GRK 1523
Quantum and
Gravitational Fields

CIÊNCIA
SEM FRONTEIRAS

PHASE DIAGRAM OF 4D FIELD THEORIES WITH CHIRAL ANOMALY FROM HOLOGRAPHY

Rodrigo Panosso Macedo
(with Julian Leiber and Martin Ammon)

JHEP **3** 1-36 (2016)
ArXiv: 1601.02125

Friedrich-Schiller-Universität Jena

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CHARGED MAGNETIC HELICAL BLACK BRANES SOLUTIONS

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OUTLINE

- **Objective**: study strongly coupled four-dimensional field theories with chiral anomaly within the framework of gauge/gravity duality (focus on the phase diagram at finite temperature, chemical potential and magnetic field)

- **Geometrical setup**

- **Previous results**

- **Numerics**

- **Results**

- **Conclusion and outlook**

GEOMETRICAL SETUP

- **Objective**: study strongly coupled **four-dimensional field theory** with **chiral anomaly** within the framework of **gauge/gravity duality** (focus on the phase diagram at **finite temperature**, **chemical potential and magnetic field**)

- Asymptotically AdS spacetime in 5D
- Gauge Field (Maxwell tensor)
- Chern-Simons term
- Black brane spacetime

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{L^2}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

GEOMETRICAL SETUP

- **Objective**: Solve Einstein-Maxwell-Chern-Simons equations in 5D with boundary conditions describing: (i) AdS and (ii) black brane
  - Einstein equations

$$R_{mn} = -\frac{4}{L^2} g_{mn} + \frac{1}{2}\left(F_{mo}F_n{}^o - \frac{1}{6} g_{mn} F_{op}F^{op}\right)$$

  - Maxwell-Chern-Simons equations

$$F = dA \qquad d\star F + \frac{\gamma}{2} F \wedge F = 0$$

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    - Maxwell-Chern-Simons equations

$$F = dA \qquad d \star F + \frac{\gamma}{2} F \wedge F = 0$$

    - **Reissner-Nordstrom black brane solution**

ELECTRICALLY CHARGED BLACK BRANE

(ADS-REISSNER-NORDSTROM BLACK BRANE SOLUTION)

- Conformal approach to GR (compact radial coordinate)
- Coordinate system: $\{t, x^1, x^2, x^3, z\}$

$$ds^2 = \frac{L^2}{z^2}\left[-U(z)dt^2 + \frac{dz^2}{U(z)} + d\vec{x}^2\right]$$

$$\mathbf{A} = -e(z)dt \quad \mathbf{F} = E(z)dt \wedge dz$$

- Coordinate freedom: $x^a = \lambda \tilde{x}^a$
- Fix the coordinate location of the horizon: $z_{\mathrm{H}} = 1$

$$U(z) = 1 - z^4\left[1 + \frac{\mu^2}{3}\left(1 - z^2\right)\right]$$

$$e(z) = -\mu\left[1 - z^2\right] \qquad E(z) = \rho z \quad \rho = 2\mu$$

ELECTRICALLY CHARGED BLACK BRANE

(ADS-REISSNER-NORDSTROM BLACK BRANE SOLUTION)

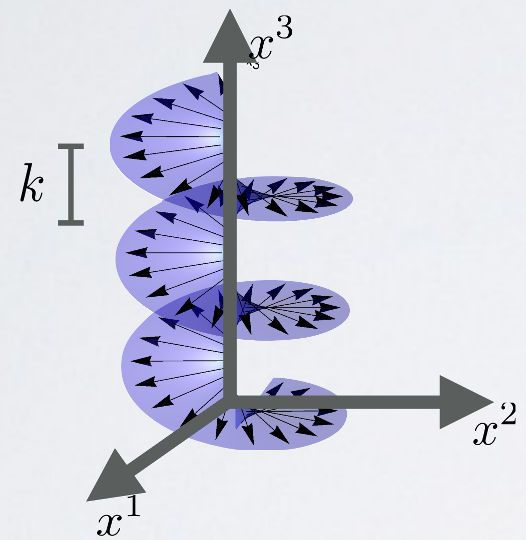
ELECTRICALLY CHARGED BLACK BRANE

(ADS-REISSNER-NORDSTROM BLACK BRANE SOLUTION)

A. Donos and J.P. Gauntlett -
PRD 86 064010 (2012)

Helical structure $\gamma > \gamma_c = 1.158$

(instabilities against spatial modulation)



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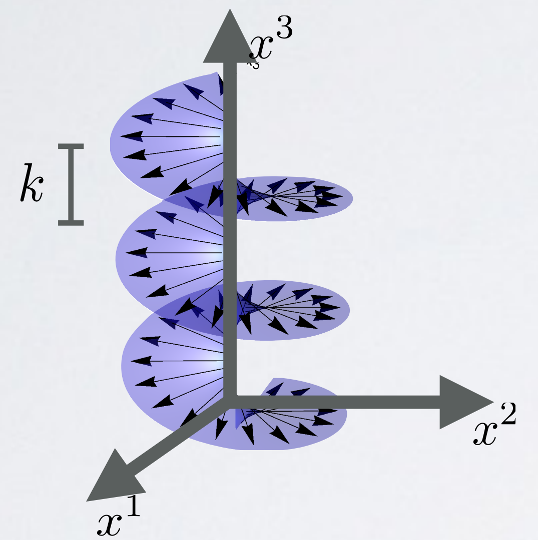

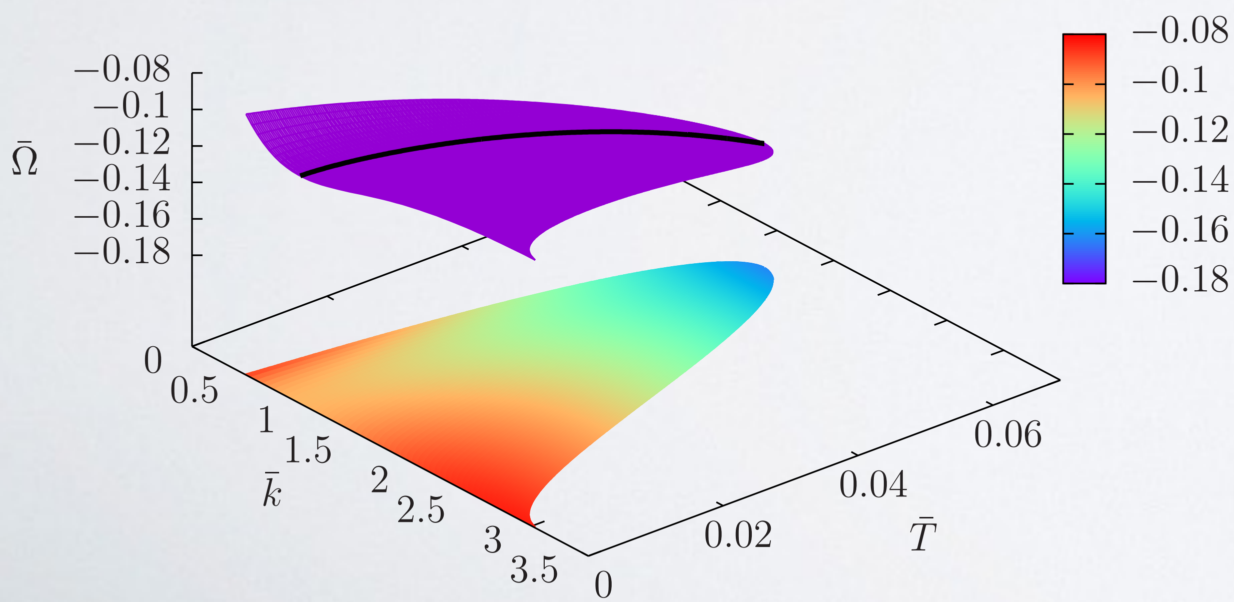

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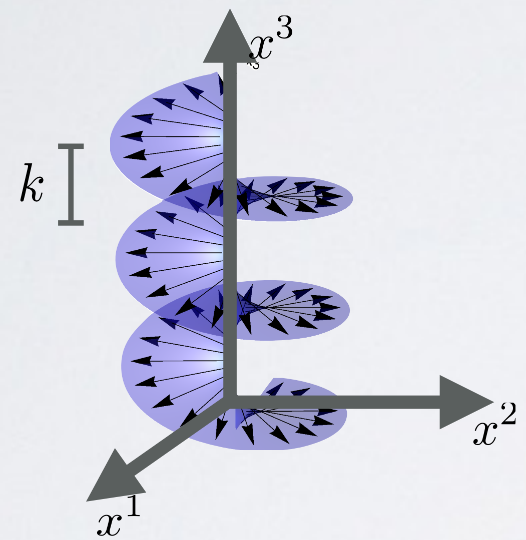
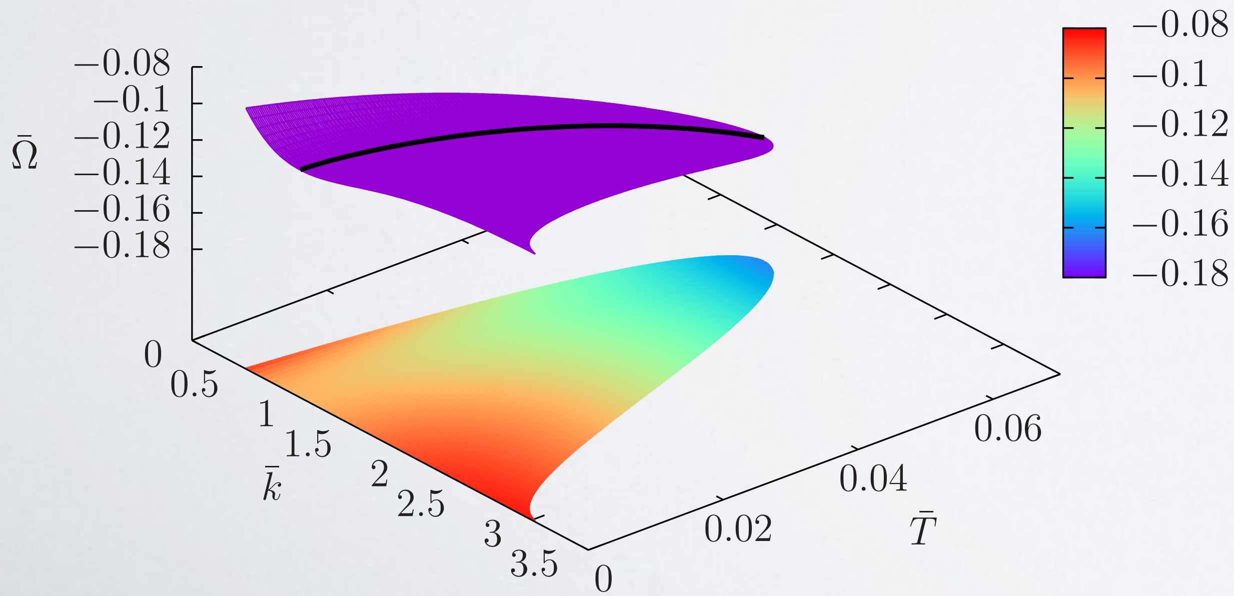
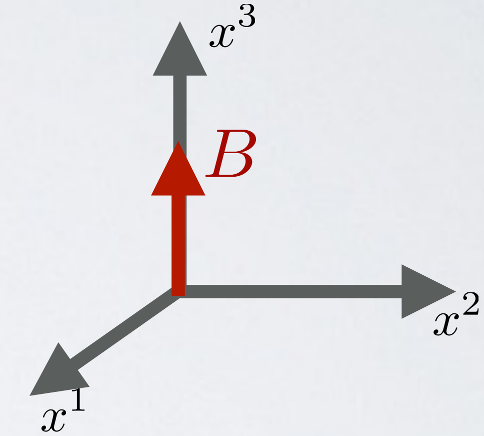




Constant magnetic field



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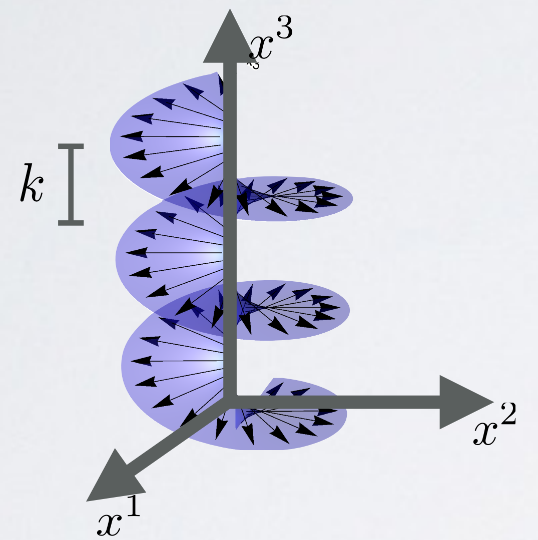

