon}$$

Speed of sound

$$u_s^2 = \frac{\partial p}{\partial \epsilon} \quad \Longrightarrow \quad \boxed{u_s = \pm\sqrt{\frac{2}{\lambda}}}$$

Values of λ

SUSY intersections

$$(n\,|\,p \perp q) \text{ with } n = \frac{p+q-4}{2} \to \lambda = q - p + 2$$

- $$Dp - D(p+4) \to (p\,|\,p \perp (p+4)) \to \lambda = 6$$

  Examples $D3 - D7$, $D2 - D6$

- $$Dp - D(p+2) \to (p-1\,|\,p \perp (p+2)) \to \lambda = 4$$

  Examples $D3 - D5$, $D4 - D6$

- $$Dp - Dp \to (p-2\,|\,p \perp p)) \to \lambda = 2$$

  Example $D3 - D3$

Non-Susy examples

Model	λ	p	q	n
Sakai-Sugimoto D4-D8/$\overline{\text{D8}}$	5	4	8	3
D3-D7'	4	3	7	2
D2-D8'	5	2	8	2

Notice that for $p = 3 \rightarrow \lambda = 2n$

Scaling behavior

Energy-radius relation $\mathcal{E} \sim \rho^{\frac{5-p}{2}}$

Energy rescaling

$$\mathcal{E} \to \Lambda\, \mathcal{E} \qquad \rho \to \Lambda^{\Delta_\rho}\, \rho \qquad \Delta_\rho = \frac{2}{5-p}$$

Density&magnetic field

$$d \to \Lambda^{\Delta_d}\, d \qquad B \to \Lambda^{\Delta_B}\, B \qquad \Delta_d = \frac{\lambda}{5-p} \qquad \Delta_B = \frac{7-p}{5-p}$$

$\lambda \to$ related to the scaling dimension of d

$$p = 3 \to \lambda = 2n \quad \Longrightarrow \quad \Delta_d = n \quad \Delta_B = 2$$

canonical dimensions

SUSY intersections

$$\Delta_d^{SUSY} = \frac{q-p+2}{5-p} = \frac{2}{5-p}\,(n+3-p)$$

Excitations → Poles of the retarded Green's functions → quasinormal modes → density waves in the dual field theory

Perturb as $A_\nu = A_\nu^{(0)} + a_\nu(\rho, x^\mu)$

Define $\mathcal{G}$ and $\mathcal{J}$ as $\left(g^{(0)} + F^{(0)}\right)^{-1} = \mathcal{G}^{-1} + \mathcal{J}$

$\mathcal{G} \rightarrow$ open string metric (symmetric part) $\quad \mathcal{J} \rightarrow$ antisymmetric part

Lagrangian

$$\mathcal{L} \sim \frac{\rho^\lambda + B^2\,\rho^{\lambda+p-7}}{\sqrt{\rho^\lambda + B^2\,\rho^{\lambda+p-7} + d^2}}\left(\mathcal{G}^{ac}\mathcal{G}^{bd} - \mathcal{J}^{ac}\mathcal{J}^{bd} + \frac{1}{2}\mathcal{J}^{cd}\mathcal{J}^{ab}\right) f_{cd} f_{ab}$$

Take $a_\nu = a_\nu(\rho, t, x)$ and Fourier transform $\quad a_\nu(\rho, t, x) = \int \frac{d\omega dk}{(2\pi)^2} a_\nu(\rho, \omega, k) e^{-i\omega\, t + ikx}$

Solve the equations with the conditions:

- In-falling boundary conditions at the horizon
- No sources at the UV boundary
- Low ω, k

For $T = 0, \ B = 0$ → Holographic zero sound

Take ω, k small and of the same order $\quad \omega \sim \mathcal{O}(\epsilon), \ k \sim \mathcal{O}(\epsilon)$

$$\omega(k) = \omega_R(k) \ - \ i\Gamma(k)$$

$\omega_R(k) \rightarrow$ real part

$\Gamma(k) \rightarrow$ attenuation (decay rate)

$$\omega_R = \pm\sqrt{\frac{2}{\lambda}}\, k$$

Same speed as the first sound

$$\Gamma = \frac{\pi}{2\mu}\, \frac{(5-p)^{\frac{p-3}{5-p}}}{\left[\Gamma\left(\frac{1}{5-p}\right)\right]^2} \left(\frac{2}{\lambda}\right)^{\frac{7-p}{2(5-p)}} k^{\frac{7-p}{5-p}}$$

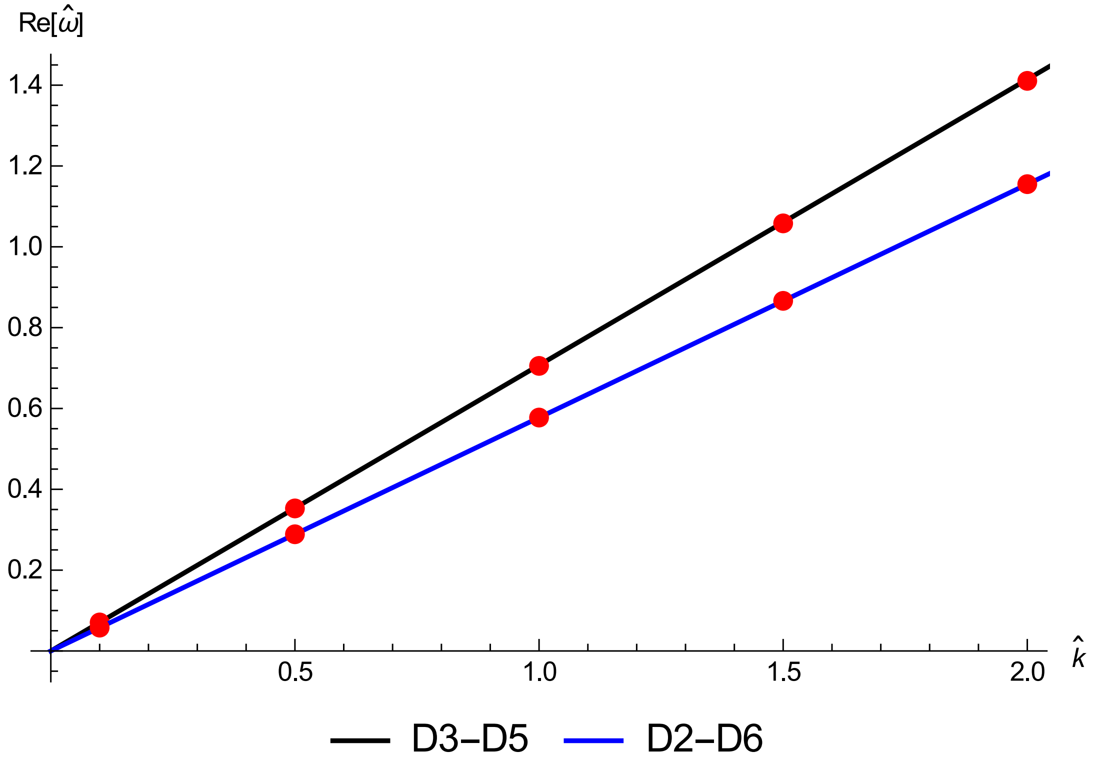
Zero sound for D3-D7 ($\lambda = 6$, $p = 3$) → $\omega = \pm\frac{k}{\sqrt{3}} - \frac{i}{6}\frac{k^2}{\mu}$

Karch, Son, Starinets

Zero sound for D3-D5 ($\lambda = 4$, $p = 3$) → $\omega = \pm\frac{k}{\sqrt{2}} - \frac{i}{4}\frac{k^2}{\mu}$

