Explorations of Holographic Kondo Physics: Quantum Quenches

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June, 2016

Based on works:

J. Erdmenger, C. Hoyos, A. O'Bannon, JW
JHEP 1312, 086 (2013), arXiv:1310.3271

J. Erdmenger, M. Flory, M. Newrzella, M. Strydom, JW
In progress

Advertisement

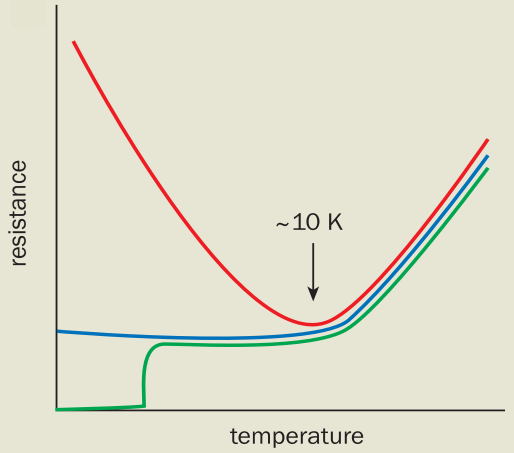
"Quasinormal modes, charged magnetic brane in AdS_5, and (chiral) transport"

M. Ammon, M. Kaminski, R. Koirala, J. Leiber, JW

Outline

- Introduction
- The holographic Kondo model
- Quantum quenches
- Discussion

The Kondo Effect



The Kondo Effect

A **many-body** state (Kondo resonance) formed when conduction electrons hybridize with the (magnetic) impurity:

- Impurity screened
- IR physics changed

Modern perspective:

- RG flow triggered by a marginally relevant operator
- **Single-impurity problem**: rare example of flow between two trivial fixed points
- **Generalised Kondo problem**: non-Fermi liquid IR FP!

Motivation

Many techniques developed to study Kondo physics:

- NRG, DMRG, Quantum Monte Carlo
- Bethe ansatz/Integribility
- Large-N
- CFT

Non-equilibrium aspects not easy to access with traditional techniques

Holography provides new/alternative method of study

Holographic Description

Essential ingredients of the Kondo problem:

- Chiral fermions: $J^a = \psi_L^\dagger \Gamma^a \psi_L$
- Impurity: $S^a = \chi^\dagger T^a \chi$ (slave fermion representation)
- Kondo interaction: $J \cdot S = |\mathcal{O}|^2 + O(1/N)$, $\mathcal{O} = \psi_L^\dagger \chi$

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Single-impurity problem effectively 1D: CFT approach

- J^a satisfy Kac-Moody algebra
- $\chi^\dagger \chi = Q \leftrightarrow R_{imp}$
- $\langle \mathcal{O} \rangle \neq 0 \Rightarrow$ impurity screened

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How to describe these holographically?

Top-Down View

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N_c D3	•	•	•	•	–	–	–	–	–	–
N_7 D7	•	•	–	–	•	•	•	•	•	•
N_5 D5	•	–	–	–	•	•	•	•	•	–

Maldacena limit: $N_c \to \infty$, $g_s \to 0$, $\lambda \equiv 4\pi g_s N_c$ fixed; $\lambda \to \infty$

Probe limit: $\lambda_{5,7} \propto N_{5,7}/N_c \to 0$

- 3-7 strings: (1+1)-dim. chiral fermions ψ_L
- 3-5 strings: (0+1)-dim. slave fermions χ
- 5-7 strings: tachyon

Gravity Dual

Near horizon limit:

- D3: $AdS_5 \times S^5$ background
- D7 along $AdS_3 \times S^5$: Chern-Simons gauge field A
- D5 along $AdS_2 \times S^4$: gauge field a, bifundamental scalar Φ

Duality dictionary:

- AdS_3 CS gauge field $A \leftrightarrow$ chiral fermion current $J = \psi_L^\dagger \Gamma \psi_L$
- AdS_2 gauge field $a \leftrightarrow$ impurity spin $S = \chi^\dagger T \chi$
- AdS_2 bifundamental scalar $\Phi \leftrightarrow$ Kondo interaction $\mathcal{O} = \psi_L^\dagger \chi$

