“Holographic matter at finite chemical potential”

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Base on work with:

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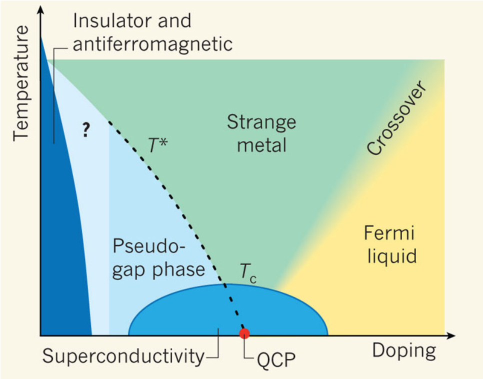
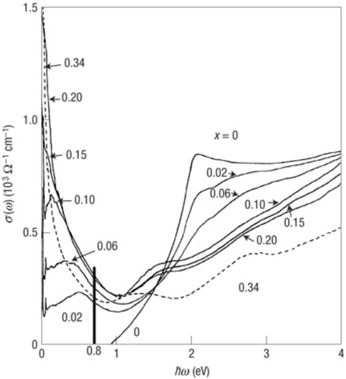
Outline

Outline

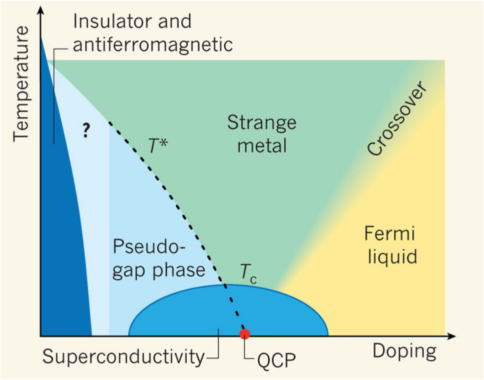
The Cuprates

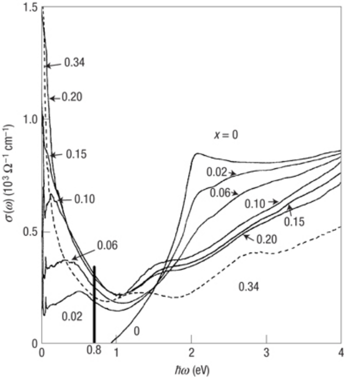




- The Cuprates are real life example of strong coupling
- Intriguing phase diagram
- Related to strong coupling?

The Cuprates





- Incoherent transport
- Anomalous scaling of conductivity and Hall angle with T

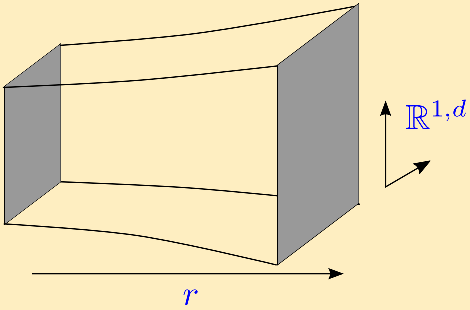
$$\sigma_{DC}^{B=0} \propto T^{-1}, \quad \theta_H \propto T^{-2}$$

AdS/CMT

To model these systems in holography:

- Consider a CFT in $d+1$ dimensions with a $U(1)$
- Finite temperature T
- Deform by chemical potential μ and magnetic field field B
- Deform by relevant operators to model
  - Lattices and Momentum dissipation (More on that later...)
  - Impurities
  - Random Disorder
- Introduce sources to study two point functions (e.g. transport properties)

AdS/CMT



The CFT vacuum is modelled by AdS_{d+2}

$$ds^2 = r^2 \left(-dt^2 + d\mathbf{x}_d^2\right) + \frac{dr^2}{r^2}$$

AdS/CMT

Schematically the bulk action is

$$\mathcal{L} = R_{d+2} + \Lambda - \frac{1}{4}F^2 + \text{matter}$$

- The vacuum is a solution with trivial matter fields
- Different microscopic theories correspond to different matter content
- Each bulk field corresponds to different boundary operator $\mathcal{O}$
- Use the relevant/marginal ones to deform the boundary theory

$$S = S_{CFT} + \int d^{d+1}x \, \phi(x) \, \mathcal{O}(x)$$

- Introduce a black hole (brane) horizon in the bulk to raise temperature T

AdS/CMT

Universal deformations are

- The stress tensor

$$ds^2 = r^2 \left(-dt^2 + d\mathbf{x}_d^2 + \delta g_{\mu\nu}(\mathbf{x}) dx^\mu dx^\nu\right) + \frac{dr^2}{r^2} + \cdots$$

