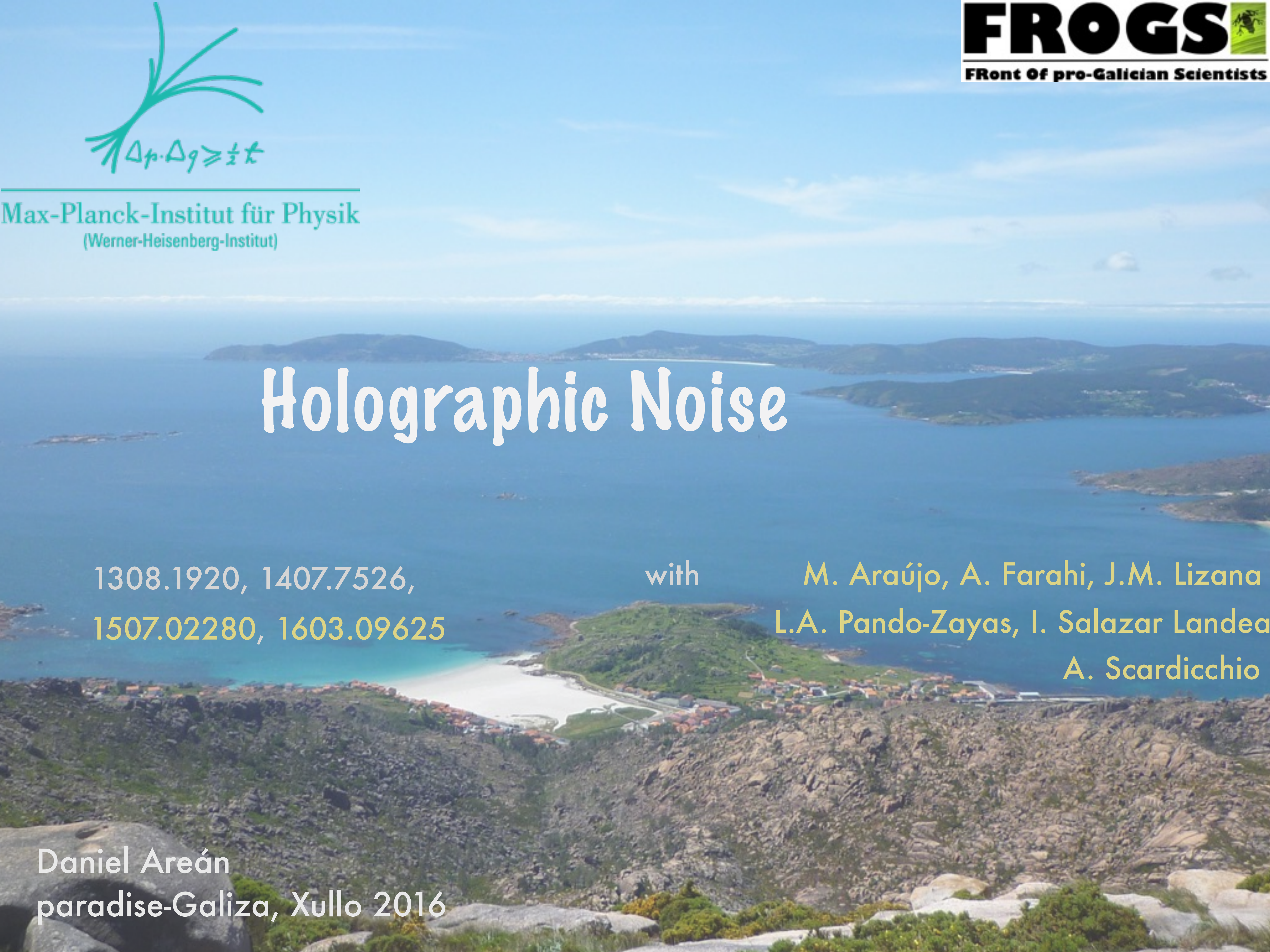


Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

FROGS
FRont Of pro-Galician Scientists

Holographic Noise

1308.1920, 1407.7526,
1507.02280, 1603.09625

with

M. Araújo, A. Farahi, J.M. Lizana
L.A. Pando-Zayas, I. Salazar Landea
A. Scardicchio

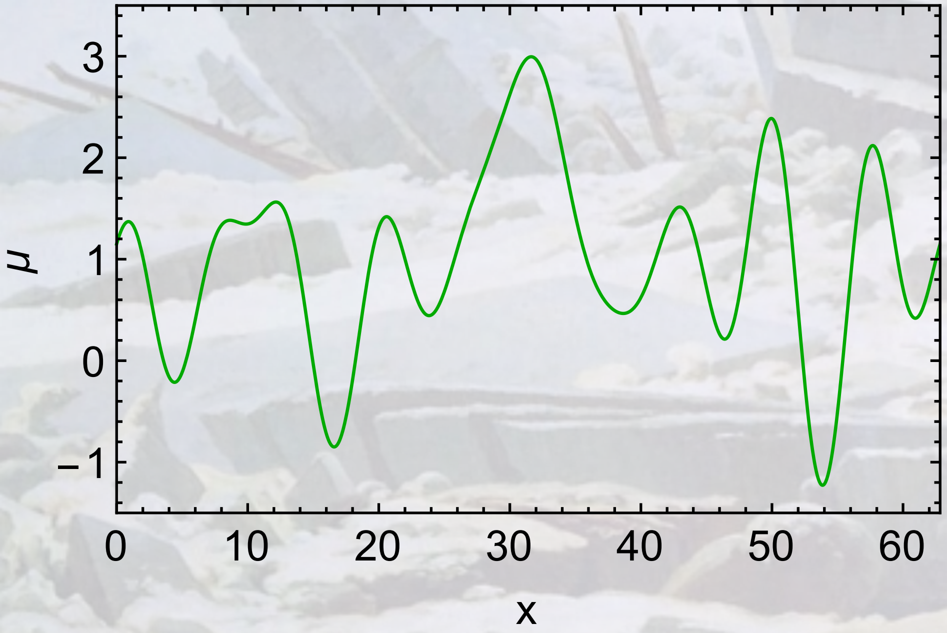
Daniel Areán
paradise-Galiza, Xullo 2016

Disorder rocks!

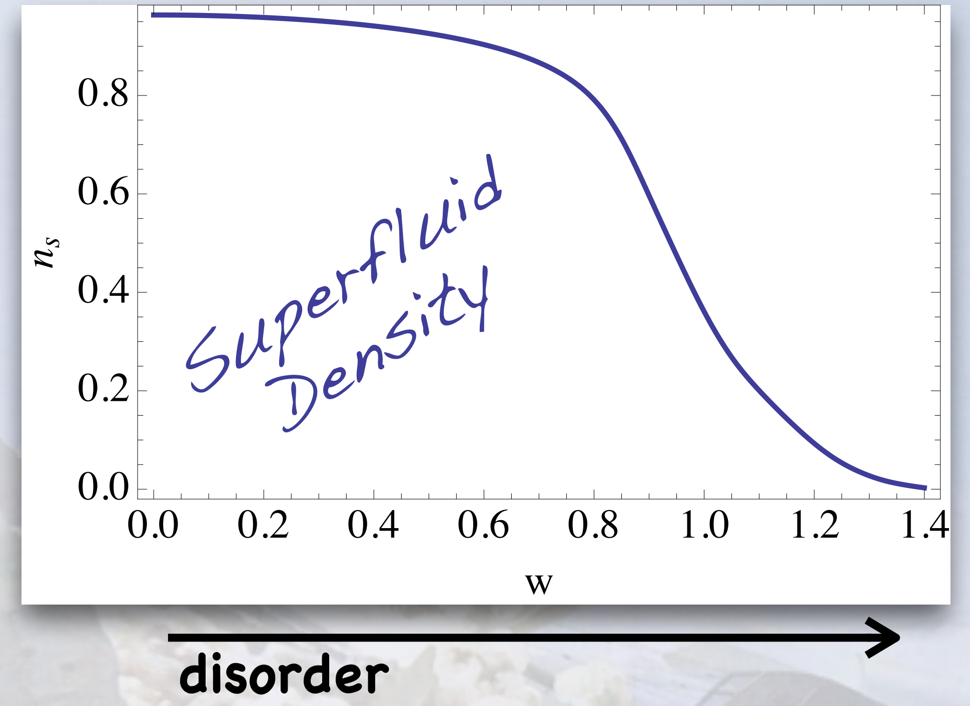
Gauge/Gravity

+

Disordered chemical potential

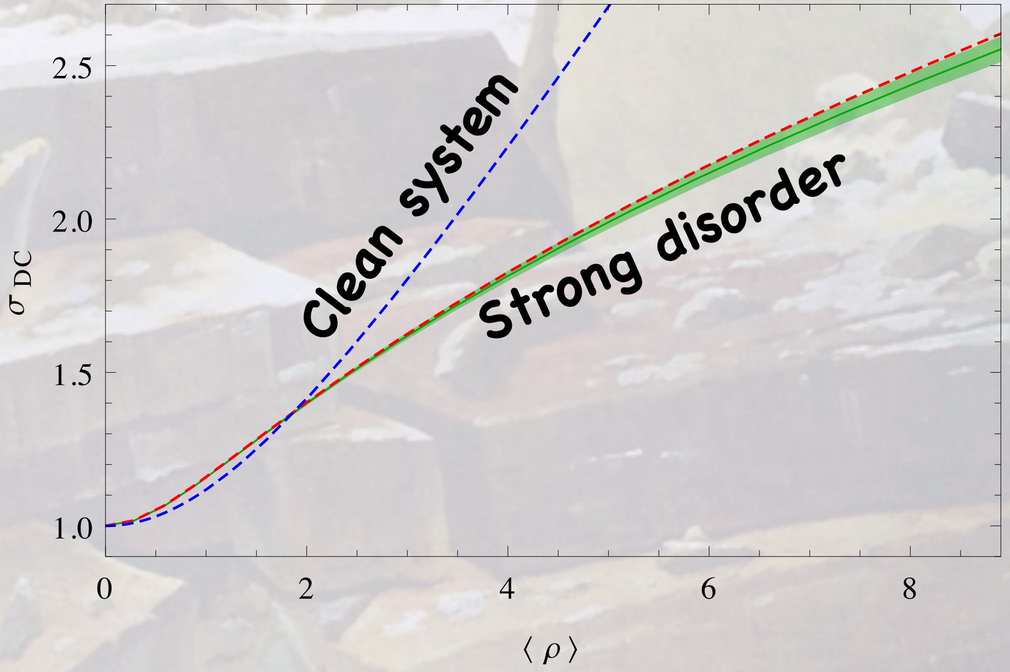


Disordered holographic Superconductors



[→ High-Tc Superconductors]

Disordered brane intersections (D3/D5)

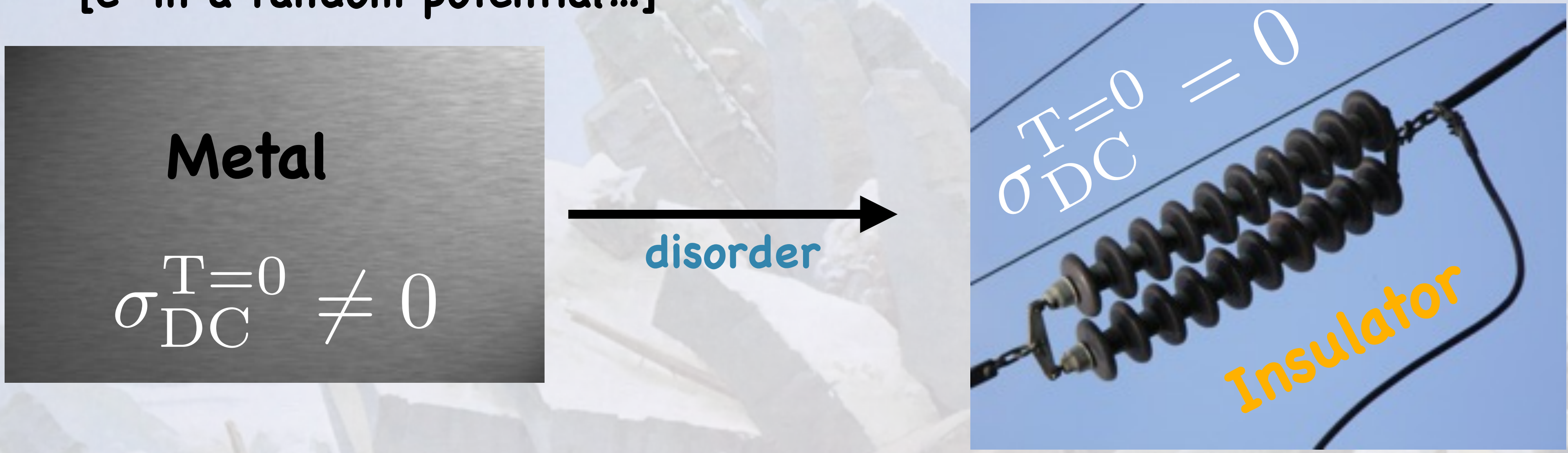


[→ Graphene]

Disorder is cool

- Anderson Localization (1958):

[e⁻ in a random potential...]



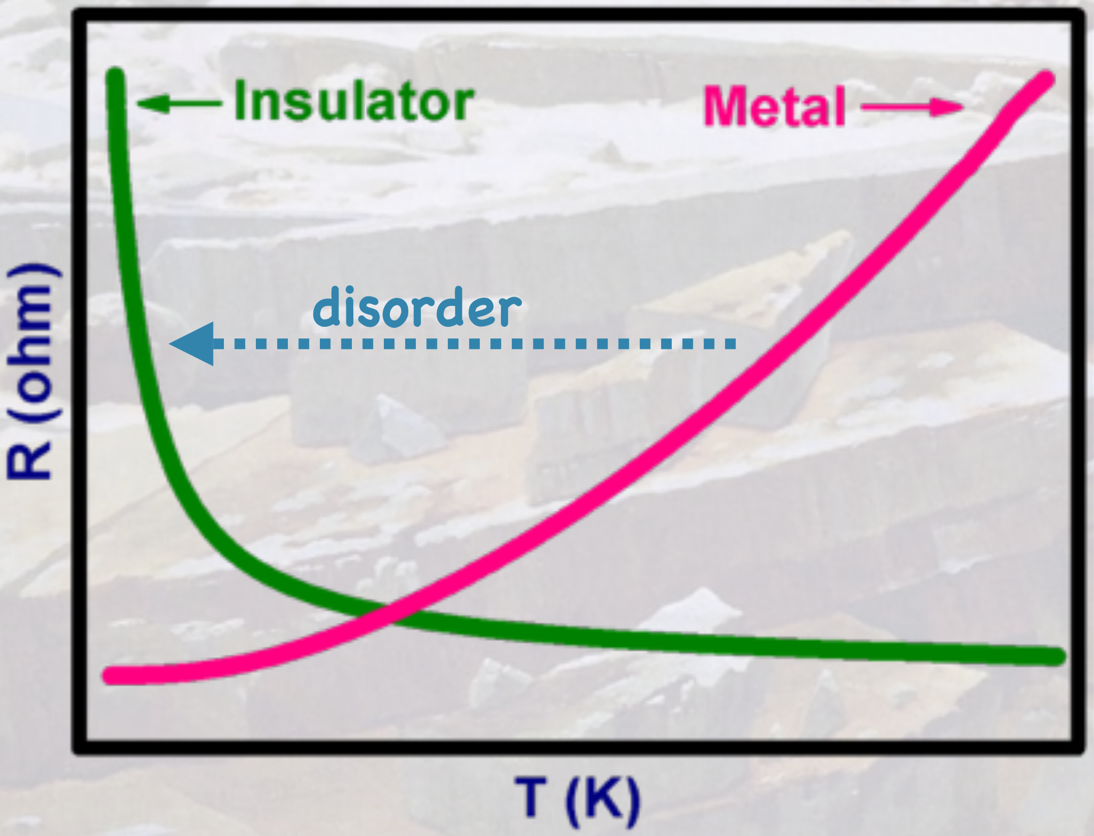
Metal

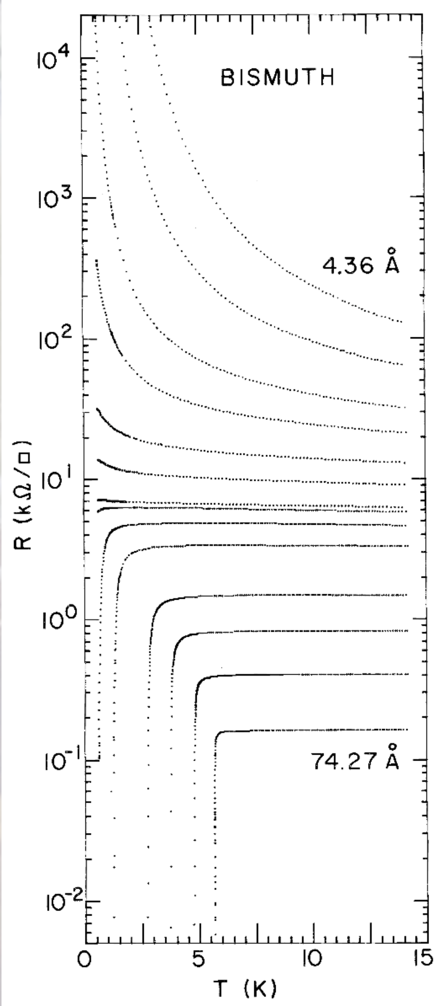
$\sigma_{\mathrm{DC}}^{\mathrm{T=0}} \neq 0$

disorder

$\sigma_{\mathrm{DC}}^{\mathrm{T=0}} = 0$

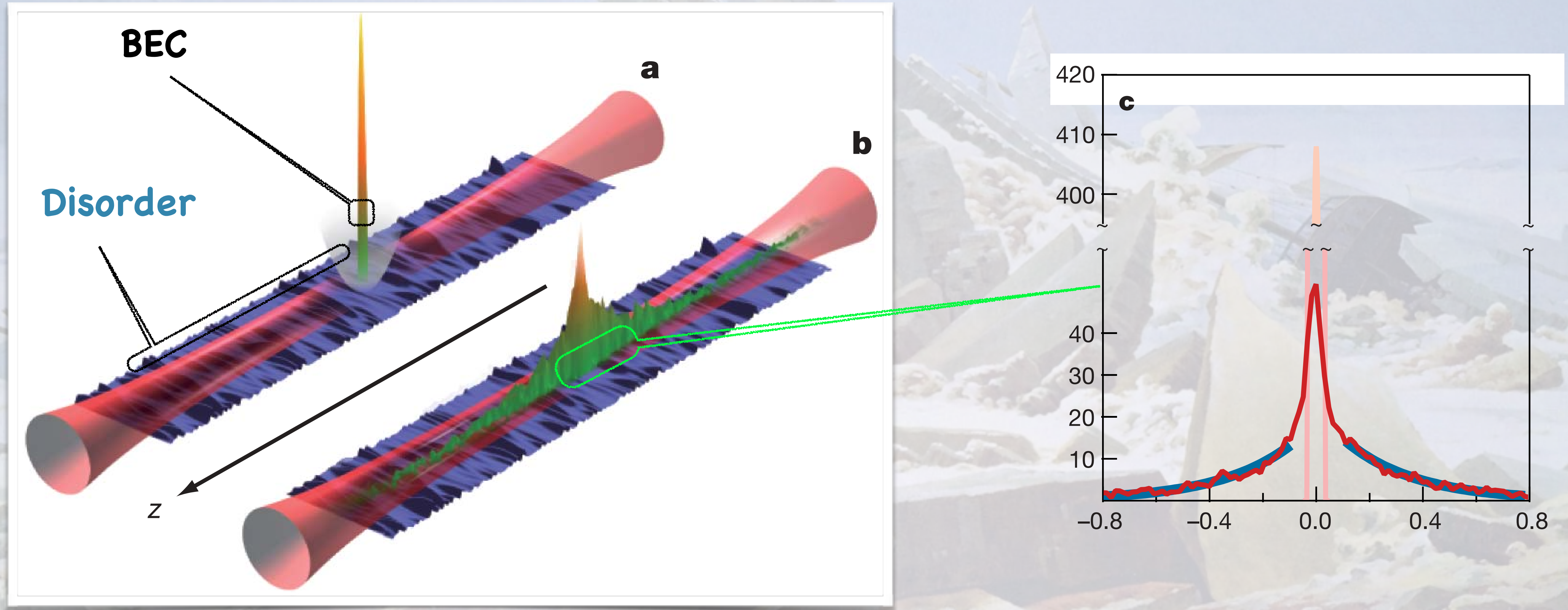
Insulator





Disorder is cool

- Anderson Localization (1958).