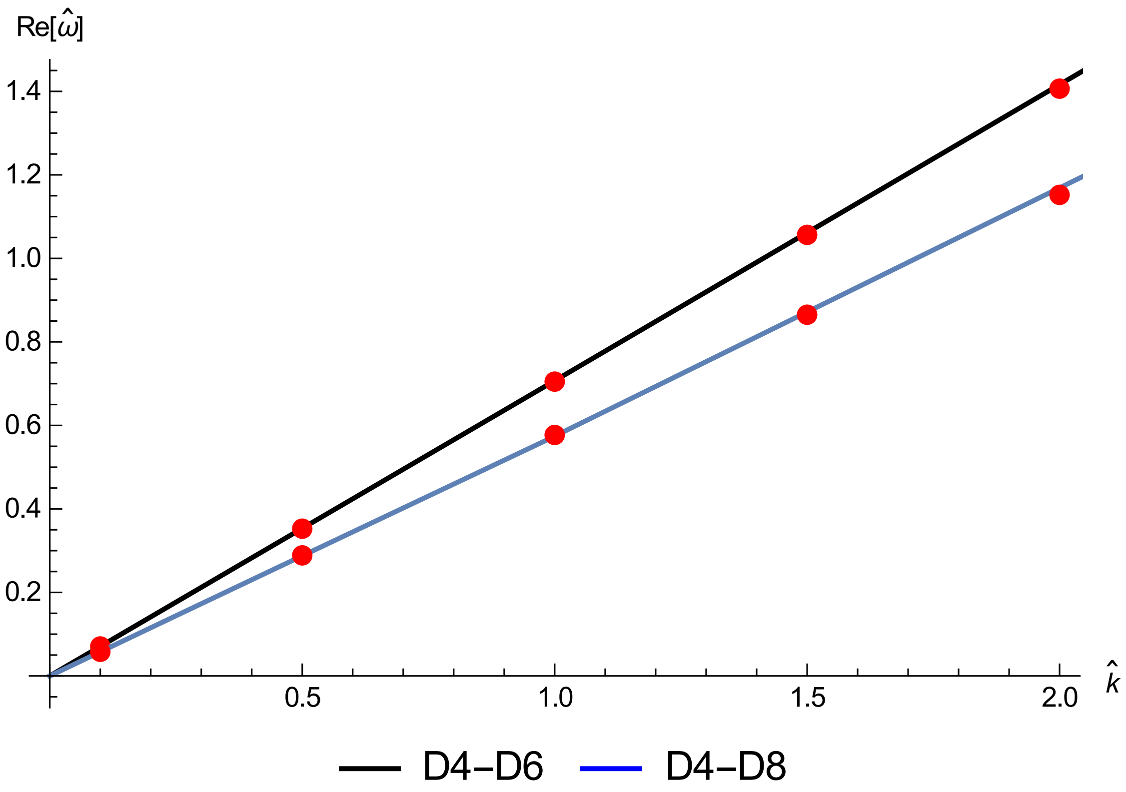
Different cases considered in Brattan et al., Kulaxizi &Parnachev, Goykhman et al, ...

Speed of zero sound for different λ and p



$$D3 - D5 \;\rightarrow\; p = 3, \lambda = 4$$

$$D2 - D6 \;\rightarrow\; p = 2, \lambda = 6$$



$$D4 - D6 \;\rightarrow\; p = 4, \lambda = 4$$

$$D4 - D8 \;\rightarrow\; p = 4, \lambda = 6$$

Same curves for the same λ

For $T \neq 0, \; B = 0$ → Hydrodynamic charge diffusion mode

Purely imaginary pole with $\omega \sim \mathcal{O}(\epsilon^2)$, $k \sim \mathcal{O}(\epsilon)$

$$\omega = -i\, D\, k^2$$ → Fick's law

$D \longrightarrow$ diffusion constant

$$D = \frac{7-p}{2\pi(\lambda-2)} \frac{(1+\hat{d}^{\,2})^{\frac{1}{2}}}{T} F\Big(\frac{3}{2}, \frac{1}{2}-\frac{1}{\lambda}; \frac{3}{2}-\frac{1}{\lambda}; -\hat{d}^2\Big)$$

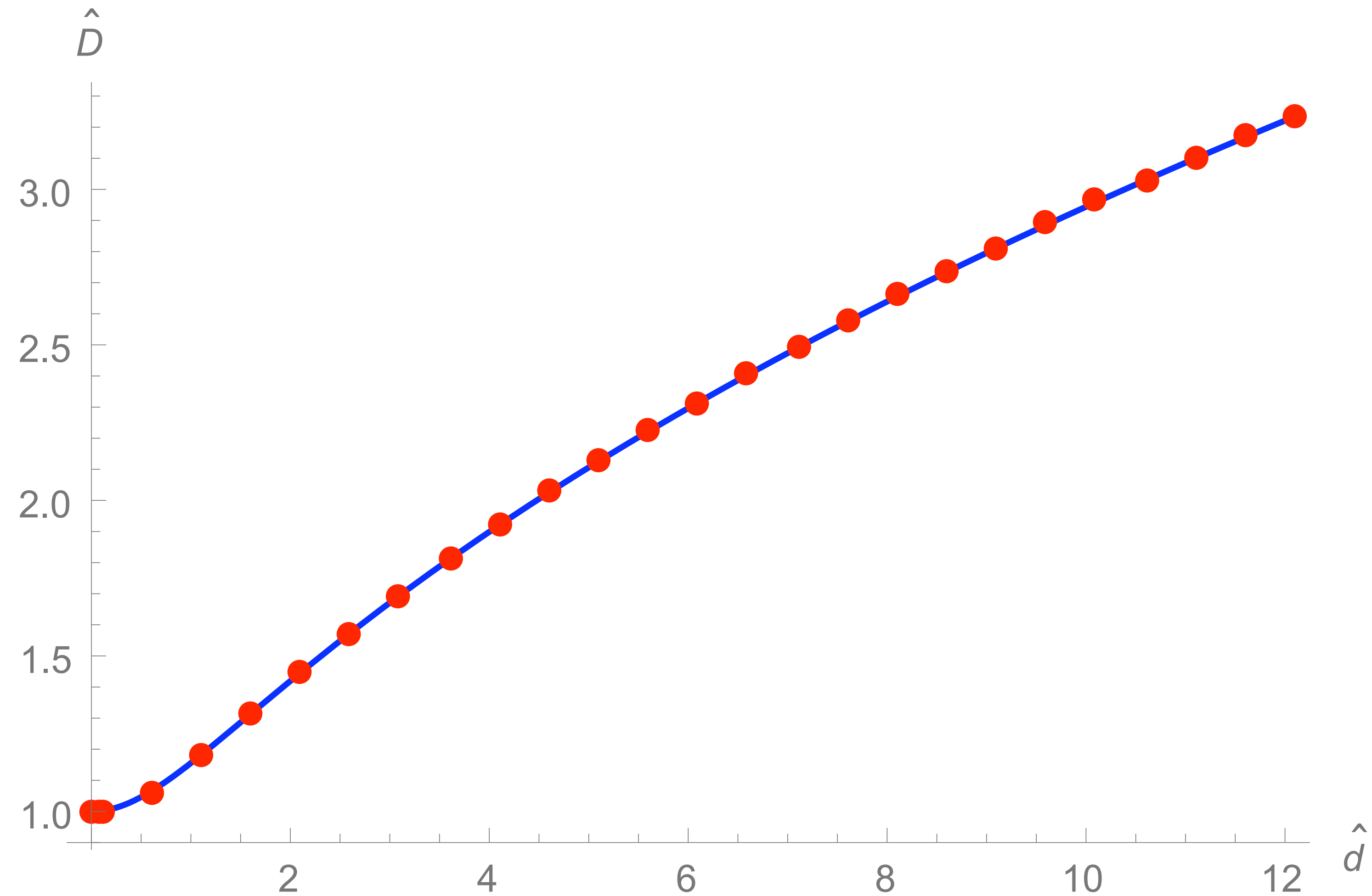
$$\hat{d} = \frac{d}{r_h^{\frac{\lambda}{2}}} = \Big(\frac{7-p}{4\pi T}\Big)^{\frac{\lambda}{5-p}} d$$