- The chemical potential

$$A = \mu(\mathbf{x})\, dt + \cdots$$

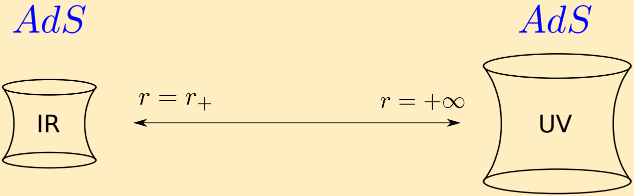
- Subleading terms give the VEVs
- The dual action is now

$$S = S_{CFT} + \int \mu\, J^t + \frac{1}{2} \delta g_{\mu\nu} T^{\mu\nu}$$

AdS/CMT

The bulk spacetime geometrizes the resulting RG flow:

- At finite T the geometry the BH horizon serves as an IR cutoff
- At $T = 0$ the bulk geometry becomes an interpolation between fixed point solutions e.g.



- Each fixed point has its own spectrum: $(\mathcal{O}_{UV}, \Delta_{UV})$, $(\mathcal{O}_{IR}, \Delta_{IR})$

AdS/CMT

To probe the IR/UV part of the geometry:

- Introduce perturbative source $\delta S = \int \delta\phi(\omega, \mathbf{k})\mathcal{O}_\phi$
- Extract the two point function $G(\omega, \mathbf{k})$
- See how it scales, e.g.
  - $\mathrm{Im}\, G(\lambda\omega, \lambda\mathbf{k}) = \lambda^{2\Delta_{UV}-d-1}\, \mathrm{Im}\, G(\omega, \mathbf{k})$, for $\lambda >> 1$
  - $\mathrm{Im}\, G(\lambda\omega, \lambda\mathbf{k}) = \lambda^{2\Delta_{IR}-d-1}\, \mathrm{Im}\, G(\omega, \mathbf{k})$, for $\lambda << 1$
- The low T horizon describes the ground state of holographic theories

Outline

Finite chemical potential

Possible (unbroken phase) ground states at finite chemical potential?

- Preserve time translations T_t and Euclidean group $\mathbb{E}_d$
- Semi-local critical $t \to \lambda\, t$, $AdS_2 \times \mathbb{R}^d$

$$ds^2 = -r^2\, dt^2 + d\mathbf{x}_d^2 + dr^2/r^2$$

- Lifshitz $t \to \lambda^z\, t, \quad \mathbf{x} \to \lambda\, \mathbf{x}$

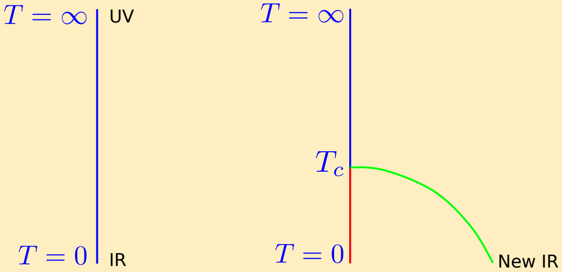
$$ds^2 = -r^{2z}\, dt^2 + r^2\, d\mathbf{x}_d^2 + dr^2/r^2$$

- Hyper-scale violating

$$ds^2 = r^{-2\theta/d}(-r^{2z}\, dt^2 + r^2\, d\mathbf{x}_d^2 + dr^2/r^2)$$

Finite chemical potential

Phase transitions

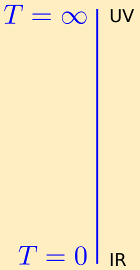


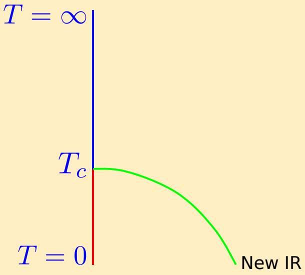
Theory can develop symmetry breaking instabilities:

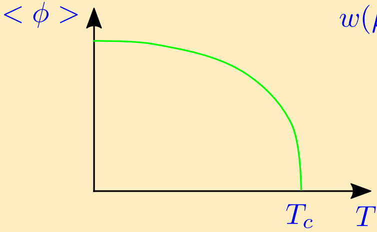
- At $T >> \mu$ the normal phase bh's are stable
- At $T < T_c$ there exist tachyonic modes
- Zero mode at $T = T_c$ gives rise to broken phase black hole branch
- Dual operator ϕ takes a VEV $< \phi >$

