Holographic Entanglement Chemistry

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The 1^{st} law for Black Holes: where is $V\delta P$?

mass M $\longleftrightarrow$ Internal Energy U

area A $\longleftrightarrow$ Entropy S

surface gravity κ $\longleftrightarrow$ Temperature T

First Law: $\boxed{dM = \frac{\kappa}{8\pi G} dA}$

$(dU = T\partial S)$

What about Pressure-Volume?

Use cosmological constant:

$$P = \frac{-\Lambda}{8\pi G}$$

$$\Rightarrow dM = \frac{\kappa}{8\pi G} dA + \underbrace{\left(\frac{\partial M}{\partial \Lambda}\right)_A d\Lambda}_{\boxed{VdP!}}$$

$\boxed{dH \text{ (enthalpy!)}}$

. For Schwarzschild:

$$\boxed{V = \mathrm{Vol}(S_{d-1}) \frac{r_+^d}{d}} \leftarrow \text{volume of a ball...}$$

② Holographic BH Chemistry

How are we to interprete $P-V$ in the dual field thy?

- Λ is <u>not</u> field theory pressure $\langle T_{ij} \rangle$, and the conjugate volume is <u>not</u> the volume of the field theory either...

- Holographic Dictionary: $\boxed{\dfrac{L^{d-1}}{G} \sim N^2}$

$\Rightarrow$ Varying L at fixed G changes N !

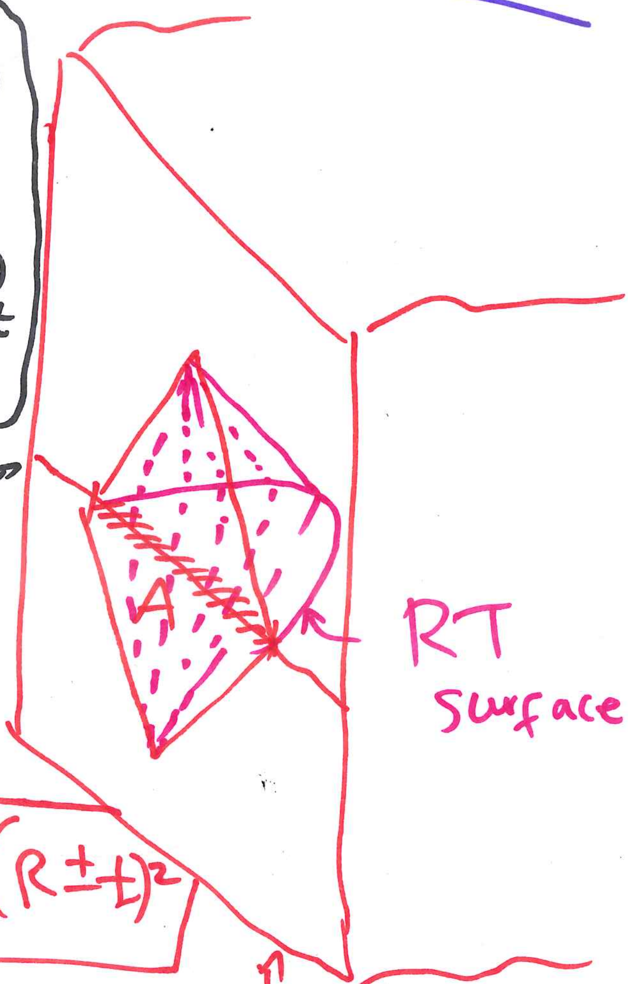
- The volume variable is a <u>chemical potential</u> associated to N.

The 1^{st} law for entanglement

. Recently, it was noticed that the RT surface in AdS is the bifurcation surface of a Killing horizon.

$$\boxed{\xi = -\frac{2\pi}{R} t (z\partial_z + x^i\partial_i) + \frac{\pi}{R}(R^2 - z^2 - t^2 - \vec{x}^2)\partial_t}$$

Generates flow confined to wedge. $t=0$



$\xi^2 = 0$ at $\boxed{x^2 + z^2 = (R \pm t)^2}$

Killing horizon

Bifurcation surface at $t=0$

$$\Rightarrow \boxed{\vec{x}^2 + z^2 = R^2} \Rightarrow \text{RT surface!}$$

36 1^{st} Law of entanglement (cont.)

. Field theory side:

Since reduced density matrix is Hermitian, pos-def:

$$\rho_A = \frac{e^{-H_A}}{\mathrm{Tr}(e^{-H_A})}$$

Modular Hamiltonian...