$D \sim T^{-1}$ $\qquad T$ large

$D \sim T^{-\frac{7-p}{5-p}}$ $\qquad T$ small

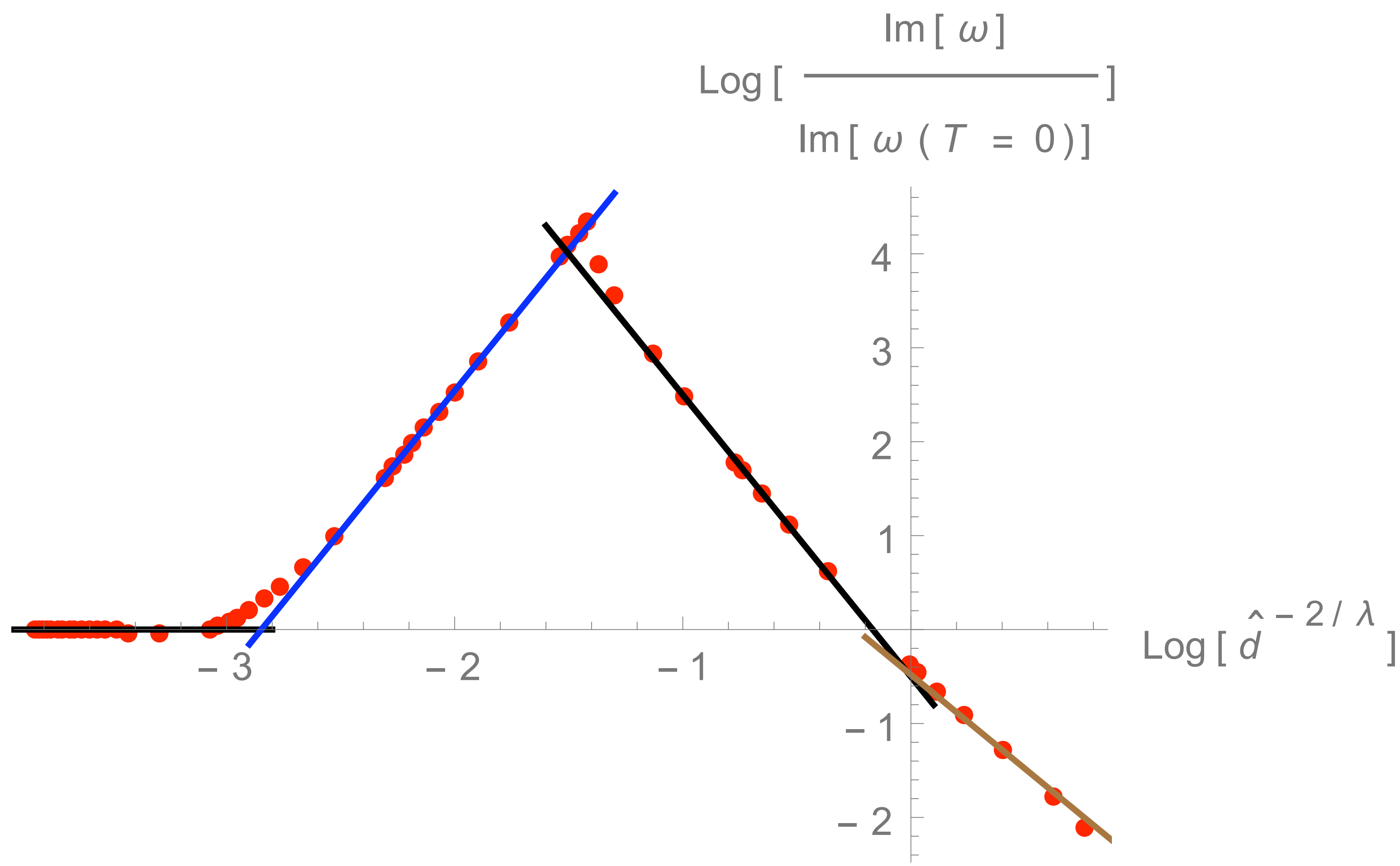
Diffusion constant for D2-D6

$$\hat{D} = r_h^{\frac{5-p}{2}} D = \frac{4\pi T}{7-p} D$$



$$\hat{d} = \frac{d}{r_h^{\frac{\lambda}{2}}} = \left(\frac{7-p}{4\pi T}\right)^{\frac{\lambda}{5-p}} d$$

Lowest excitations in D1-D5



Colisionless quantum regime → Hydrodynamic diffusive regime

$$\omega_{cr} \sim \frac{T^{\frac{7-p}{5-p}}}{\mu} \qquad k_{cr} \sim \frac{T^{\frac{7-p}{5-p}}}{\mu}$$

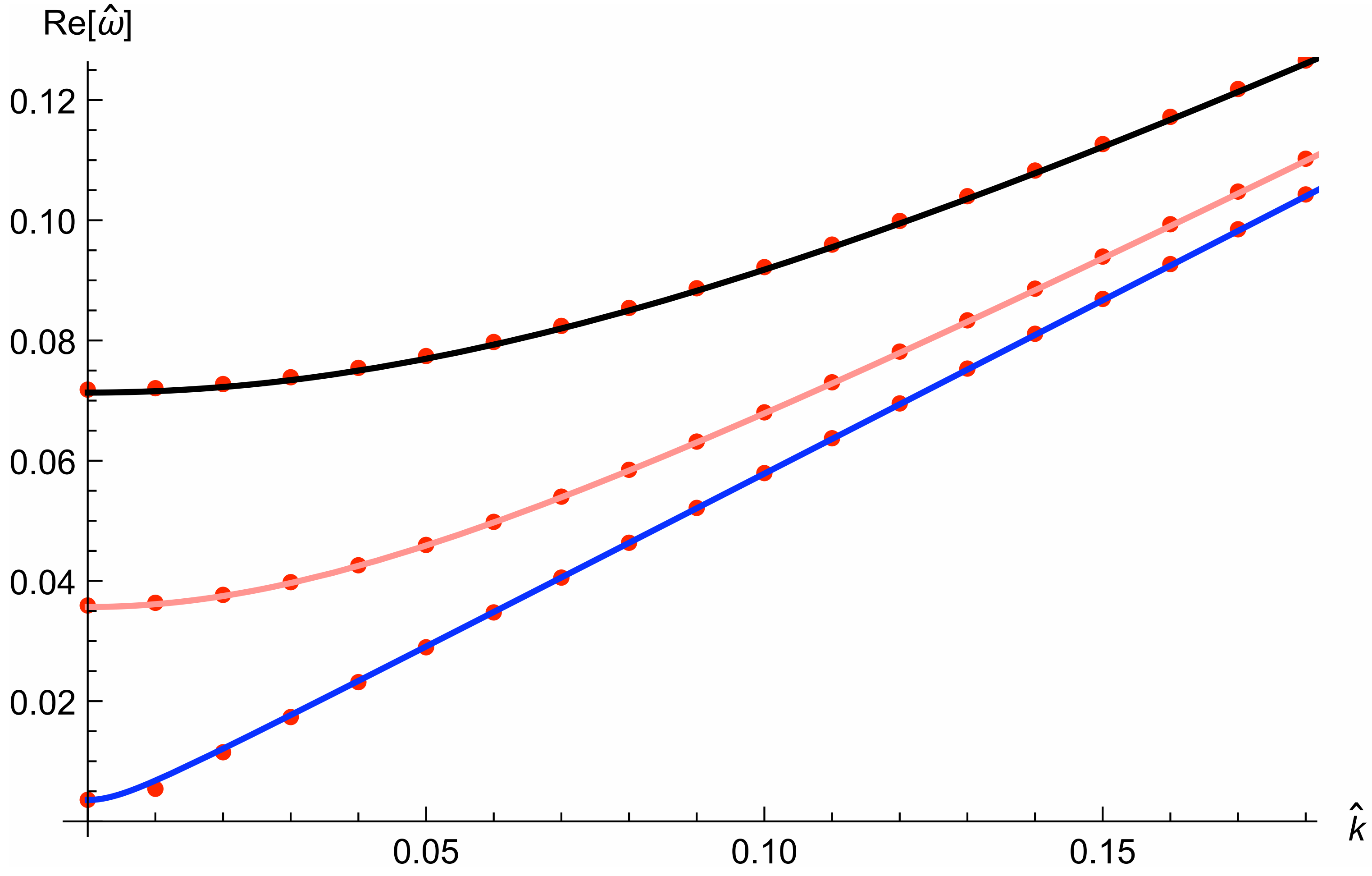
$T = 0$ and $B \neq 0$ → Zero sound with B field

Gapped dispersion relation

$$\omega_R = \pm\sqrt{\frac{2}{\lambda}\,k^2 + \frac{B^2}{\mu^2}}$$

$$\Gamma = \frac{\pi}{\mu}\,\frac{(5-p)^{\frac{p-3}{5-p}}}{\left[\Gamma\left(\frac{1}{5-p}\right)\right]^2}\left(\frac{2}{\lambda}\,k^2 + \frac{B^2}{\mu^2}\right)^{\frac{p-3}{2(5-p)}}\left[\frac{k^2}{\lambda} + \frac{B^2}{\mu^2}\right]$$

ω_R for D2-D6 with B field



Massive quarks in SUSY intersections

The speed&attenuation of the zero sound depends on the reduced mass **m** → $\mathbf{m} = \frac{m}{\mu}$