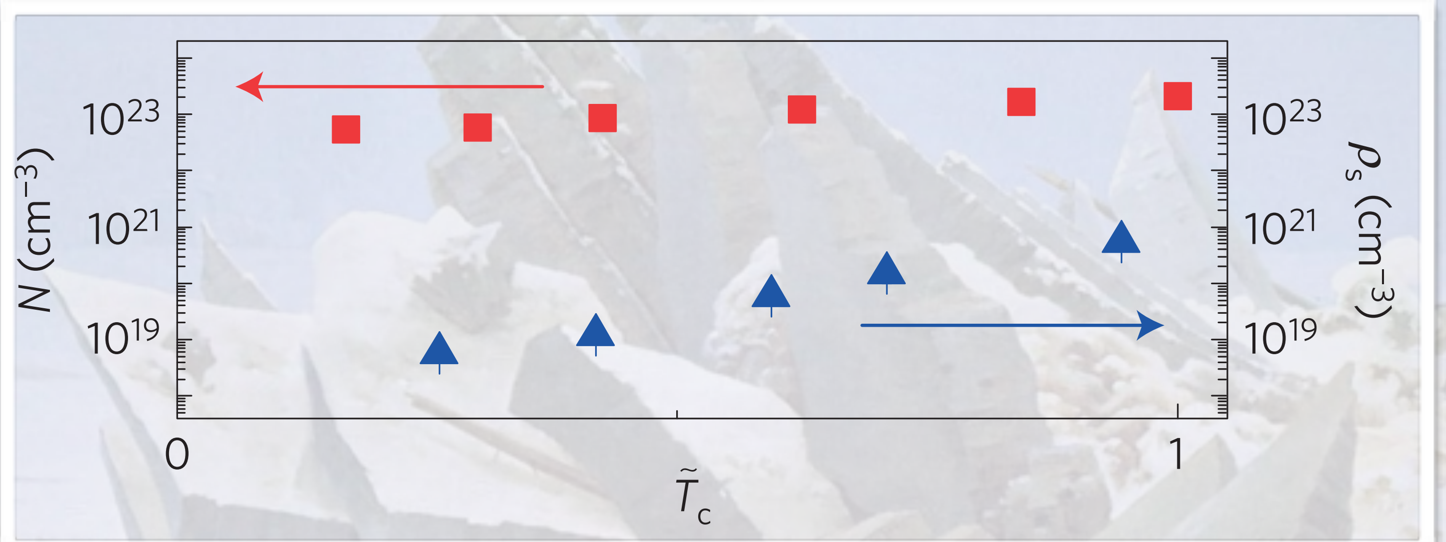
[Billy et al, Nature 453(2008)]

Disorder is cool

- SC to insulator disorder-induced phase transition

[Sherman et al, Nature Phys 11, 188–192 (2015)]

- Experiment

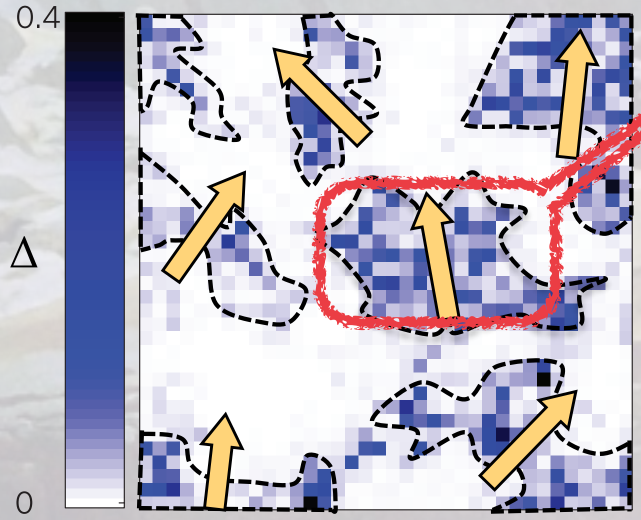


▲ Superfluid Density

disorder

- Theory (quantum Montecarlo)

'Condensate'

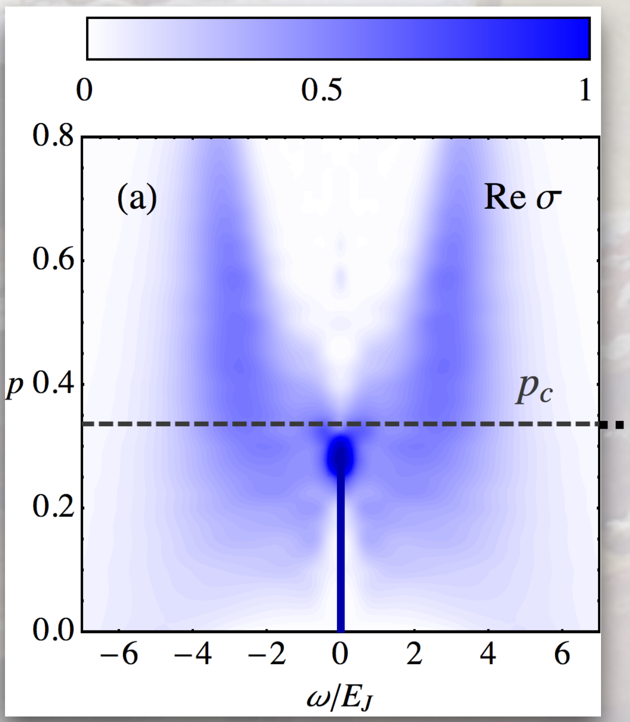


Islands

[Swanson et al, 1310.1073]

Conductivity

disorder

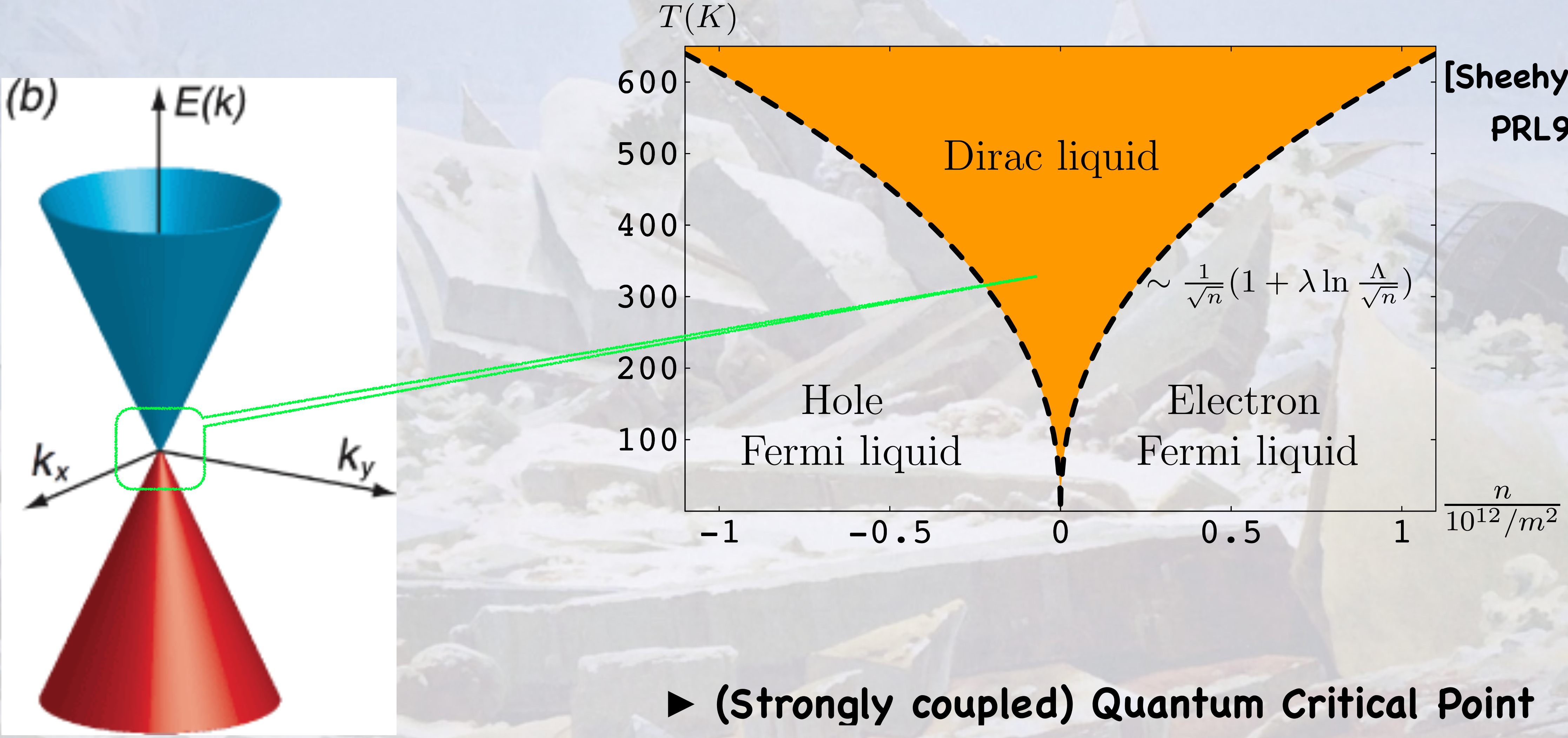


INSULATOR

SUPERCONDUCTOR

Disorder is cool

> **Graphene** (near charge neutrality)



[Sheehy&Schmalian, PRL99(2007)]

▶ (Strongly coupled) Quantum Critical Point

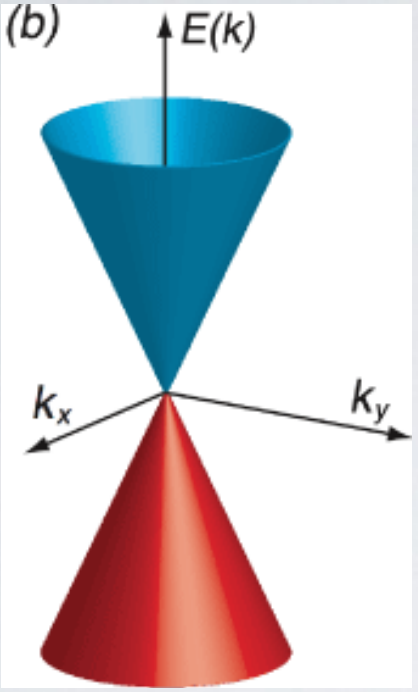
➥ Graphene as strongly coupled relativistic Dirac fluid

[Observed in Crossno et al, Science'16, 1509.04713]

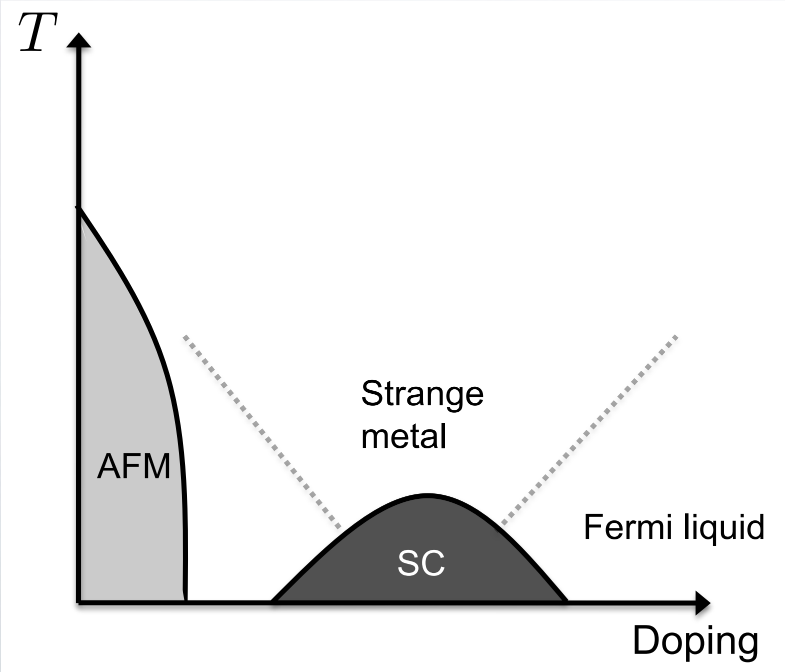
[Hydro model in Lucas et al, Science'16, 1510.01738]

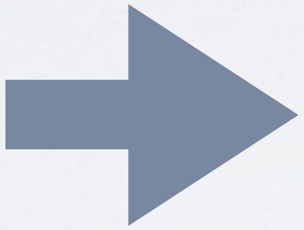
Disorder + strong interactions = Challenge!

> **Graphene**

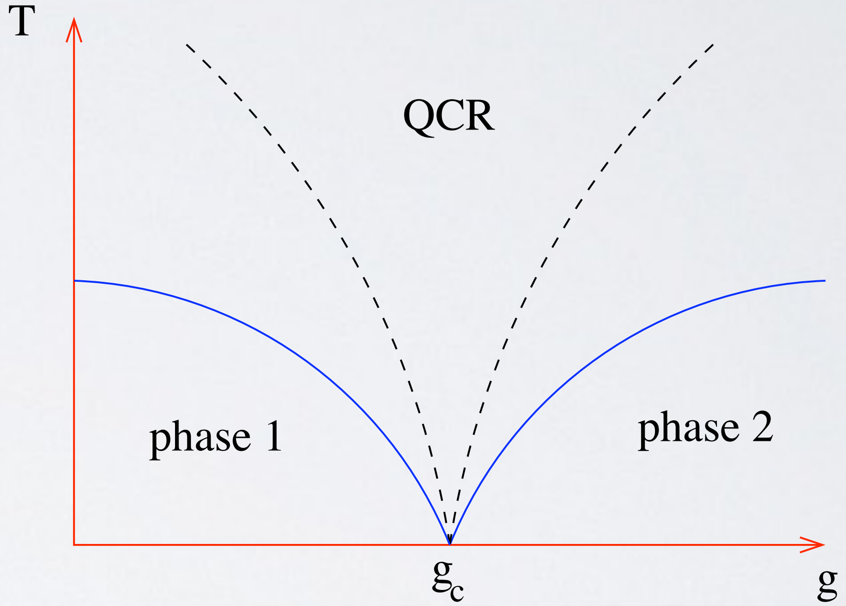


> **High Tc Superconductors**





- **(Strongly coupled) Quantum Critical Point**



[from Herzog]

@ QCP...

$$\xi \sim (g-g_c)^{-\nu}$$

$$\Delta \sim (g-g_c)^{\nu\, z}$$

→ **Strongly coupled scale invariant FT**

Use Holography!

Holographic Disorder

> DISORDERED HOLOGRAPHIC SUPERCONDUCTORS

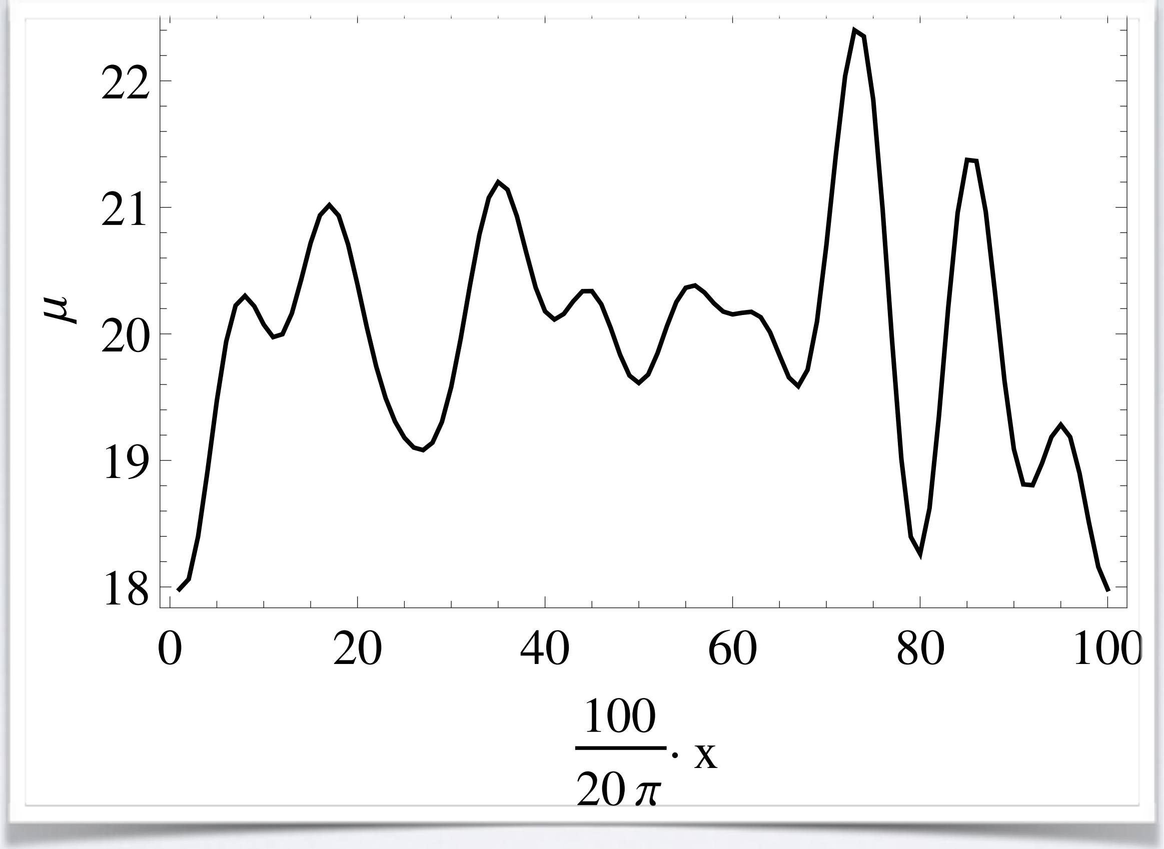
> DISORDERED TRANSPORT (STRANGE METALS): LUCAS, SACHDEV, SCHALM'14; DONOS, GAUNTLETT'14, O'KEEFFE, PEET'14; HARTNOLL, RAMIREZ, SANTOS'15

> DISORDERED FIXED POINTS: HARTNOLL, SANTOS'14; GARCIA, LOUREIRO'15

> DISORDERED HYDRO (-> GRAPHENE): LUCAS ET AL'15

> DISORDERED BRANES

Disordered Chemical Potential



Periodicity

$[x \sim x + L]$

1d-noise(x)

$$\boxed{\mu(x) = \mu_0 + \frac{\mu_0}{25}\, w \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)}$$

$k_* \to \infty$

$\Rightarrow$ Gaussian noise

- w **Noise strength**
- $k_0 \sim$ **1/(System Size). [IR Scale]**
- $k_* \sim$ **1/Correlation length [UV Scale]**

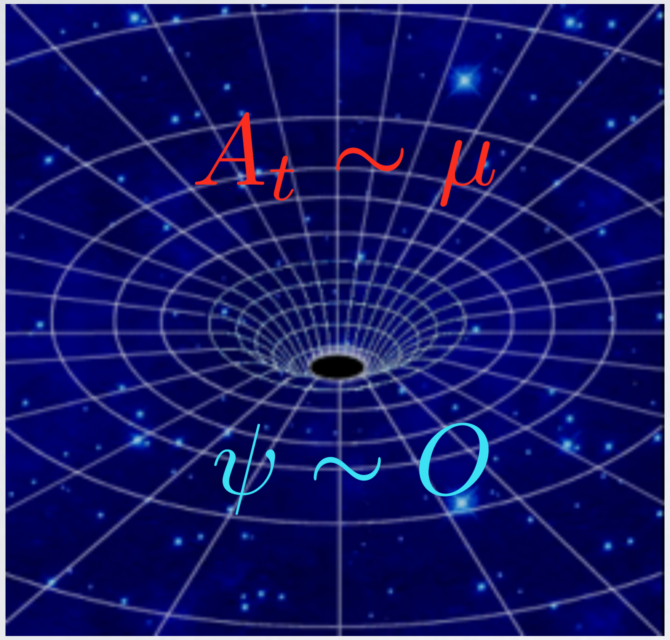
* [in units of T]

We set: $k_0 = 1/10\,,\quad \text{and}\quad 10\ \text{modes}$

Dirty Holographic Superconductors

[1308.1920]