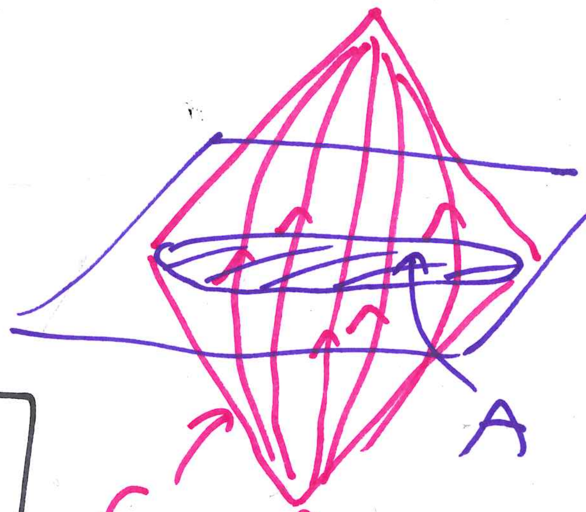
Spherical A: H_A is local, corresponds to flow along conformal Killing vector:

$$\zeta = \lim_{z\to 0} \xi = \boxed{-\frac{2\pi t}{R} x^i \partial_i + \frac{\pi}{R}(R^2 - t^2 - \vec{x}^2)\partial_t}$$

First law:

Under $\rho_A \to \rho_A + \delta\rho_A$,

$$\boxed{\delta S_{EE} = \delta \langle H_A \rangle}$$



Causal development of A.

3c) 1^{st} law of entanglement: Bulk side (Iyer-Wald formalism)

Ingredient: 1) A diffeomorphism-invariant theory of gravity

2) A bifurcate Killing horizon of some ξ^{μ}.

$\Rightarrow$ Construct the Noether charge diff. form

$$\boxed{Q \doteq -\frac{1}{16\pi G} \nabla^a \xi^b \varepsilon_{ab}}$$

only for Einstein gravity!

and the symplectic potential associated to a perturbation δg:

$$\boxed{H \doteq \frac{1}{16\pi G} g^{ac} g^{bd} (\nabla_b \delta g_{cd} - \nabla_c \delta g_{bd}) \varepsilon_a}$$

only for Einstein gravity!

For on-shell perturbations, the 1^{st} law is equivalent to:

$\boxed{\int d\chi = 0}$ with $\boxed{\chi = \delta Q - \xi \cdot H}$

④ First Law of entanglement: varying the couplings ...

Following the BH chemistry program, we can start varying L in the 1^{st} law of entanglement...

But it's also important to vary G_N!

Back to the basic dictionary:

$$\frac{L^{d-1}}{G} \sim N^2 \Rightarrow \boxed{\text{Varying } G \text{ at fixed } L \text{ also changes } N!}$$

Moreover, varying the pressure L (ironically) changes the field theory volume (in global AdS)!

We can simply differentiate:

$$\delta S_{EE} = \delta E + \left(\frac{\partial S_{EE}}{\partial L}\right)\delta L + \left(\frac{\partial S_{EE}}{\partial G}\right)\delta G$$

(5) Iyer-Wald with variable couplings...

- More interesting to apply Iyer-Wald formalism to see how $\left(\frac{\partial S}{\partial L}\right)$, $\left(\frac{\partial S}{\partial G}\right)$ arise geometrically $\Rightarrow$ $\boxed{\text{Holographic interpretation?}}$

- The usual Iyer-Wald:

$$\int_{\Sigma} d\chi = 0 = \int_{\partial\Sigma} \chi = \underbrace{\int \chi}_{\text{horizon}} - \underbrace{\int \chi}_{\text{infinity}}$$

Σ ← Cauchy surface...

horizon ↑ "$T\delta S$"   infinity ↑ "δE"

- With variable couplings:

$$\boxed{\delta c \int_{\Sigma} \xi \cdot \left(\frac{\partial L}{\partial c}\right) + \int_{\Sigma} d\chi = 0}$$

c ← coupling being varied.

⑥ Example: Einstein Gravity with varying L

- $$\boxed{\mathbf{L} = \left(R + \frac{d(d-1)}{L^2}\right)\varepsilon}$$ ← Lagrangian $(d+1)$-form

- Unperturbed Noether charge:

$$\boxed{Q|_\Sigma = -\frac{1}{16\pi G}\left(\frac{4\pi z^2 x^i}{R L^2}\varepsilon_{ti} + \frac{2z^2}{L^2}\left(\frac{2\pi z}{R} + \frac{\xi^t}{z}\right)\varepsilon_{tz}\right)}$$

$$Q|_\Sigma \propto L^{d-1} \Rightarrow \boxed{\delta Q = \left(\frac{d-1}{L}\right) Q\, \delta L}$$