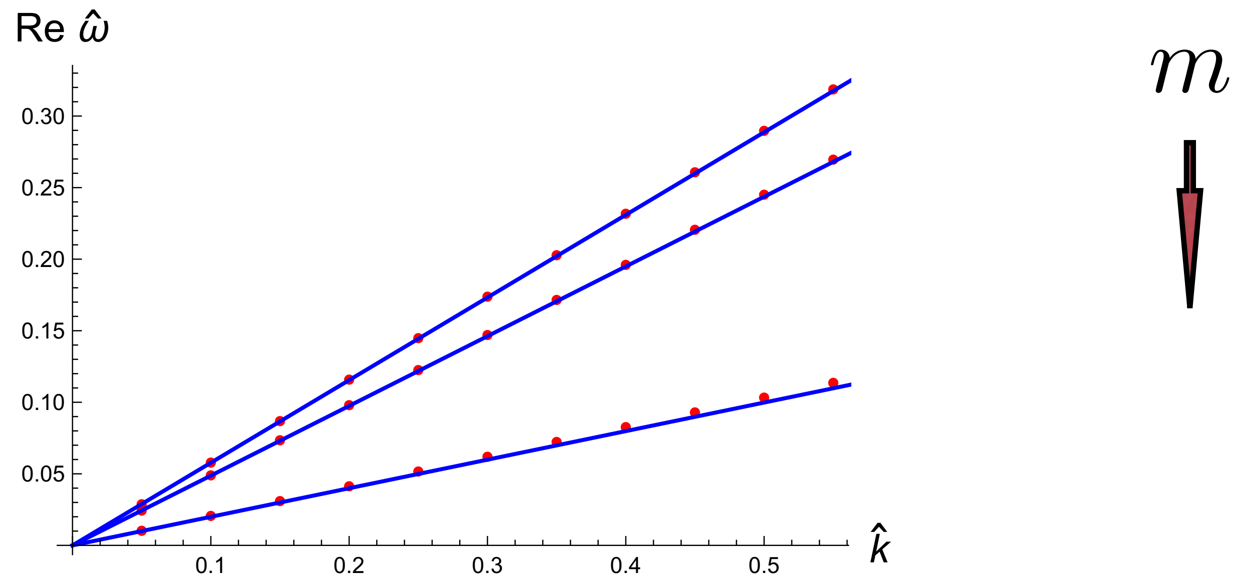
$$\omega_R^2 = \frac{2}{\lambda} \frac{1-\mathbf{m}^2}{1 - \frac{2\,\mathbf{m}^2}{\lambda}} k^2$$

$$\Gamma = \frac{\pi}{2\mu} \frac{(5-p)^{\frac{p-3}{5-p}}}{\left[\Gamma\left(\frac{1}{5-p}\right)\right]^2} \left(\frac{2}{\lambda}\right)^{\frac{7-p}{2(5-p)}} \frac{(1-\mathbf{m}^2)^{\frac{6-p}{5-p}-\frac{1}{\lambda}}}{(1-\frac{2\mathbf{m}^2}{\lambda})^{\frac{7-p}{2(5-p)}+1}} k^{\frac{7-p}{5-p}}$$

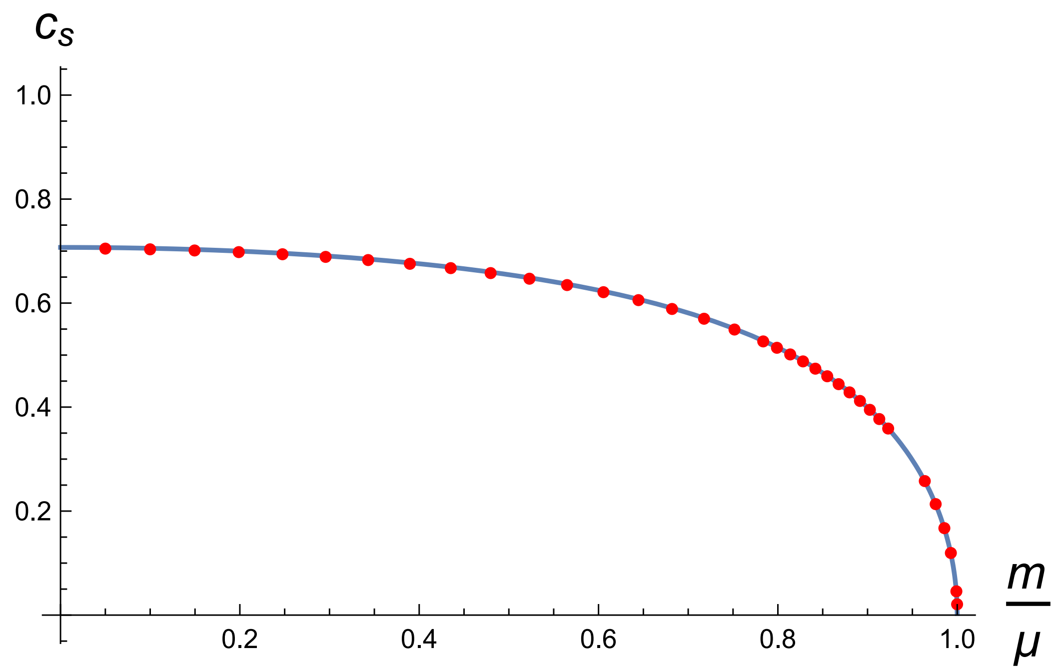
Kulaxizi &Parnachev, Davison&Starinets

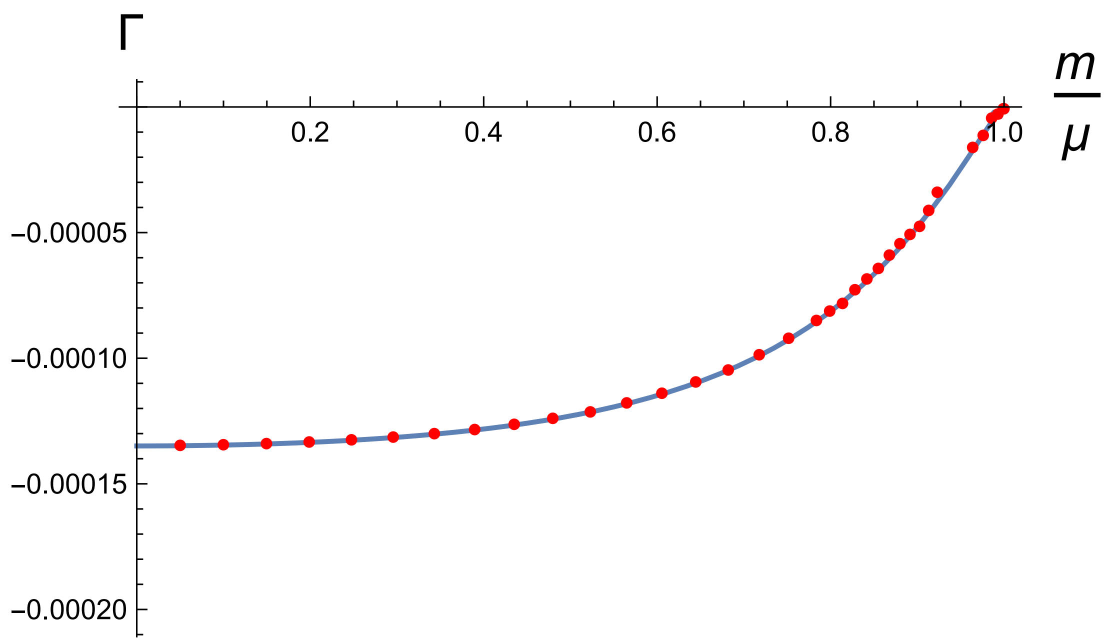
- Speed of the zero sound=speed of the first sound
- The speed of the zero-sound vanishes at $m = \mu$