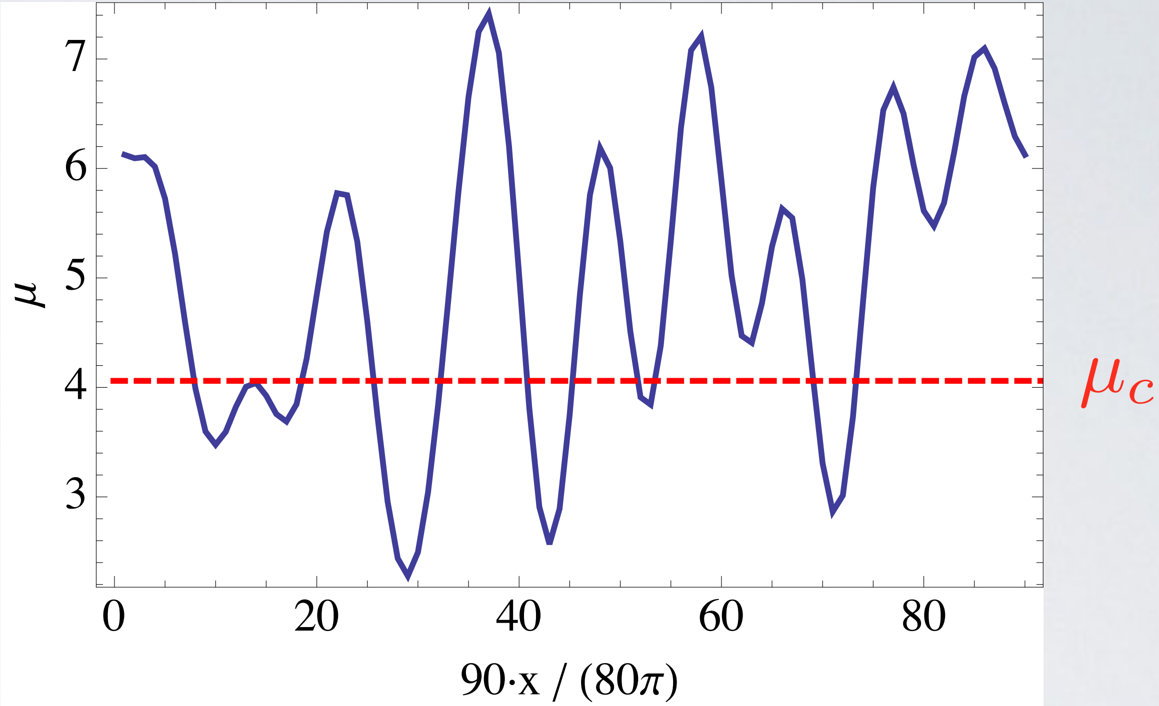
Holo SC



[Hartnoll et al'08]

+

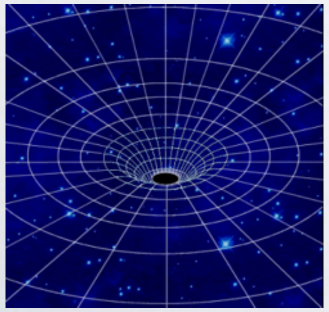
Disordered $\mu(x)$



Noisy chemical potential

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{ab} F^{ab} - (D_\mu \Psi)(D^\mu \Psi)^\dagger - m^2 \Psi^\dagger \Psi \right)$$

in



(3+1) Neutral Black Brane

[Probe Limit]

⟹

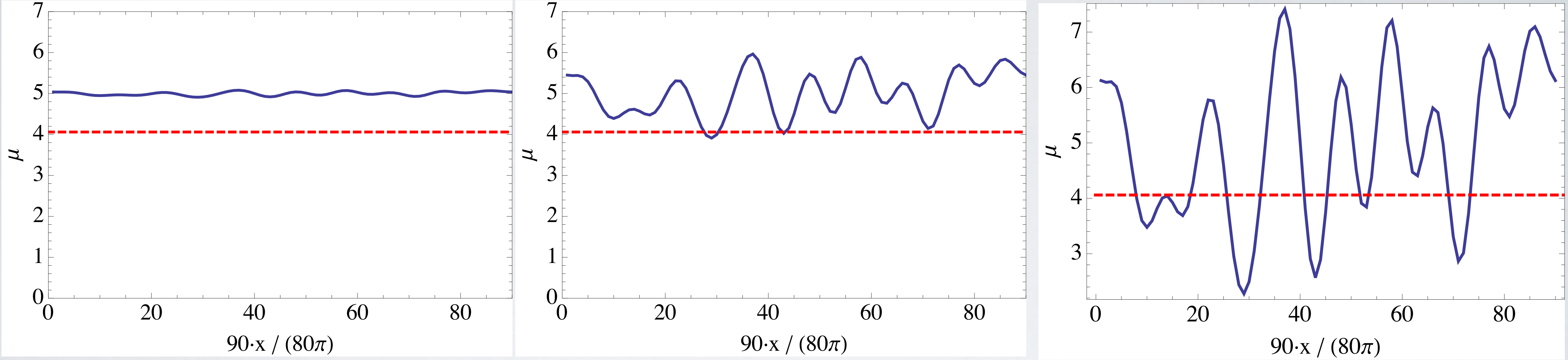
2 PDEs

> Results: The Inhomogeneous Condensate [1507.02280]

$\mu = 5 \rightarrow T \sim 0.8\, T_c$

$L_x = 80\pi\ ,\ 9\ \text{modes}$

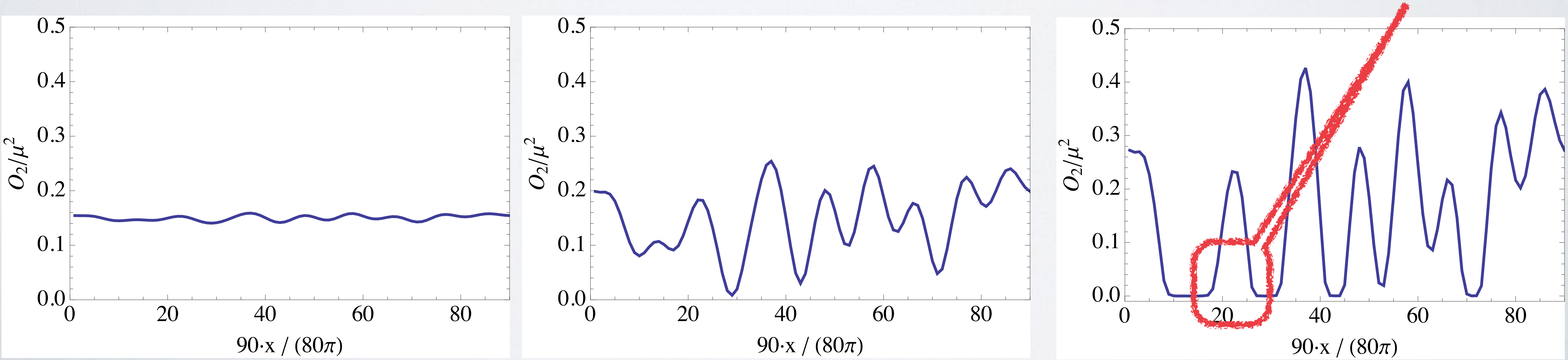
Chemical potential



$w = 0.004$ $w = 0.048$ $w = 0.12$

disorder

Condensate



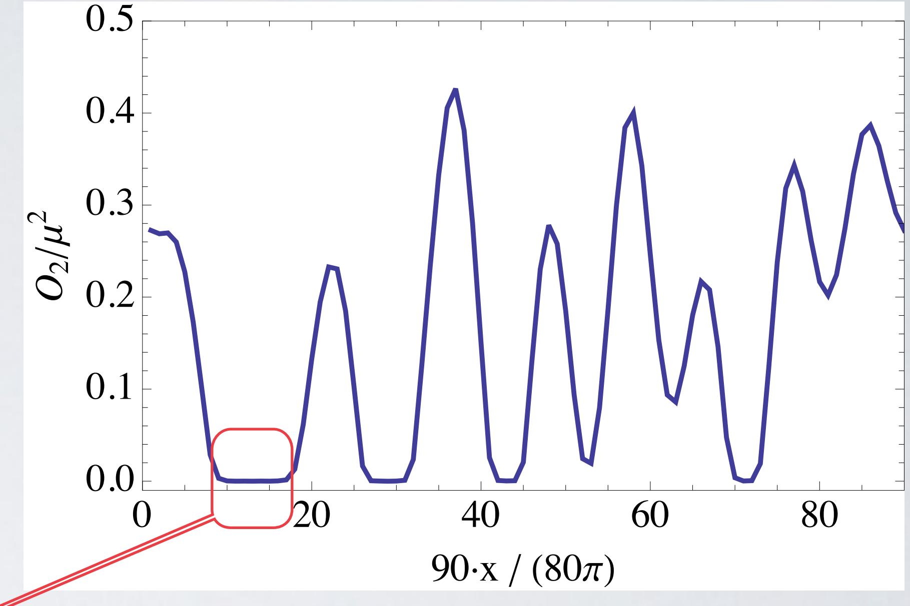
Islands?

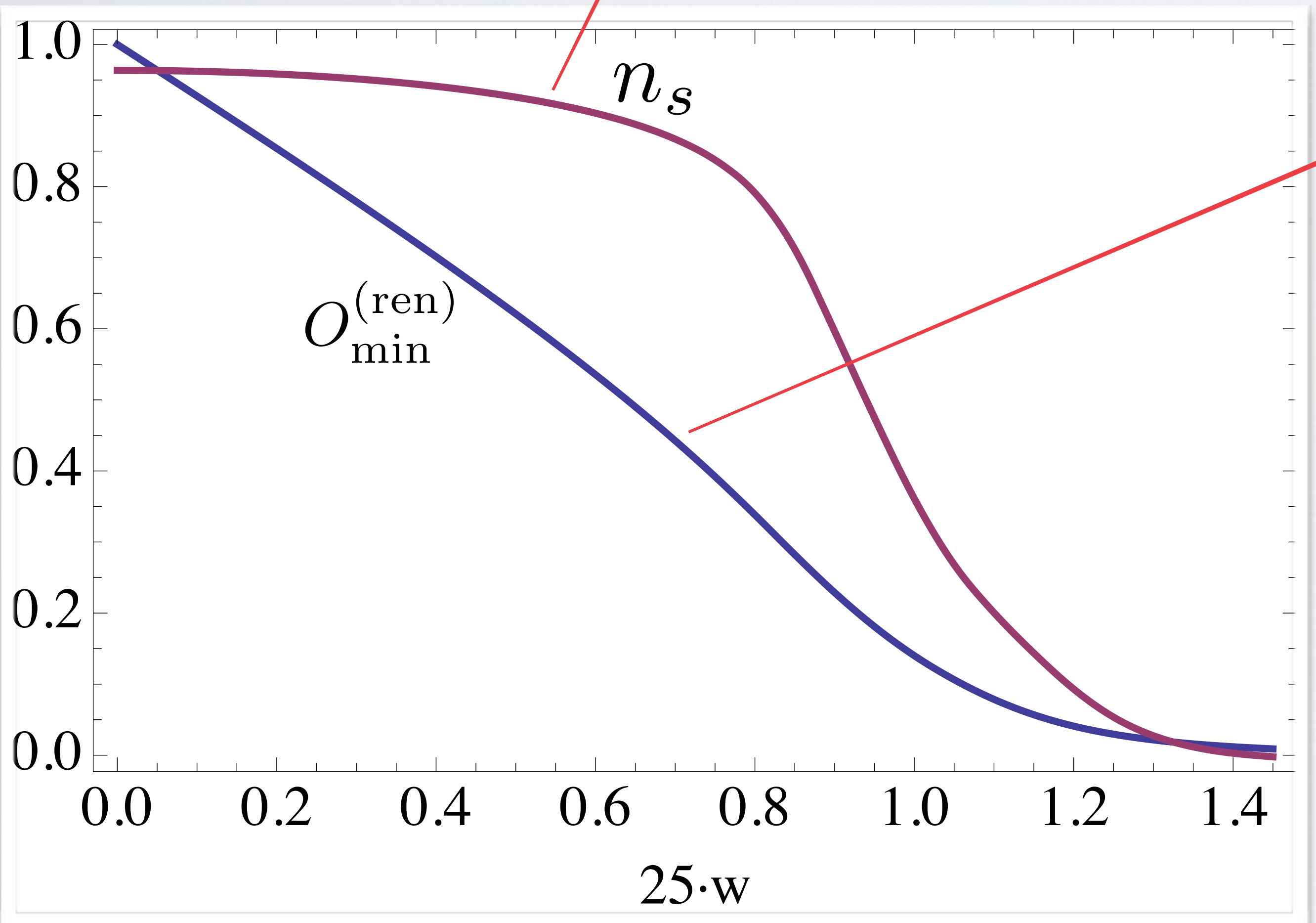
> Results: Disorder-induced Phase Transition

Superfluid density

Conductivity ⇒ $\mathrm{Im}(\sigma) = \frac{n_s}{\omega}$

Condensate

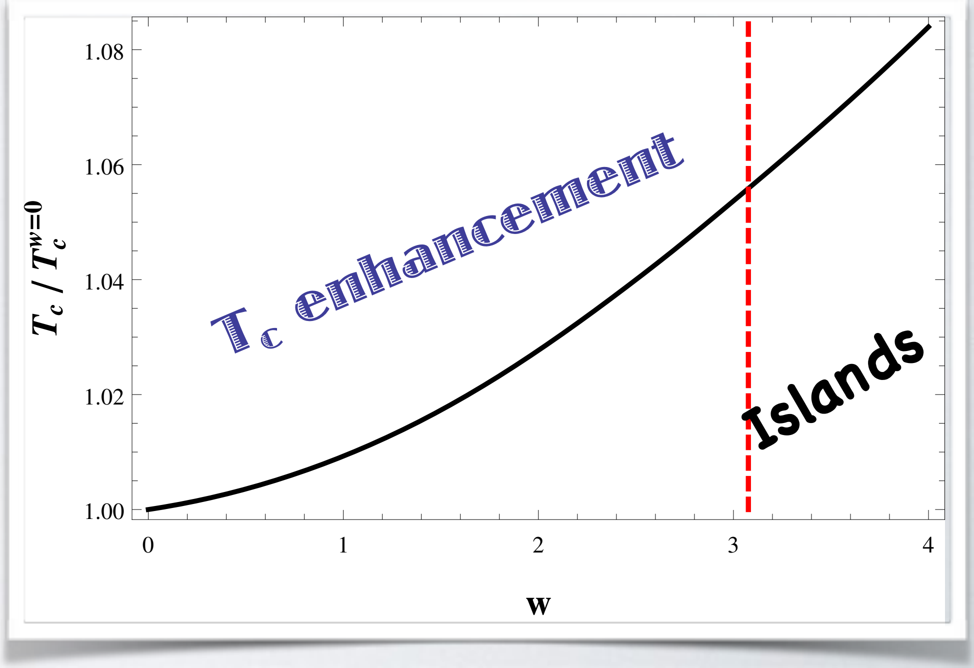




disorder

➥ Superfluid density follows O_{min} and dies away (exp-like)

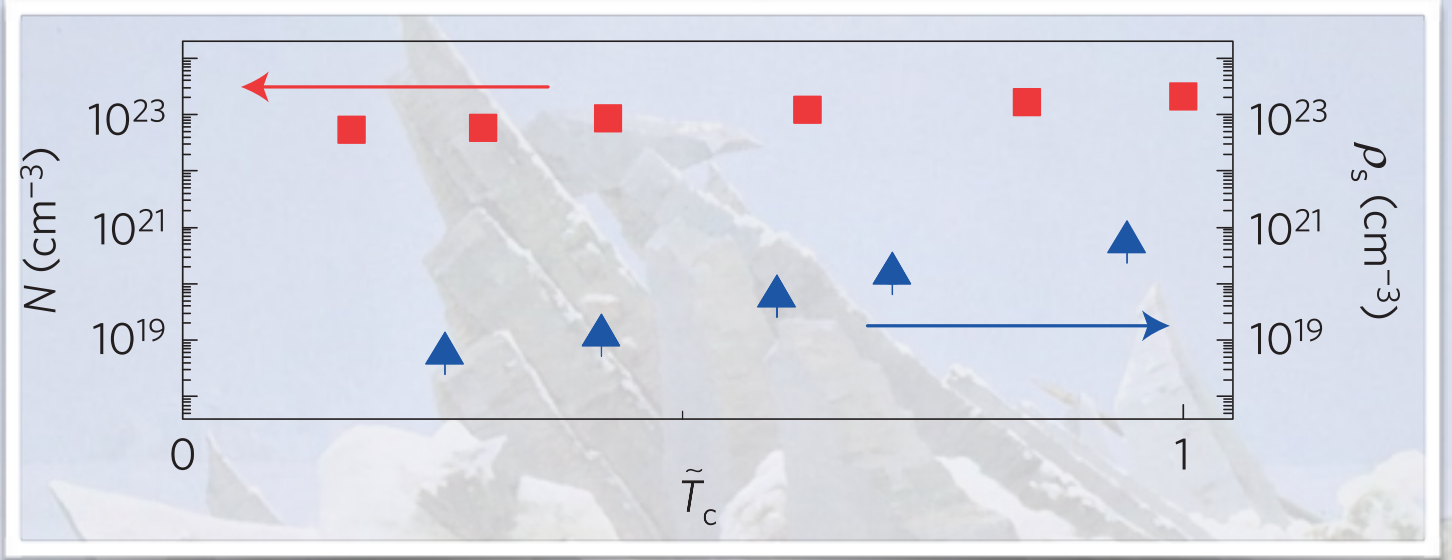
meanwhile the spatial average...



> SC to insulator disorder-induced phase transition

[Sherman et al, Nature Phys 11, 188-192 (2015)]

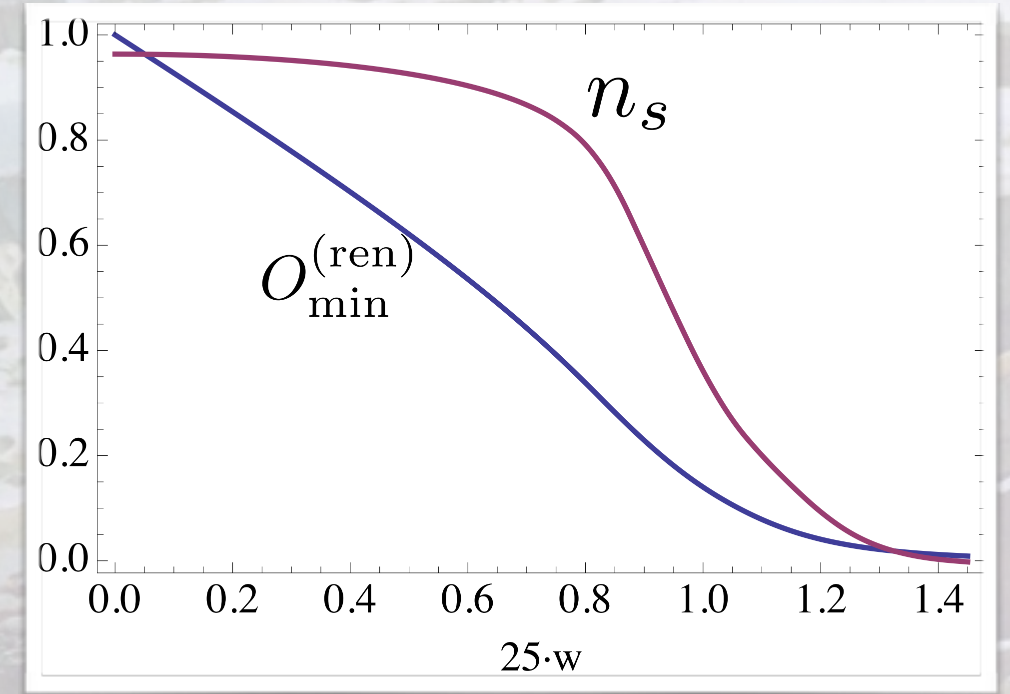
> Experiment



▲ Superfluid Density

disorder

> Holography: Disorder-induced Phase Transition



disorder

+

- ENHANCEMENT OF <CONDENSATE>
- (HINTS OF) HIGGS MODE
- NON MEAN-FIELD PHASE TRANSITION
- ....

