Holographic fermions in helical lattice

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Lattice in CMT

Lattice is fundamental in CMT

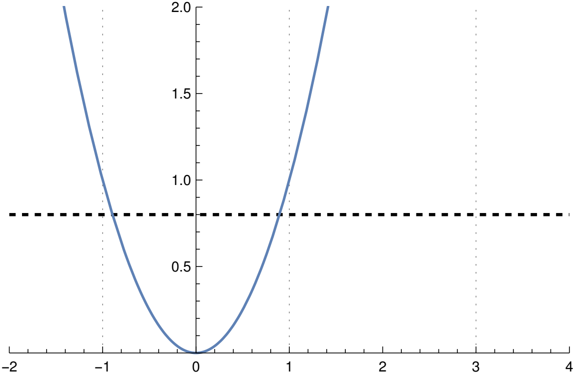
In metals

- Bloch states
- Brillouin zone
- Band structure
- UV cut off
- ...

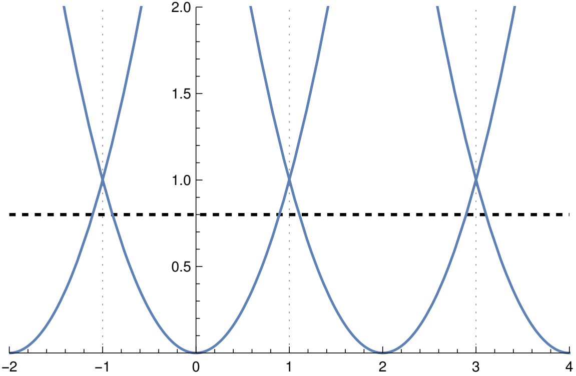
In insulators

- Tight binding
- Hubbard model
- AF state
- Mott insulator
- ...

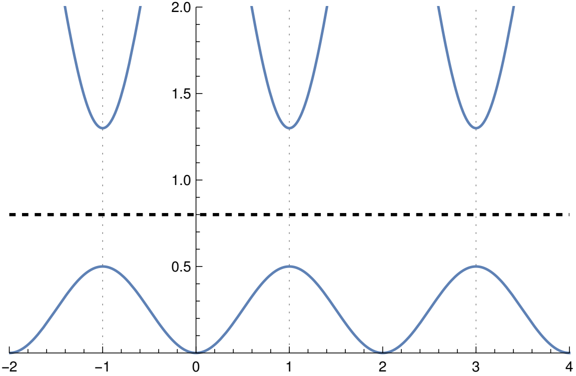
Insulator: Umklapp scattering



Insulator: Umklapp scattering



Insulator: Umklapp scattering



Lattice in holography

Holography doesn't come with lattice included

Crystal lattice $\equiv$

- Translation symmetry breaking
- Finite resistivity
- Metal/Insulator phase transition

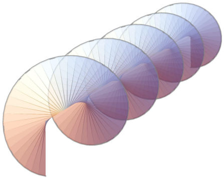
Homogeneous lattices

- Linear Axions $\psi \to \psi + \delta\psi$ $\delta\psi \sim x$
- Q-lattices $\phi e^{i\alpha} \to \phi e^{i(\alpha+\delta\alpha)}$ $\delta\alpha \sim x$
- Bianchy VII helix $B_\mu \to R(\delta\theta)_\mu^\nu B_\nu$ $\delta\theta \sim x$

Are also closely connected to massive gravity approach

Helical background

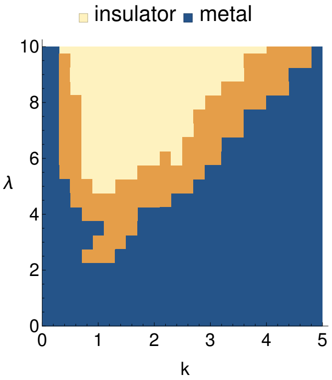
$$\omega_1 = dx,$$
$$\omega_2 = \cos(px)dy - \sin(px)dz,$$
$$\omega_3 = \sin(px)dy + \cos(px)dz.$$



$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2v_1}{\omega_1}^2 + e^{2v_2}{\omega_2}^2 + e^{2v_3}{\omega_3}^2,$$

$$\mathbf{e}^a = \Big(U(r)^{1/2}dt,\ e^{v_1(r)}\omega_1,\ e^{v_2(r)}\omega_2,\ e^{v_3(r)}\omega_3,\ U(r)^{-1/2}dr \Big)$$

Metal/insulator transition



$$\sigma_{DC} = \frac{1}{2} e^{-v_1+v_2+v_3} \left(1 + \frac{e^{2(v_1+v_2+v_3)} a'^2}{p^2[(e^{2v_2} - e^{2v_3})^2 + e^{2v_2} w^2]} \right) \Bigg|_{r=r_h}$$