Dispersion relation for D3-D7

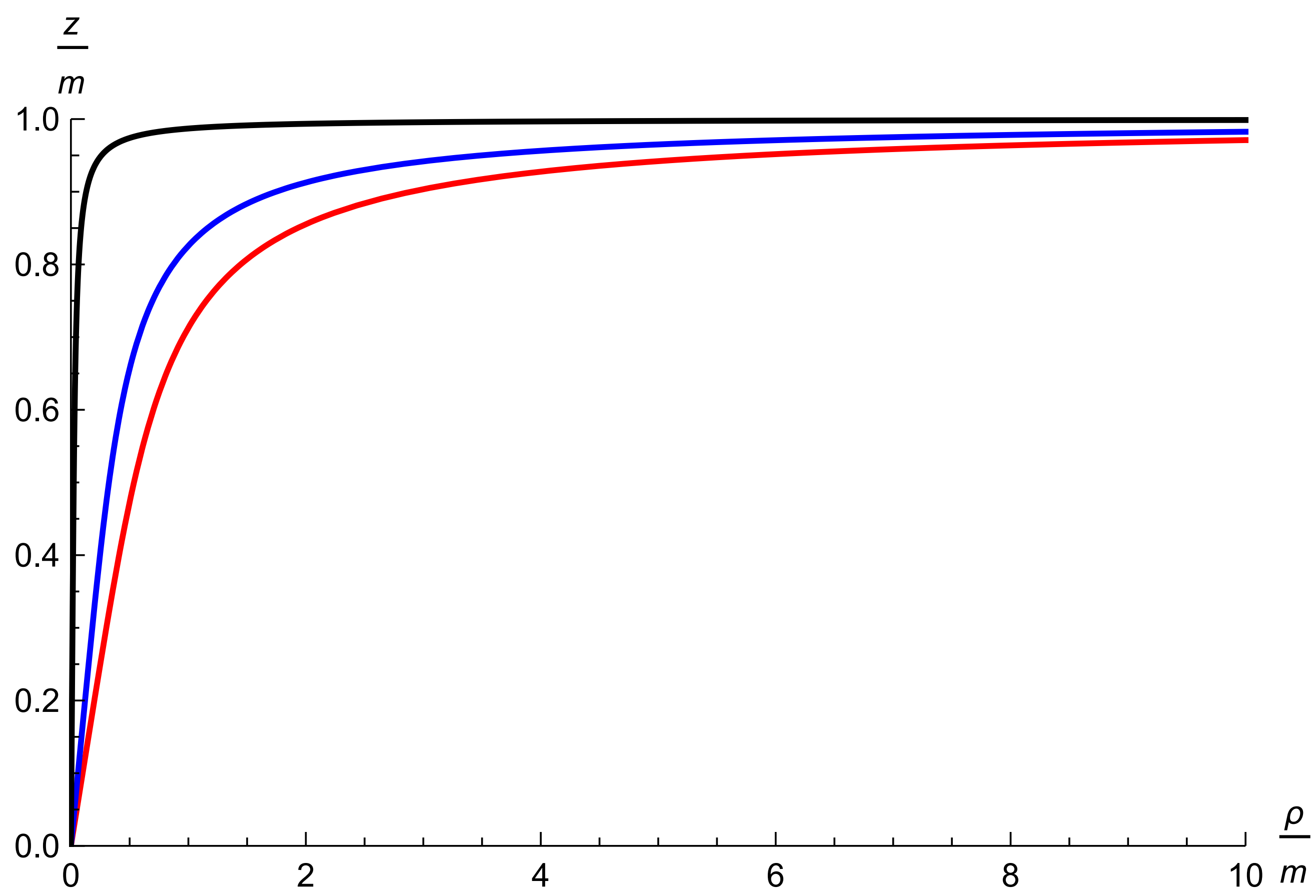


Speed of sound&attenuation for D3-D5





Massive embeddings at different densities



$$d \neq 0 \ (\mu > m) \rightarrow \text{black hole embedding}$$

zero density limit $(m = \mu)$ ➡ $z = m \quad \rightarrow$ Minkowski embedding

There is a quantum phase transition when $m \to \mu$ (Ammon et al.)

Non-relativistic energy density $\to$ $$e = \epsilon - d_{phys}\, m = \epsilon - \mathcal{N}\, d\, m$$

near the quantum critical point

$$e = \frac{n-\theta}{z} P$$

$z \to$ dynamical critical exponent

$\theta \to$ hyperscaling violating exponent

In our case near $m = \mu$

$$e = \frac{\lambda - 2}{4} P$$

Exponents $\to$ $z = 2$ $\qquad \theta = p - 2$

Non-relativistic free energy at $T \neq 0$

$$f_{non-rel}(\mu, m, T) \,=\, f(\mu, m, T) \,-\, d_{phys}\, m \,=\, e \,+\, \pi\, d_{phys}\, T \,+\, \mathcal{O}(T^2)$$

Near the critical point it should scale as

$$f_{non-rel} \sim (\bar{\mu})^{2-\alpha}\, g\Big(\frac{T}{\bar{\mu}^{\nu z}}\Big) \qquad \bar{\mu} = \mu - m$$

For our system we get

$$\alpha \,=\, \frac{6-\lambda}{4} \,=\, 1 \,-\, \frac{q-p}{4} \qquad \nu \,=\, \frac{1}{2}$$

The exponents satisfy the hyperscaling violating relation

$$(n+z-\theta)\,\nu \,=\, 2-\alpha$$

Anyonic excitations in 2+1 d

Anyons → Charges with a magnetic flux attached in 2+1d

Fractional statistics by Aharonov-Bohm effect

Holographic realization

Alternative quantization with Dirichlet-Neumann b. c. →

$n \to \text{constant}$

$$\lim_{\rho\to\infty} \left[n\, \rho^{\frac{\lambda}{2}}\, f_{\rho\,\mu} - \frac{1}{2}\, \epsilon_{\mu\alpha\beta}\, f^{\alpha\beta} \right] = 0$$

$$\lim_{\rho\to\infty} E = -i\,n \lim_{\rho\to\infty} \left[\rho^{\frac{\lambda}{2}}\, a_y' \right] \qquad \lim_{\rho\to\infty} a_y = i\,\frac{n}{\omega^2-k^2} \lim_{\rho\to\infty} \left[\rho^{\frac{\lambda}{2}}\, E' \right]$$

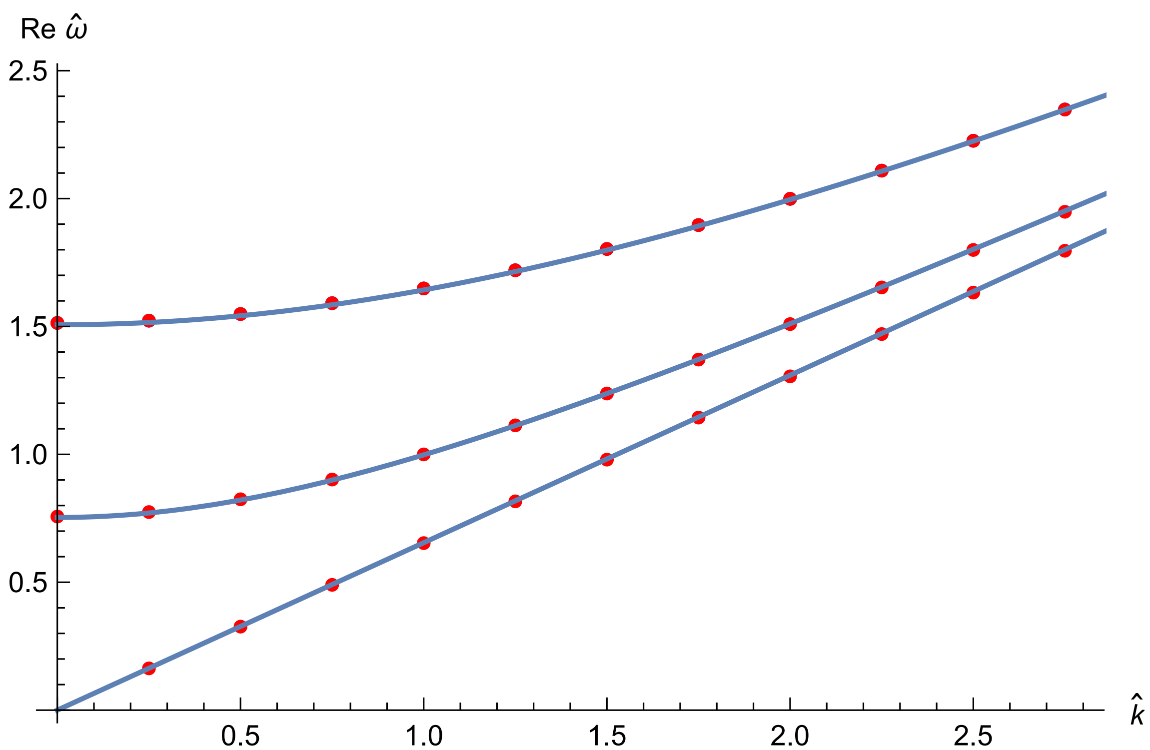
Zero sound spectrum

$$\omega_0^2 = 2\,\frac{1-\mathbf{m}^2}{\lambda - 2\,\mathbf{m}^2}\,k^2 + \frac{1}{\mu^2}\,(d\,n - B)^2$$