Noisy Branes [1603.09625]

D3/D5 Intersection

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X	–	–	–	–	–	–
D5	X	X	X	–	X	X	X	–	–	–

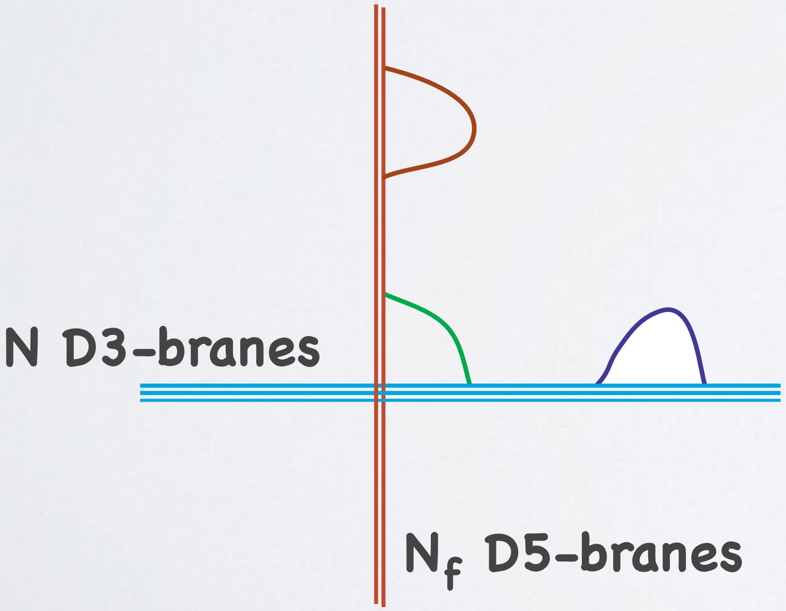
➡

2d (strongly coupled) matter

➥ **Graphene?**

3+1 N=4 SYM + (2+1) hypermultiplet

> Gravity Dual



$g_s\, N \ll 1$

$N \gg N_f$

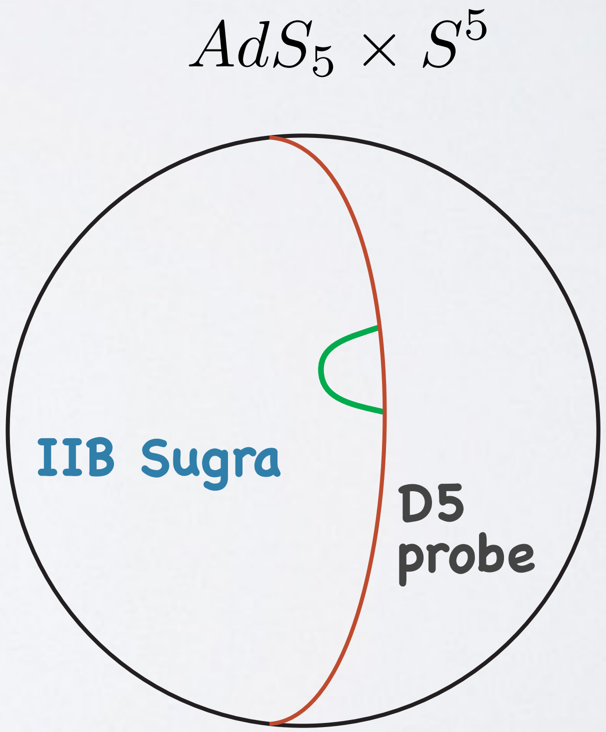
[Probe Limit]

$\sim$

$(\alpha' \to 0)$

$(g_s \to 0)$

(Karch&Randall'01)



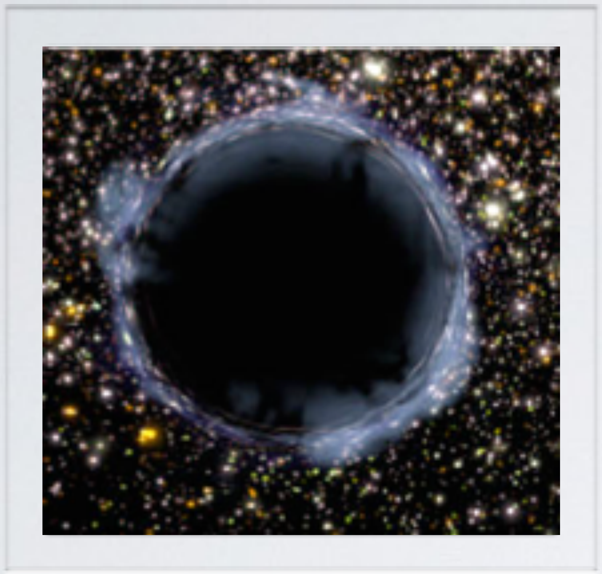
$g_s\, N \gg 1$

$g_s\, N_f \ll 1$

> Setup: D3/D5 @ finite T and μ

★ **Nonzero Temperature**

N_c Black D3-branes



$(\mathrm{Sch} - AdS_5) \times S^5$

★ **Charge Density (matter in the fdtal.):**

$A_\mu \sim J_\mu$ **U(1) Worldvolume gauge field**

$$A_t = \underset{\text{Chemical potential}}{\boxed{\mu}} - \underset{\text{Charge density}}{\boxed{\rho}}/r + O(r^{-2})$$

★ **D5 along (x,y) and wrapping** $S^2 \subset S^5$

$$\boxed{\begin{array}{l}\chi \equiv \cos\theta = \chi(z,x) \\ A_t(z,x)\end{array}} \longrightarrow$$

DBI (D5-brane)

$$\boxed{S = -N_f T_{D5} \int d^6x \sqrt{-\det(P[g] + 2\pi\,\alpha'\,F)}}$$

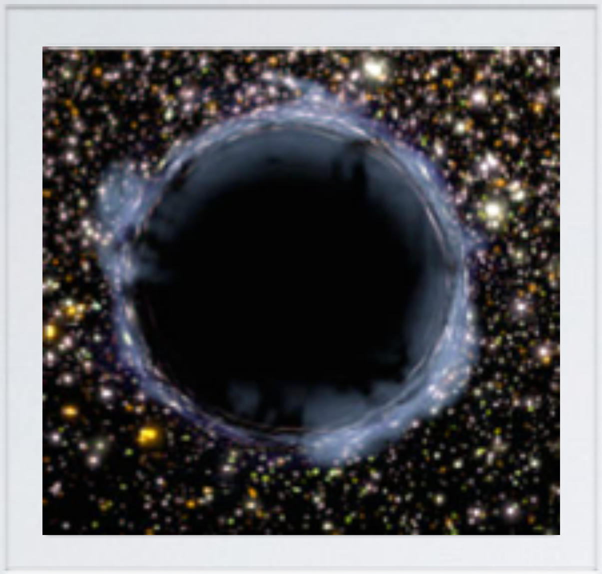
$$ds^2 = \frac{L^2}{z^2}\left(-\frac{f(z)^2}{h(z)}\,dt^2 + h(z)\,d\vec{x}^2 + dz^2\right) + L^2\,d\Omega_5^2$$

and $d\Omega_5^2 = d\theta^2 + \sin^2\theta\,d\Omega_2^2 + \cos^2\theta\,d\tilde{\Omega}_2^2$

> Setup: D3/D5 @ finite T and μ

★ **Nonzero Temperature**

N_c Black D3-branes



$(\mathrm{Sch} - AdS_5) \times S^5$

★ **Charge Density (matter in the fdtal.):**

$A_\mu \sim J_\mu$ **U(1) Worldvolume gauge field**

$$A_t = \boxed{\mu} - \boxed{\rho}/r + O(r^{-2})$$

Chemical potential

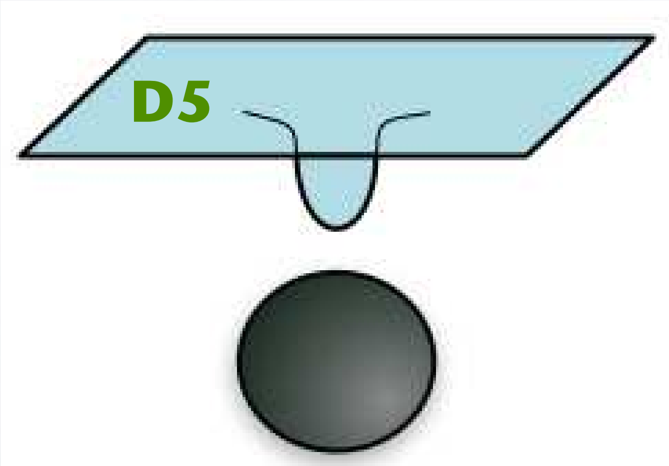
Charge density

Only BH embeddings at ρ≠0

[Kobayashi et al'06]

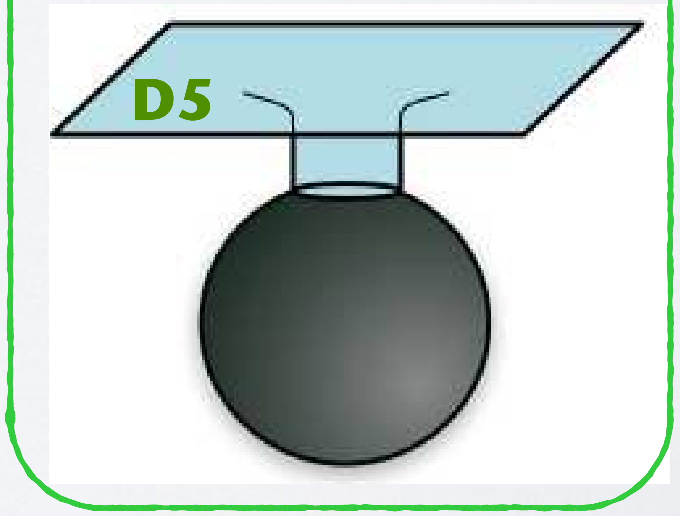
★ **Phase Diagram**

Minkowski



[hep-th/0611099]

Black Hole



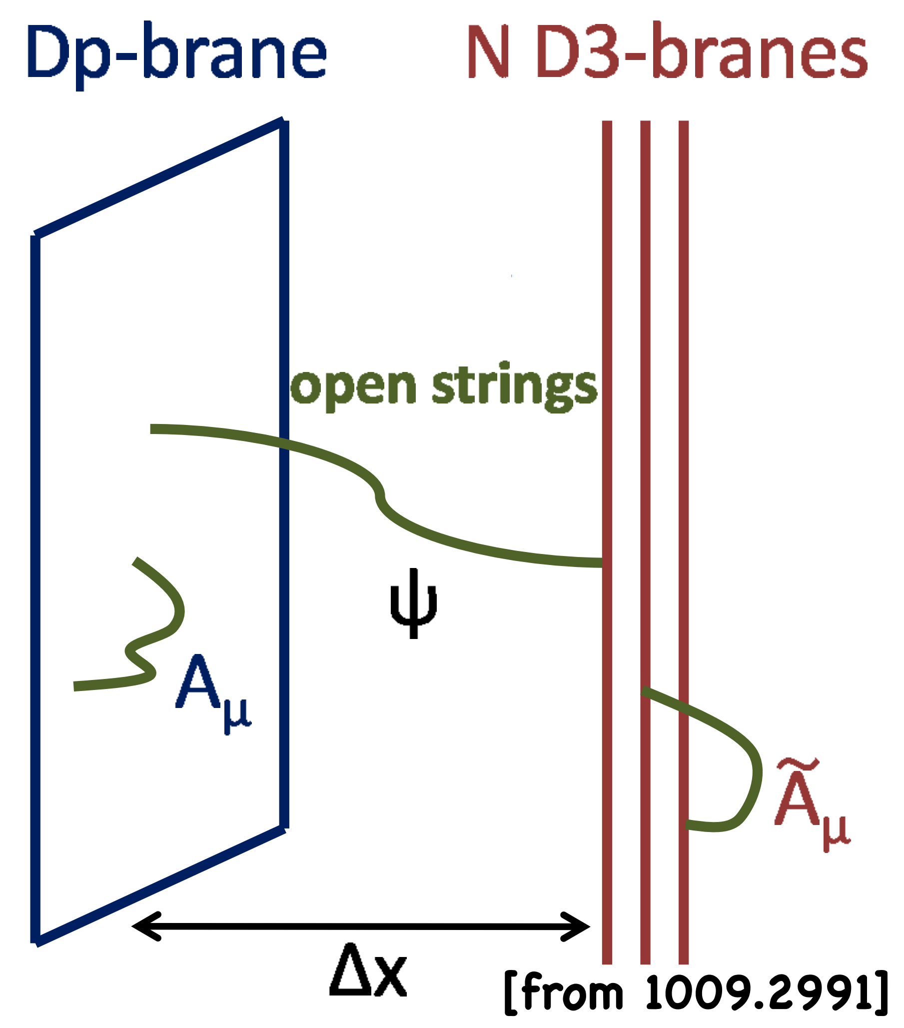
[Meson melting: Hoyos et al'06]

Noisy Branes

[1603.09625]
[See also: Ryu, Takayanagi, Ugajin'11;
Ikeda, Lucas, Nakai'16]

> Adding noise…

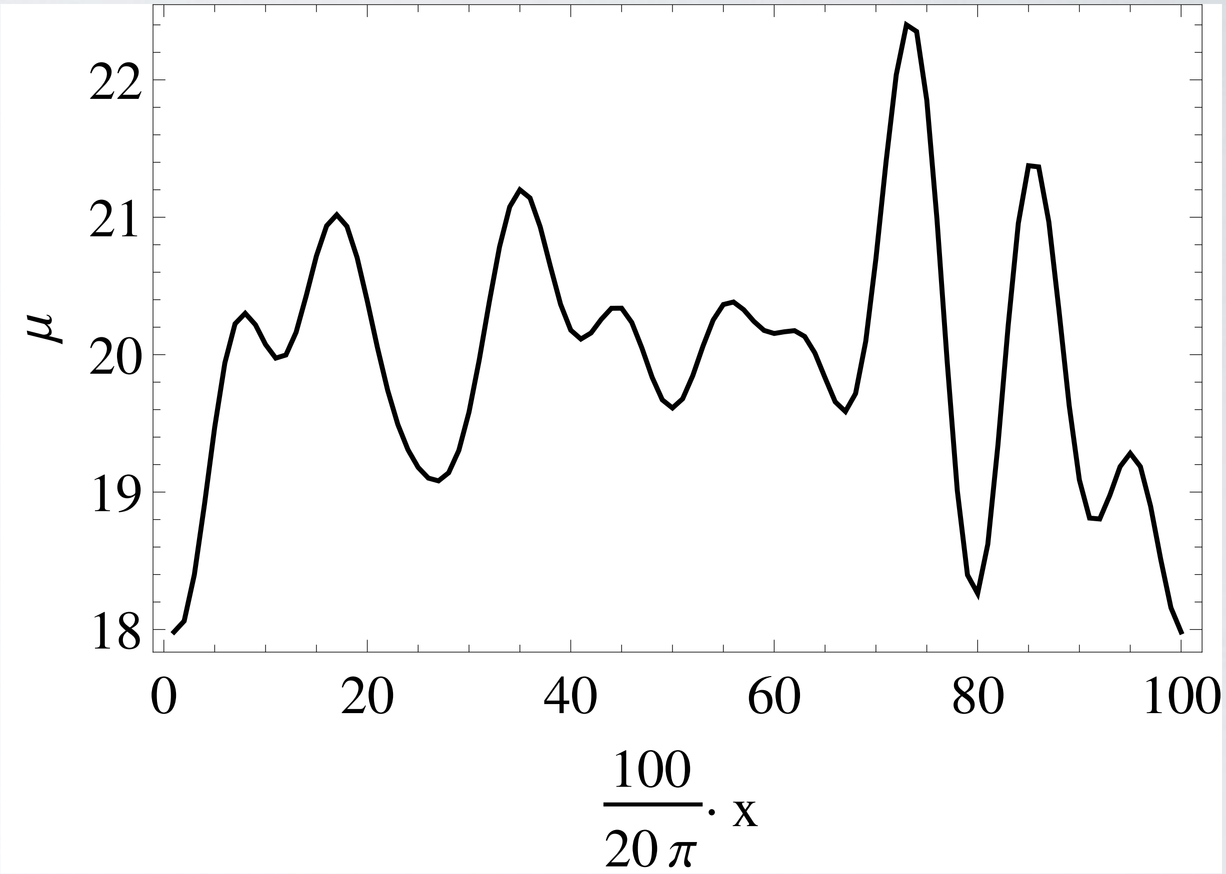
D3/D5



[from 1009.2991]

$$\boxed{\mathcal{S} = \mathcal{S}_{\text{DBI}}(\text{D5} - \text{brane})}$$

(1d-)Noisy μ(x)



$$\boxed{\mu(x) = \mu_0 + \frac{\mu_0}{25}\, w \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)}$$

→ 2 PDEs (A_t,χ) →

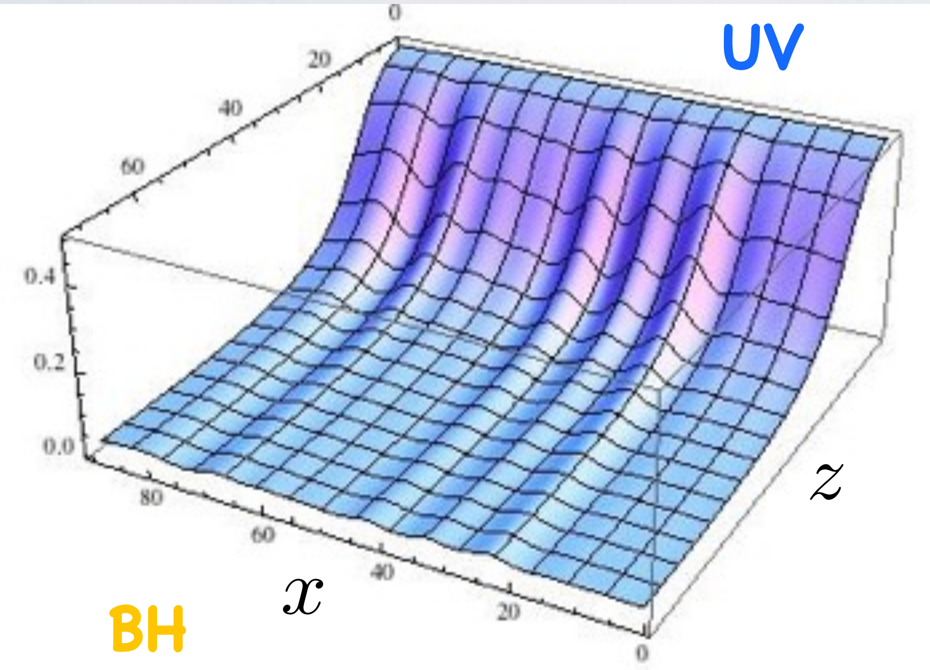
Numerics

> D3/D5. Results

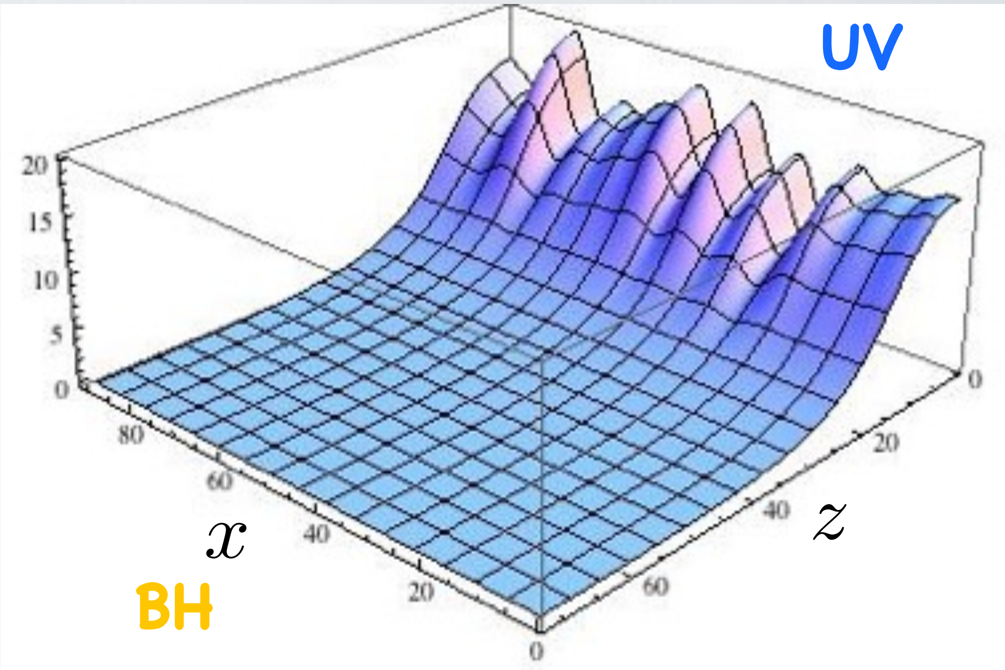
$$m = 0.5$$
$$\mu(x) = 15 + \text{noise}(x)$$