T. Andrade, A. K., arXiv:1512.02465

Fermions

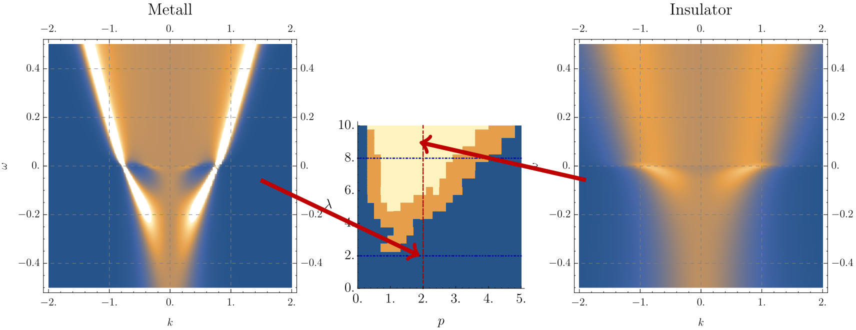
$$\left[-i\gamma^f \mathbf{e}_f^\mu \left(\partial_\mu + \frac{1}{4}\omega_{b\mu}^a \sigma_a{}^b + ieA_\mu\right) + m\right]\Psi = 0$$

$$d\omega_1 = 0, \qquad d\omega_2 = -p\,\omega_1\wedge\omega_3, \qquad d\omega_3 = p\,\omega_1\wedge\omega_2$$

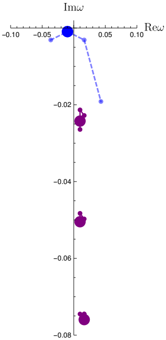
$$\left[\partial_r - \frac{m}{\sqrt{U}}\sigma_3 - \frac{\frac{p}{2}[\mathrm{ch}(v_2 - v_3) - 1] \mp k_x}{e^{-v_1}\sqrt{U}}\sigma_1 + \frac{\omega + qA_t}{U}\; i\sigma_2\right]\phi^R_{\uparrow,\downarrow}$$
$$+\left[\frac{1 + e^{v_2 - v_3}}{2e^{v_2}\sqrt{U}}(k_y \pm ik_z) + \frac{1 - e^{v_2 - v_3}}{2e^{v_2}\sqrt{U}}e^{\mp 2ipx}(k_y \mp ik_z)\right]\sigma_1\phi^R_{\downarrow,\uparrow} = 0$$

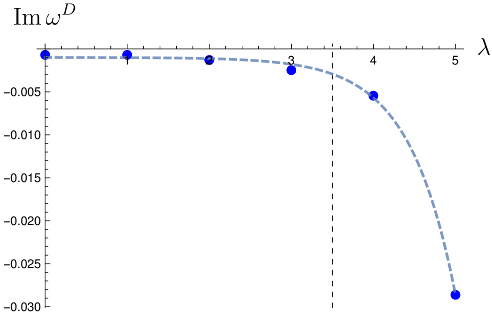
A.Bagrov, N.Kaplis, A.K., K.Schalm, J.Zaanen, arXiv:1606.XXXX

Spectral function



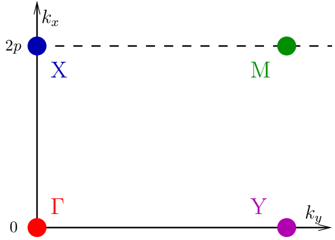
Quasinormal modes



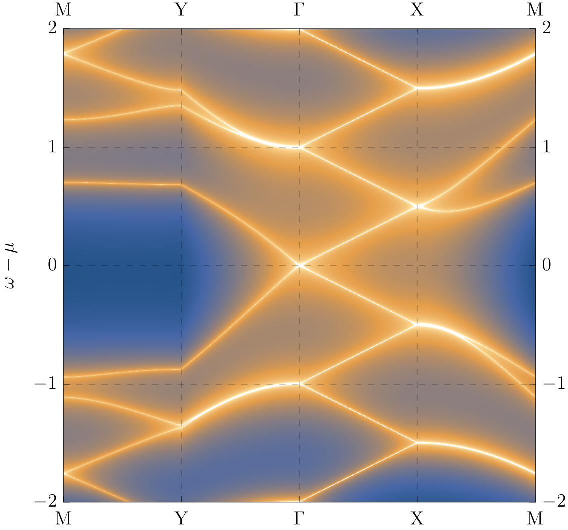


Transversal motion

$$\left[\partial_r - \frac{m}{\sqrt{U}}\sigma_3 - \frac{\frac{p}{2}[\mathrm{ch}(v_2 - v_3) - 1] \mp k_x}{e^{-v_1}\sqrt{U}}\sigma_1 + \frac{\omega + qA_t}{U}\ i\sigma_2\right]\phi^R_{\uparrow,\downarrow}$$
$$+\left[\frac{1 + e^{v_2 - v_3}}{2e^{v_2}\sqrt{U}}(k_y \pm ik_z) + \frac{1 - e^{v_2 - v_3}}{2e^{v_2}\sqrt{U}}e^{\mp 2ipx}(k_y \mp ik_z)\right]\sigma_1\phi^R_{\downarrow,\uparrow} = 0$$



Band Structure



Conclusion

- Helical background has insulating state with **unique** features
- The band structure, gap, umklapp, etc. are completely absent for parallel motion.
- The Fermi surface is **destroyed** by the momentum relaxation

- Interesting perspective arises for the non-parallel motion.
- PDEs are inevitable the study of the realistic gapped insulator.