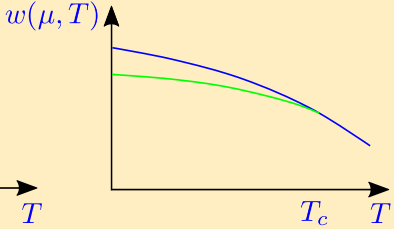
Finite chemical potential

Phase transitions





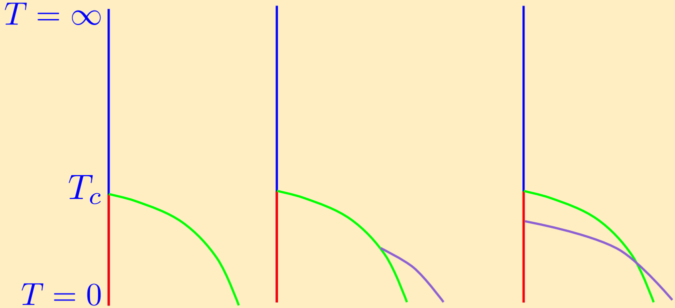




Finite chemical potential

Competing orders

More complicated possibilities lead to "competing orders":



- Additional deformations (doping, B, ...) change the diagram
- Holographic theories can lead to intricate phase diagrams
- At $T = 0$ they provide excellent models for Quantum Phase Transitions (doping)

Finite chemical potential

S-Superfluidity

Bulk theory with charged scalar under $U(1)$

[Gubser] [Hartnoll, Herzog, Horowitz]

$$\mathcal{L} = R - \frac{1}{4}F^2 - D_\mu\psi\, D^\mu\psi^* - V(|\psi|^2)$$

$$D_\mu\psi = \nabla_\mu\psi - \imath q A_\mu\psi$$

- Broken phase black holes have $\psi \neq 0$
- Spontaneous breaking of $U(1)$
- Goldstone mode responsible for delta function in conductivity
- Ground states have similar symmetries with normal phase but without the $U(1)$.

Finite chemical potential

Helical order

Bulk Einstein-Maxwell + CS term

[Nakamura, Ooguri, Park] [AD, Gauntlett]

$$\mathcal{L} = R + \Lambda - \frac{1}{4}F^2 + \lambda\,\varepsilon^{\alpha\beta\gamma\delta\epsilon} A_\alpha F_{\beta\gamma} F_{\delta\epsilon}$$

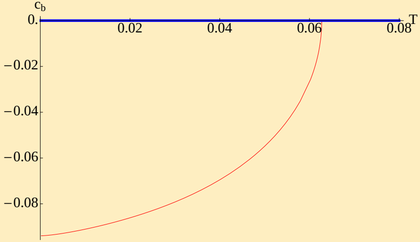
- Broken phase develops helical current density/magnetisation e.g. long wavelength helimagnetism in MnSi

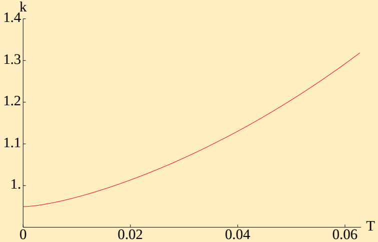
$$< J_y >= c_b \sin(kx), \quad < J_z >= c_b \cos(kx)$$

- Spontaneous breaking of translations
- Ground states with broken translations have significant impact on transport
- Similar story with p-wave order parameter in superfluids

Finite chemical potential

Helical order





- Helical black holes come with the period k as a modulus
- Fix k by minimising the free energy density
- Preferred k changes with temperature

Finite chemical potential

Simple ground state geometries:

$$ds^2 = -r^{2z}\,dt^2 + dx^2 + r^2\,(\omega_1^2 + \omega_2^2) + dr^2/r^2$$
$$\omega_1 + i\omega_2 = e^{ikx}\,(dy + idz)$$

- Scaling $t \to \lambda^z\, t, \quad y \to \lambda\, y, \quad z \to \lambda\, z$
- Helical symmetry (Bianchi VII_0)
- Also hyper-scaling violating version of those

