Spatially modulated magnetic blak branes

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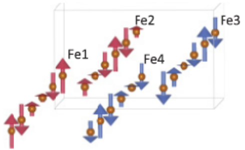
Spatially modulated phases

Spatially modulated phases are phases with *spontaneously* broken translational invariance. Realised in various configurations, e.g. stripes, checkerboards and helices.

- Spin Density Wave
- Charge Density Wave
- Current density wave



The modulation is fixed by an operator with non-zero momentum that spontaneously gets an expectation value, $< O(k) > \neq 0$.

Spatially modulated phases

Why should we care about SM?
→ Very common in CM: more the rule than the exception.
→ Connections with QCD at high density? "chiral density wave" [Deryagin, Grigoriev,Rubakov]
→ Involved in understanding the pseudogap region of the high-Tc superconductor phase diagram. [Sachdev]

Holographically, SM phases are dual to black hole that break spontaneously translation invariance. Many such examples exist.

We are interested in spatially modulated phases in the presence of external magnetic field, with no charge density. Instabilities found:

- in D=4,5 with mixing term $\phi * F \wedge G$ [Donos,Gauntlett,CP]
- in $U(1)^3$ and $U(1)^4$ sugra. Interesting interplay with susy solutions. [Almheiri,Polchinski] [Donos,Gauntlett,CP]

In this talk, we construct the fully backreacted SM back holes related to the instabilities of [Donos,Gauntlett,CP].

- Is there a phase transition?
- Proterties of these solutions?
- Ground state?

The theory

Consider $D = 4$ Einstein Maxwell theory coupled to a scalar and an additional gauge field: $F = dA$, $G = dB$

$$\begin{aligned}\mathcal{L} =& \tfrac{1}{2} R * 1 - V(\phi) * 1 - \tfrac{1}{2} * d\phi \wedge d\phi - \tfrac{1}{2} t(\phi) * F \wedge F \\ & - \tfrac{1}{2} v(\phi) * G \wedge G - u(\phi) * F \wedge G\end{aligned}$$

where

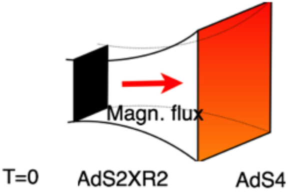
$$\begin{aligned} V(\phi) &= -6 + \tfrac{1}{2} m_s^2 \phi^2 + \cdots , \qquad & t(\phi) &= 1 - n\phi^2 + \cdots , \\ u(\phi) &= s\phi + \cdots , & v(\phi) &= 1 + \cdots . \end{aligned}$$

Direct connection with top-down theories, eg $(m_s^2, n, s) = (-4, -1, \sqrt{2})$ corresponds to $SU(3) \subset SO(8)$ $D = 4$ gauged sugra.

Solutions:

- Unit radius AdS_4 vacuum solution, with $\phi, A, B = 0$, which is dual to a $d = 3$ CFT with two conserved $U(1)$ currents.

- Magnetically charged AdS-RN black brane, $\phi, B = 0$, $A = \beta x dy$: [D'Hoker,Kraus]



This corresponds to the high temperature, spatially homogeneous and isotropic (diamagnetic) phase of the dual CFTs when held in a uniform magnetic field.

Instabilities

Step 1: [Donos,Gauntlett,CP]
Search for instabilities in the near-horizon, $AdS_2 \times R^2$.

$$\phi = \delta\phi\,(t,r)\,\cos\,(kx)\,,$$
$$B = \delta B\,(t,r)\,\sin\,(kx)\,\,dy,$$

The equations of motion take the form: $\mathbf{v} = (\delta\phi, \delta B)$

$$\Box_{AdS_2}\mathbf{v} - L^2 M^2 \mathbf{v} = 0, \quad M^2 = \begin{pmatrix} m_s^2 - 12n + k^2 & \sqrt{12}s\,k \\ \sqrt{12}s\,k & k^2 \end{pmatrix}.$$

Comments:

- For $s = 0$ (no mixing term $-u(\phi) * F \wedge G$), the most unstable mode has $k = 0$. The finite temprature instability sets in along a "bell curve" with maximum at $k = 0$. Thus, SM is surpressed.
- For $s \neq 0$ (non-zero mixing), minimise the mass eigenvalues as functions of k and compare them with the BF bound: possible violation depending on choice of parameters, centered at $k \neq 0$.
- No instabilities of the RN detected in top-down theories using this method.

Step 2: [Donos,Gauntlett,CP]

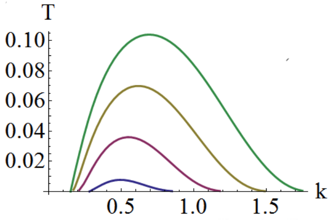
Construct zero modes for $m_s^2 = -4, n = -1$ and various s:

$\rightarrow$ Enough to consider the same perturbations as before. Solve eom subject to boundary conditions.

$\rightarrow$ They appear along a bell curve $T_c(k)$: at T_c^{max} there will be new BHs dual to

$$< O_\phi > \sim \cos(kx)\,, \quad < J_B > \sim \sin(kx)\ dy$$

dual to CFTs with current density wave, but no charge density wave.