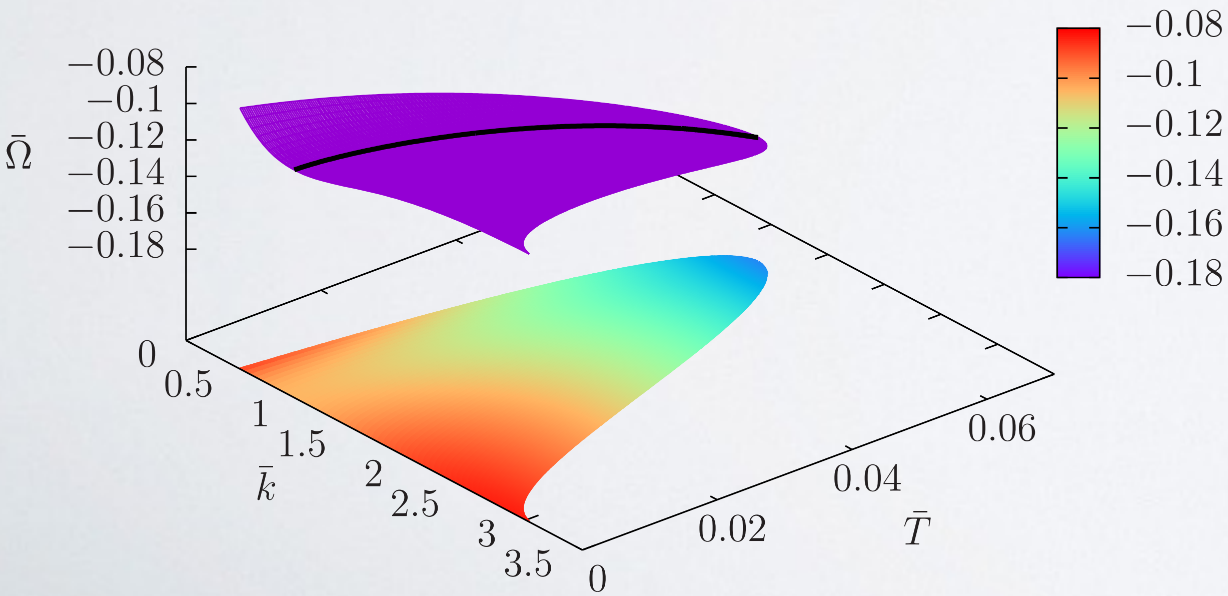

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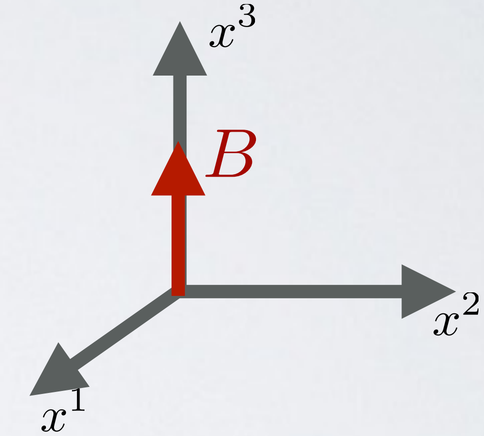

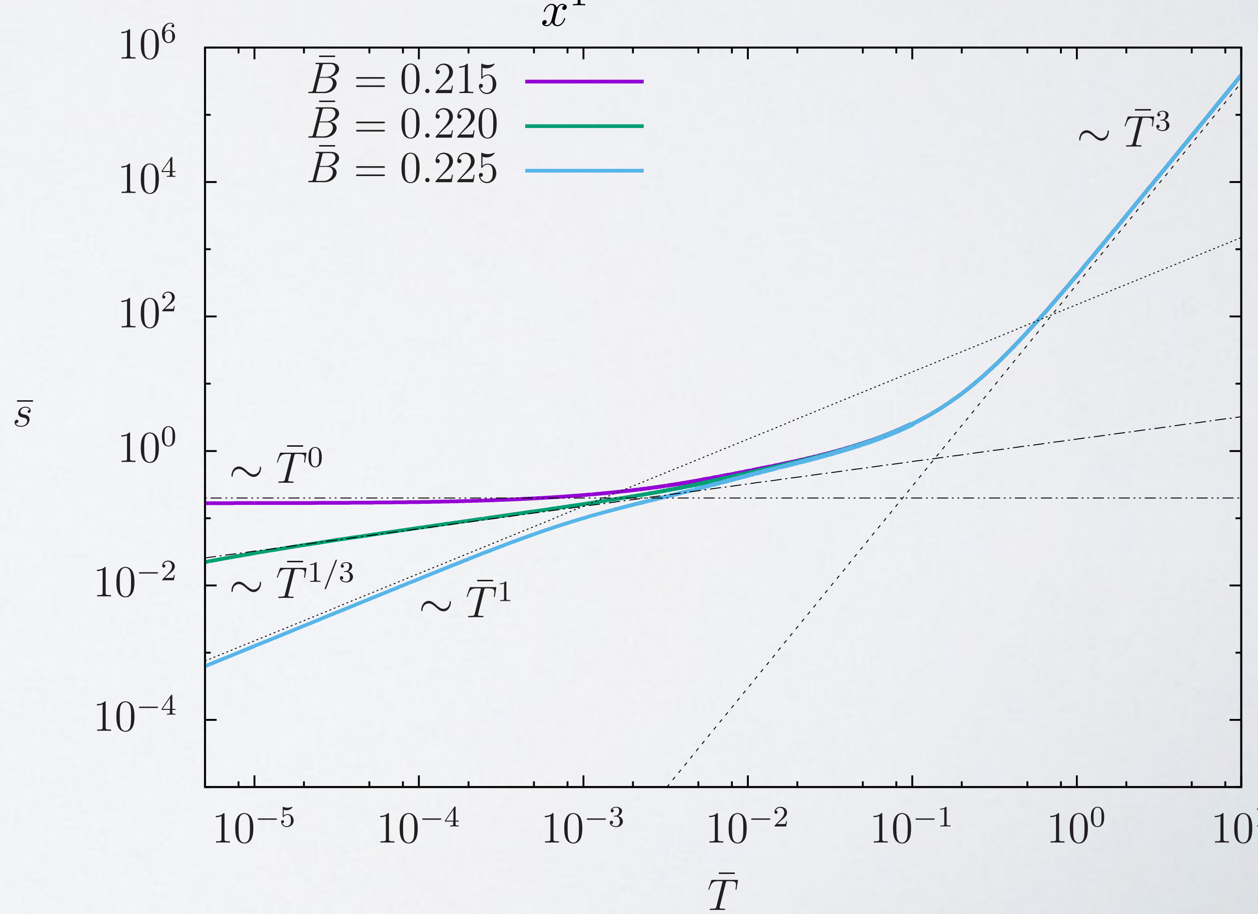

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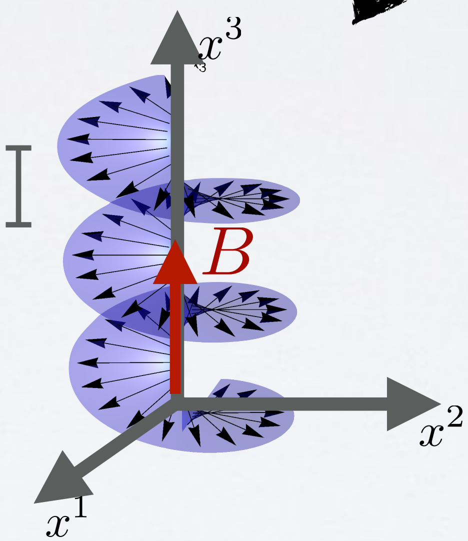

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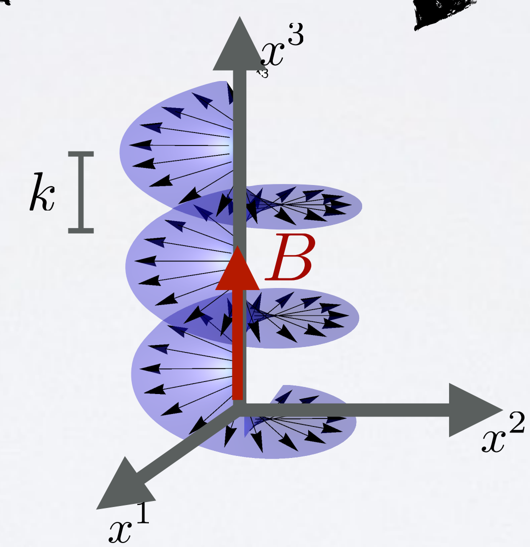
- **Does the helical phase extend to the regime $B \neq 0$?**



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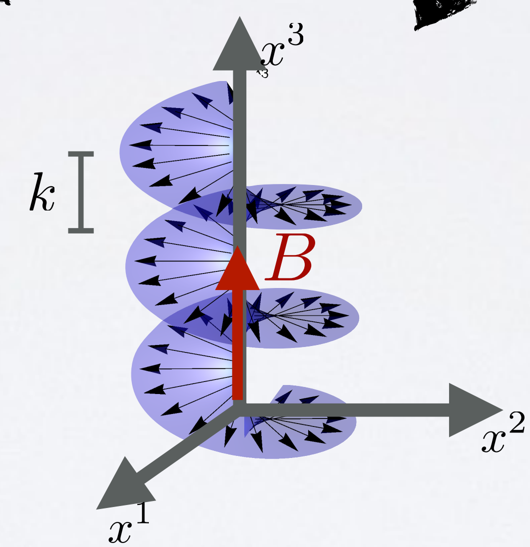

- **How does the entropy behave for $\bar{B} < \bar{B}_{\mathrm{C}}$?**

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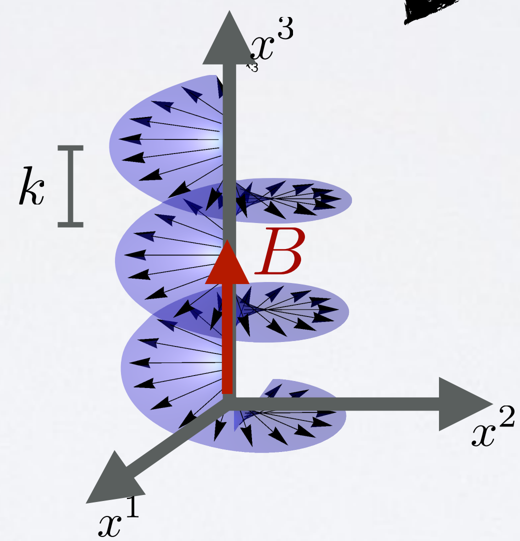

- **How does the entropy behave for $\bar{B} < \bar{B}_C$?**

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- **Thermodynamics: critical exponents**

Charged Magnetic Branes

(D'Hoker and Kraus)

$$ds^2 = \frac{L^2}{z^2}\left\{-U(z)dt^2 + \frac{dz^2}{U(z)} + V(z)^2\left[(dx^1)^2 + (dx^2)^2\right] + W(z)^2\left[dx^3 + C(z)dt\right]^2\right\}$$

$$\mathbf{A} = -e(z)dt - Bx^2dx^1 + p(z)dx^3$$

$$\mathbf{F} = E(z)dt \wedge dz + Bdx^1 \wedge x^2 + P(z)dz \wedge dx^3$$

with boundary/regularity conditions:

- Horizon:

$$U(1) = 0 \qquad e(1) = 0$$
$$\mathrm{EQ}_V(1) = 0 \qquad \mathrm{EQ}_p(1) = 0$$
$$\mathrm{EQ}_W(1) = 0$$
$$C(1) = 0$$

- AdS:

$$\frac{d}{dz}U(0) = 0 \qquad e(0) = \mu$$
$$V(0) = 1 \qquad p(0) = 0$$
$$W(0) = 1$$
$$C(0) = 0$$

Charged Magnetic Branes

(D'Hoker and Kraus)

$$ds^2 = \frac{L^2}{z^2}\left\{-U(z)dt^2 + \frac{dz^2}{U(z)} + V(z)^2\left[(dx^1)^2 + (dx^2)^2\right] + W(z)^2\left[dx^3 + C(z)dt\right]^2\right\}$$

Coordinate freedom: $x^a = \lambda \tilde{x}^a$

$$\tilde{B} = \lambda^2 B \qquad \bar{B} = B/\mu^2 = \tilde{B}/\tilde{\mu}^2$$

$\mathbf{A} = -e(z$

$\mathbf{F} = E(z)d$

with boundary/regularity conditions:

- Horizon:

$$U(1) = 0 \qquad e(1) = 0$$
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Charged Helical black branes

(Donos and Gauntlett)

$$ds^2 = \frac{L^2}{z^2}\left\{-U(z)dt^2 + \frac{dz^2}{U(z)} + V(z)^2\alpha(z)^{-2}\left[\omega^2\right]^2 + V(z)^2\alpha(z)^2\left[\omega^1 + Q(z)dt\right]^2 + W(z)^2\left[dx^3\right]^2\right\}$$

$$\mathbf{A} = -e(z)dt + b(z)\omega^1$$

$$\omega^1 = \cos(kx^3)dx^1 - \sin(kx^3)dx^2$$

$$\omega^2 = \sin(kx^3)dx^1 + \cos(kx^3)dx^2$$

$$\mathbf{F} = E(z)dt \wedge dz + \frac{d}{dz}b(z)dz \wedge \omega^1 + b(z)d\omega^1$$

with boundary/regularity conditions:

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$$EQ_\alpha(1) = 0$$
$$Q(1) = 0$$

- AdS:

$$\frac{d}{dz}U(0) = 0 \qquad e(0) = \mu$$
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$$\omega^1 = \cos(kx^3)dx^1 - \sin(kx^3)dx^2$$

$$\mathbf{A} = -e(z)dt + b(z)\omega^1$$

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$$\mathbf{F} = E(z)dt \wedge dz + \frac{d}{dz}b(z)dz \wedge \omega^1 + b(z)d\omega^1$$

with bou

- Horizo

Coordinate freedom: $x^a = \lambda\tilde{x}^a$

$$\tilde{k} = \lambda k \qquad \bar{k} = k/\mu = \tilde{k}/\tilde{\mu}$$

$U(1) =$

$\mathrm{EQ}_V(1) = 0$ $\quad EQ_b(1) = 0$

$\mathrm{EQ}_W(1) = 0$

$EQ_\alpha(1) = 0$

$Q(1) = 0$

$b(0) = 0$

$V(0) = 1$

$W(0) = 1$

$\alpha(0) = 1$

$Q(0) = 0$

Charged Magnetic Helical black branes

$$ds^2 = \frac{L^2}{z^2}\left\{-U(z)dt^2 + \frac{dz^2}{U(z)} + V(z)^2\alpha(z)^{-2}\left[\omega^2\right]^2 + V(z)^2\alpha(z)^2\left[\omega^1 + g(z)dx^3 + Q(z)dt\right]^2 + W(z)^2\left[dx^3 + C(z)dt\right]^2\right\}$$

$$\mathbf{A} = -e(z)dt - Bx^2dx^1 + p(z)dx^3 + b(z)\omega^1$$

$$\mathbf{F} = E(z)dt\wedge dz + Bdx^1\wedge x^2 + P(z)dz\wedge dx^3 + \frac{d}{dz}b(z)dz\wedge\omega^1 + b(z)d\omega^1$$

with boundary/regularity conditions

- Horizon:

$U(1) = 0$ $\quad C(1) = 0$

$\mathrm{EQ}_V(1) = 0$ $\quad EQ_g(1) = 0$

$\mathrm{EQ}_W(1) = 0$ $\quad e(1) = 0$

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$Q(1) = 0$ $\quad \mathrm{EQ}_p(1) = 0$

- AdS:

$\frac{d}{dz}U(0) = 0$ $\quad C(0) = 0$

$V(0) = 1$ $\quad g(0) = 0$

$W(0) = 1$ $\quad e(0) = \mu$

$\alpha(0) = 1$ $\quad b(0) = 0$

$Q(0) = 0$ $\quad p(0) = 0$

Charged Magnetic Helical black branes

asymptotic expansions: AdS boundary

$$U(z) = 1 + \frac{B^2}{6} z^4 \ln z + z^4 u_4 + \mathcal{O}(z^6)$$

$$W(z) = 1 + \frac{B^2}{12} z^4 \ln z + z^4 w_4 + \mathcal{O}(z^6)$$

$$V(z) = 1 - \frac{B^2}{24} z^4 \ln z - z^4 \frac{w_4}{2} + \mathcal{O}(z^6)$$

$$\alpha(z) = 1 + z^4 a_4 + \mathcal{O}(z^6)$$

$$C(z) = z^4 c_4 + \mathcal{O}(z^6)$$

$$Q(z) = z^4 q_4 + \mathcal{O}(z^6)$$

$$g(z) = z^4 \frac{B}{2k} b_2 + \mathcal{O}(z^6)$$

$$e(z) = \mu + \frac{\rho}{2} z^2 + \mathcal{O}(z^4)$$

$$p(z) = p_2 z^2 + \mathcal{O}(z^4)$$

$$b(z) = b_2 z^2 + \mathcal{O}(z^4)$$

Charged Magnetic Helical black branes

asymptotic expansions: AdS boundary

$$U(z) = u_H(1-z) + \mathcal{O}((1-z)^2)$$

$$W(z) = w_H + \mathcal{O}(1-z)$$

$$V(z) = v_H + \mathcal{O}(1-z)$$

$$\alpha(z) = a_H + \mathcal{O}(1-z)$$

$$C(z) = c_H(1-z) + \mathcal{O}((1-z)^2)$$

$$Q(z) = q_H(1-z) + \mathcal{O}((1-z)^2)$$

$$g(z) = g_H + \mathcal{O}(1-z)$$

$$e(z) = e_H(1-z) + \mathcal{O}((1-z)^2)$$

$$p(z) = p_H + \mathcal{O}(1-z)$$

$$b(z) = b_H + \mathcal{O}(1-z)$$

Charged Magnetic Helical black branes

Thermodynamics:

- **Temperature:** $T = -\frac{u_H}{4\pi}$ - **Entropy density:** $s = 4\pi v_H^2 w_H$
- **Energy-momentum tensor of dual theory:**

$$\langle T_{t\omega_1} \rangle = \langle T_{\omega_1 t} \rangle = 4\, q_4 \qquad \langle T_{tt} \rangle = -3u_4 \qquad \langle T_{tx_3} \rangle = \langle T_{x_3 t} \rangle = 4\, c_4$$

$$\langle T_{\omega_1\omega_1} \rangle = -\frac{B^2}{4} + 8\, a_4 - u_4 - 4\, w_4 \quad \langle T_{\omega_2\omega_2} \rangle = -\frac{B^2}{4} - 8\, a_4 - u_4 - 4\, w_4$$

$$\langle T_{\omega_1 x_3} \rangle = \langle T_{x_3\omega_1} \rangle = \frac{2\, B\, b_2}{k} \qquad \langle T_{x_3 x_3} \rangle = 8\, w_4 - u_4$$

- **Current of dual theory:**

$$\langle J_t \rangle = -\rho \qquad \langle J_{\omega_1} \rangle = -2\, b_2 \qquad \langle J_{x_3} \rangle = -p_1$$

- **Grand canonical potential:**

$$\Omega = \langle T_{tt} \rangle - sT - \mu \langle J^t \rangle + \frac{1}{3} B\gamma \int_0^1 dz e(z) P(z)$$

Charged Magnetic Helical black branes

Thermodynamics:

- **Temperature:** $T = -\frac{u_H}{4\pi}$ - **Entropy density:** $s = 4\pi v_H^2 w_H$

- **Energy-**

$\langle T_{t\omega_1} \rangle = \langle T$ … $4\, c_4$

$\langle T_{\omega_1 \omega_1} \rangle = -$ … $4\, w_4$

$\langle T_{\omega_1 x_3}$ …

Coordinate freedom: $x^a = \lambda \tilde{x}^a$

$$\bar{T} = \frac{T}{\mu} \qquad \bar{s} = \frac{s}{\mu^3} \qquad \langle \bar{J}_\mu \rangle = \frac{\langle J_\mu \rangle}{\mu^3}$$

$$\bar{\Omega} = \frac{\Omega}{\mu^4} \qquad \langle \bar{T}_{\mu\nu} \rangle = \frac{\langle T_{\mu\nu} \rangle}{\mu^4}$$

- **Current**

$\langle J_t \rangle = -\rho \qquad \langle J_{\omega_1} \rangle = -2\, b_2 \qquad \langle J_{x_3} \rangle = -p_1$

- **Grand canonical potential:**

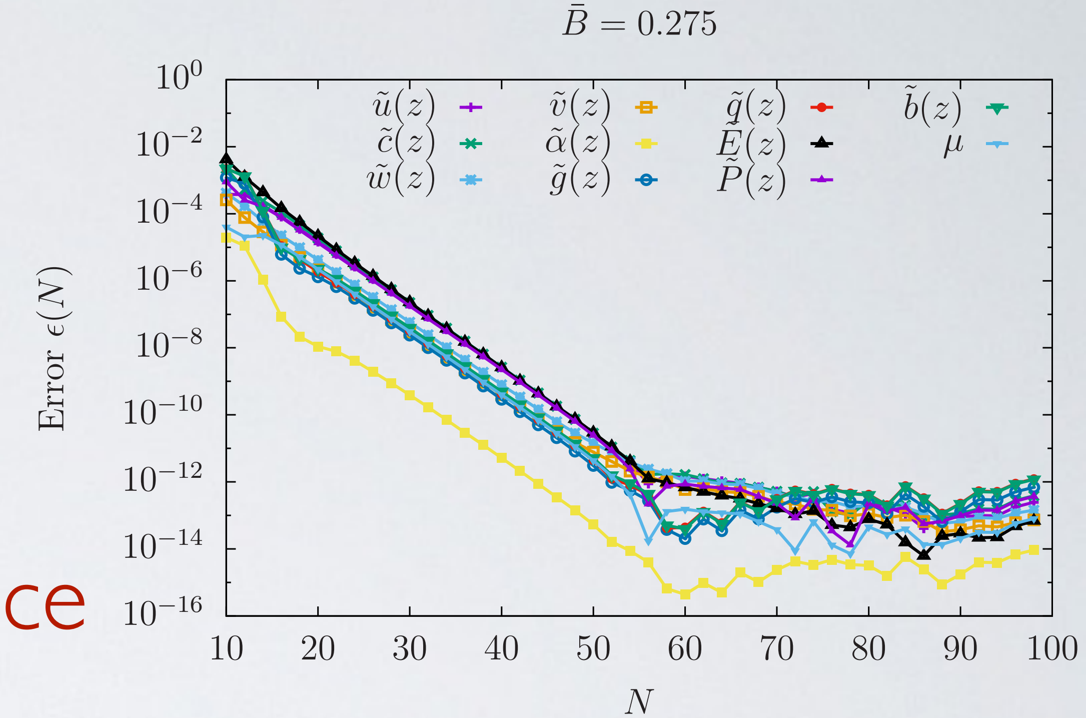
$$\Omega = \langle T_{tt} \rangle - sT - \mu \langle J^t \rangle + \frac{1}{3} B\gamma \int_0^1 dz e(z) P(z)$$