$k_0 = \frac{1}{10}$, 10 modes

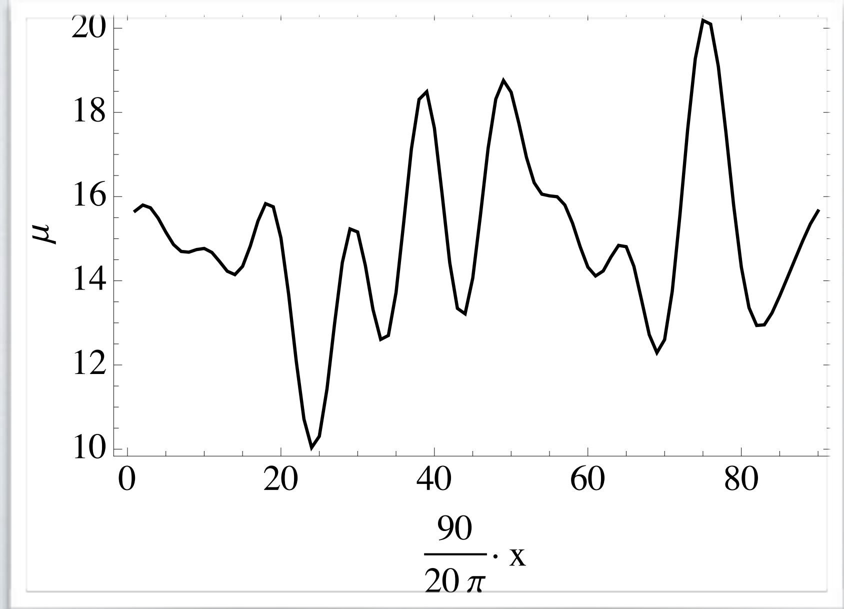
$\chi(z,x)$



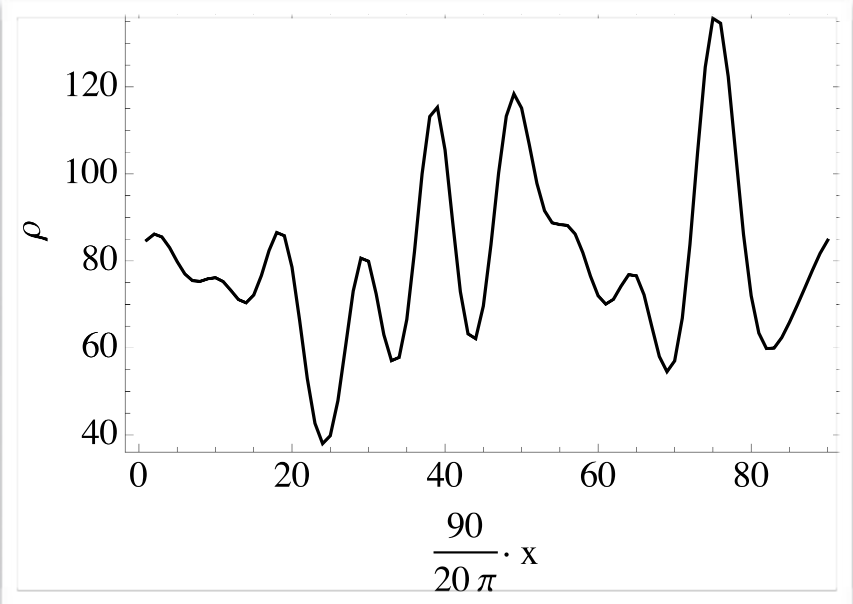
$\phi(z,x)$



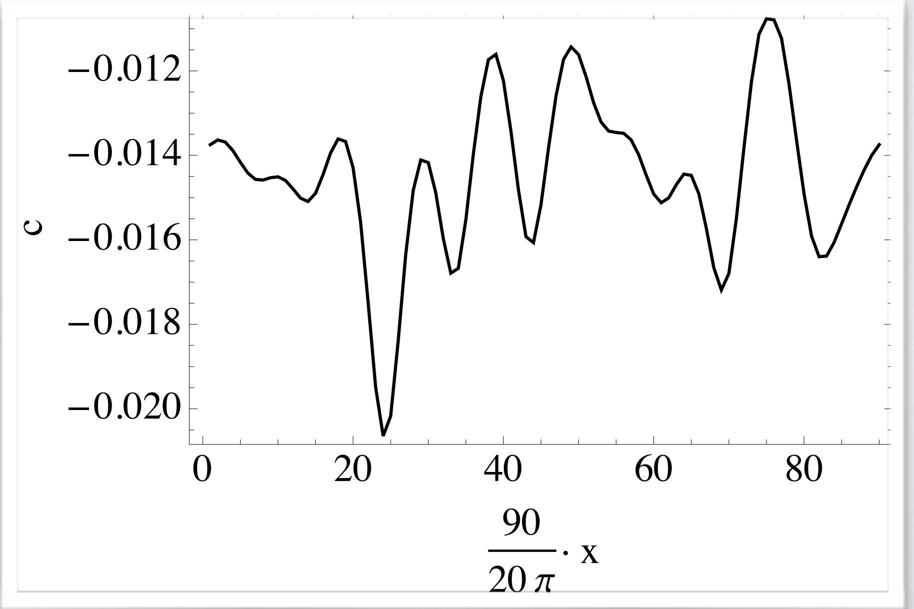
Chemical potential



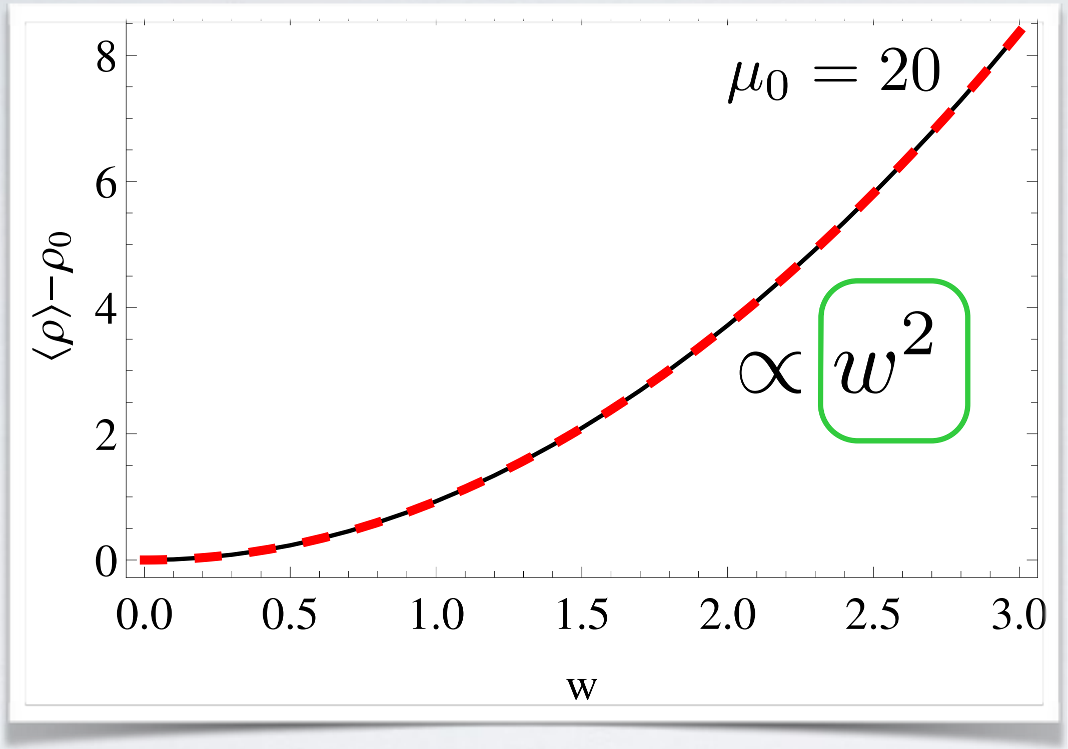
Charge density



Quark condensate



Charge density enhancement

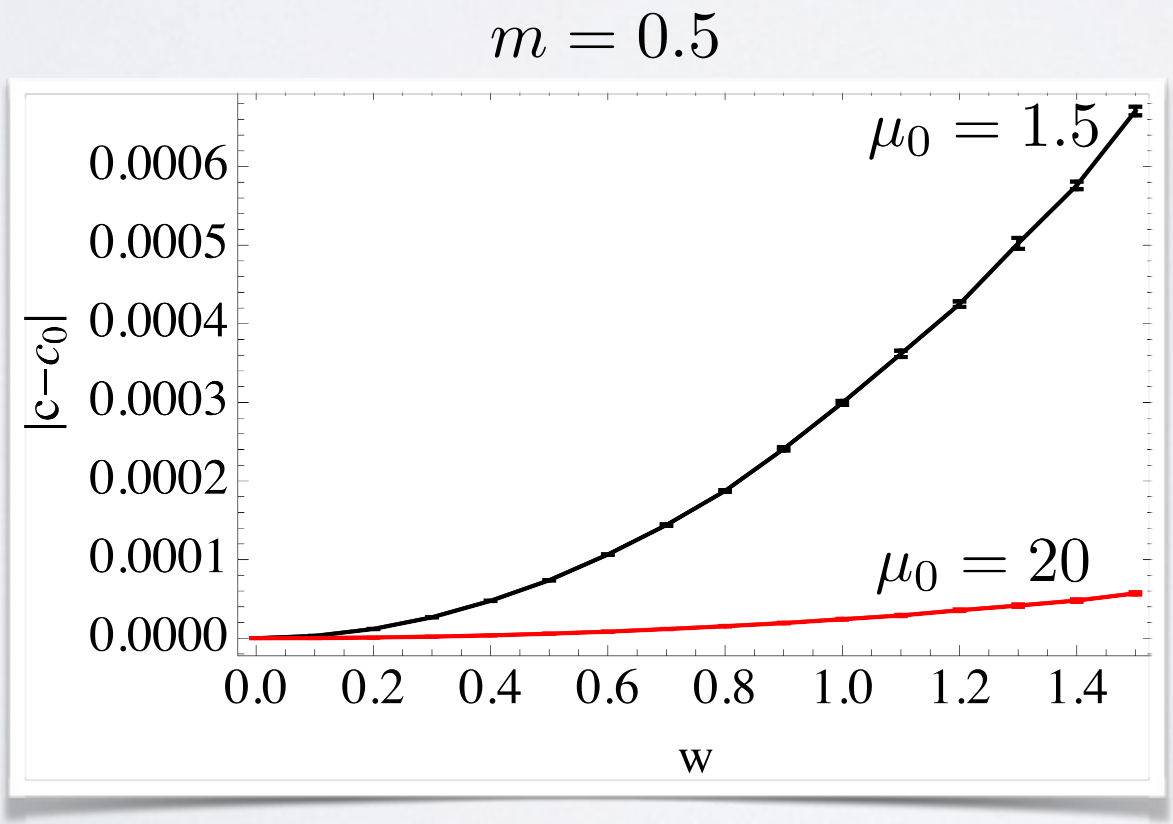


$$\langle\rho\rangle \neq \langle\rho\rangle(w)\,; \qquad (\rho \ll 1)$$

$$\langle\rho\rangle \propto w^2\,; \qquad (\rho \gg 1)$$

Can be shown analytically (as perturbation of clean system)

Quark condensate

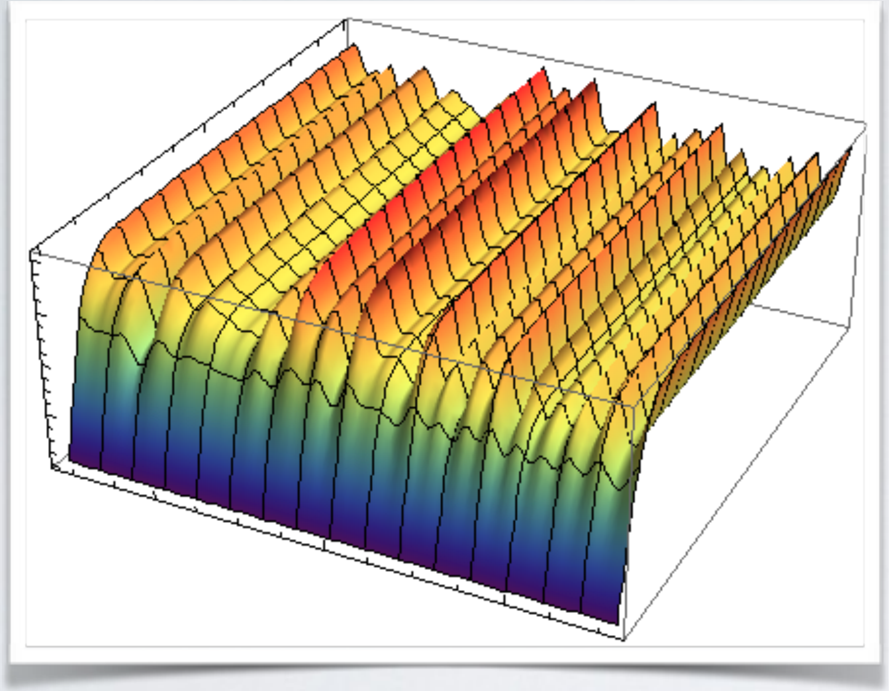


$$\langle c\rangle \neq \langle c\rangle(w)\,; \qquad (\rho \gg 1)$$

$$\langle c\rangle \propto w^2\,; \qquad (\rho \ll 1)$$

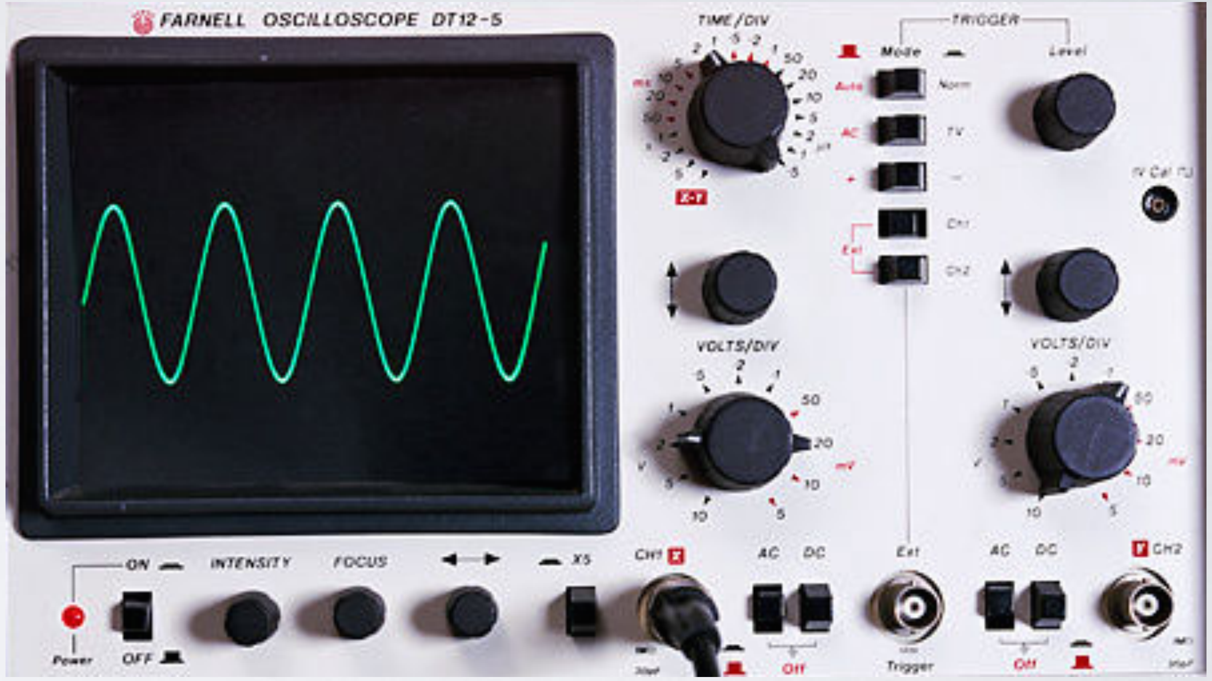
Noisy Conductivity

Noisy charged D5



+

Electric field (along x)



Fluctuations ⇒ Constant E-field ($\mathfrak{e}_i = 1$) @ bdry

$$\boxed{A_i(z,x,t) = a_i(z,x)\, e^{i\omega\, t}\,,\ \ F_{ti}(0,x) = (i\omega\, \mathfrak{e}_i)\, e^{i\omega\, t}\,; \qquad (i = x, y)}$$

$$\boxed{\sigma_i(\omega, x) = \frac{j_i}{i\omega\, \mathfrak{e}_i} = \lim_{z\to 0} \frac{f_{iz}}{f_{ti}}\,;\ \ (i = x, y)}$$

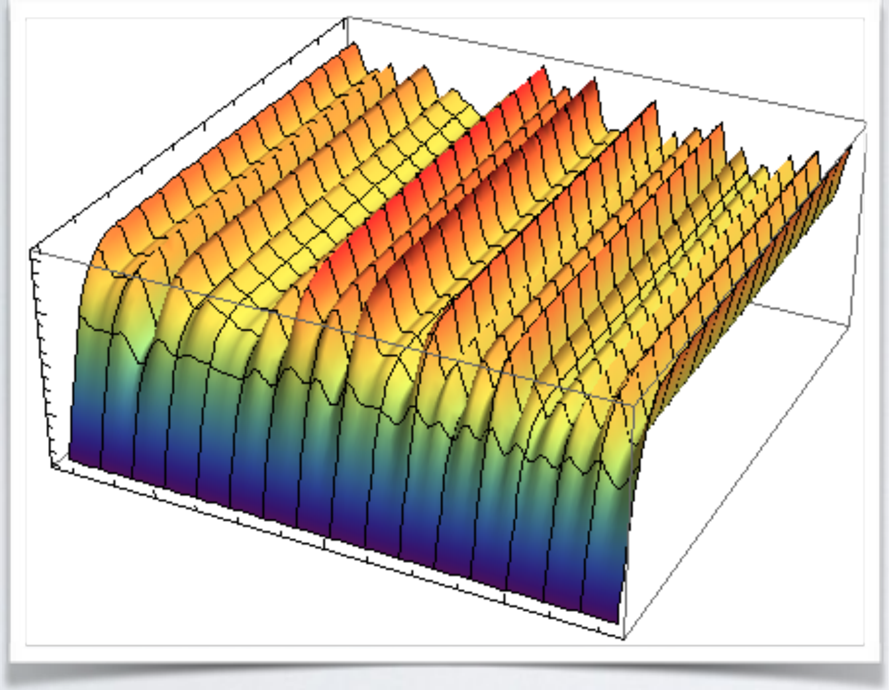
linear response

$\boxed{\sigma_x \equiv \sigma}$ → $(a_x, a_t, \delta\chi \equiv c)$ **coupled** → **3 linear PDEs**

[$a_z = 0$ **gauge]**

Noisy Conductivity

Noisy charged D5



+

Electric field DC (along x)

- $\sigma_{\mathrm{DC}} \iff$ Horizon data **[Iqbal&Liu'08; Ryu et al'12]**

$$\sigma_x^{\mathrm{DC}} = \frac{2L}{\int_{-L}^{L} \frac{dx}{\mathcal{F}(1,x)}} \quad \textbf{with} \quad \mathcal{F}(z=1,x) = \frac{2\left(C^{(0)}(x)^2 - 1\right)^{3/2}}{\sqrt{\left(a^{(2)}(x)^2 - 2\right)\left(2 - 2C^{(0)}(x)^2 + C^{(0)\prime}(x)^2\right)}}$$

linear response

$$\boxed{\text{DC Conductivity} \Longleftarrow \left(\chi(z=1,x)\,,\phi''(z=1,x)\right)}$$

[*Periodic system]

DC Conductivity vs Charge Density

> In Graphene...