Bottom-Up Model

$$S = S_G + S_{CS} + S_{AdS_2}$$

$$S_G = \frac{1}{2\kappa_N^2}\int d^3x\,\sqrt{-g}\left(R + \frac{2}{L^2}\right)$$

$$S_{CS} = -\frac{\mathcal{N}}{4\pi}\int tr\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

$$S_{AdS_2} = -\mathcal{N}\int d^3x\,\delta(x)\,\sqrt{-\gamma}\left[\frac{1}{4}tr\,f^2 + |\mathcal{D}\Phi|^2 + V(\Phi^\dagger\Phi)\right]$$

$$\mathcal{D}\Phi \equiv \partial\Phi + i\,q\,A\,\Phi - i\,q\,a\,\Phi\,, \quad \Phi = \phi e^{i\psi}\,, \quad f = da$$

Bottom-up: Choose $V(\Phi^\dagger\Phi) = M^2\Phi^\dagger\Phi$

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Finite temperature: BTZ black hole

$$ds^2 = \frac{1}{z^2} \left(\frac{dz^2}{h(z)} - h(z)\, dt^2 + dx^2 \right), \quad h(z) = 1 - \frac{z^2}{z_H^2}$$

Field Equations

The equations of motion (EOMs) are

$$\begin{aligned} \epsilon^{n\mu\nu} F_{\mu\nu} &= -\frac{4\pi}{N}\delta(x) J^n \\ \partial_m \left(\sqrt{-g}\, g^{mp} g^{nq} f_{pq}\right) &= -J^n \\ \partial_m \left(\sqrt{-g}\, g^{mn} \partial_n \phi\right) &= \sqrt{-g}\, \Delta^m \Delta_m \phi + \frac{1}{2}\sqrt{-g}\, \frac{\partial V}{\partial \phi} \\ \partial_n J^n &= 0 \end{aligned}$$

where we parametrise $\Phi = \phi e^{i\psi}$, and define

$$J^n \equiv 2\sqrt{-g} g^{mn} \Delta_m \phi^2 \,, \qquad \Delta_m \equiv A_m - a_m + \partial_m \psi$$

Static Solution

Ansatz ($a_z = 0$ gauge): $a_t(z)$, $\phi(z)$, $A_x(z) \neq 0$

Near the boundary $z \to 0$:

$$a_t(z) \sim \frac{Q}{z} + \mu\,, \quad \phi(z) \sim \sqrt{z}\,[\alpha \log(\Lambda z) + \beta]\ , \quad A_x(z) \to 0$$

$|\mathcal{O}|^2$ marginal $\leftrightarrow M^2 - Q^2 = -1/4 = AdS_2$ BF bound

Double trace coupling [Witten 01]: $\alpha = \kappa\beta \propto \langle\mathcal{O}\rangle$

- κ Kondo coupling
- κ runs: $\phi(z)$ independent of Λ

Running of Kondo Coupling

$\mathbf{T = 0}$: Under scale change $\Lambda \to \Lambda'$,

$$\kappa(\Lambda') = \frac{\kappa(\Lambda)}{1 + \kappa(\Lambda)\log(\Lambda/\Lambda')}$$

For $\kappa < 0$:

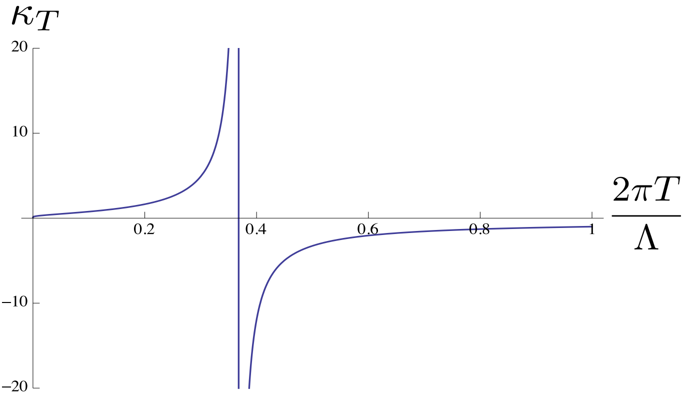
- Asymptotic freedom: $\Lambda'/\Lambda \to \infty$, $\kappa \to 0$
- IR divergence: $\Lambda'/\Lambda \to 0$, $\kappa \to \infty$ at $\Lambda' = \Lambda e^{1/\kappa(\Lambda)}$