Finite chemical potential

Inhomogeneous phases in $D = 4$

$$\mathcal{L} = R - \frac{1}{2}\partial\phi^2 - V(\phi) - \frac{1}{4}Z(\phi)\,F^2 + \frac{1}{4}\vartheta(\phi)\,\epsilon_{abcd}F^{ab}F^{cd}$$

- There terms lead to charge/magnetisation density wave phases of electric and magnetic branes
- Standard terms of $N = 2$ SUGRA in $D = 4 \rightarrow$ appear in top-down models

For the record:

$$V = -6\,\cosh\left(\frac{\phi}{\sqrt{3}}\right), \quad Z = \frac{1}{\cosh\left(s\sqrt{3}\,\phi\right)}, \quad \vartheta = \chi\,\tanh\left(\sqrt{3}\,\phi\right)$$

Choose $s = 1$, $\chi = 3/2$

Finite chemical potential

- Normal phase ansatz

$$ds_4^2 = r^{-2}\left(-g(r)dt^2 + f(r)\,dr^2 + dx^2 + dy^2\right)\,,$$
$$A = a_t(r)\,dt + \frac{B}{2}\,(x\,dy - y\,dx)\,,\quad \phi = \phi(r)$$

- Set deformations of scalar operator to zero
- Fix chemical potential $a_t(0) = \mu$
- For $B = 0$ it is just the electric AdS RN black brane

Finite chemical potential

Examine zero modes of:

$$\delta g_{tt} = g_{tt}(r)\, h_{tt}(r)\, \cos(\mathbf{k}\cdot\mathbf{x}), \quad \delta g_{rr} = -g_{rr}(r)\, h_{tt}(r)\, \cos(\mathbf{k}\cdot\mathbf{x})\,,$$

$$\delta g_{ti} = \hat{n}_i\, h_{t\perp}(r)\, \sin(\mathbf{k}\cdot\mathbf{x}),$$

$$\delta g_{ij} = \left(\delta_{ij} - \hat{k}_i \hat{k}_j\right)\, h_{\perp\perp}(r)\, \cos(\mathbf{k}\cdot\mathbf{x}) + \hat{k}_i \hat{k}_j\, h_{\|\|}(r)\, \cos(\mathbf{k}\cdot\mathbf{x})\,,$$

$$\delta a_t = h_t(r)\, \cos(\mathbf{k}\cdot\mathbf{x}), \qquad \delta a_i = \hat{n}_i\, h_{\perp}(r)\, \sin(\mathbf{k}\cdot\mathbf{x}),$$

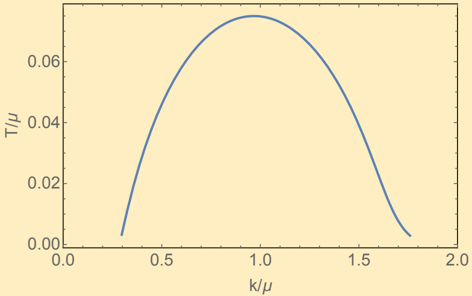
$$\delta\phi = h(r)\, \cos(\mathbf{k}\cdot\mathbf{x})$$

$$\hat{k} = \mathbf{k}/\|\mathbf{k}\|, \quad \hat{n}\cdot\hat{k} = 0, \quad \|\hat{n}\| = 1$$

Wants to modulate:

- Charge, current/magnetisation
- Momentum/thermal magnetisation
- Energy density
- Involves longitudinal + transverse sector

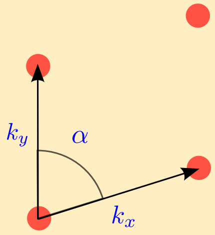
Finite chemical potential



- Zero modes appear at $T(\|\mathbf{k}\|/\mu, B/\mu^2)$
- Plot for $B/\mu^2 = 0.05$
- Combine zero modes of different directions $\hat{k}$ to construct periodic ones
- At low temperatures need to do backreaction

Finite chemical potential

Asymptotics:



$$(x', y') \equiv \left(x' + \frac{L_x}{\sin\alpha}\, n_1 - \cot\alpha\, L_y\, n_2, y' + L_y\, n_2 \right)$$

$$ds_3^2 = -dt^2 + dx'^2 + dy'^2$$
$$A = \mu\, dt + B\, x\, dy$$

A bit unnatural for numerics. . . Change coordinates. . .