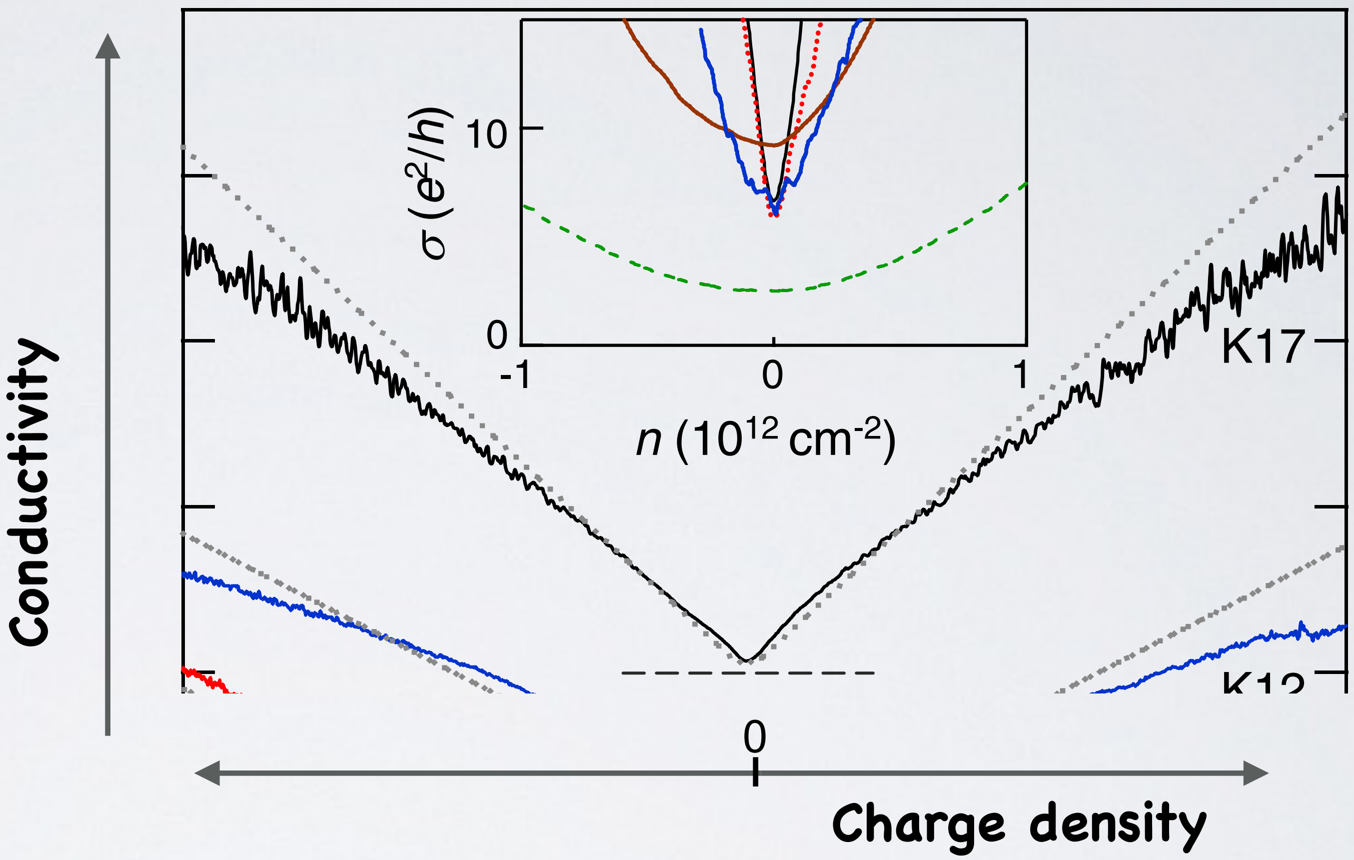
- Experimental Conductivity **[Tan et al, PRL'99]**

i) ∃ σ **minimal @ CNP**

ii) $\sigma \propto n$ **up to n***

iii) σ **sublinear for high n**

[Hwang et al, PRL'98]



> Clean D3/D5

$$\sigma = \sqrt{2 + \rho^2/2}$$

$$\sigma \sim \rho^2 \, , \; (\rho \sim 0)$$

$$\boxed{\sigma \sim \rho \, , \; (\rho \to \infty)}$$

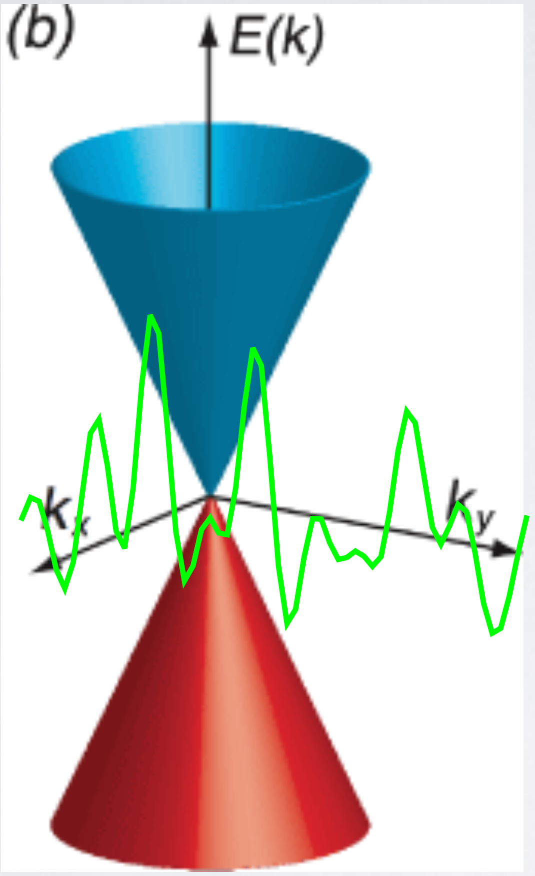
linear conductivity

➡ **What if we add disorder?**

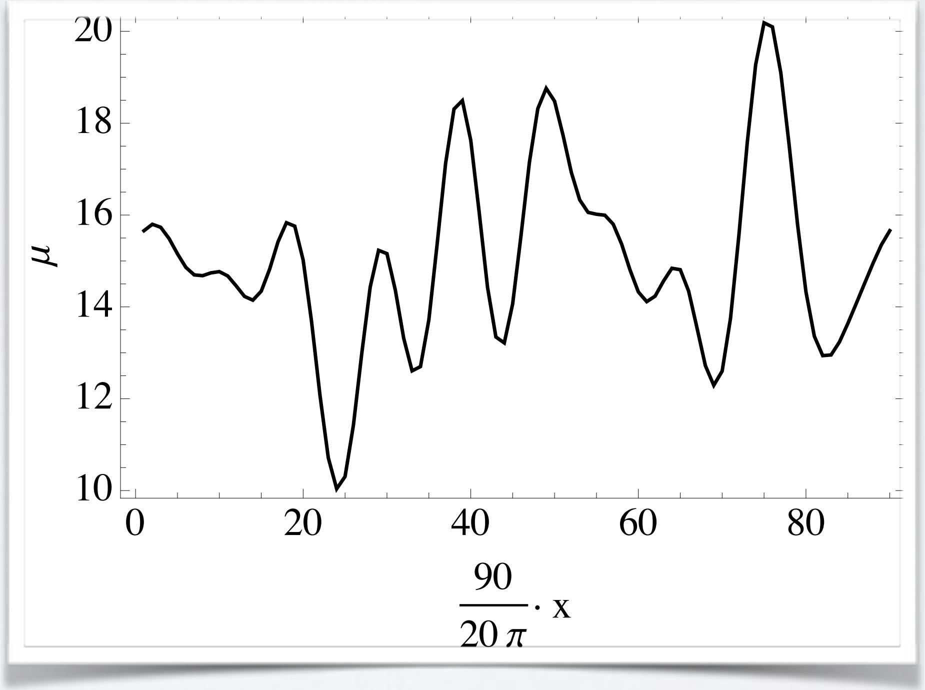
DC Conductivity vs Charge Density

[from here on: massless 'quarks']

Moderate Noise ($\mu(x)>0$)



with



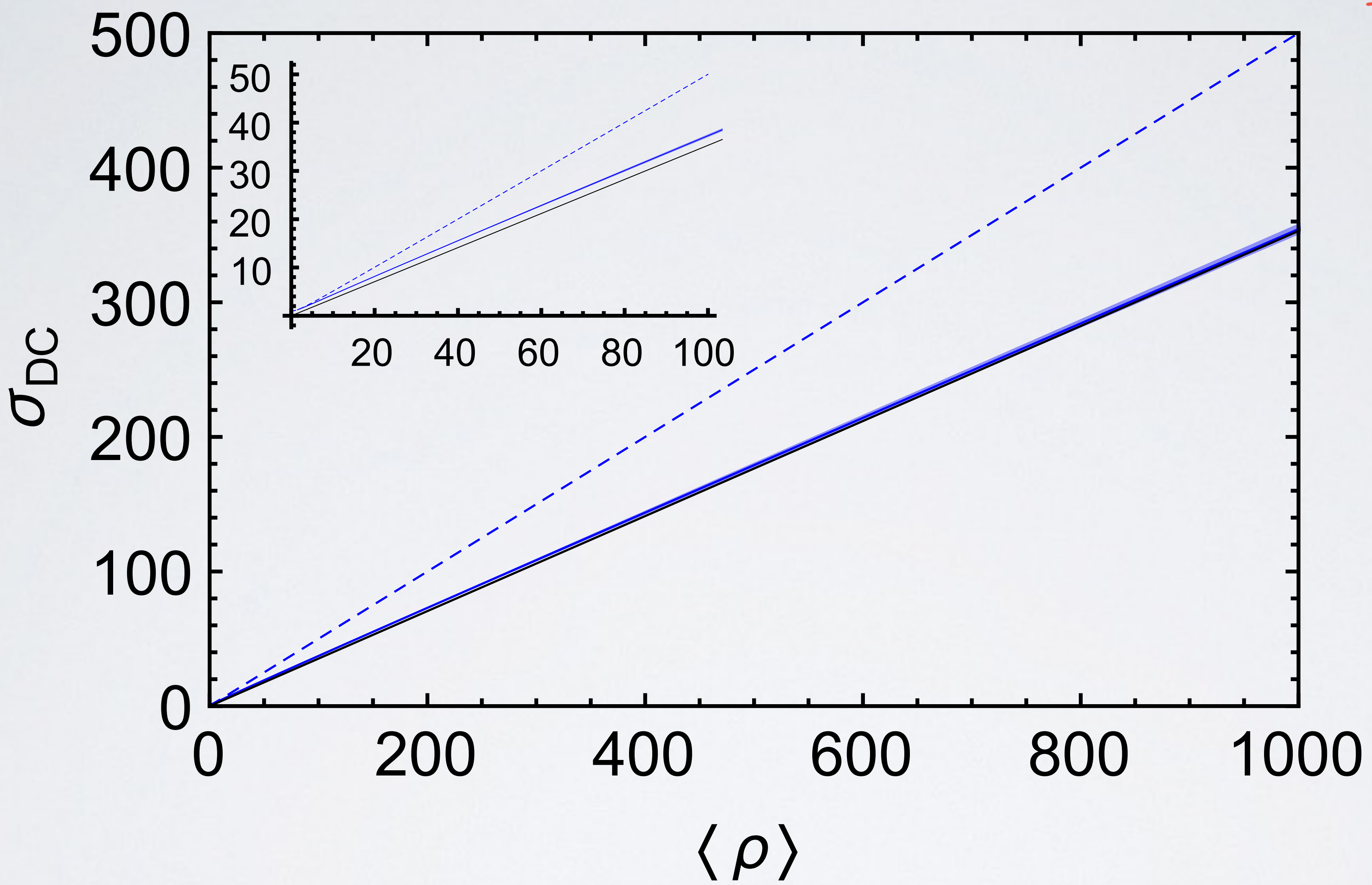
➥ *Non-linear conductivity?*

$k_0 = \frac{1}{10}$, 10 modes

> DC Conductivity vs Charge Density

Moderate Noise
($\mu(x) > 0$)

[w=3 ➡ Charge always > 0]



σ linear

Clean system

'Weak noise' prediction (w = 3)

Numerics (w = 3)

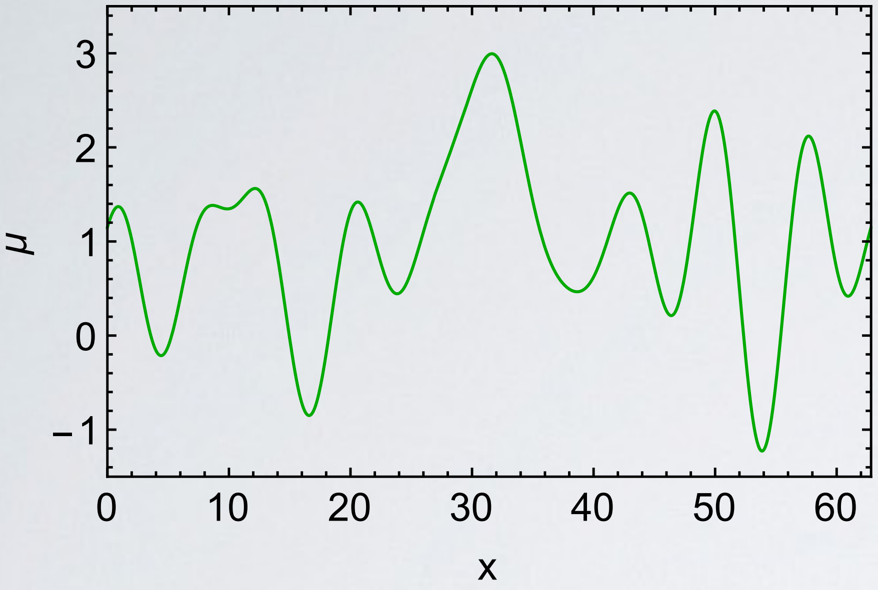
$\boxed{\sigma \sim \rho}$

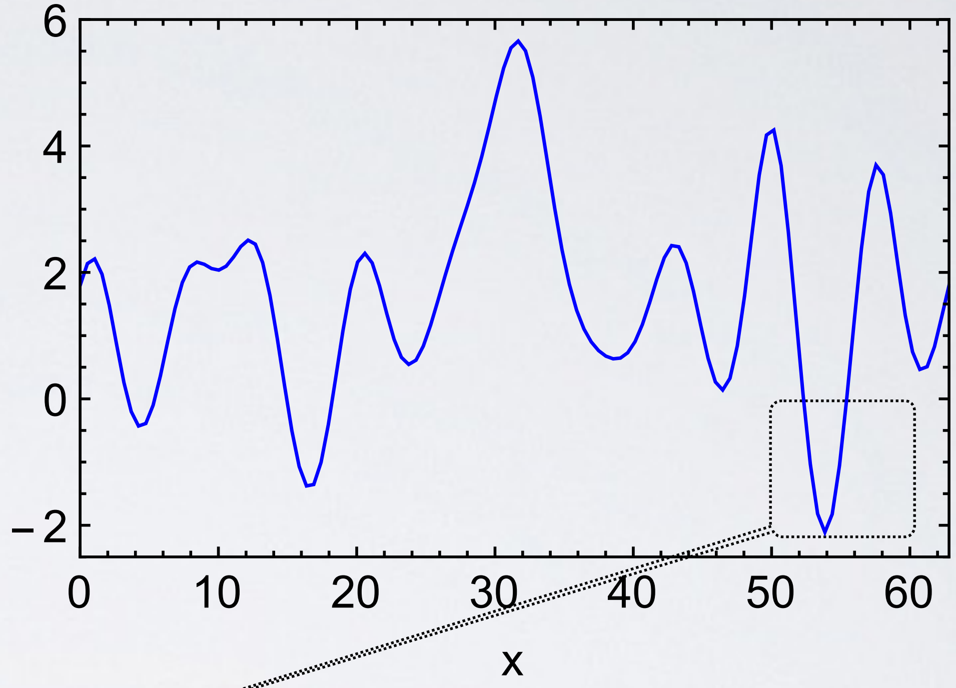
can be proven analytically (for moderate noise)

DC Conductivity at Strong Disorder

Increase noise... → What if $\mu(x) < 0$?

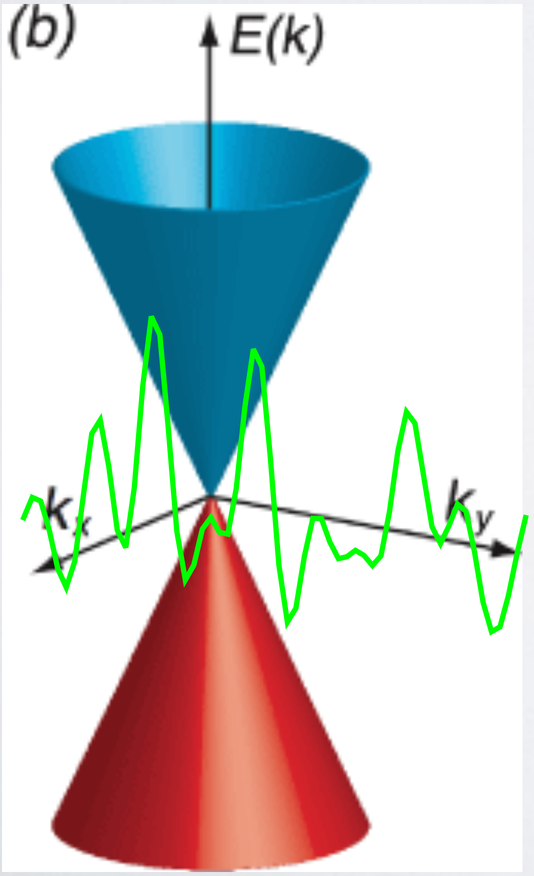
$\mu_0 = 1\,, \quad w = 10$





$\langle\rho\rangle = 1.6$

regions of 'negative' charge



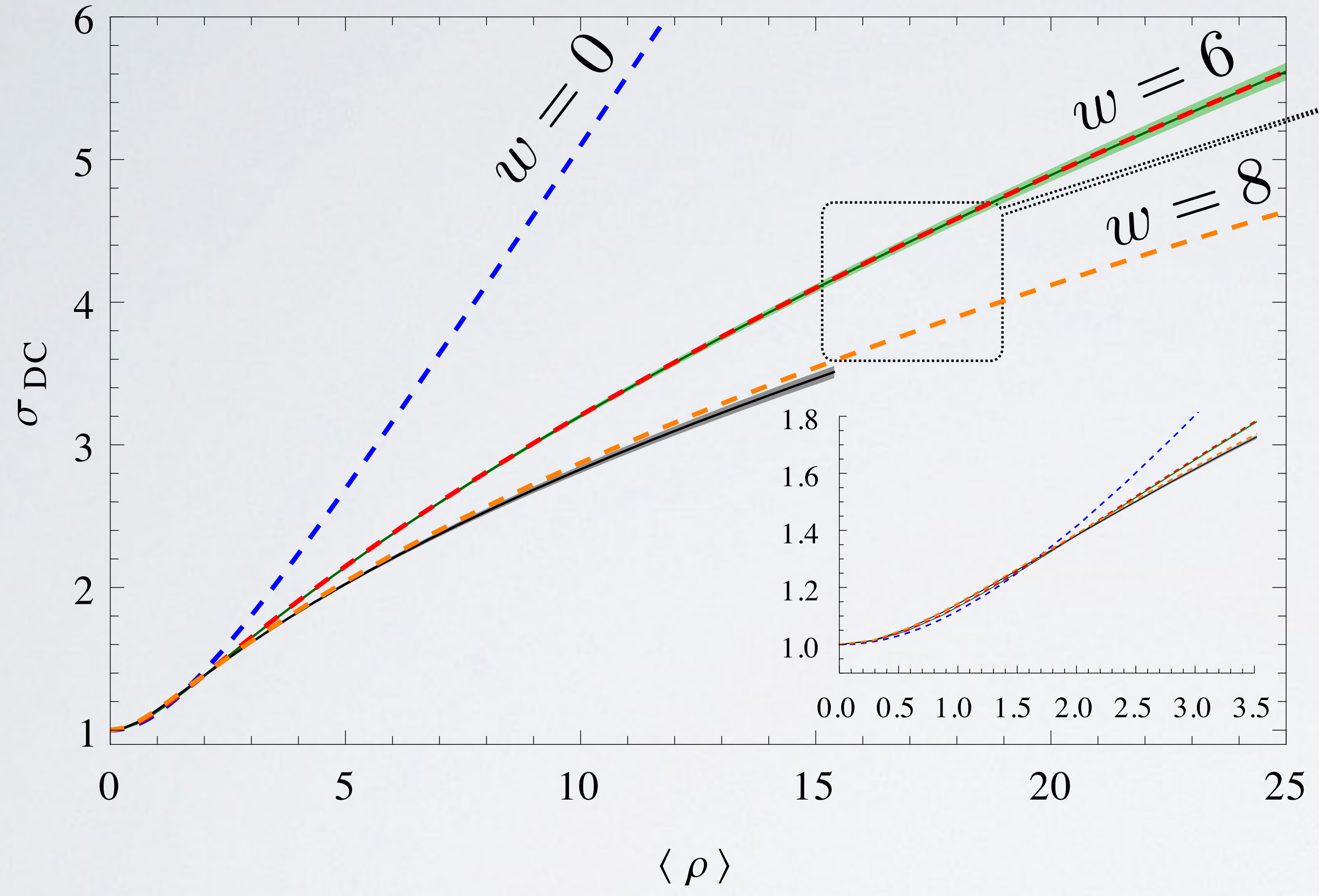
Expected in graphene (e$^-$ – hole puddles present)

➡ Non-linear conductivity?