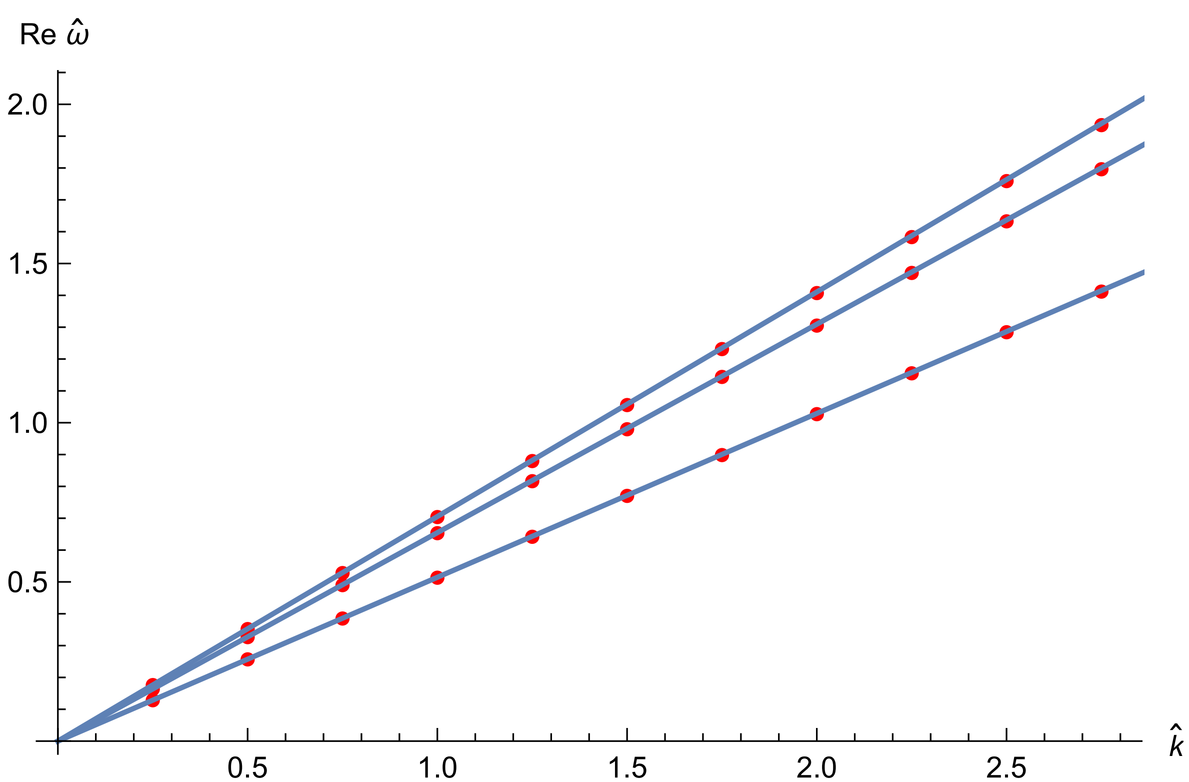
The alternative quantization is like an internal magnetic field

gappless spectrum → $n_{crit} \equiv \frac{B}{d}$

Comparison with numerics for D3-D5



$$n = 0\,,\ \frac{1}{2}n_{crit}\,,\ n_{crit}$$



$$n = n_{crit}$$

$$\frac{m}{\mu} = 0.1\,,\ 0.5\,,\ 0.8$$

How universal are our results?

Let us consider cold flavors in the ABJM model

ABJM

ABJM Field Theory → Chern-Simons-matter theories in 2+1 dimensions

gauge group: $U(N)_k \times U(N)_{-k}$

The ABJM model has $\mathcal{N} = 6$ SUSY in 3d

It has two parameters

$N \rightarrow$ rank of the gauge groups

$k \rightarrow$ CS level ($1/k \sim$ gauge coupling)

't Hooft coupling $\lambda \sim \frac{N}{k}$

It is a CFT in 3d with very nice properties

It is the 3d analogue of N=4 SYM

Sugra description in type IIA

$$AdS_4 \times \mathbb{CP}^3 + \text{ fluxes}$$

$$ds^2 = L^2\, ds^2_{AdS_4} + 4\, L^2\, ds^2_{\mathbb{CP}^3}$$

$$L^4 = 2\pi^2 \frac{N}{k}$$

$$F_2 = 2k\, J \qquad F_4 = \frac{3\pi}{\sqrt{2}} \left(kN \right)^{\frac{1}{2}} \Omega_{AdS_4}$$

$$e^{\phi} = \frac{2L}{k} = 2\sqrt{\pi} \left(\frac{2N}{k^5} \right)^{\frac{1}{4}}$$

Effective description for $N^{\frac{1}{5}} << k << N$

Flavor branes

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Hohenegger&Kirsch 0903.1730 Gaiotto&Jafferis 0903.2175

Worldvolume action of flavor D6-branes in ABJM

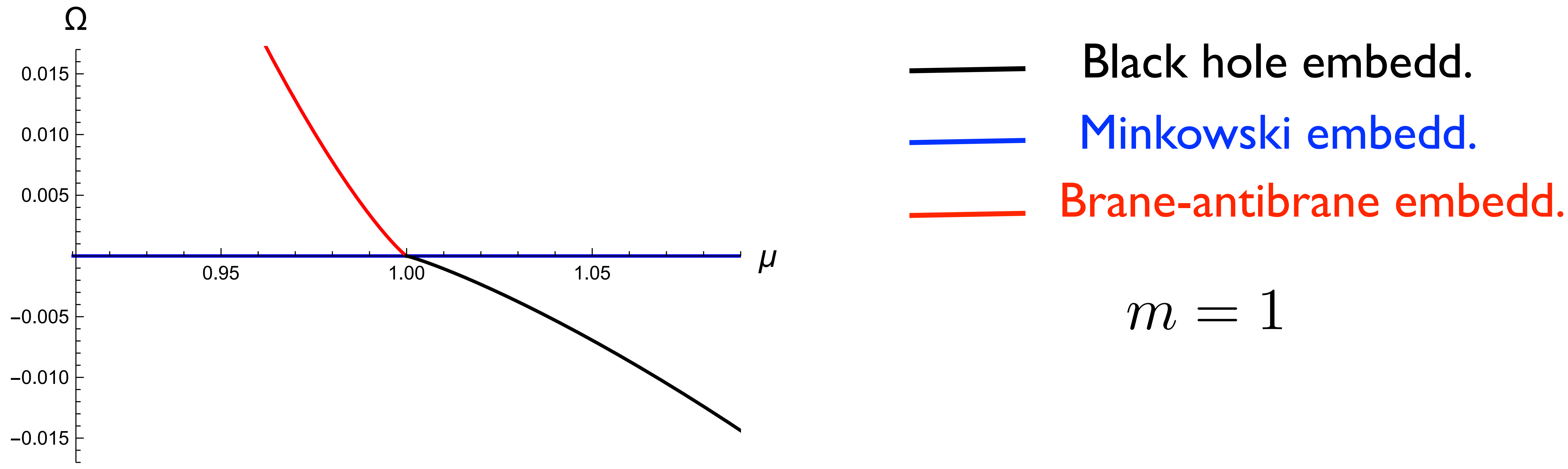
$$S \;=\; S_{DBI} \;+\; S_{WZ}$$

$$S_{DBI} = -T_{D6} \int_{\mathcal{M}_7} d^7\zeta \, e^{-\phi} \sqrt{-\det(g+F)}$$

$$S_{WZ} = T_{D6} \int_{\mathcal{M}_7} \left(\hat{C}_7 + \hat{C}_5 \wedge F + \frac{1}{2} \hat{C}_3 \wedge F \wedge F + \frac{1}{6} \hat{C}_1 \wedge F \wedge F \wedge F \right)$$