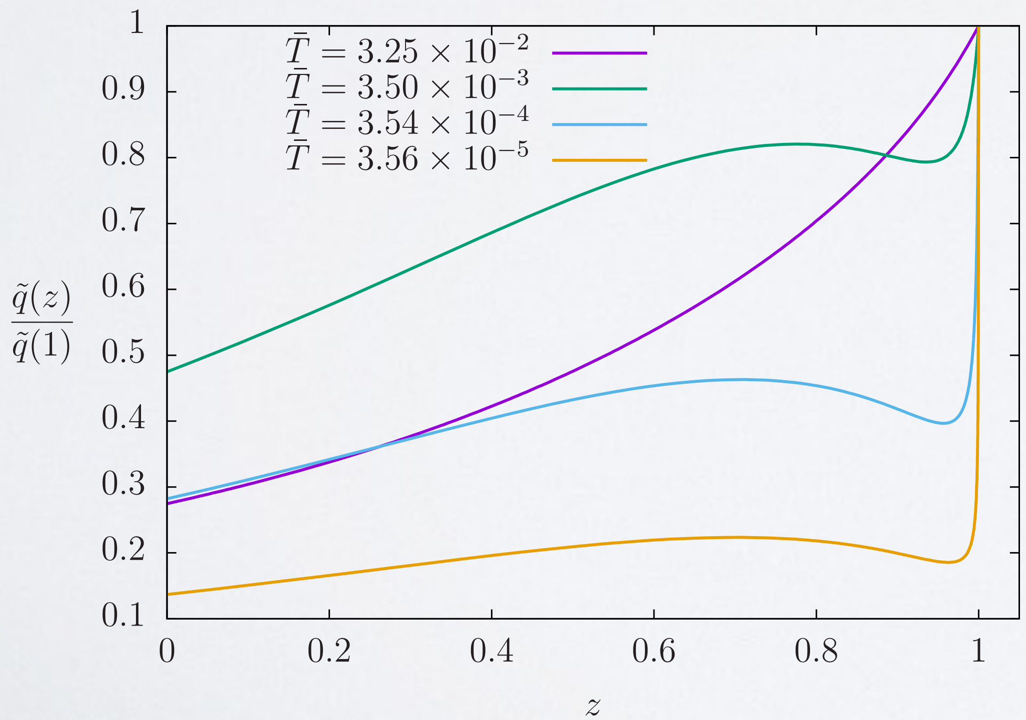
NUMERICS

- Boundary value problem (1-D)
- Spectral methods
- Highly accurate solution

✦ log terms spoils spectral convergence

✦ Strong gradients for low temperature

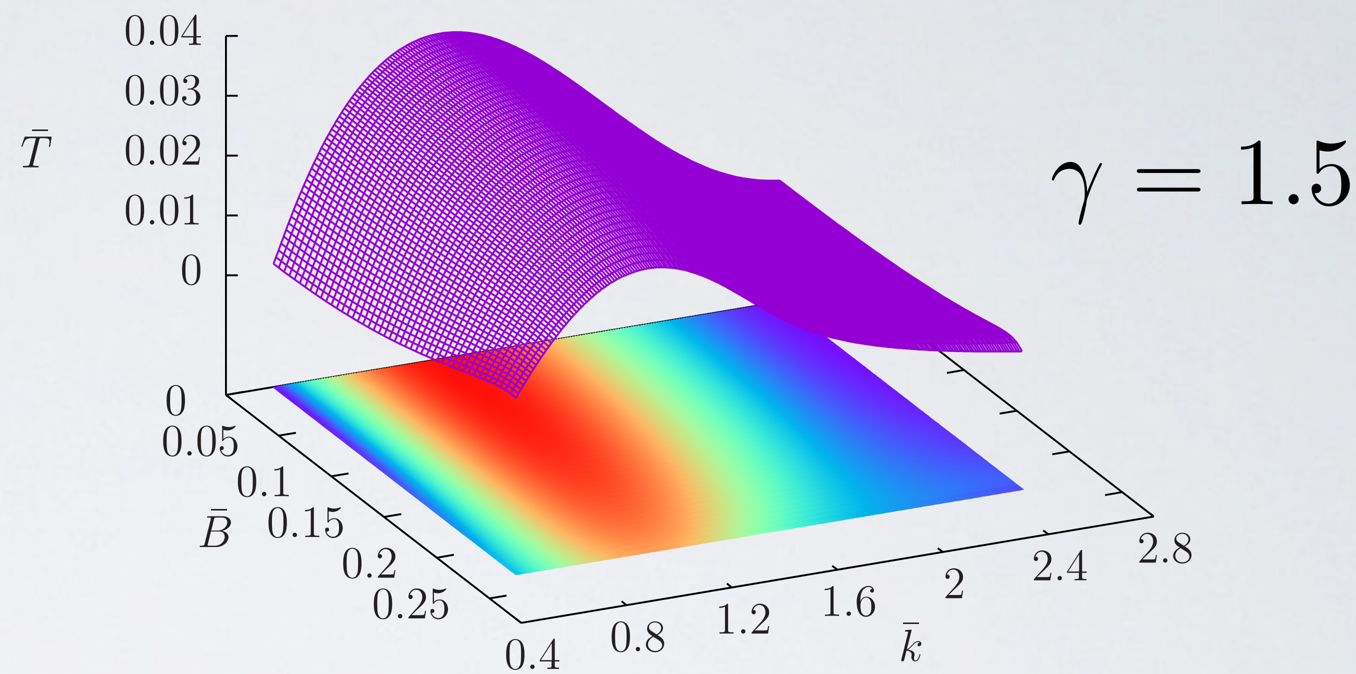




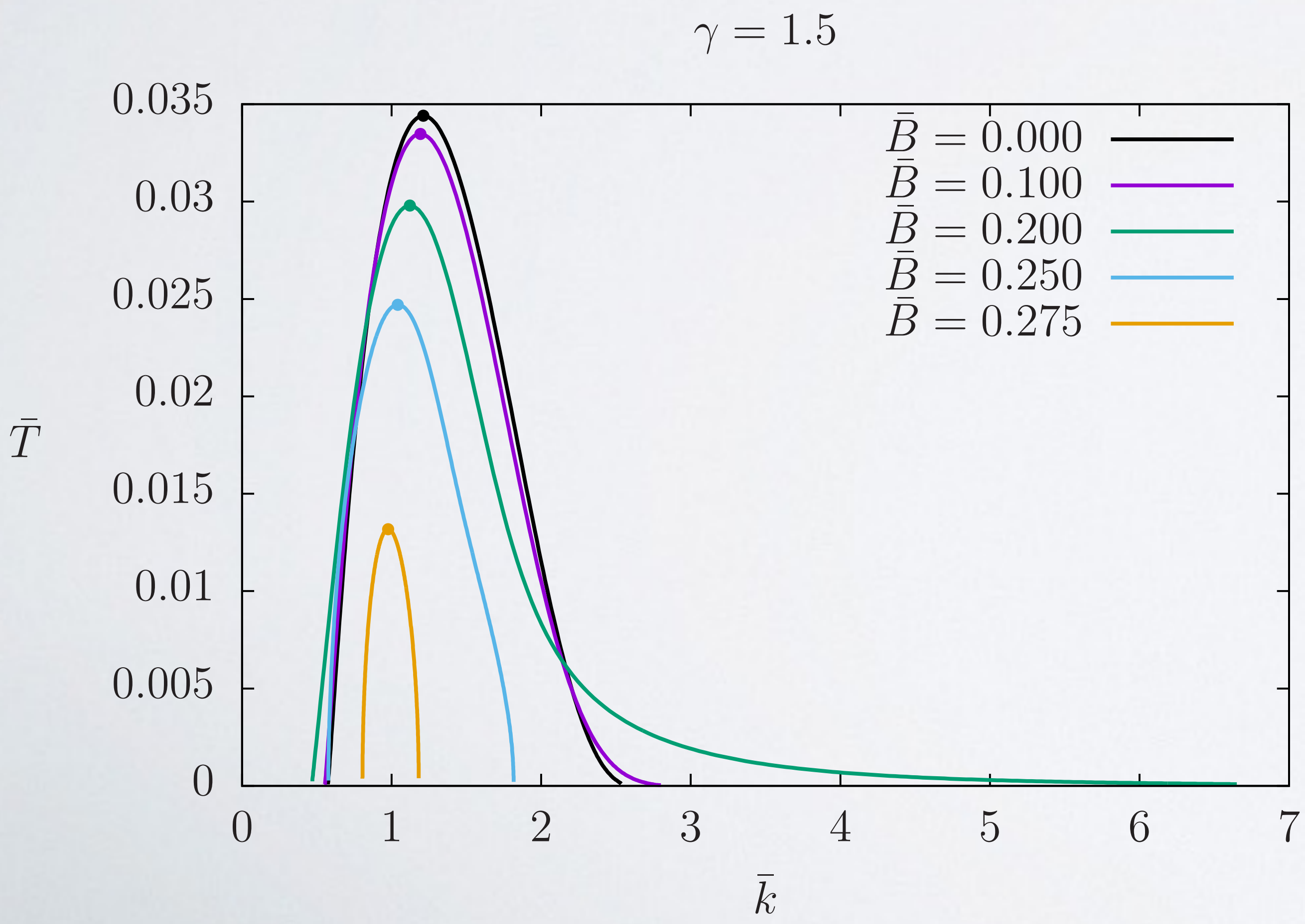
Charged Magnetic Helical black branes

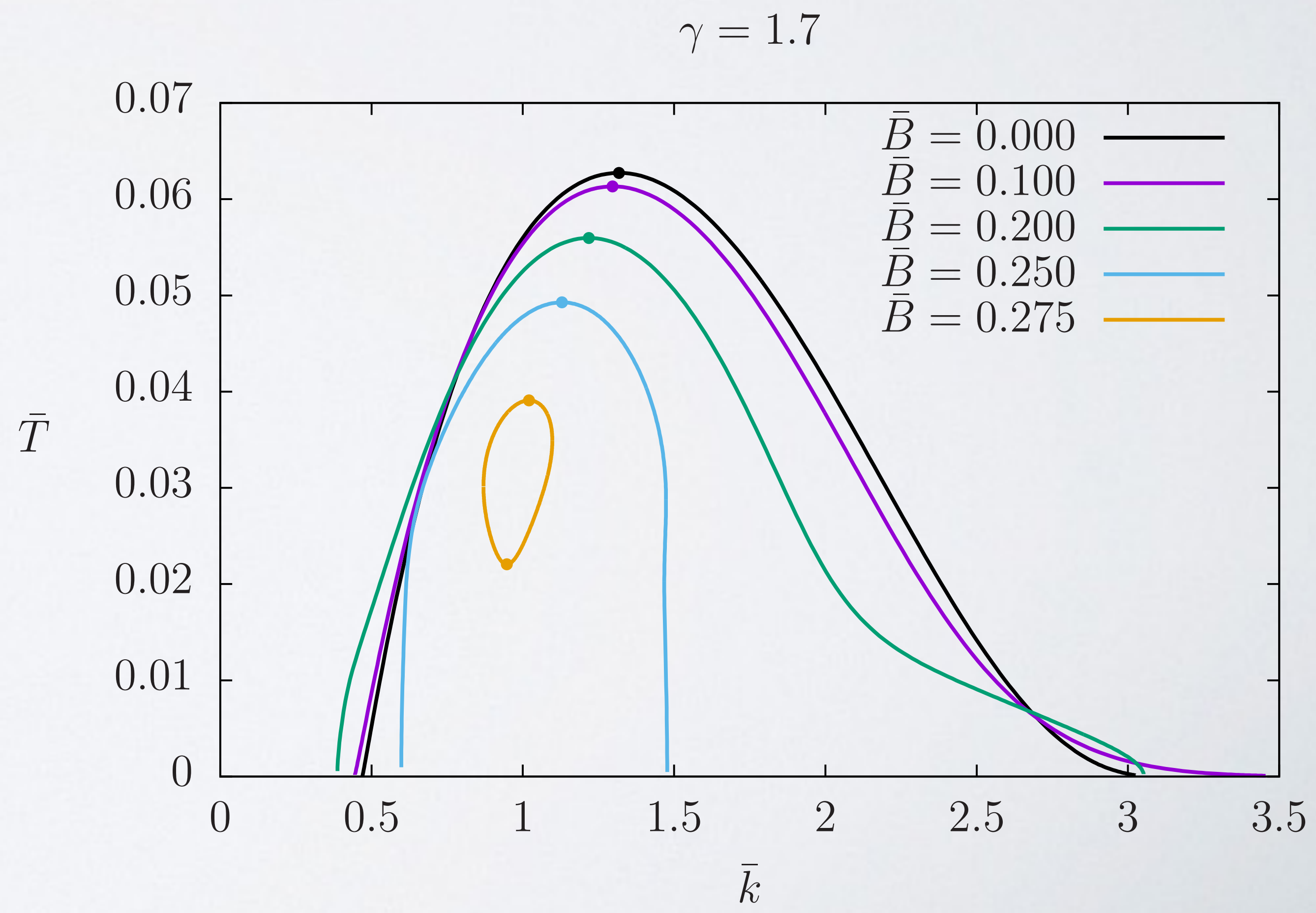
- **Does the helical phase extend to the regime $B \neq 0$?**

Yes!



- The $\bar{T} \times \bar{k}$ plane:

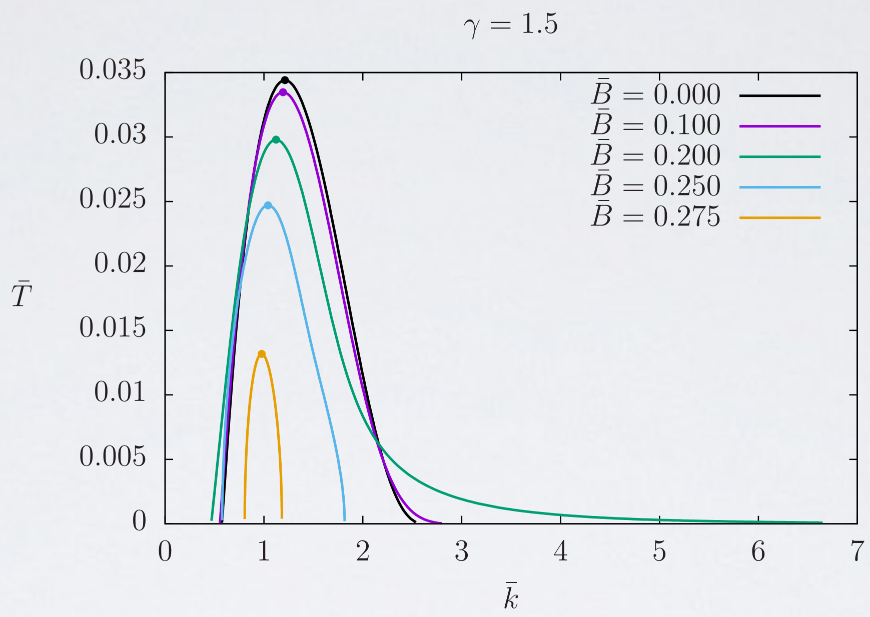


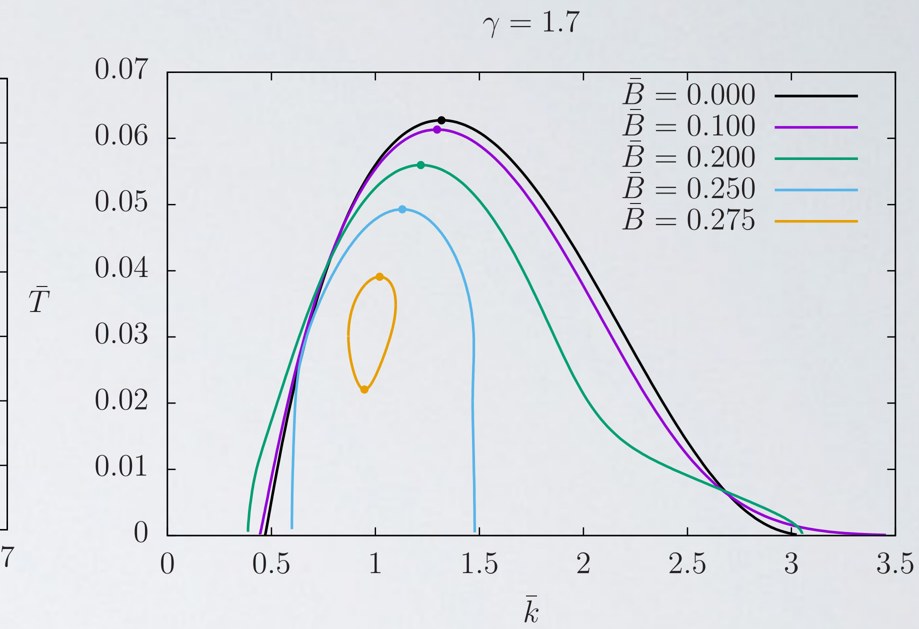


Charged Magnetic Helical black branes

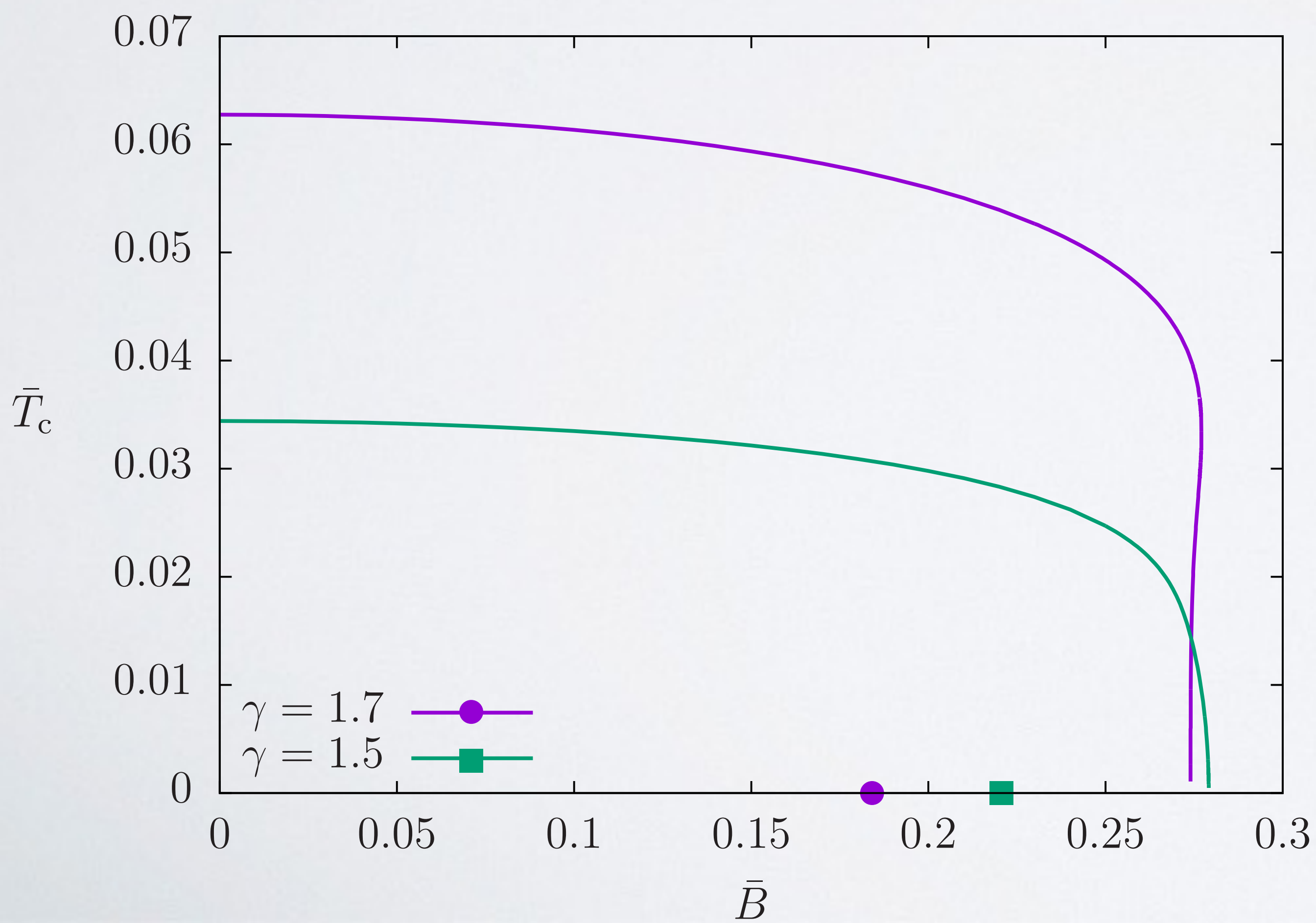
Critical Temperature

$$\bar{T}_{\mathrm{C}}(\bar{B}) = \max_{\bar{k}} \bar{T}(\bar{k}, \bar{B})$$





- The $\bar{T} \times \bar{B}$ plane:

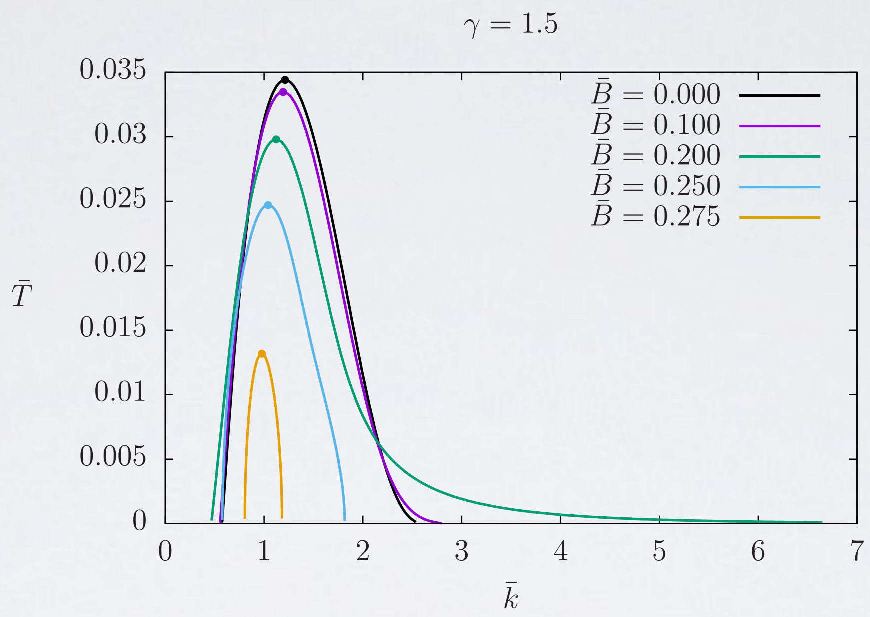


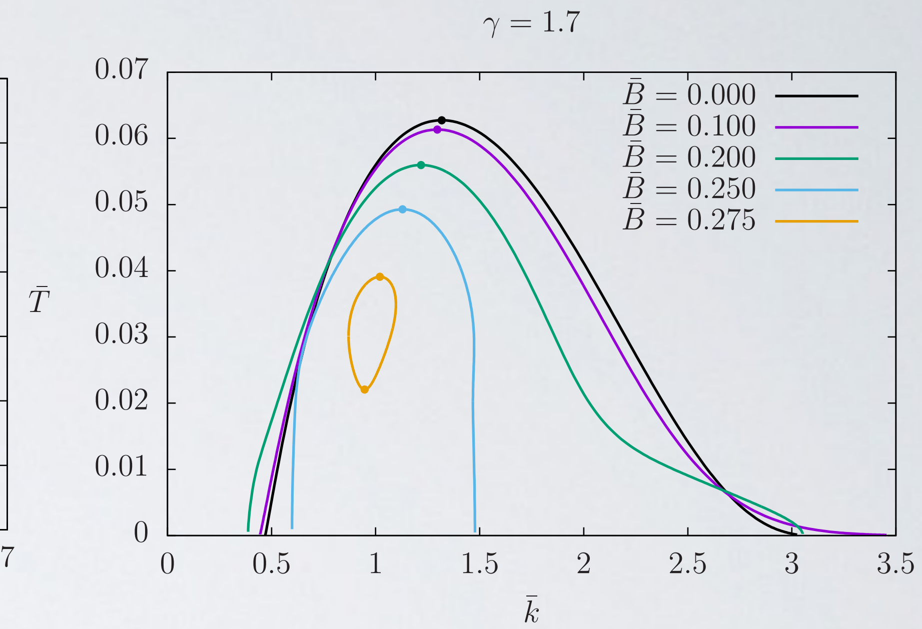
Quantum critical point $\bar{B}_{\mathrm{C}}$ lies within the new helical phase.

Charged Magnetic Helical black branes

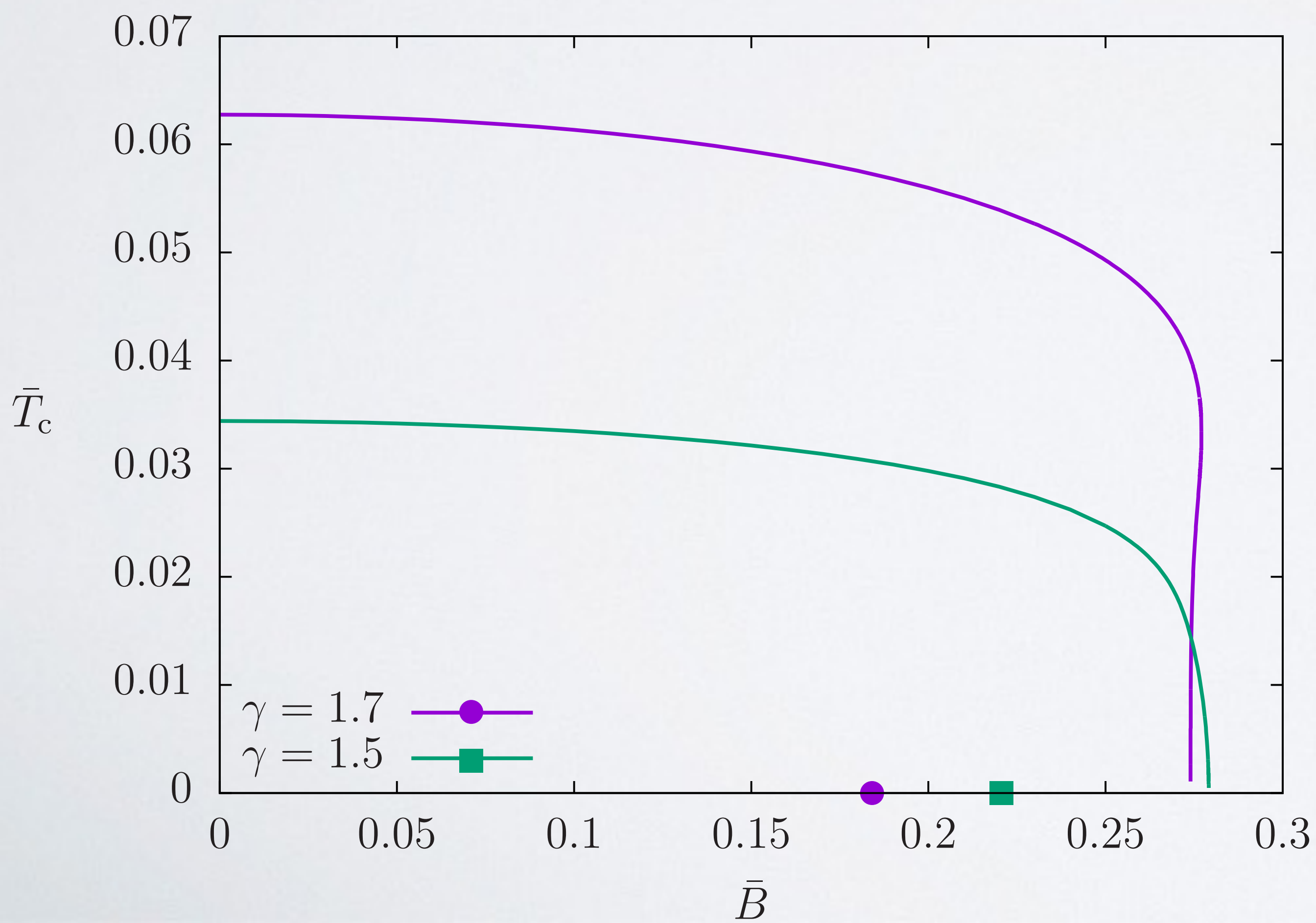
Critical Temperature

$$\bar{T}_{\mathrm{C}}(\bar{B}) = \max_{\bar{k}} \bar{T}(\bar{k}, \bar{B})$$





- The $\bar{T} \times \bar{B}$ plane:



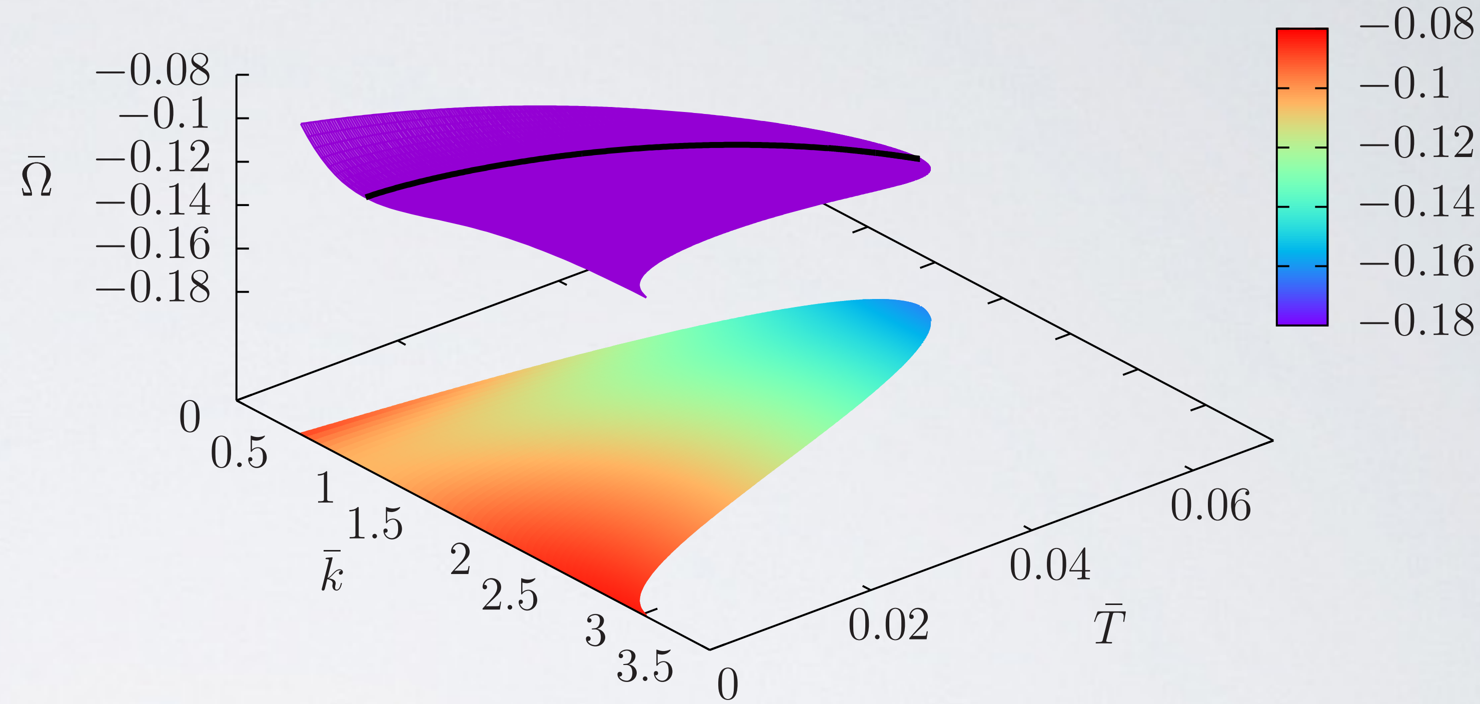
Quantum critical point $\bar{B}_{\mathrm{C}}$ lies within the new helical phase.