DC Conductivity at Strong Disorder

$k_0 = \frac{1}{10}\,,\ 10\ \text{modes}$

> ‘Strong disorder’ (w ≳ 5) ➡ e⁻ – hole puddles appear



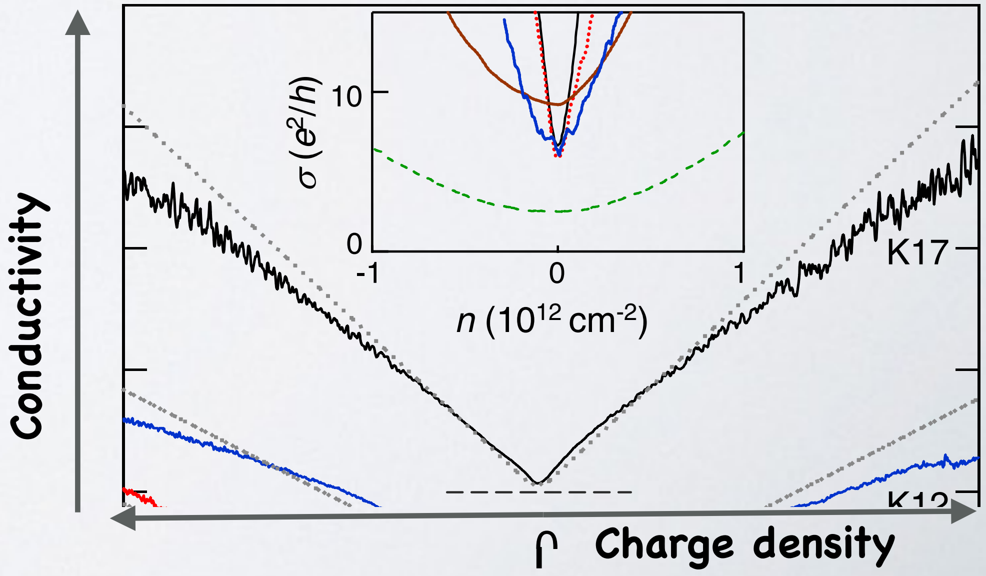
σ sublinear?!

Semi-analytic (w=6)

Semi-analytic (w=8)

➥ **Remember graphene …**

σ sublinear…!

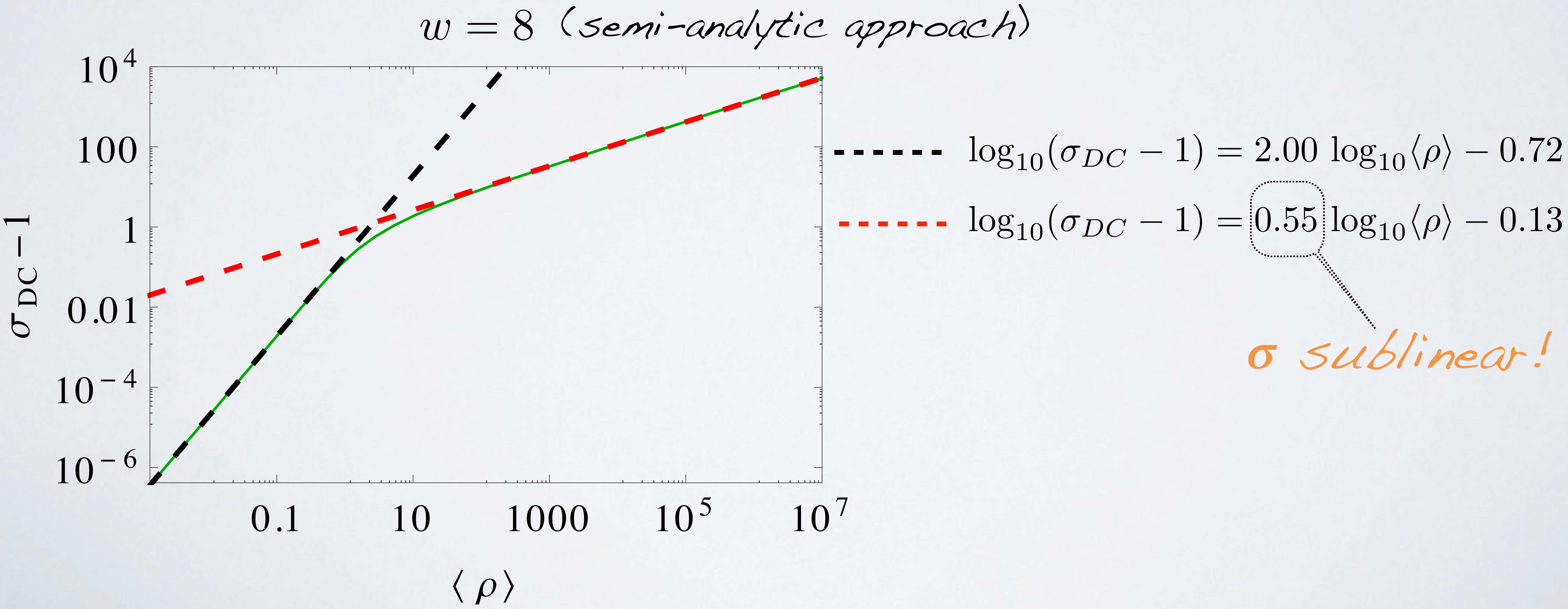


DC Conductivity at Strong Disorder

> **Semi-analytic approximation @ strong disorder**

(approximating the setup by a succession (along x) of homogeneous systems → gradients neglected)

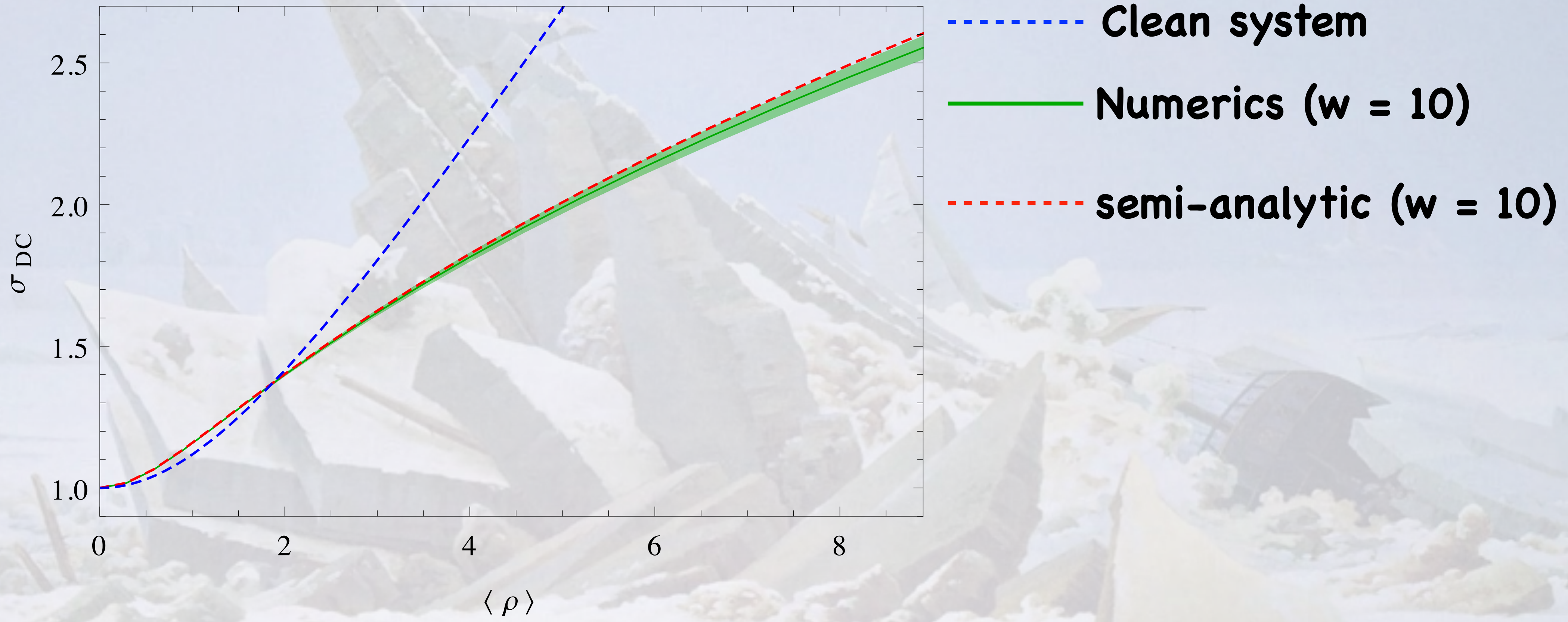
$$\boxed{\sigma_{\rm DC} = \left\langle \frac{1}{\frac{1}{L}\int_0^{L_x} dx\, \frac{2}{\sqrt{4+\rho(x)^2}}} \right\rangle_{\rm noise}} \xrightarrow{\text{'strong noise'}} \boxed{\begin{array}{ll} \sigma_{\rm DC} \propto \langle\rho\rangle^2\,, & (\langle\rho\rangle \ll 1) \\ \boxed{\sigma_{\rm DC} \propto \sqrt{\langle\rho\rangle}\,,} & (\langle\rho\rangle \to \infty) \end{array}}$$


$w = 8$ (semi-analytic approach)

σ sublinear!

DC Conductivity vs Charge Density

> Noisy branes @ strong disorder



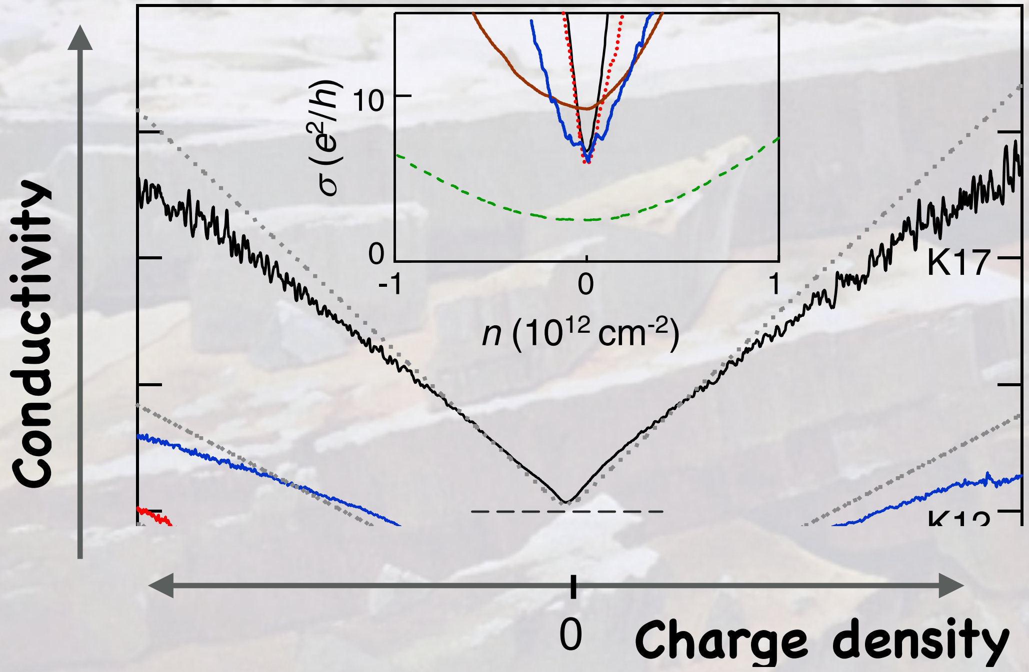
> In Graphene...

i) $\exists$ σ minimal @ CNP

ii) $\sigma \propto n$ up to n*

iii) σ sublinear for high n

[Hwang et al, PRL'98]



See also: σ vs noise strength

- σ ENHANCEMENT @ LOW ρ
- σ DECREASE @ HIGH ρ

OUTLINE

> CHALLENGE: DISORDER + STRONG INTERACTIONS ✔

> DIRTY HOLO SCs: islands of SC -> Disorder-induced PT

> NOISY BRANES: 2+1 matter + disorder

> NOISY Conductivity (strong noise): Sub-linear at ‘large ρ’ ✔

> Epic Challenges: backreaction (smearing?), 2d noise (thin films, graphene, disorder SC fixed points, QHE models?

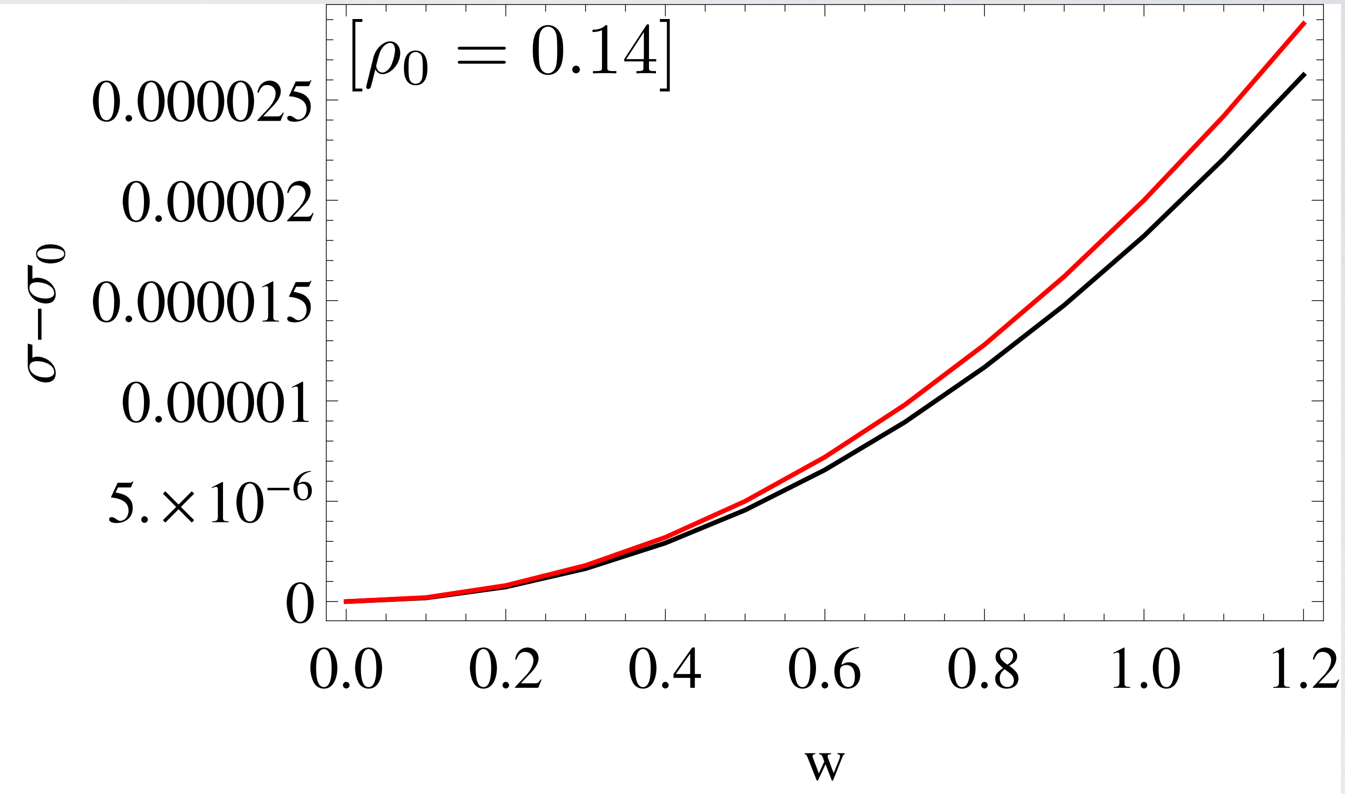
and now... some **SUPPLEMENTARY SLIDES**

> Conductivity. `Weak Noise'

$k_0 = \frac{1}{10}$, 10 modes

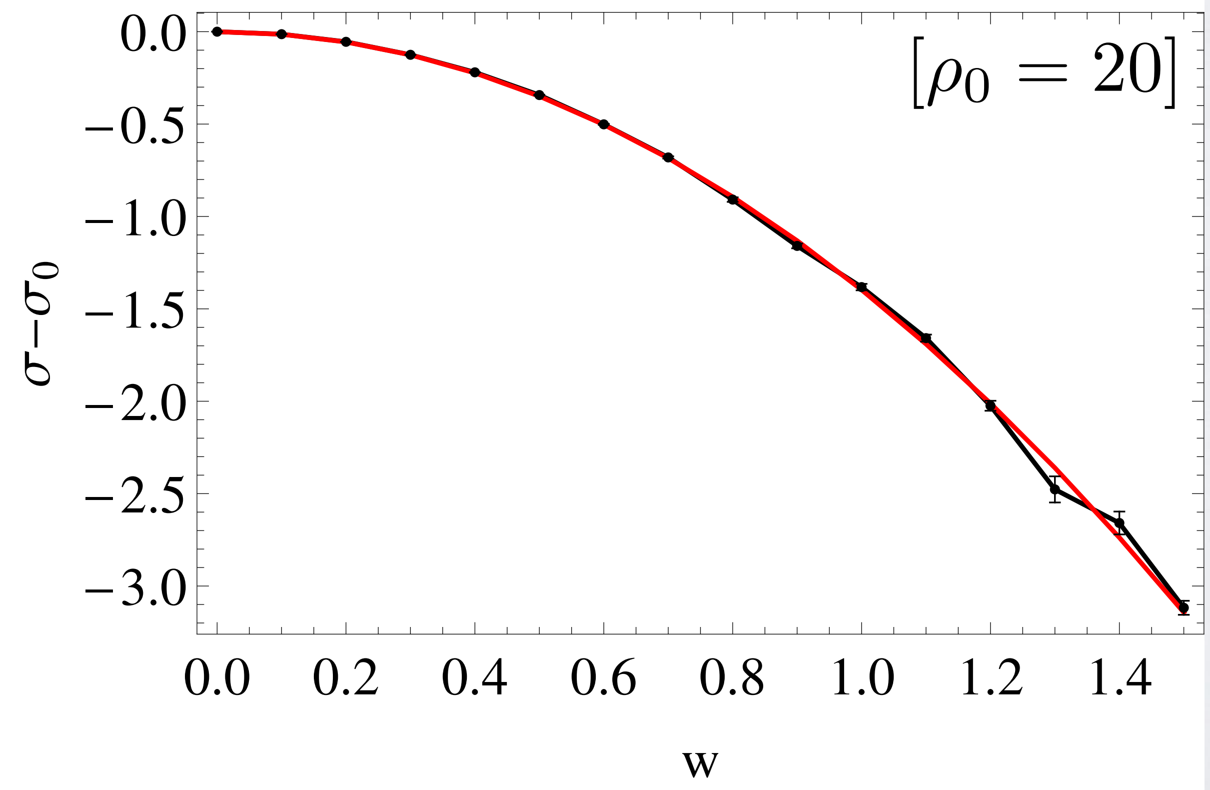
σ - Enhancement

- Small $\boldsymbol{\rho}$. Numerics



σ - Decrease

- Large $\boldsymbol{\rho}$. Numerics



> Conductivity. 'Weak Noise'

(~ ignoring gradients along x)

- Homogeneous limit: $\xrightarrow[a^{(2)} = \frac{\sqrt{2}\rho}{\sqrt{4+\rho^2}}]{}$ $\sigma_{\mathrm{DC}} = \left\langle \frac{1}{\frac{1}{L}\int_0^{L_x} dx \, \frac{2}{\sqrt{4+\rho^2}}} \right\rangle_{\text{noise}}$

- Noise strength expansion:

$$\mu = \mu_0 \left(1 + \frac{w}{25} n(x)\right) \to \rho(n(x)) \quad \textbf{in} \quad \sigma_{\mathrm{DC}} = \left\langle \frac{1}{\frac{1}{L}\int_0^{L_x} dx \, \frac{2}{\sqrt{4+\rho(x)^2}}} \right\rangle_{\text{noise}}$$