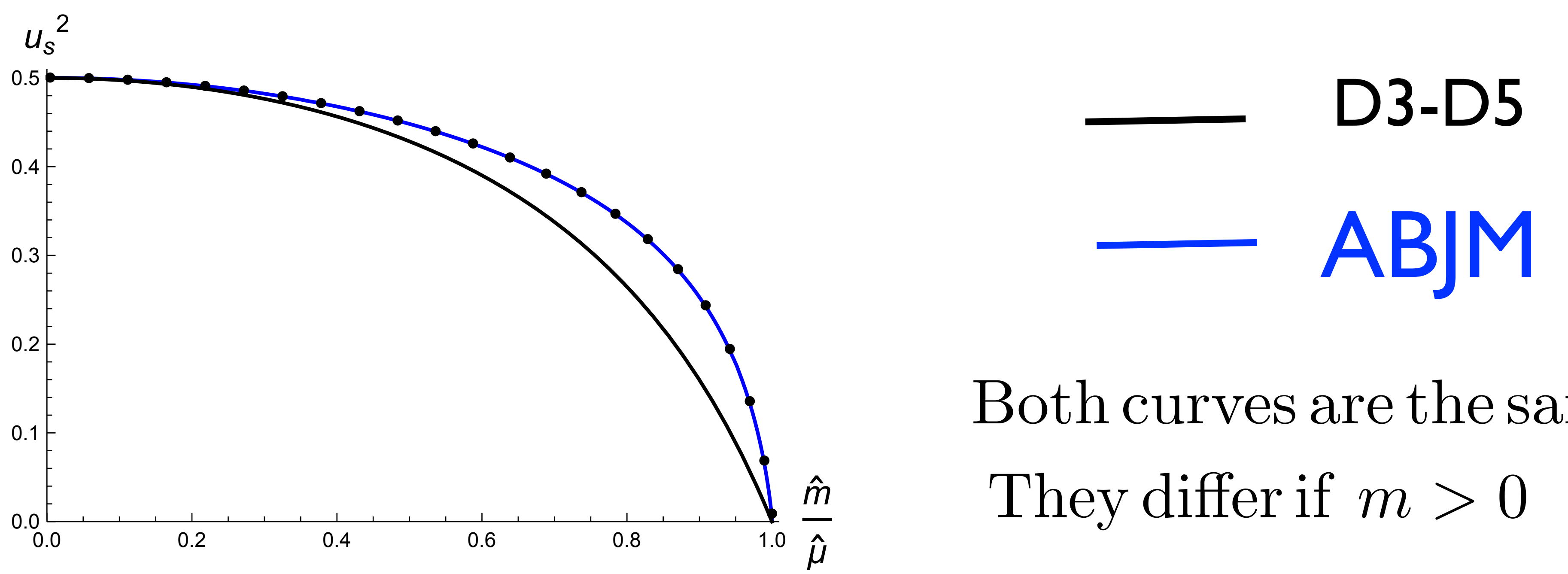
Now the WZ term contributes and produces non-trivial effects when the quarks are massive

Grand canonical potential at T=0



$$m = 1$$

Speed of first&zero sound



Both curves are the same if $m = 0$

They differ if $m > 0$

Quantum critical behavior

$$\bar{\mu} = \mu - m \quad \Longrightarrow \quad \Omega \approx -C\,\bar{\mu}^{\frac{n+z-\theta}{z}}\left(\left|\log\frac{\bar{\mu}}{m}\right|\right)^{-\zeta} \qquad \text{near } \bar{\mu} = 0$$

$\zeta \rightarrow$ new exponent $\qquad n = 2$ in ABJM

Charge density near $\bar{\mu} = 0$

$$\rho_{ch} \approx C\,\bar{\mu}^{\frac{n-\theta}{z}}\left(\left|\log\frac{\bar{\mu}}{m}\right|\right)^{-\zeta}\left[1 + \frac{n-\theta}{z} + \frac{\zeta}{\left|\log\frac{\bar{\mu}}{m}\right|}\right]$$

e/P ratio

$$\frac{e}{P} \approx \frac{n-\theta}{z} + \frac{\zeta}{\left|\log\frac{\bar{\mu}}{m}\right|}$$

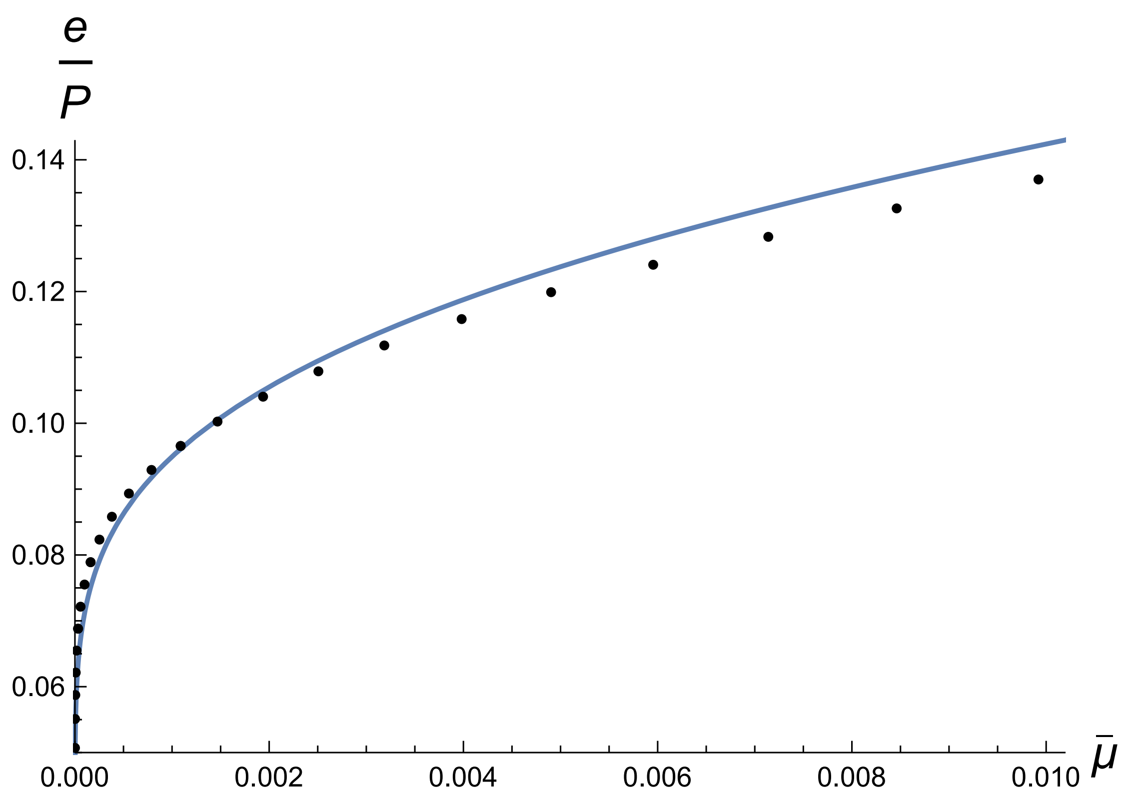
The numerical results show that $e/P \rightarrow 0$ near $\bar{\mu} = 0$ $\Longrightarrow$ $\theta = n = 2$

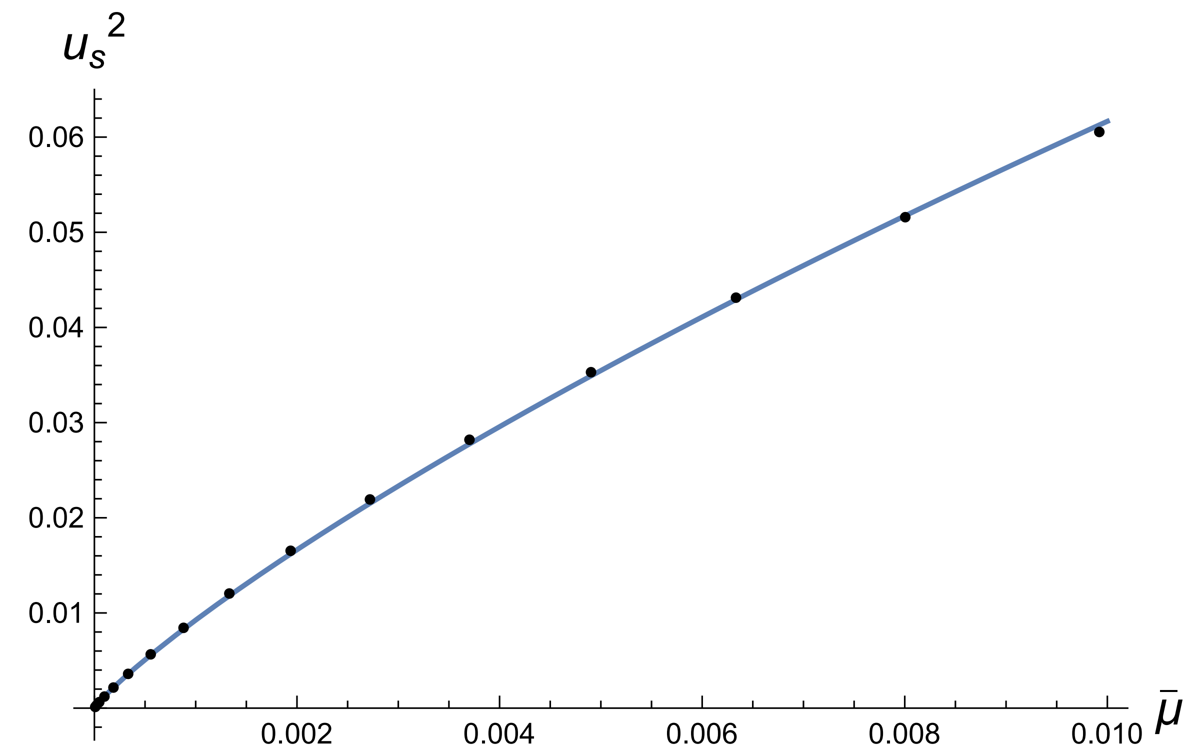
$\zeta \neq 0$ since $\rho_{ch} = 0$ at the critical point