- **How does the entropy behave for $\bar{B} < \bar{B}_{\mathrm{C}}$?**

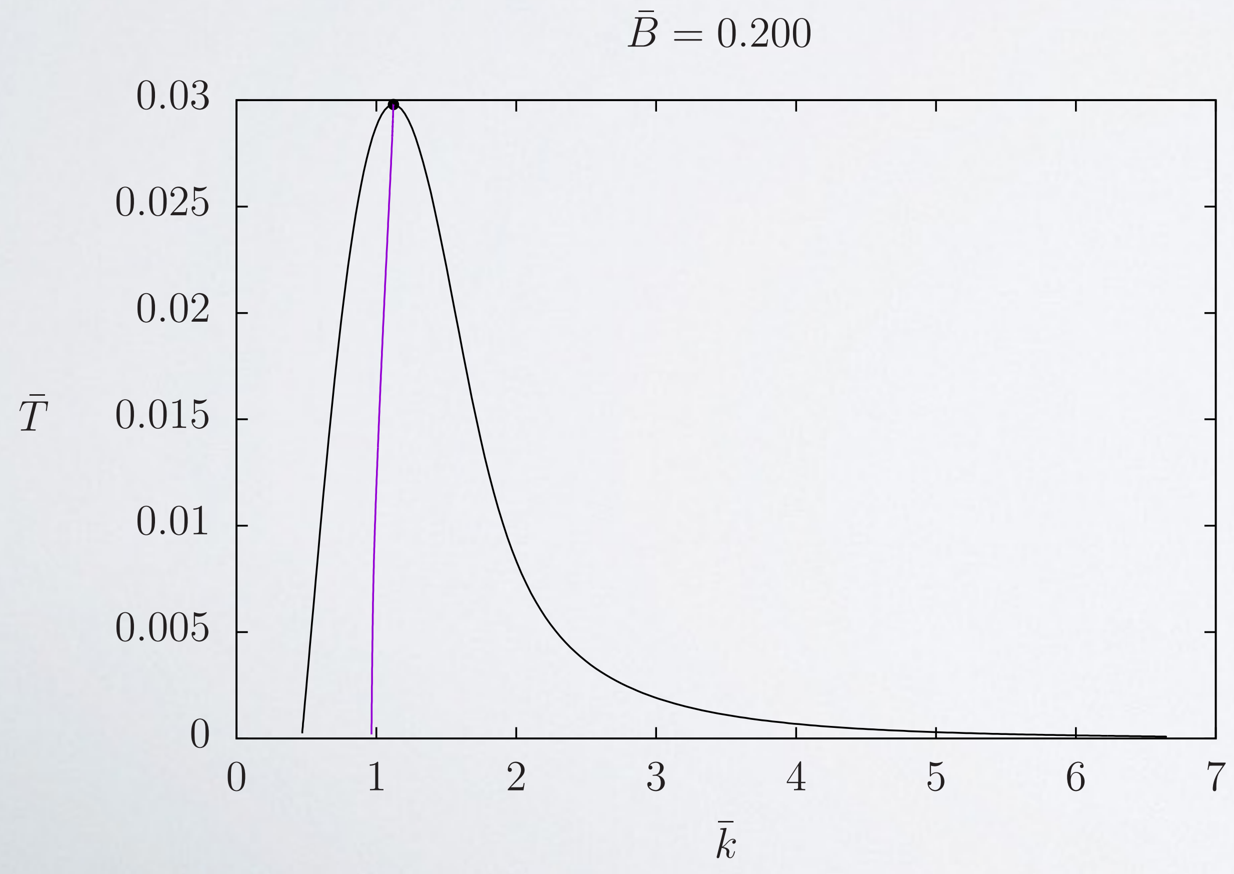
Charged Magnetic Helical black branes

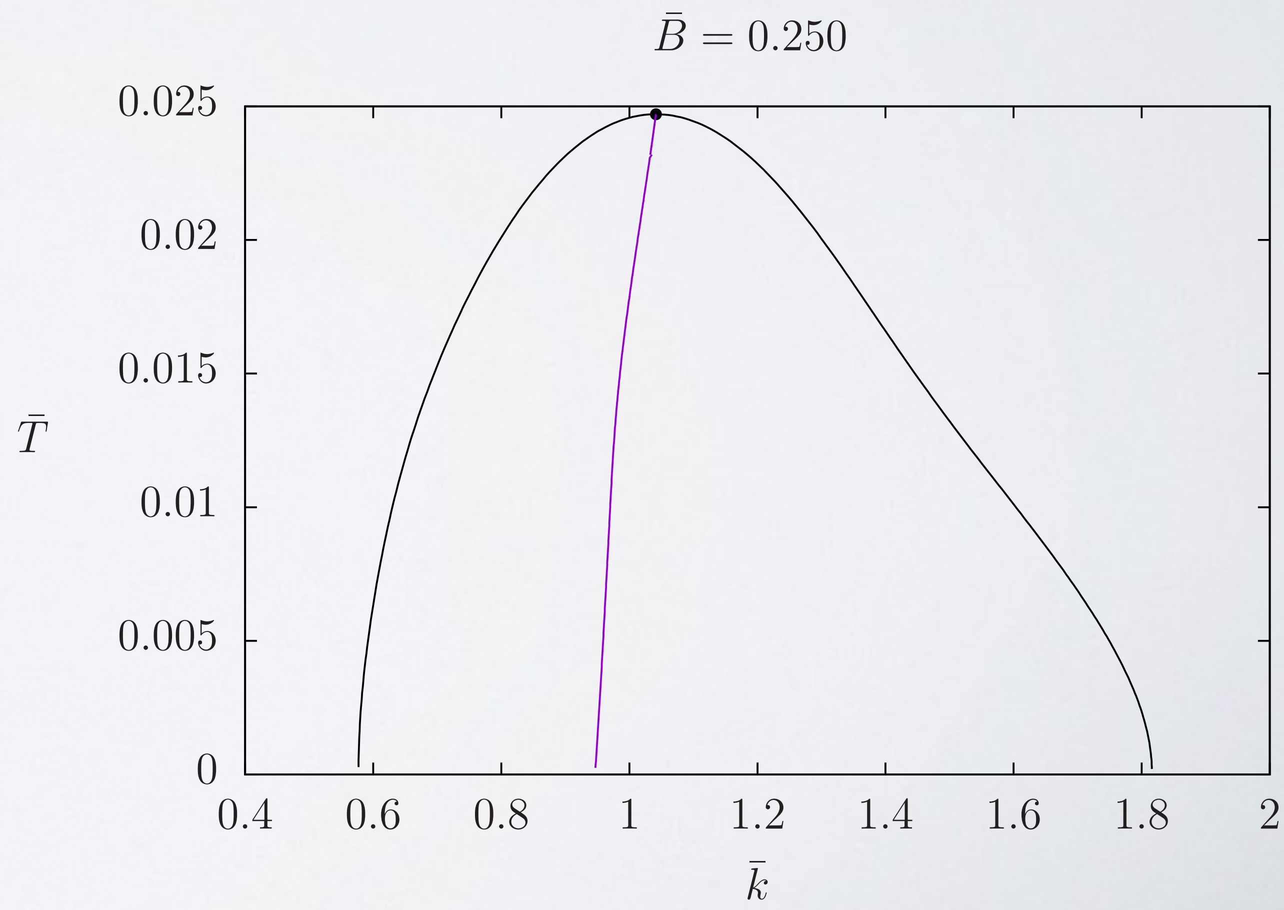
Thermodynamically preferable states:

$$\min_{\bar{k}}[\bar{\Omega}(\bar{k})]\Big|_{\bar{T},\bar{B}} \Rightarrow \bar{k} = \bar{k}^*(\bar{T})$$

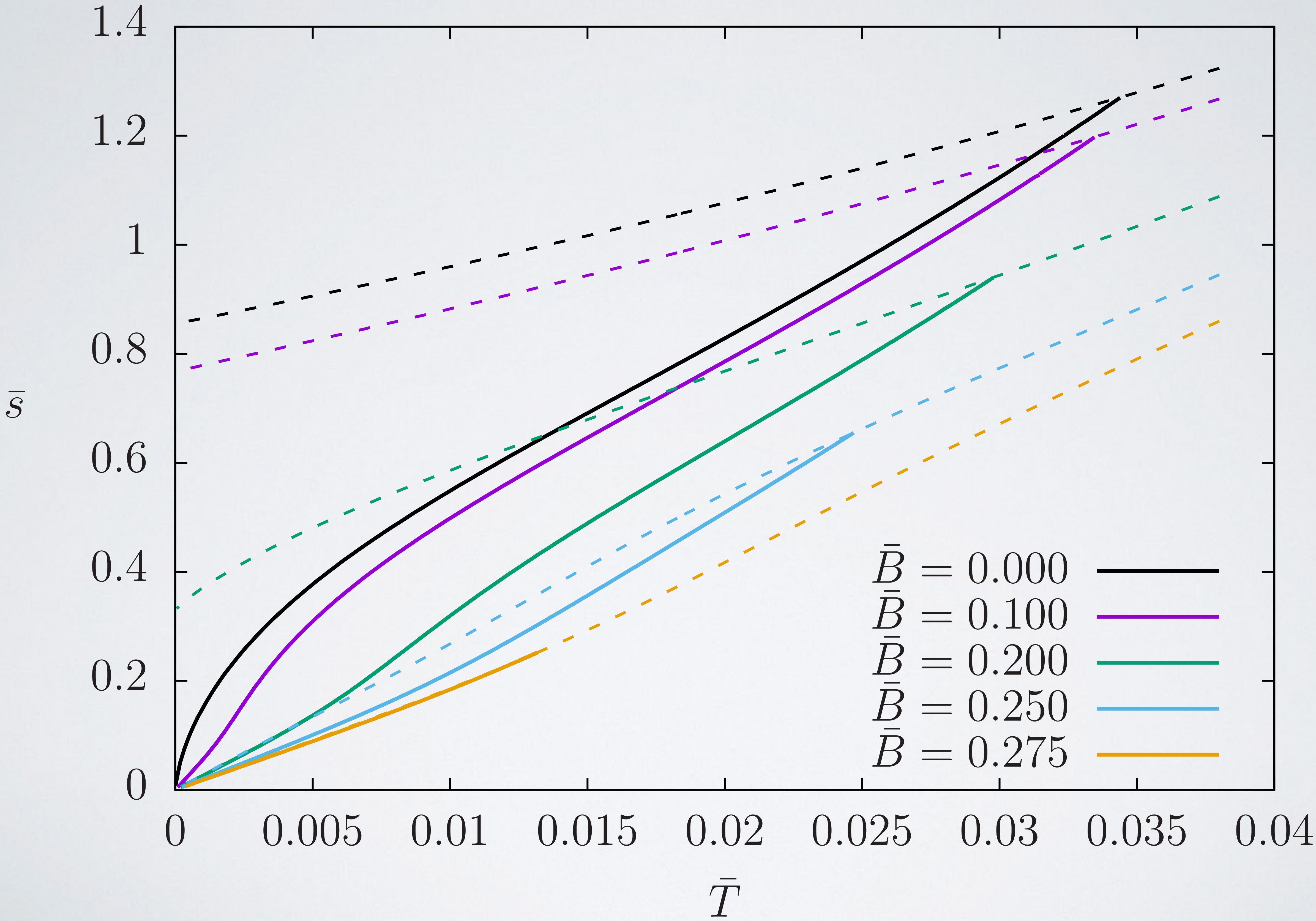


- The $\bar{T} \times \bar{k}$ plane:





Charged Magnetic Helical black branes $\gamma = 1.5$



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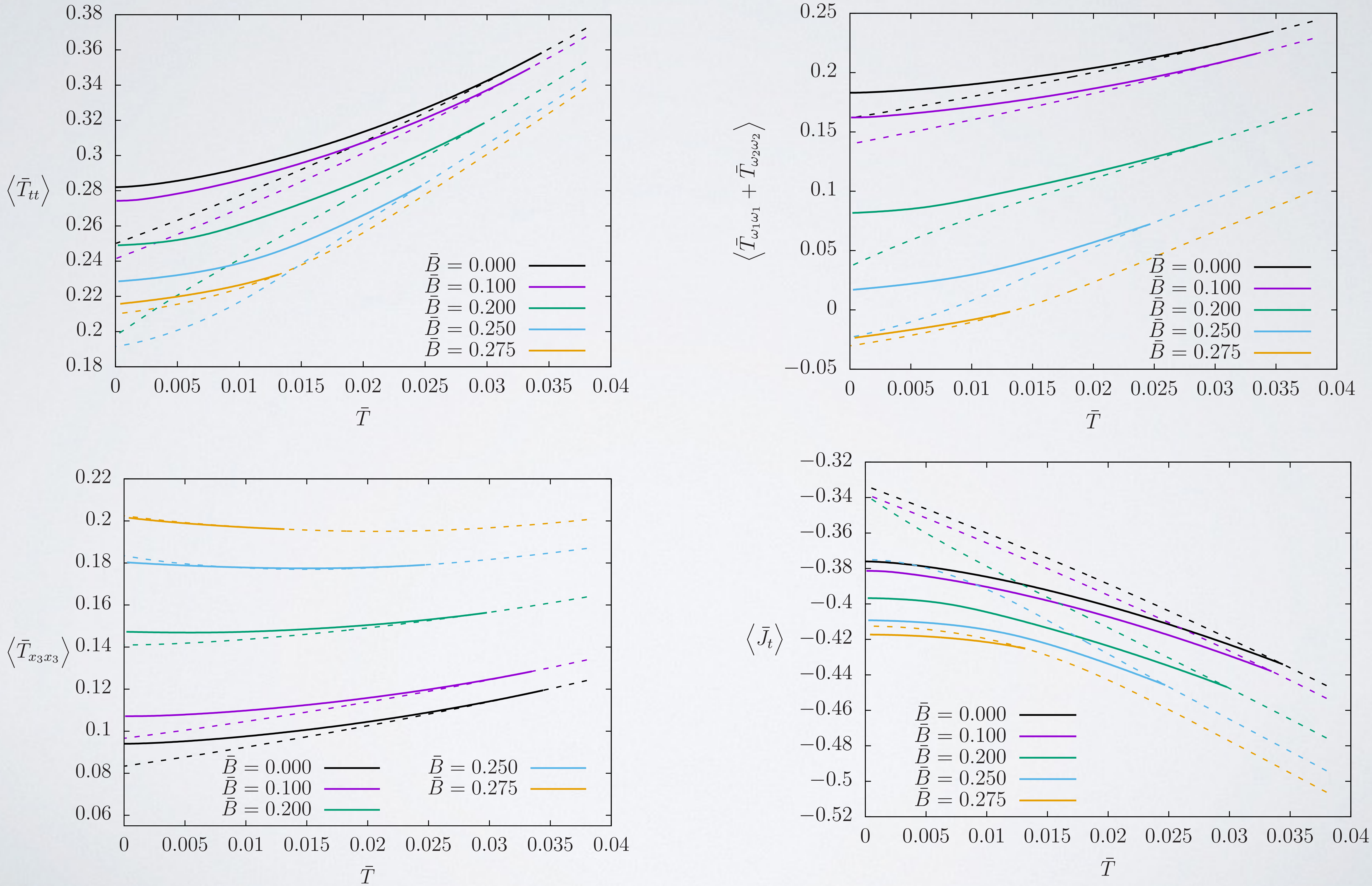
- **Thermodynamics:**