Finite chemical potential

Choosing

$$x = L_x^{-1}\,\left(\cos\alpha\, y' + \sin\alpha\, x'\right), \quad y = L_y^{-1}\, y'$$

The asymptotics become

$$ds_3^2 = -dt^2 + \frac{1}{\sin^2\alpha}\,\left(L_x^2\, dx^2 + L_y^2\, dy^2 - 2\, L_x\, L_y\, \cos\alpha\, dx\, dy\right)$$
$$A = \mu\, dt + B\,\frac{L_x\, L_y}{\sin\alpha}\, x\, dy$$

But now $x \sim x + 1$, $y \sim y + 1$

Finite chemical potential

- Periodic x and y
- Regular horizon at $z = 1$
- Conformal boundary at $z = 0$
- Fix Goldstone modes
- Chemical potential μ and magnetic field B
- Use DeTurck method to solve
  [Headrick, Kitchen, Wiseman], [Figueras, Lucietti, Wiseman]

Finite chemical potential

For fixed T, μ and B minimise free energy density

$$w = -sT - \mu\,\bar{J}^t + \bar{T}^{tt}$$

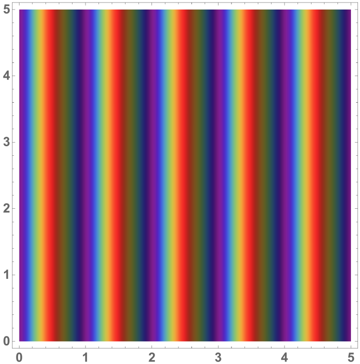
with respect to L_x, L_y and α

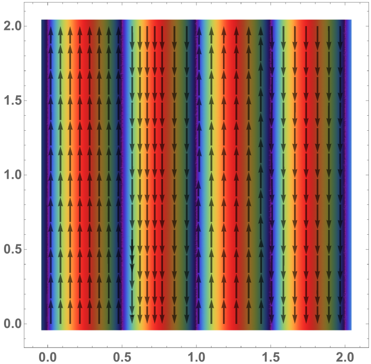
$$-L_x\frac{\delta w}{\delta L_x} = w + \bar{m}B + \bar{T}^x{}_x, \quad -L_y\frac{\delta w}{\delta L_y} = w + \bar{m}B + \bar{T}^y{}_y$$
$$\frac{\delta w}{\delta \alpha} = \cot\alpha(w + \bar{m}B + \bar{T}^x{}_x + \bar{T}^y{}_y) - \frac{L_x L_y}{\sin\alpha}\bar{T}^{xy}$$

It will look like a perfect fluid on average

$$w = -\left(\bar{m}\,B + p\right)$$
$$\bar{T}^{ij} = p\,\gamma^{ij}$$

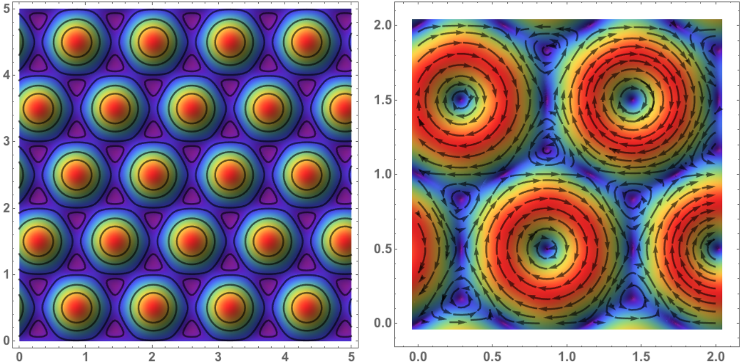
Finite chemical potential





- For $B = 0$ striped structures seem preferred
- Charge density gets modulated
- Spontaneous electric/heat magnetisation current densities appear