$$\rho_{ch} \approx \frac{C}{\left(\left|\log \frac{\bar{\mu}}{m}\right|\right)^{\zeta}}$$

$$\frac{e}{P} \approx \frac{\zeta}{\left|\log \frac{\bar{\mu}}{m}\right|}$$

$$u_s^2 \approx \frac{1}{\zeta}\,\frac{\bar{\mu}}{m}\left|\log \frac{\bar{\mu}}{m}\right|$$





$$\zeta = 0.65 - 0.75$$

Other critical exponents

$$z = 2 \qquad \alpha = 1 \qquad \nu = \frac{1}{2}$$

Including the backreaction

Consider ABJM+unquenched smeared flavors (m=0)

$$AdS_4 \times \text{(squashed)}\, \mathbb{C}\,\mathbb{P}^3$$

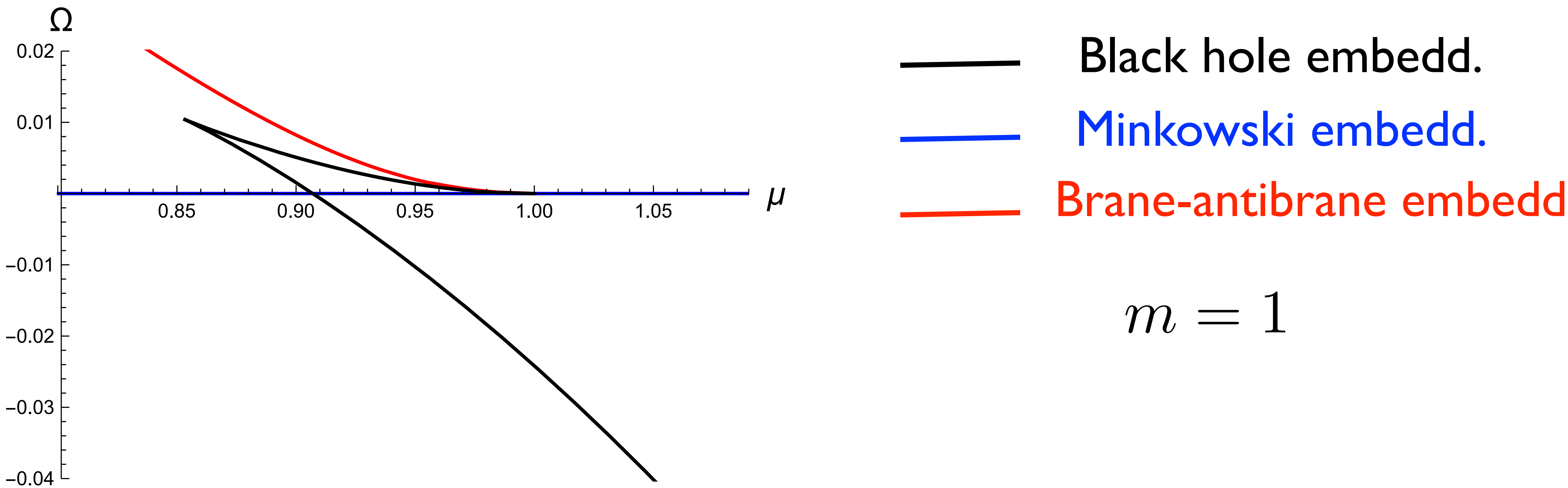
(E. Conde, AVR)

The backreaction is a very mild deformation which does not include the effects of the charge density

Drastic effects in the BH-Mink. transition →

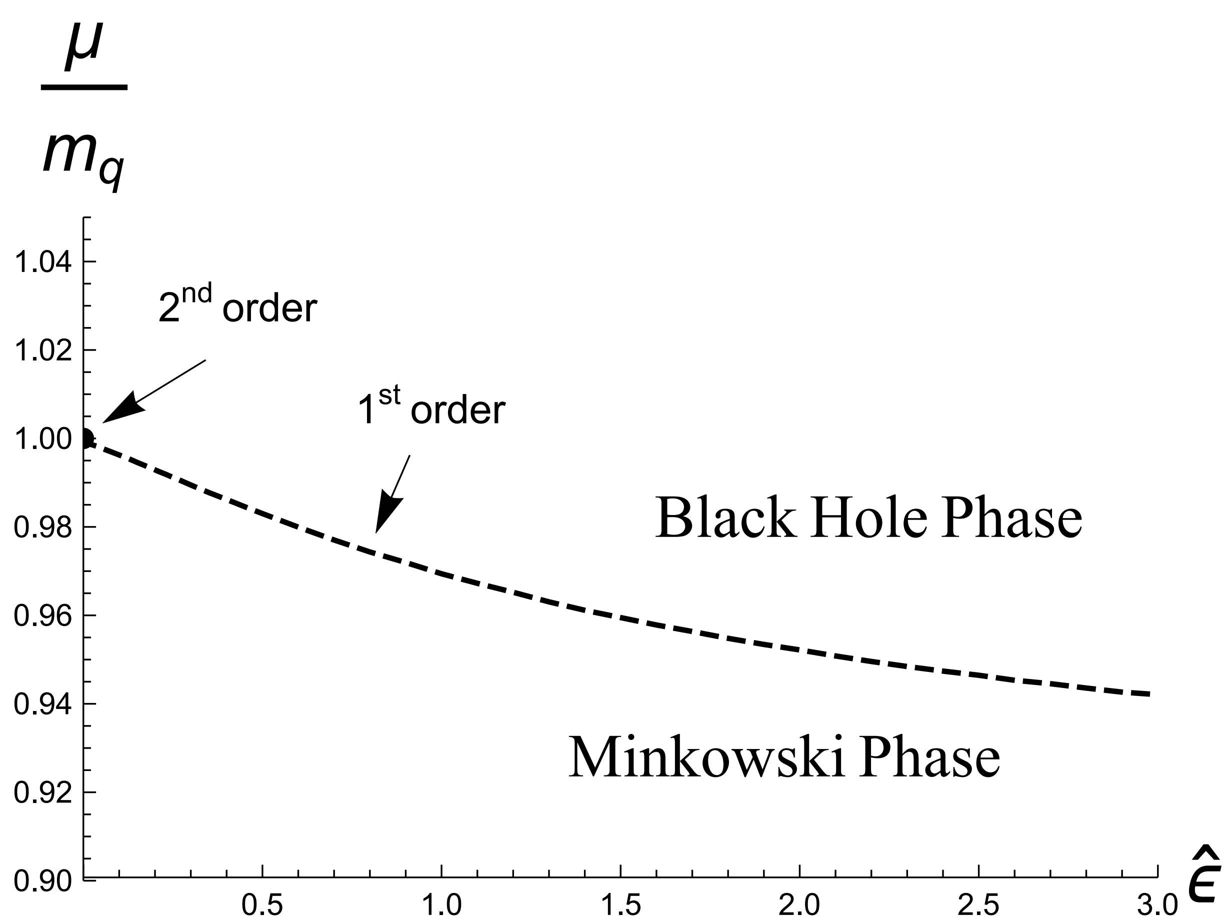
- Occurs at non-zero density
- It is of first-order

Grand canonical potential with unquenched flavor



The transition occurs at $\mu < m_q$ with $\rho_{ch} \neq 0$

Phase diagram at $T \neq 0$



$$\hat{\epsilon} = \frac{3}{4}\frac{N_f}{N}\lambda$$

Collective excitations in other probe brane systems

- Higgs branch for Dp-D(p+2) intersections with flux

  (G. Itsios, N. Jokela, AVR, 1505.02629)

- Anyons and magnetic fields in non-relativistic Lifshitz systems

  (J. Järvelä, N. Jokela, AVR, 1605.09156)

- Anisotropic backgrounds

  (in progress)

- Collective excitations with full backreaction at non-zero density


Thank You