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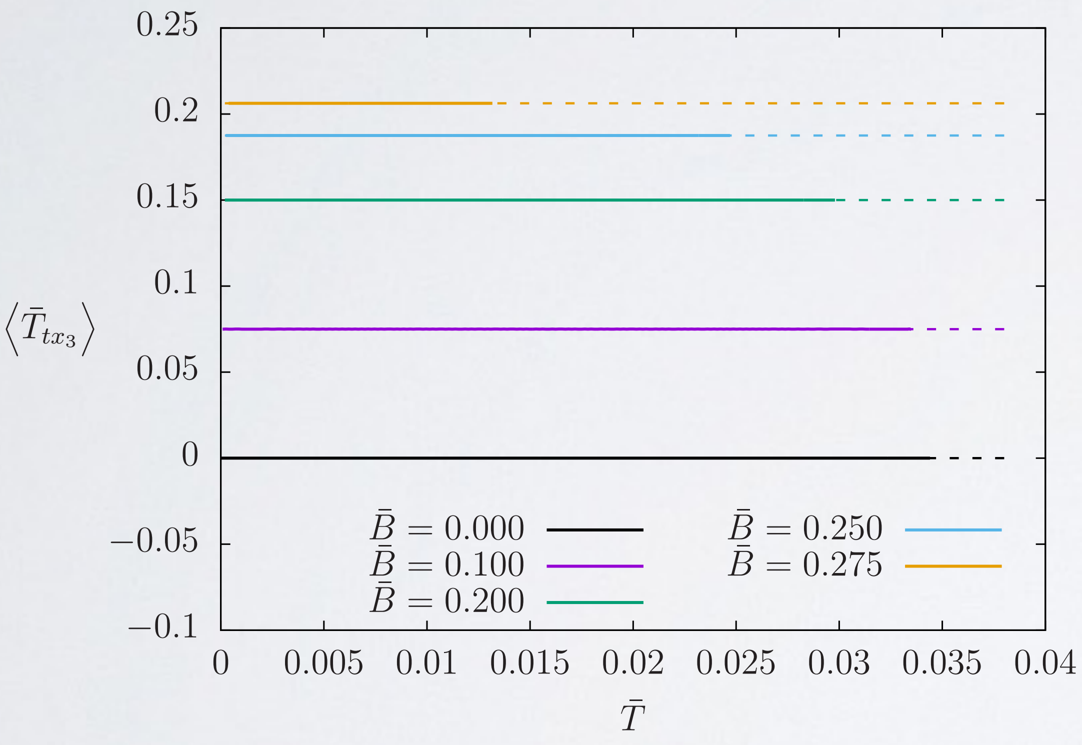
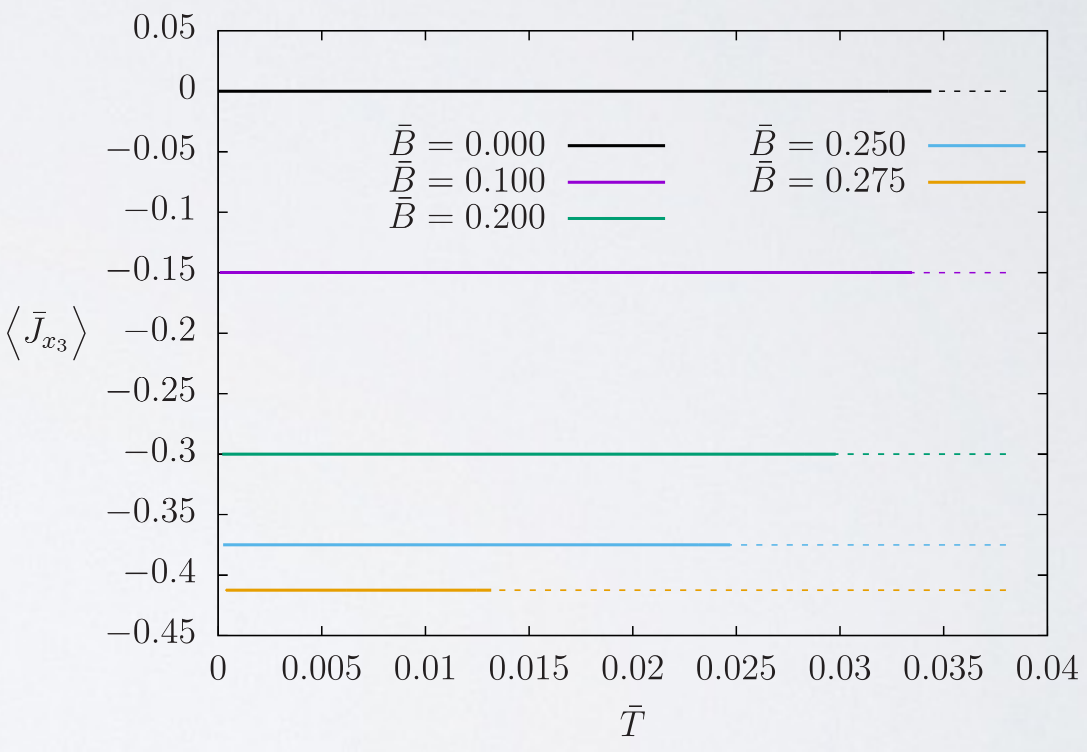
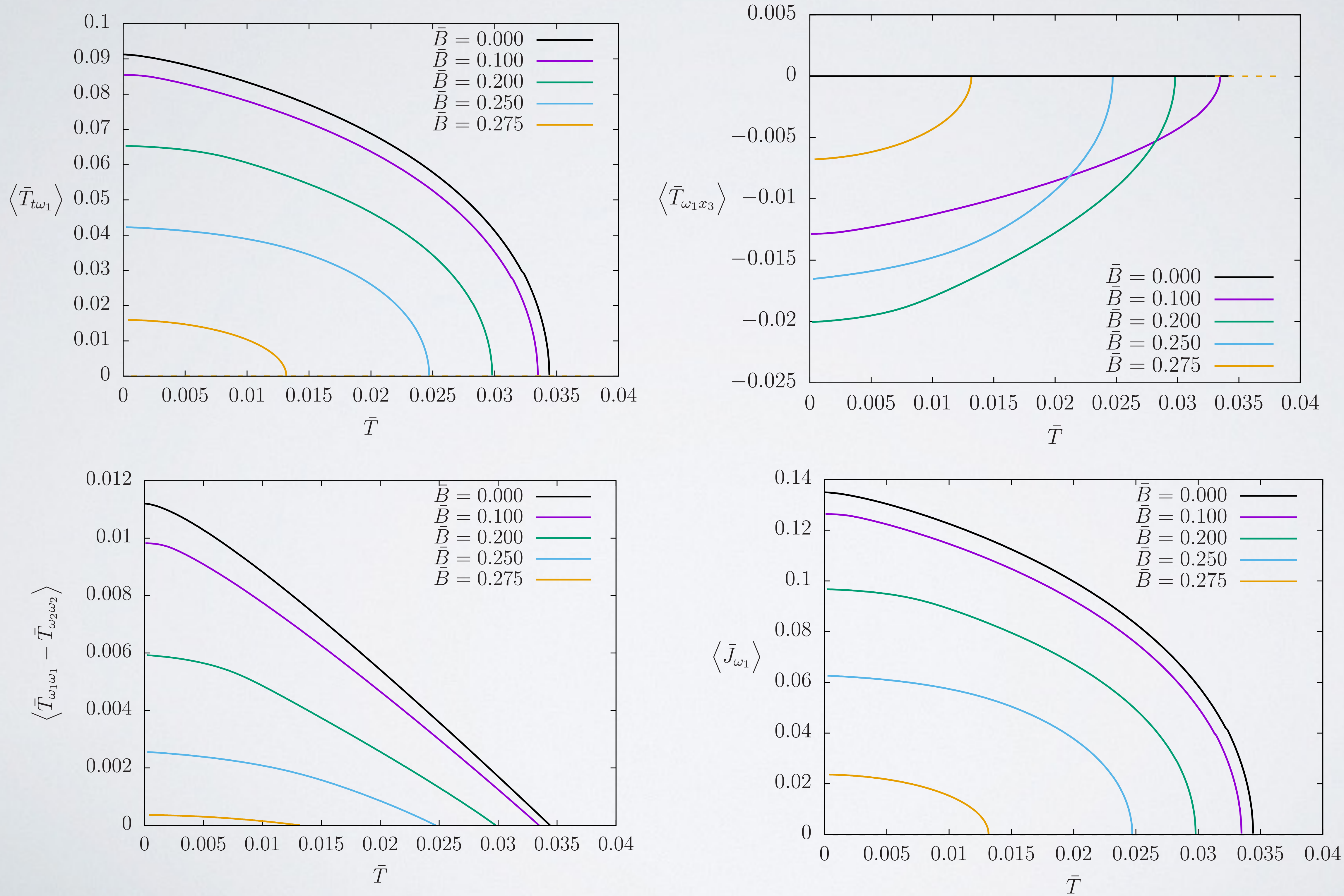




Charged Magnetic Helical black branes $\gamma = 1.5$

- **Thermodynamics: critical exponents**



Charged Magnetic Helical black branes $\gamma = 1.5$

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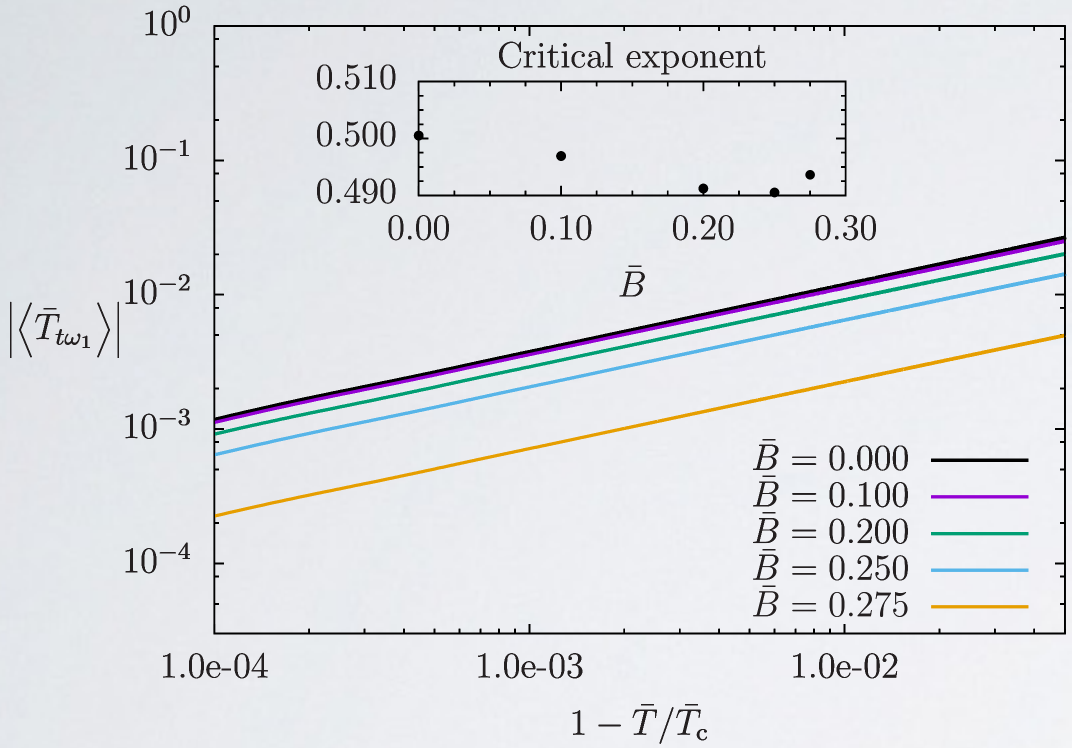