Step 3: [Donos,CP]
Construction of back-reacted solutions by solving PDEs. Expect to find a two-parameter family of solutions labeled by (T, k), compatible with the bell curve.

To proceed, one needs to identify the non-linear terms in the lagrangian: $s = 2, n = -1, m_s^2 = -4$

$$V(\phi) = -6 - 2\phi^2 + \phi^4,$$
$$t(\phi) = 1 + \phi^2, \quad u(\phi) = 2(\phi - \phi^3), \quad v(\phi) = 1.$$

Disclaimer: Final conclusions do depend highly on the non-linear terms. Results only valid for this model.

Back-reacted black branes

Consider the following **ansatz**: $z\epsilon[0,1]$ and $x\epsilon[0,1)$.

$$ds^2 = \frac{1}{z_h^2 z^2}\left[-f\,Q_{tt}\,dt^2 + \frac{z_h^2\,Q_{zz}}{f}\,dz^2 + Q_{xx}\,(L\,dx + z^2 Q_{zx} dz)^2 + Q_{yy}\,dy^2\right]$$

$$A = (\beta L\,x + a_y)dy\,, \quad B = b_y dy\,, \quad \phi = z\,h\,,$$

- Eight unknown functions: $Q_{tt}, Q_{zz}, Q_{xx}, Q_{yy}, Q_{zx}, a_y, b_y, h$
- β is the magnetic field, $L = 2\pi/k$ is the modulation period, z_h horizon radius.
- General enough to contain the normal phase and zero modes.

Back-reacted black branes

DeTurck trick:

- The nature of these PDEs is weakly elliptic: not suitable for numerics.
- Use the DeTurck trick to dynamically fix the gauge: promote the constraint to an eom [Headrick, Kitchen,Wiseman].

$$R_{\mu\nu} \to R_{\mu\nu} + \nabla_\mu \xi_\nu \,, \quad \xi^\mu = g^{\nu\lambda}(\Gamma^\mu_{\nu\lambda}(g) - \bar{\Gamma}^\mu_{\nu\lambda}(\bar{g}))$$

- Check apostriori that $\xi_\mu = 0$.

Boundary conditions:

- In the UV, we impose AdS asymptotics compatible with spontaneous symmetry breaking: no sources
- In the IR, impose regularity of the solution at the horizon

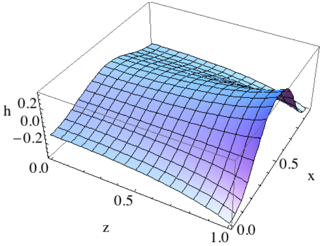
Numerics:

We proceed by discretising our domain to form a lattice:

→ Chebyshev lattice in z, equispaced in x.

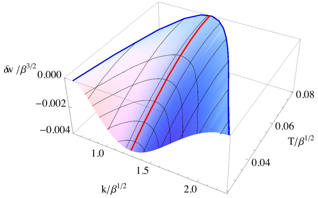
→ Approximate derivatives using pseudospectral methods.

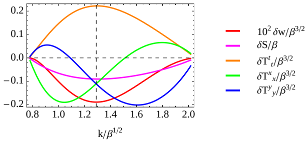
→ Use the Newton-Raphson to iteratively improve your intitial seed until you hit a solution.



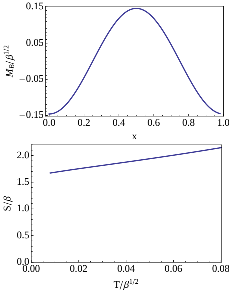
Plot of the scalar field for $(T, k) = (0.5T_c, k_c)$, $\beta = 1$.

- Study the whole two dimensional moduli of solutions, specified by (T, k).
- All the solutions have a lower free energy than AdS-RN: the CFT undergoes a second order transition to a SM phase.
- The thermodynamic ensemble is dominated by the red locus: k is increasing for $T > 0.3\,T_c$.
- Along the preferred branch, the the stress tensor is that of a perfect fluid.





- The magnetisation, M_B, is now non-zero: order parameter.
- Following the preferred branch to low temperature: hints of a ground state with non-zero k, magnetisation and entropy: AdS_2? Model dependent?



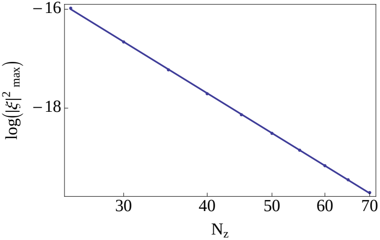
Further remarks

- Backreacted SM phases in top-down models?
- Repeat the analysis allowing for modulation in two dimensions: computationally more demanding.
  - Preferred lattice? Triangular lattice?
- Phase diagram of CFTs in external magnetic field?
  -Competing phases? eg superconducting Vs SM
- Long standing problem: identify the ground states.
  - What are the possible ground states? classification.
- Using linear response, analyse transport properties, fermion spectral functions etc

Thank you!

Convergence

Convergence is power law and not exponential as expect for pseudospectral calculations!!! This is due to non-analytic terms in the UV expansion: $\log z$, $z^{(3+\sqrt{33})/2}$ (manifestation of De Truck trick). [Donos,Gauntlett]



Other tests: first law, ward itentity,. . .