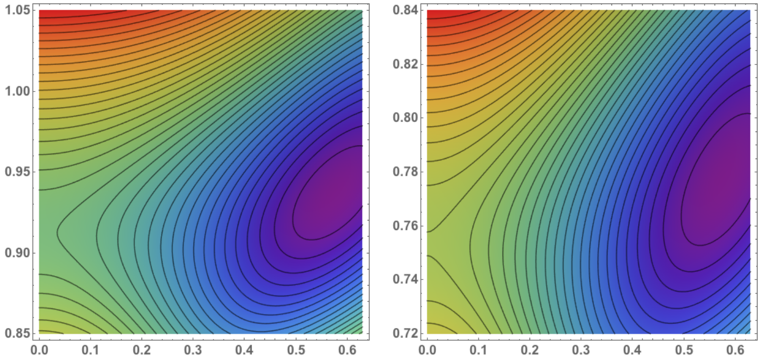
Finite chemical potential



Density waves

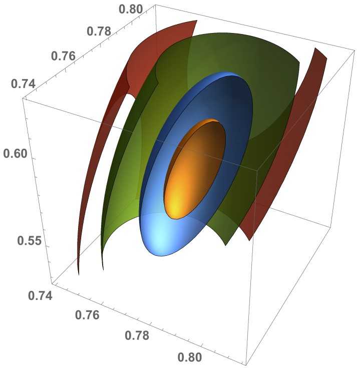
- Switching on magnetic field makes other structures preferred [AD, Gauntlett]
- Triangular lattice formations seem to win

Finite chemical potential



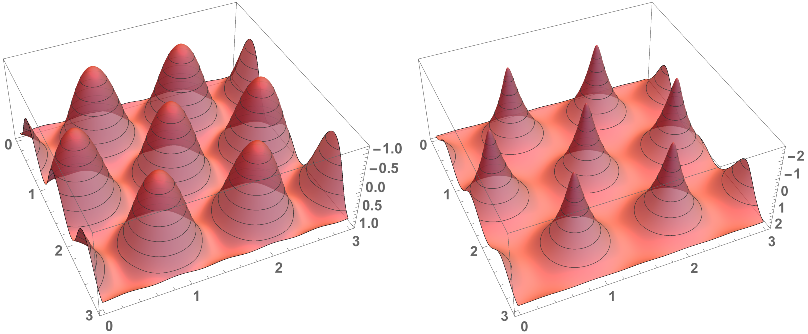
- Set $B = 0.05\,\mu^2$
- Free energy as a function of $k = k_x = k_y$ and $R = -\cot\alpha$ for $T = 0.85\,T_c$ and $T = 0.5\,T_c$.
- Global minimum at $R = 1/\sqrt{3}$, $\alpha = 2\pi/3$
- Triangular lattices seem preferred

Finite chemical potential



- Set $B = 0.05\,\mu^2$
- Free energy as a function of k_x, k_y and $R = -\cot\alpha$ for $T = 0.5\,T_c$.
- $R = 1/\sqrt{3}$ is actually a minimum

Finite chemical potential



Density waves

- At lower temperatures translations breaking effects become stronger
- IR Theory develops point-like defect structure
- New ground states to be found!
- Competing order with superfluidity [AD, Gauntlett, Sonner, Withers]

Outline

Transport in Holography

Hydro approach to transport

How do strongly coupled systems conduct electricity and heat?

- Assume existence of stress tensor $T_{\mu\nu}$ and electric current J_μ
- Ward identities imply

$$\nabla_\mu < T^\mu{}_\nu >= F_{\nu\mu} < J^\mu >, \quad \nabla_\mu < J^\mu >= 0$$

- To make progress assume hydro description:
  - Express $T_{\mu\nu}$ and J_μ in terms of fluid velocity v^μ, $\delta\mu$ and δT (Constitutive relations)
  - Assume weak momentum relaxation through:
    - Modified Ward identities
    - Transport coefficients which explicitly break translations

Transport in Holography

Transport in holography

- Introduce momentum relaxation mechanism
- Use UV relevant operators to introduce periodic sources

$$S = S_{CFT} + \int \mu(\mathbf{x}) J^t + \phi(\mathbf{x})\, \mathcal{O}$$

$$\mu = < \mu(\mathbf{x}) >, \quad E = -\nabla \mu(\mathbf{x})$$

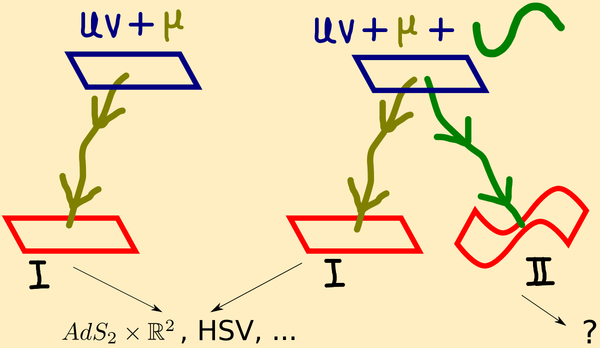
- Find new deformed black hole backgrounds
- Perturb by δE and $\delta \nabla T$

$$\delta A_i = \delta E_i e^{-i\omega t} + \cdots, \quad \delta g_{ti} = \delta \nabla_i T\, e^{-i\omega t} + \cdots$$