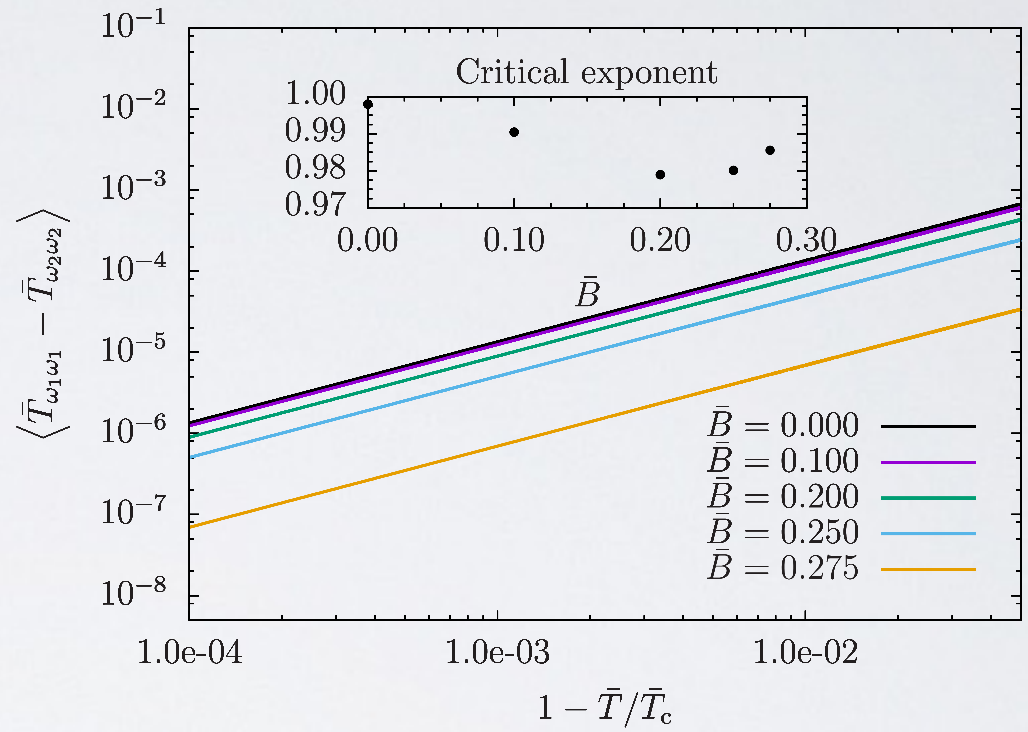

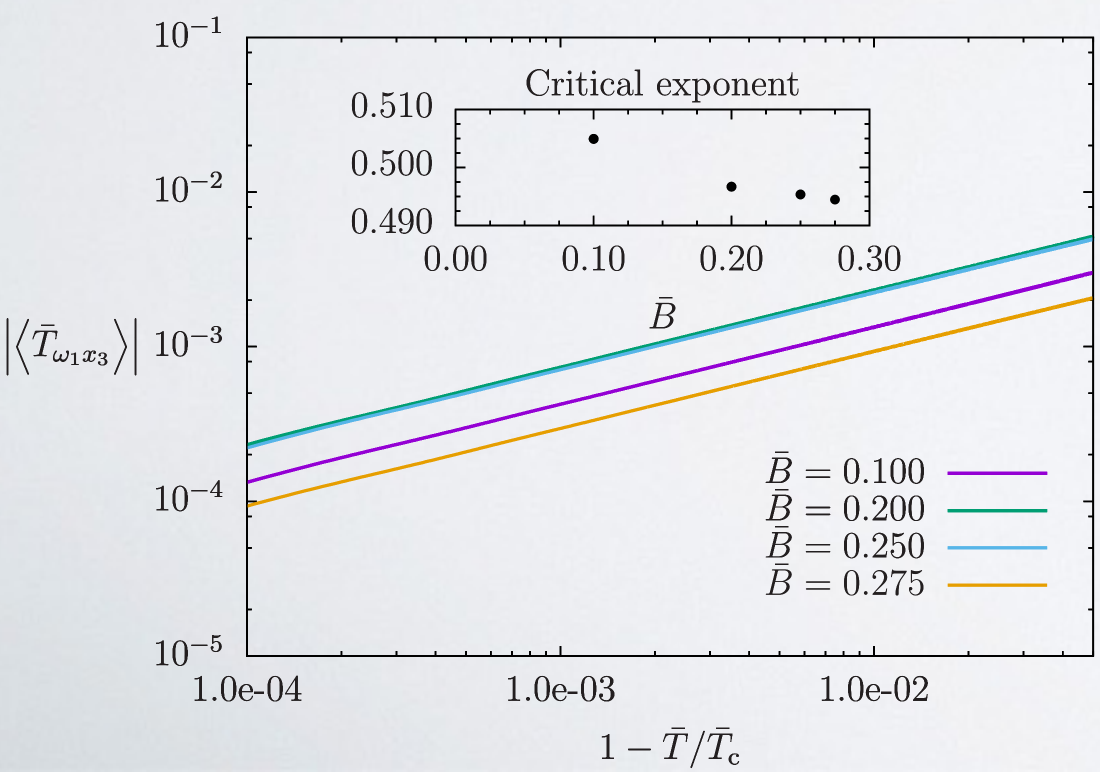

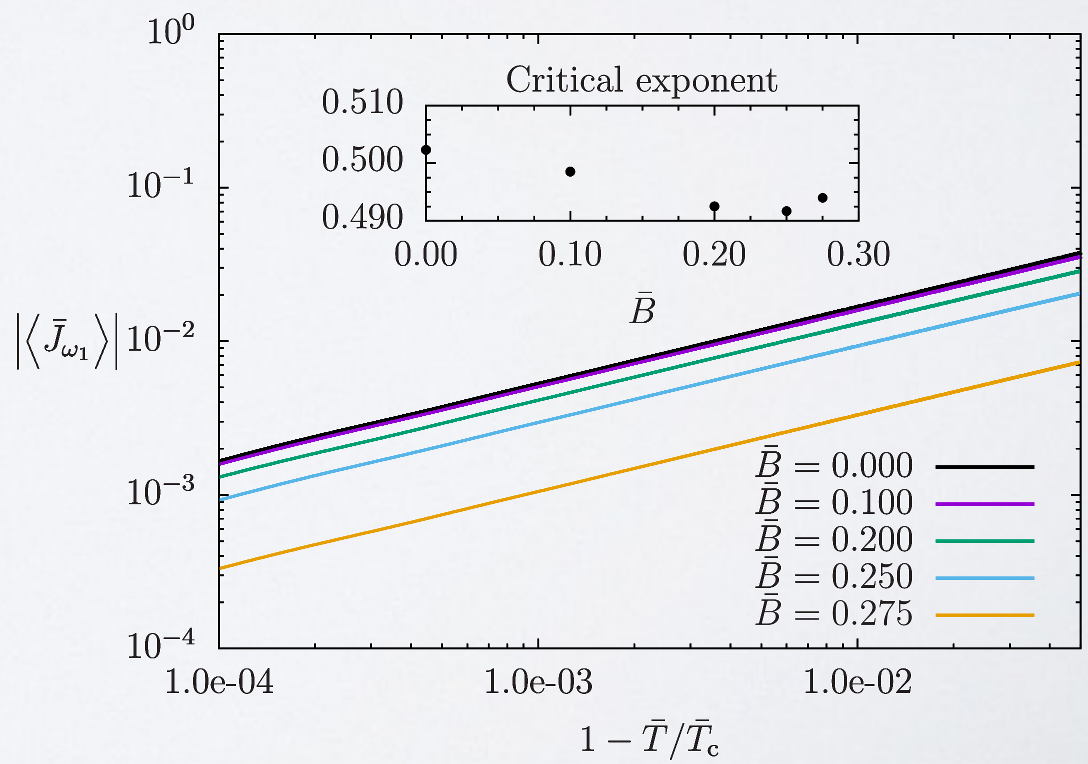

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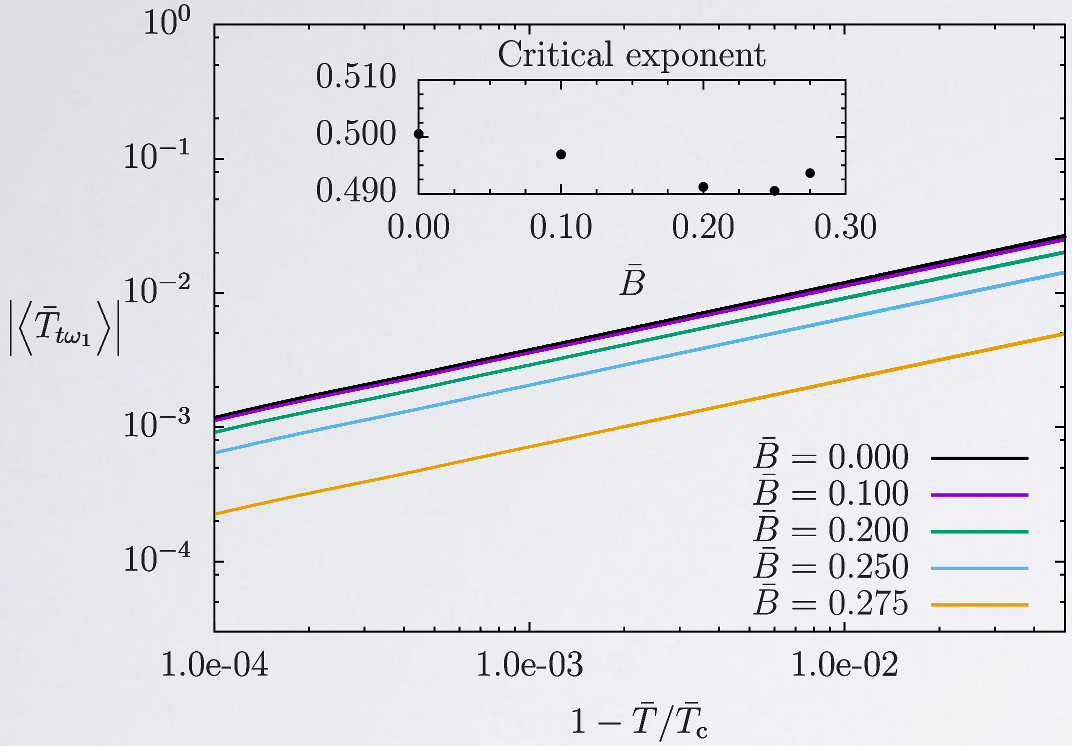

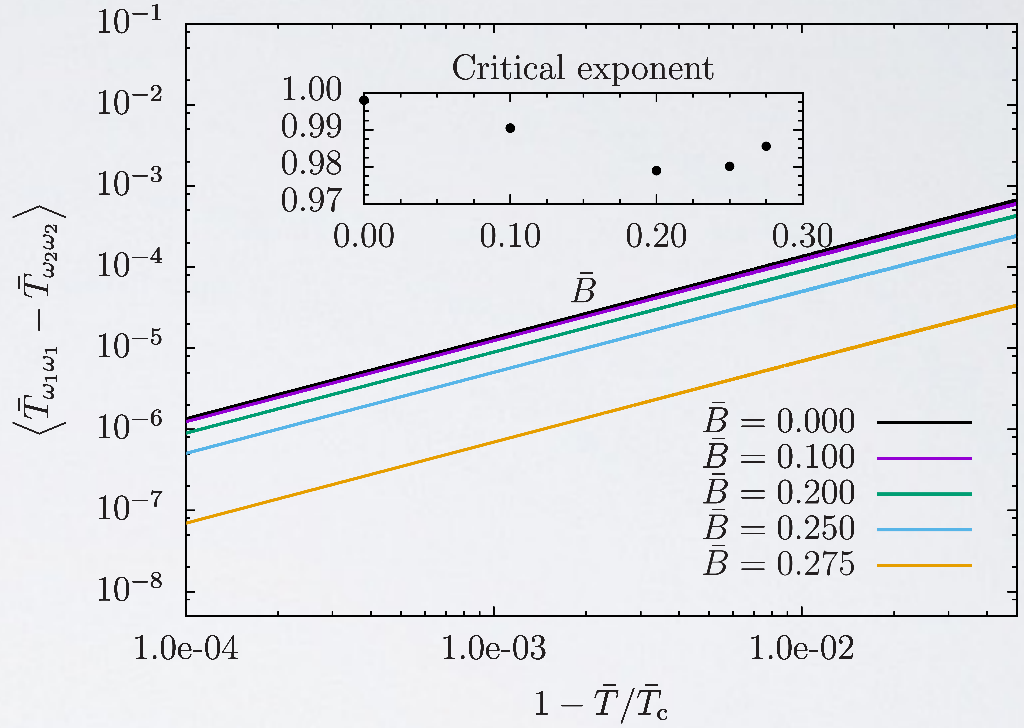

- No systematic dependence of critical exponent with $\bar{B}$

- Values are the one expected from mean field theory

$$\langle \bar{T}_{t\omega_1} \rangle \sim \left| 1 - \frac{\bar{T}}{\bar{T}_C} \right|^{1/2} \qquad \langle \bar{J}_{\omega_1} \rangle \sim \left| 1 - \frac{\bar{T}}{\bar{T}_C} \right|^{1/2} \qquad \langle \bar{T}_{\omega_1 x_3} \rangle \sim \left| 1 - \frac{\bar{T}}{\bar{T}_C} \right|^{1/2}$$

$$\left[\langle \bar{T}_{\omega_1\omega_1} \rangle - \langle \bar{T}_{\omega_2\omega_2} \rangle \right] \sim \left| 1 - \frac{\bar{T}}{\bar{T}_C} \right|^{1}$$

CONCLUSION

- Highly accurate numerical solution for the Einstein-Maxwell-Chern-Simons system in AdS_5 describing **charged magnetic helical black brane**
- Strongly coupled 4-D CFT's with chiral anomaly: phase diagram at finite temperature, chemical potential and magnetic field shows of the new spatially modulated phase for low temperatures and small magnetic fields
    - Quantum critical point is hidden within the new phase
    - Second order phase transition with mean field critical exponents
    - Entropy vanishes in the limit of zero temperature

OUTLOOK

- Further explorer the dependence on the chiral anomaly coefficient γ
- Relation between B_{C} (quantum critical point) and B_0 (phase boundary) ?
- Magnetic field not aligned with helical structure (solution of PDE's)
- Linear perturbations on this background: QNM's and transport coefficients