$\mathbf{T \neq 0}$: Under rescaling $x^\mu \to \tilde{x}^\mu z_H$, $a_t \to \tilde{a}_t/z_H$, $z_H = (2\pi T)^{-1}$

$$\Phi(z) \to \tilde{\Phi}(\tilde{z}) \sim \alpha_T \sqrt{\tilde{z}} \log(\tilde{z}) + \beta_T \sqrt{\tilde{z}}$$

$$\kappa_T \equiv \frac{\alpha_T}{\beta_T} = \frac{\kappa}{1 + \kappa \log(\Lambda z_H)} \quad \textbf{(RG invar.)}$$

Dynamical Scale Generation

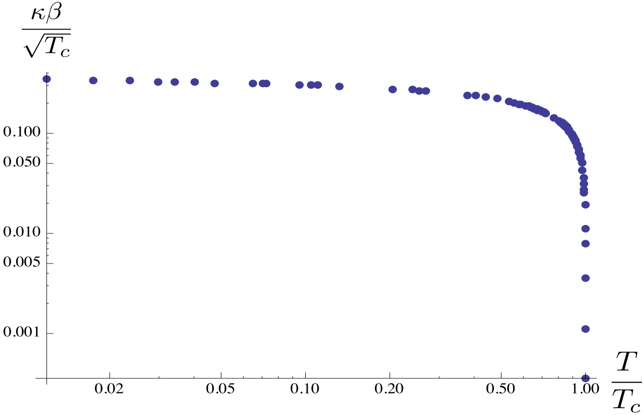


$\kappa(\Lambda) = -1$

Divergence $\Rightarrow T_K = \frac{1}{2\pi}\Lambda e^{1/\kappa(\Lambda)},\ T/T_K = e^{-1/\kappa_T}$ **(RG invar.)**

Phase Transition

Large-N Kondo effect



$$\langle \mathcal{O} \rangle \propto \kappa\beta \propto \left(1 - \frac{T}{T_c}\right)^{1/2}, \ T \lesssim T_c \approx 0.89\, T_K \ , \ \kappa_{T_c} \approx 8.98$$

Quantum Quenches

Study the response of the holograhic system as the Kondo coupling is varied in time

Quench protocols:

- Gaussian pulse
- Constant slope, erf, tanh

Consider:

- Quench within same phase
- Quench across phases

Eddington-Finkelstein Coordinates

The numerical problem is most conveniently solved in the Eddington-Finkelstein (EF) coordinates where perturbations can travel from the boundary to the horizon in finite coordinate time.

Define EF time coordinate by

$$dv = dt - \frac{dz}{h(z)} \Rightarrow v = t - \mathrm{arctanh}\, z$$

The background metric becomes

$$ds^2 = \frac{1}{z^2}\left(-h(z)dv^2 - 2dvdz + dx^2\right)$$

EF Coordinate EOMs

Ansatz: $A_x(z,v),\ a_t(z,v),\ a_z(z,v),\ \phi(z,v),\ \psi(z,v) \neq 0$

$$0 = \partial_v A_x - 4\pi\delta(x)\phi^2\left(\partial_v\psi - h\partial_z\psi\right)$$

$$0 = 4\pi\delta(x)\phi^2\left(\partial_z\psi - \frac{a_v}{h}\right) + \partial_z A_x$$

$$0 = -\frac{\partial_v^2 a_v}{h} + \partial_z\partial_v a_v - \frac{2\phi^2\left(\partial_v\psi - h\partial_z\psi\right)}{z^2}$$

$$0 = \frac{\left(zh' - 2h\right)\partial_v a_v}{zh^2} - \frac{\partial_z\partial_v a_v}{h} + \frac{2\partial_z a_v}{z} + \partial_z^2 a_v - \frac{2a_v\phi^2}{z^2 h} + \frac{2\partial_z\psi\phi^2}{z^2}$$

$$0 = \phi\left(-\frac{2a_v\partial_v\psi}{h^2} + \frac{a_v^2}{h^2} + \frac{2\partial_v\psi\partial_z\psi}{h} - \partial_z\psi^2\right) + \frac{h'\partial_z\phi}{h} - \frac{2\partial_z\partial_v\phi}{h} + \partial_z^2\phi$$