- Symplectic potential current:

$$ds^2 = \frac{L^2 + 2L\delta L}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

$$\Rightarrow \boxed{H = 0} \Rightarrow \boxed{\chi = \delta Q}$$

Extended first law: $$\boxed{\frac{d(d-1)\,\delta(L^{-2})}{16\pi G}\int_\Sigma \xi\cdot\varepsilon - \int_{\partial\Sigma_\infty}\delta Q + \int_{\partial\Sigma_h}\delta Q = 0}$$

7 Einstein Gravity ~~[illegible]~~ with varying L

with

$$\boxed{\int_{\partial\Sigma_1} \delta Q} = -\frac{(d-1)R}{4G} L^{d-2}\,\delta L\,\mathrm{Vol}(S_{d-2}) \times \int_0^{\sqrt{R^2-\varepsilon^2}} \frac{r^{d-2}}{(R^2-r^2)^{d/2}}\,dr$$

$$= \left(\frac{\partial S_{EE}}{\partial L}\right)\delta L = \boxed{\delta S_{EE}}$$

$$\boxed{\int_{\partial\Sigma_\infty} \delta Q} = -\frac{(d-1)\,\delta L}{8GR} L^{d-2}\,\mathrm{Vol}(S_{d-2}) \times \int_0^{\sqrt{R^2-\varepsilon^2}} \left(\frac{1}{\varepsilon^{d-2}} + \frac{R^2-r^2}{\varepsilon^d}\right) r^{d-2}\,dr$$

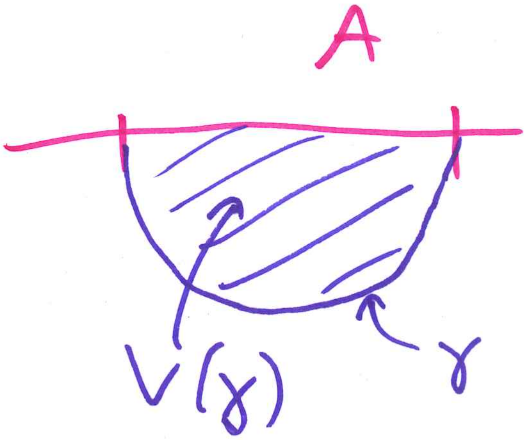
$$\boxed{\frac{d(d-1)}{16\pi G}\,\delta(L^{-2}) \int_\Sigma \xi\cdot\varepsilon} = -\frac{(d-1)\,\delta L}{8GR} L^{d-2}\,\mathrm{Vol}(S_{d-2}) \times$$

$$\int_0^{\sqrt{R^2-\varepsilon^2}} \left(\frac{R^2-r^2}{\varepsilon^d} + \frac{d}{2-d}\frac{1}{\varepsilon^{d-2}} + \frac{2}{(d-2)}\frac{1}{(R^2-r^2)^{\frac{d}{2}-1}}\right) r^{d-2}\,dr$$

$$\boxed{} + \boxed{} = \boxed{\left(\frac{\partial S_{EE}}{\partial L}\right)\delta L}$$

(8) Relation to the complexity?

. Alishahiba:



Complexity

$$\boxed{C_A = \frac{V(\gamma)}{8\pi R G}}$$

Volume between γ and boundary

. Initial hope was to relate extended 1st law to complexity: $\delta S = \left(\frac{\partial S}{\partial L}\right)\delta L$

"volume"

But $V(\gamma) \neq \left(\frac{\partial S}{\partial L}\right)$.

$$\boxed{V(\gamma)} = \int \sqrt{g_\Sigma}\, dx\, dz = \boxed{\int \left(\frac{L}{z}\right)^d d^{d-1}x\, dz}$$

$$\left(\frac{\partial S}{\partial L}\right) = \frac{d(d-1)}{16\pi G}\,\delta(L^{-2}) \underbrace{\int_\Sigma \xi \cdot \varepsilon}_{} - \int_{\partial\Sigma_\infty} \chi$$

$$= \boxed{\int (R^2 - \vec{x}^2 - z^2)\left(\frac{L}{z}\right)^{d+1} d^{d-1}x\, dz}$$

. Different power of $\frac{L}{z}$, factor $R^2 - \vec{x}^2 - z^2$...