- Read off the resulting currents and extract conductivities

Transport in Holography

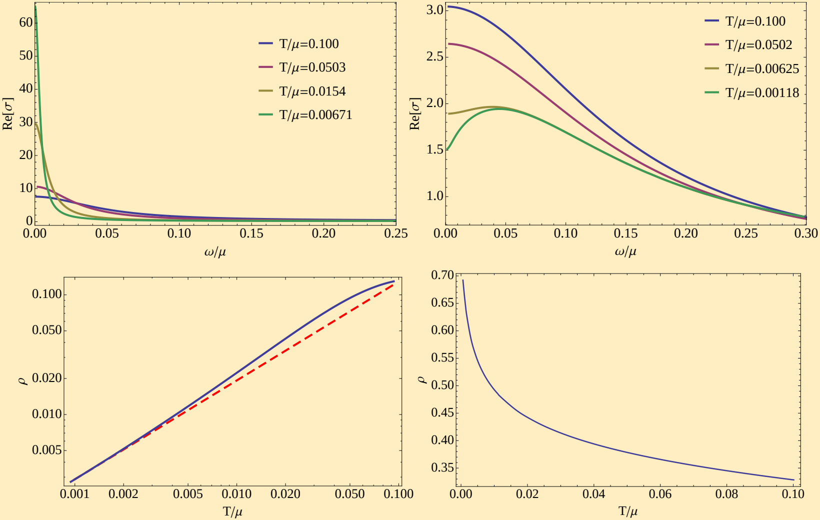
RG flow picture



- I Charge dominated RG flows, translations restored in IR $\rightarrow$ Coherent transport [Hartnoll, Hofman]
- II Lattice dominated RG flows, translations broken in IR $\rightarrow$ Incoherent transport [AD, Hartnoll] [AD, Gauntlett]

Transport in Holography

Metal/Insulator Transition



- Model for Metal - Insulator transitions

Transport in Holography

DC is fixed by the horizon geometry: [AD, Gauntlett]

- Solve linearised Navier-Stokes on curved black hole geometry

$$\nabla_i v^i = 0$$
$$2\frac{s}{4\pi}\nabla^i\nabla_{(i}v_{j)} - \rho\,\nabla_j\delta\mu - s\,\nabla_j\,\delta T = s\,\nabla_j T + \rho\,E_j$$
$$\nabla^2\delta\mu - v^i\,\nabla_i\rho = -\nabla_i E^i$$

- Plug answer in "constitutive relations" for currents

$$J^i|_{r=rh} = \frac{s}{4\pi}\left(\partial^i\delta\mu + E^i\right) + \rho\,v^i, \quad Q^i|_{r=rh} = T s\,v^i$$

- Integrate over cycles on horizon to find current fluxes
- At low T ground state geometry fixes DC behaviour of current
- Generalisation with magnetic fields [AD, Gauntlett, Griffin, Melgar]

Transport in Holography

Wiedemann-Franz law from holography

For perturbative lattices

$$\bar{\kappa} = M\,4\pi s T, \quad \alpha = \bar{\alpha} = M\,4\pi\rho, \quad \sigma = M\,\frac{4\pi\rho^2}{s}$$

- With M $(>> 1)$ fixed by perturbative modes that break translations on horizon
- Holographic version of Wiedemann-Franz law

$$\frac{\bar{\kappa}}{\sigma\,T} = \frac{s^2}{\rho^2}$$

- Not true for non-perturbative lattices!

Anomalous scaling of Hall angle

- Introduce magnetic field
- For perturbative lattices similar logic that lead to Wiedemann-Franz gives

$$\sigma \propto T^a \quad \theta_H \propto T^b$$

  with $a = b$

- Allowing for large lattices (incoherent transport) gives that $a \neq b$ in general [AD, Blake]
  - Charge conjugation even processes dominate σ_{xx}
  - Charge conjugation odd processes dominate σ_{xy}

Outline

Summary

- Modelling CMT problems in holography leads to interesting new ground states
- Rich Phase Diagrams - Competing orders
- New ground states have interesting transport properties
- Black hole horizon fluid fixes DC transport
- Physics of new ground states?
- SUSY ground states? Yes, in $N = 8$!
- Non-linear Navier-Stokes? i.e. Membrane Paradigm