Solving the PDEs

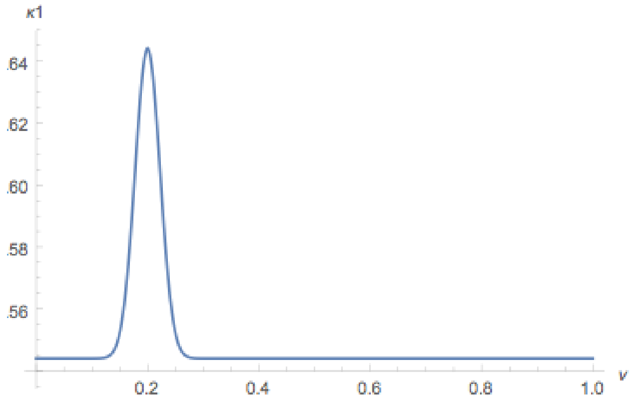
General strategy:

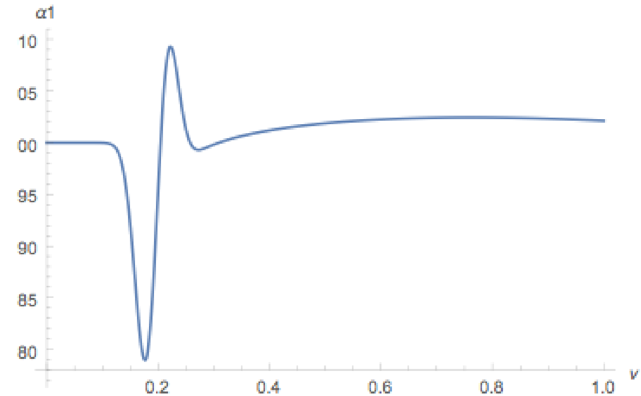
1. Solve the initial static problem
   - Find the initial static solution to the EOMs to the specified accuracy using both shooting and pseudospectral methods.
2. Time-march
   - Time-evolve the initial solution using the implicit Crank-Nicholson method.

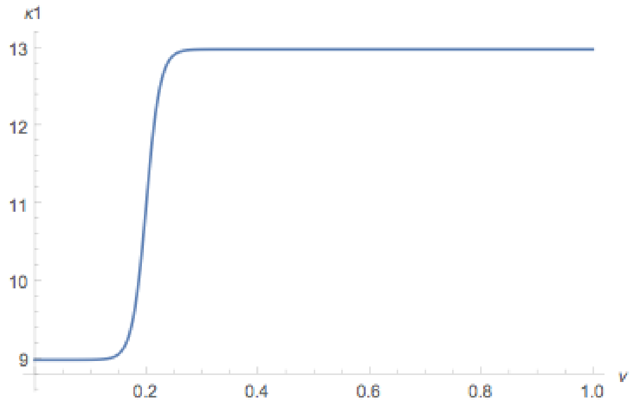
Need to regulate boundary non-analyticities:

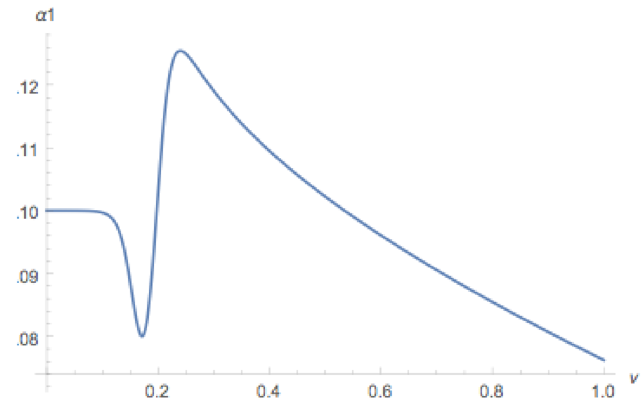
- Use change of variables and field redefinition to remove $\sqrt{z}$ and $(\log)^n$ terms so that fields are regular at the boundary up to the second derivatives.