(9) Example 2: Varying α in GB

- Lagrangian of GB gravity:

$$\mathcal{L} = \left(\frac{R-2\Lambda}{16\pi G} + \alpha\left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2\right)\right)\epsilon$$

- Symplectic potential current:

$$\Theta = \epsilon_d\Big[\left(\frac{1}{16\pi G} + 2\alpha R\right) g^{de} g^{fh}\left(\nabla_f \delta g_{eh} - \nabla_e \delta g_{fh}\right)$$
$$+\alpha\Big(-2(\nabla^e R) g^{df}\delta g_{ef} + 4R^{de}(\nabla_e \delta g_{fh}) g^{fh}$$
$$+4R^{ef}(\nabla^d \delta g_{ef}) - 8R^{ef}(\nabla_e \delta g_{fh}) g^{dh}$$
$$-4(\nabla^e R^{df})\delta g_{ef} + 4R^{defh}\nabla_h \delta g_{ef}\Big)\Big]$$

- Noether charge:

$$Q = -\epsilon_{de}\Big(\frac{1}{16\pi G}\nabla^d \xi^e + 2\alpha\big(R\nabla^d \xi^e$$
$$+4\nabla^{[f}\xi^{d]}R^e{}_f + R^{defh}\nabla_f \xi_h\big)\Big)$$

Very intimidating expressions...

10 Example 2 cont.

- GB gravity admits AdS as s/n;

but $\Lambda = \Lambda(L, G, \alpha)$:

$$\Lambda = \frac{d(d-1)}{2L^4}\left(16\pi G\alpha(d-2)(d-3) - L^2\right)$$

- HEE is computed by minimizing the functional:

$$S = \frac{1}{4G}\int_M d^{d-1}x \sqrt{h}\,[1 + 32\pi G\alpha R]$$

induced metric determinant

Ricci scalar of induced metric

$\Rightarrow$ Still a hemisphere...

$$\boxed{z = \sqrt{R^2 - \vec{x}^2}}$$

$\Rightarrow$ Still a Killing horizon $\Rightarrow$ $\boxed{\text{Iyer-Wald applies}}$

Use symmetries of AdS to simplify:

$$R_{abcd} = -\frac{1}{L^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$R_{ab} = -\frac{d}{L^2}g_{ab}$$

$$R = \frac{-d(d+1)}{L^2}$$

11 Example 2 continued

Using the maximal symmetry, Q and $\textcircled{H}$ take the form:

$$\begin{cases} Q \propto (\nabla^a \xi^b)\,\varepsilon_{ab} \\ \textcircled{H} \propto \varepsilon_d \left(g^{df}\nabla^e \delta g_{ef} - g^{ef}\nabla^d \delta g_{ef}\right) \end{cases}$$

Index structure exactly as in Einstein gravity, but factor of proportionality depend on α! (fixed L)

Extended 1st law w/ $\delta\alpha$:

$$\delta\alpha \int_\Sigma \frac{\partial \mathcal{L}}{\partial \alpha}\,\xi\cdot\varepsilon - \int_{\partial\Sigma_\infty} \chi + \underbrace{\int_{\partial\Sigma_h} \chi}_{\delta S} = 0$$

$$-\frac{1}{8\pi G}\frac{\partial\Lambda}{\partial\alpha} + \mathcal{L}_{(2)} = \frac{4d(d-1)(d-2)}{L^4}$$

$$\Rightarrow \boxed{\delta S = \left(\frac{\partial S}{\partial \alpha}\right)\delta\alpha \ldots}$$

proportional to S.

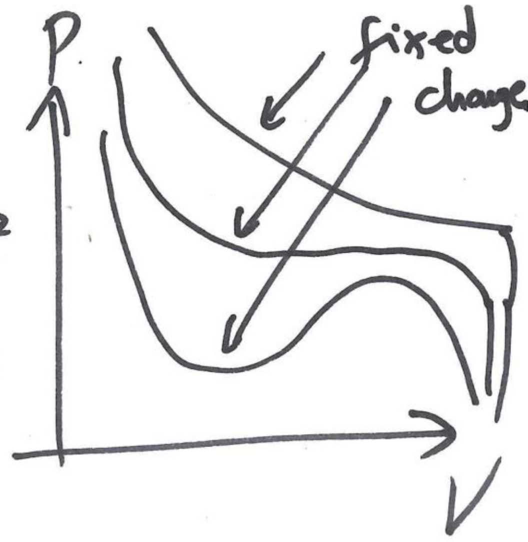
. In fact, in Lovelock gravity, any variation of any coupling $\Rightarrow$ conjugate is proportional to S...

(12) What's the hope?

- For excited states, modular Hamiltonian is nonlocal, but perhaps Iyer–Wald has something to say?
  (approximate Killing vector fields)

- Shed light on P–V criticality of charged BHs?

van der Waals-like transition



probably numerical... since RT surface is not known analytically for RN-AdS.

More about this is:
arXiv:1507.06069 [hep-th]
arXiv:1508.01955 [hep-th]