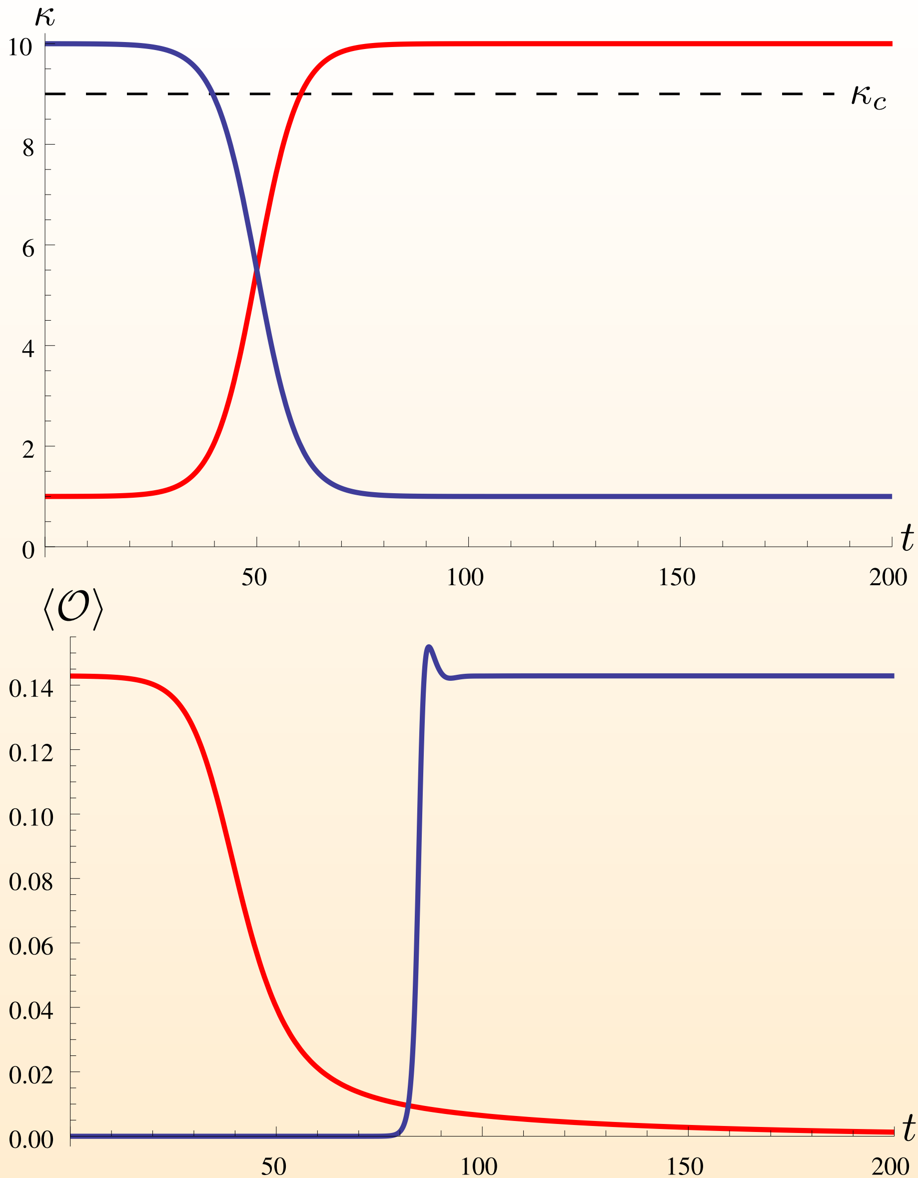
Kondo coupling → Condensate



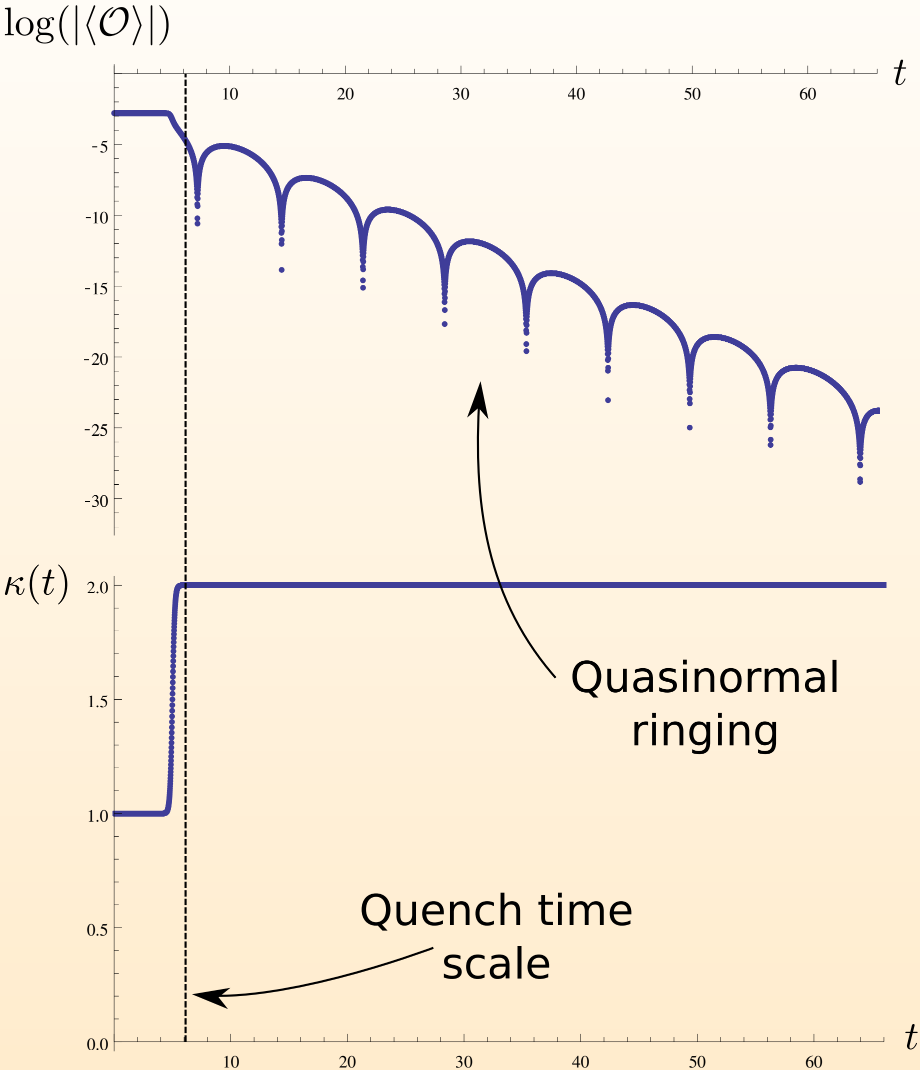








κ
10
8
6
4
2
0
κ_c
t
50
100
150
200
⟨𝒪⟩
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
t
50
100
150
200



$\log(|\langle \mathcal{O} \rangle|)$
t
10
20
30
40
50
60
-5
-10
-15
-20
-25
-30
$\kappa(t)$
2.0
1.5
1.0
0.5
0.0
Quasinormal ringing
Quench time scale

Discussion

- Holographic Kondo system relaxes back to the equilibrium state characterised by the end value of the Kondo coupling.

- Relaxation time scale depends on the final size of the condensate.

- Quasi-normal modes determine the relaxation behaviour and control the time scales of the equilibration process.

  Cf. Bayat, Bose, Johannesson, Sodano, **PRB 92, 155141 (2015)**: Late-time behviour of a two-impurity Kondo spin-chain model after a quantum quench characterised by single-frequency oscillations.