➥ **expand in powers of w** ➞ $\sigma_{\mathrm{DC}}(w)$

$\rho \approx \sqrt{2}\mu \, ; \qquad (\rho \ll 1)$

$\rho \approx 0.291 \mu^2 \, ; \qquad (\rho \gg 1)$

> Conductivity. ‘Weak Noise’

σ - Enhancement

- Small $\boldsymbol{\rho}$ $$\sigma_{\mathrm{DC}} = 1 + \frac{d^2\mu_0^2}{8}\left(1 \boxed{+ \frac{w^2}{25^2}} \frac{1}{L}\left\langle \int_0^L dx\, n(x)^2 \right\rangle_{\text{noise}}\right) + O(\mu_0^4, \tilde{w}^3)$$

➥ $$\boxed{\sigma_{\mathrm{DC}} \approx 1 + \frac{\mu_0^2}{4}\left(1 + \frac{w^2}{25^2}\frac{\#(\text{modes})}{2}\right)}$$

σ - Decrease

- Large $\boldsymbol{\rho}$ $$\sigma_{\mathrm{DC}} = \frac{c\,\mu_0^2}{2}\left(1 \boxed{- w^2} \frac{3}{25^2}\frac{1}{L}\left\langle \int_0^L dx\, n(x)^2 \right\rangle_{\text{noise}}\right) + O(\mu_0^{-2}, w^3)$$

➥ $$\boxed{\sigma_{\mathrm{DC}} \approx \frac{0.291\mu_0^2}{2}\left(1 - w^2\,\frac{3}{25^2}\frac{\#(\text{modes})}{2}\right)}$$

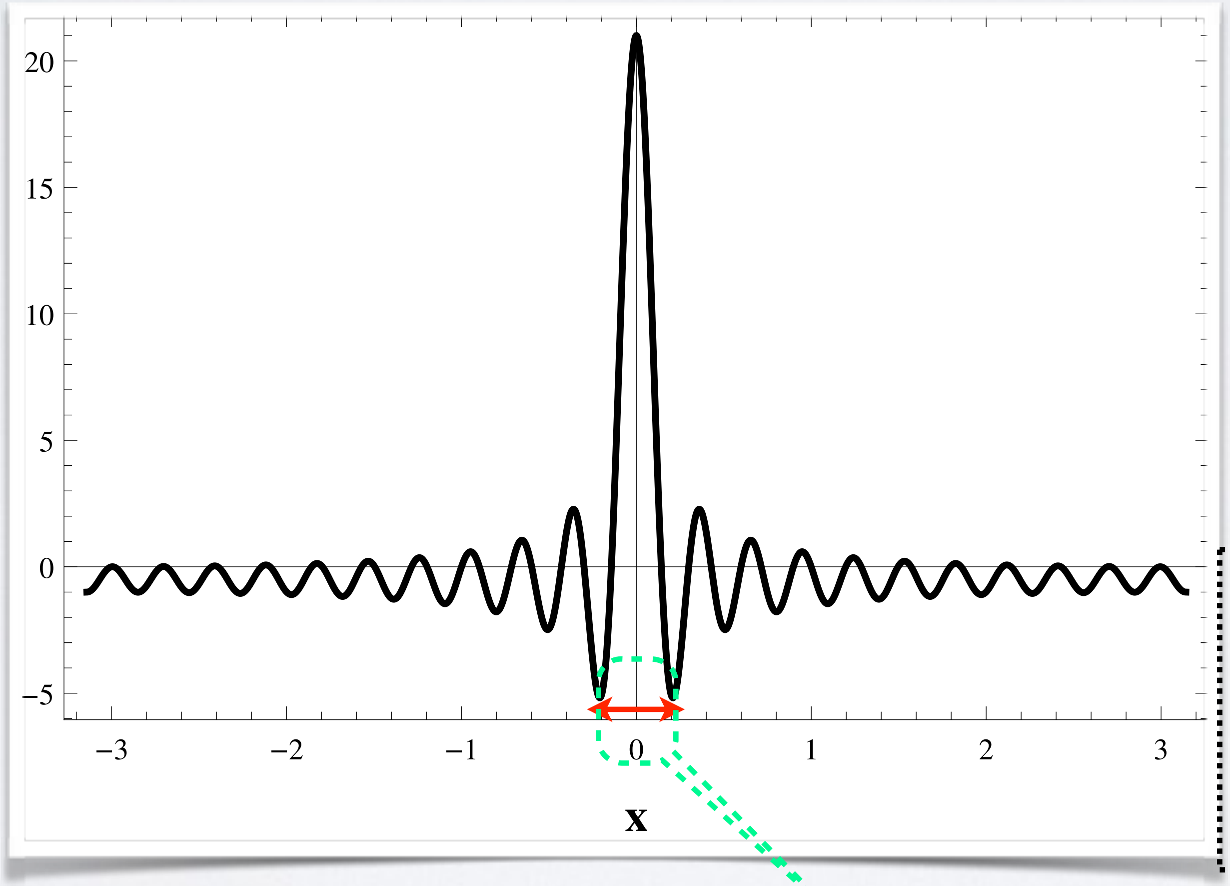
> Noise II:

$$\boxed{\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \sqrt{S(k)}\cos(k\,x + \delta_k)}$$

- **Flat Spectrum** $\boxed{S(k) = 1}$

$$\langle \mu(x)\,\mu(x')\rangle - \mu_0^2 \xrightarrow{k_* \to \infty} \boxed{(w\,\mu_0)^2\,\delta(x' - x)}$$ *Gaussian noise*

$\langle \mu(x)\,\mu(x')\rangle - \mu_0^2$

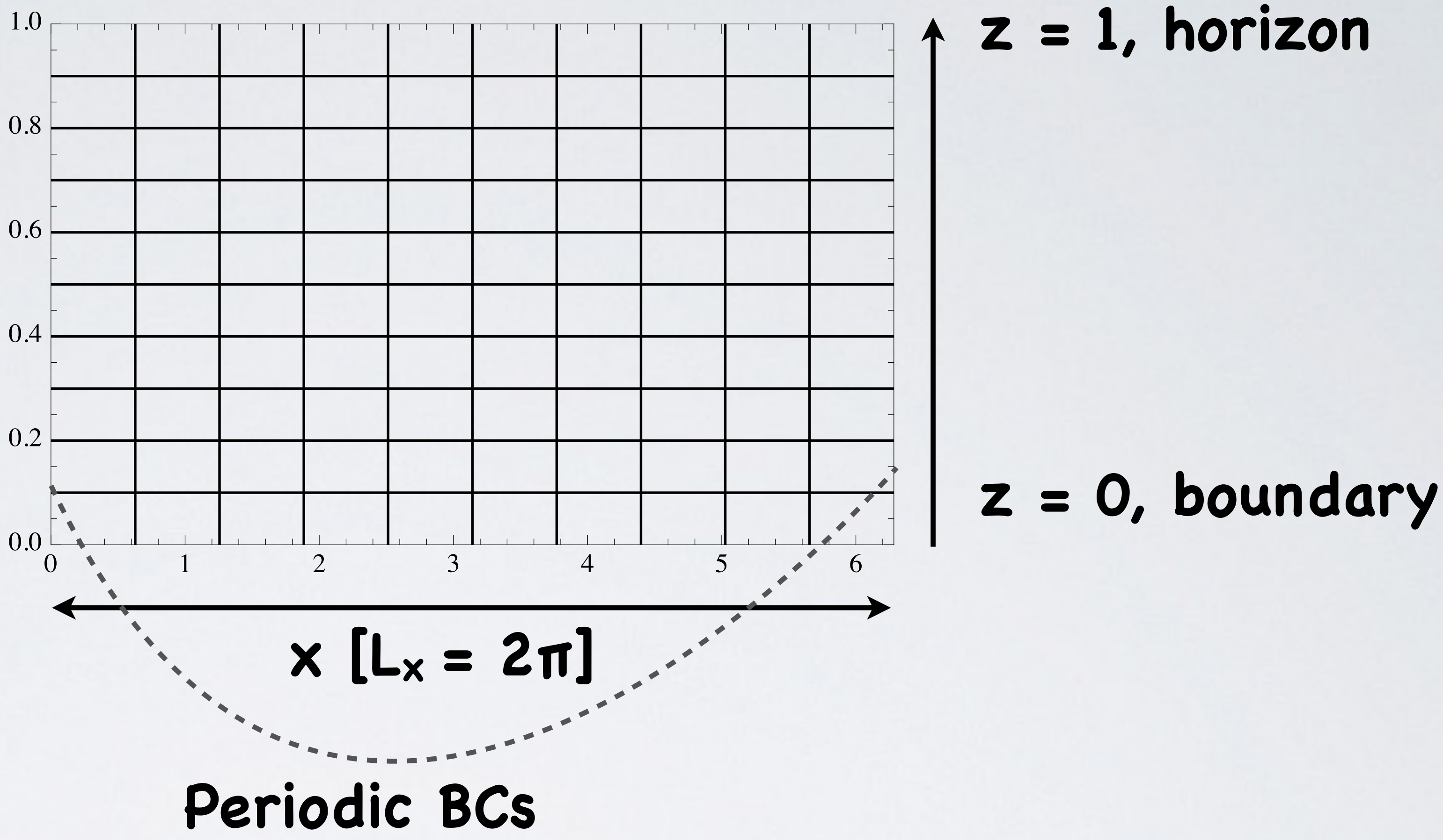


$L = 2\pi$
$k_0 = 1$
$k_* = 20$

ξ ~ *'correlation length'* $\boxed{\xi \sim 1/k_*}$

> Noise III (Solving...)

- System on a grid



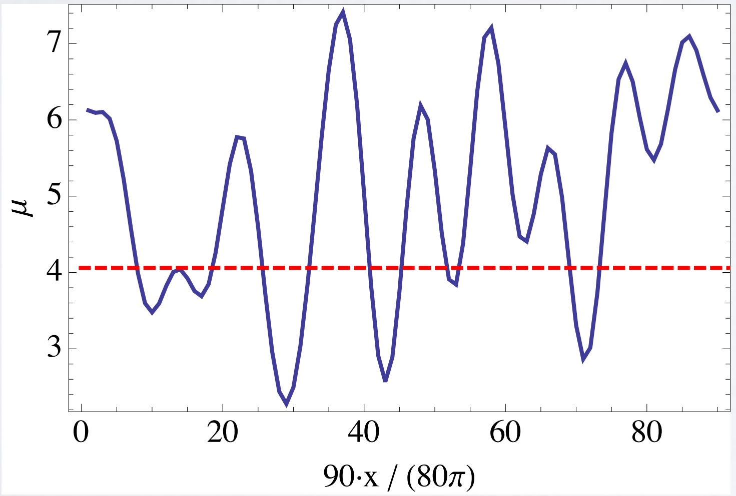
z = 1, horizon

z = 0, boundary

x [$L_x = 2\pi$]

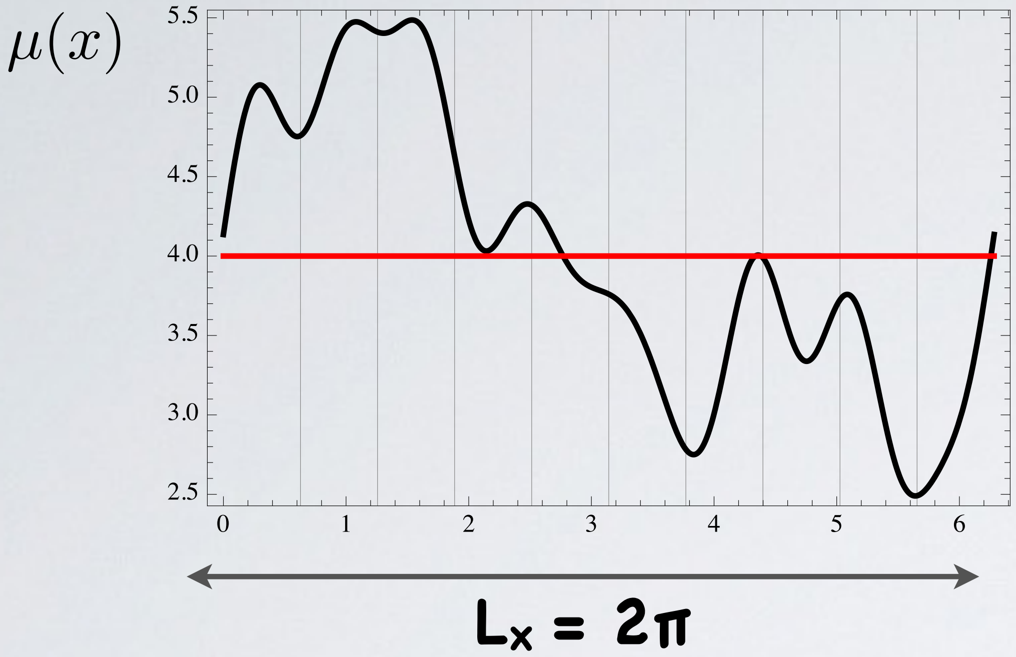
Periodic BCs

+



$$\boxed{\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)}$$

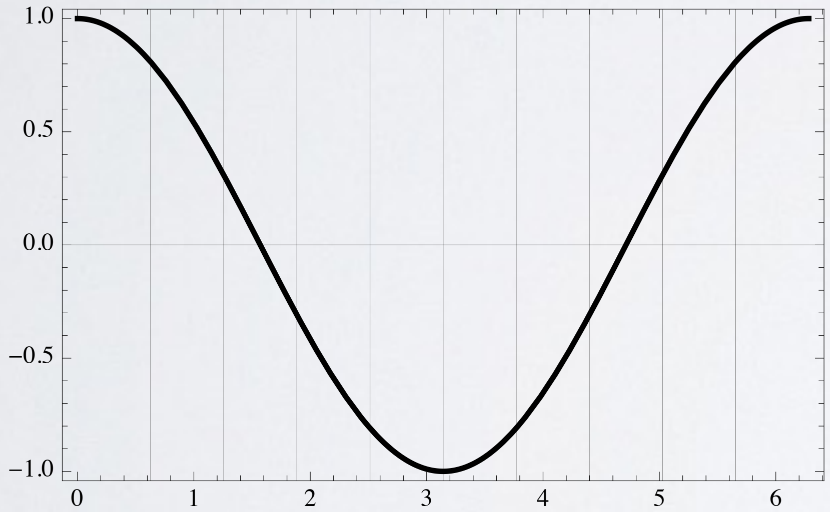
> Noise IV (UV & IR Scales)

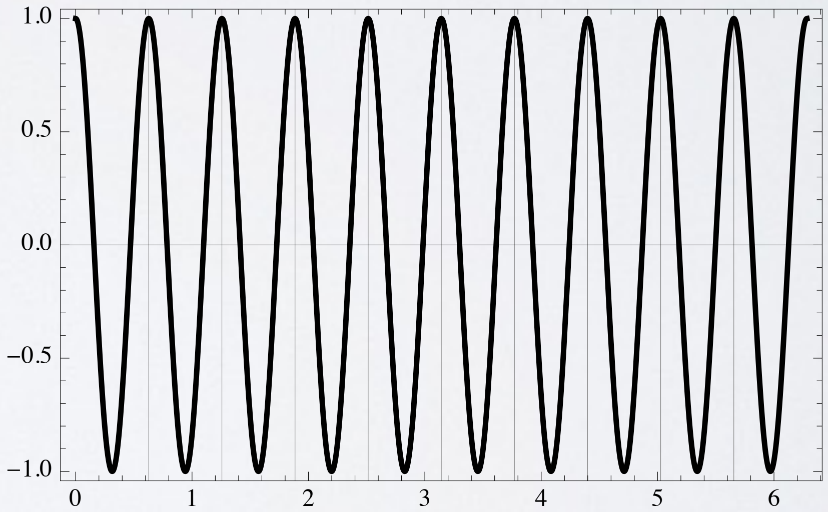


$$\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)$$

➠ **Scales:** $k_0\ ,\ k_*\ ,\ w$

- K_0 (IR scale) ⟷ system length
- K_* (UV scale) ⟷ grid





➠ TAKE (ideally): $L_x \gg 1\ [\to\ k_0 \ll 1]\ ,\ k_* \gg 1$

* [in units of temperature]

'Uncorrelated Noise'

> Noise V:

$$\boxed{\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \sqrt{S(k)}\cos(k\,x + \delta_k)}$$

- **Flat Spectrum** $\mu(x) = \mu_0 + \bar{V} \sum_{k=k_0}^{k_*} \cos(k\,x + \delta_k)$

Gaussian noise $(k_* \to \infty)$

$$\boxed{\langle \mu(x)\,\mu(x') \rangle - \mu_0^2 = \bar{V}^2\,\delta(x - x')}$$

$$\boxed{[\bar{V}] = \frac{1}{2}}$$

> Harris Criterion:

$[\bar{V}] > 0 \longrightarrow$ **1d Noisy μ(x) is relevant**

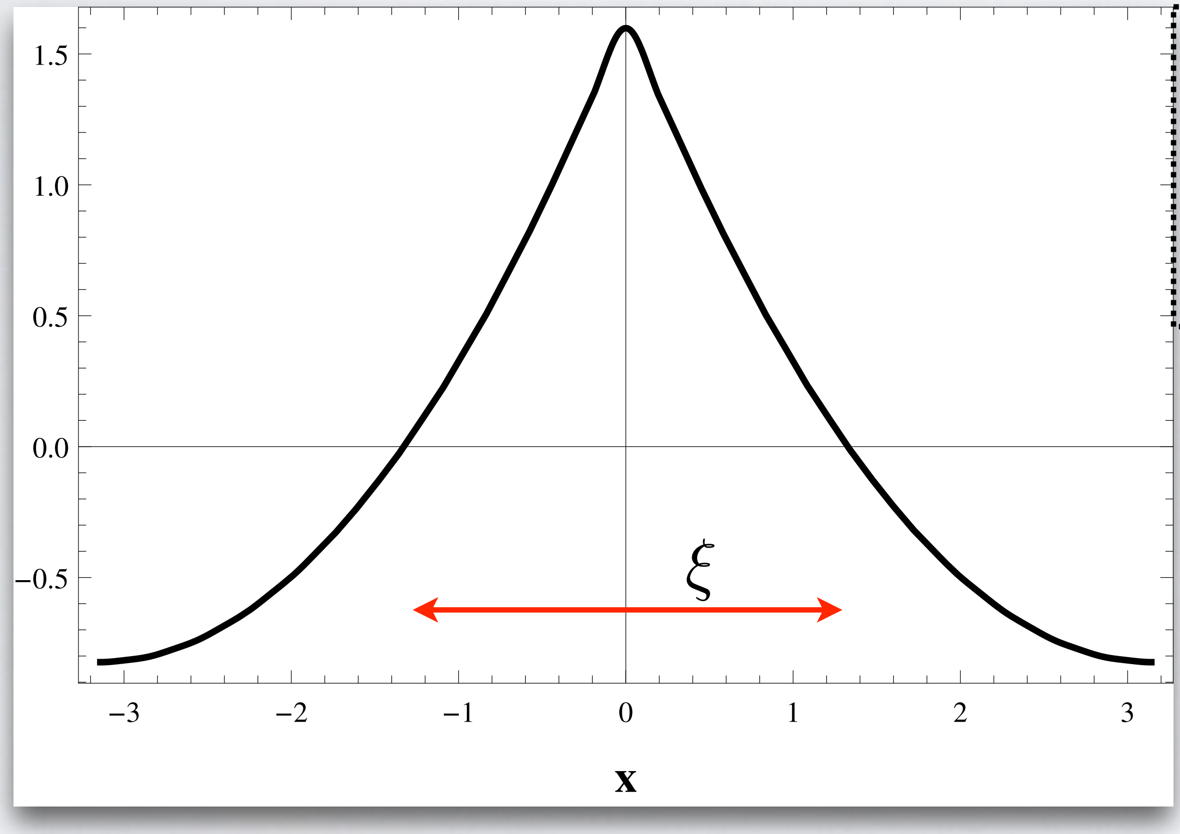
2d Noisy μ(x) $\longrightarrow [\bar{V}] = 0 \longrightarrow$ **marginal**

> Noise UI:

$$\boxed{\mu(x) = \mu_0 + w\,\mu_0 \sum_{k=k_0}^{k_*} \sqrt{S(k)}\cos(k\,x + \delta_k)}$$

- **Power Spectrum** $\boxed{S(k) = \dfrac{1}{k^{2\alpha}}}$

$\langle \mu(x)\,\mu(x')\rangle - \mu_0^2$



$$L = 2\pi$$
$$k_0 = 1$$
$$k_* = 20$$

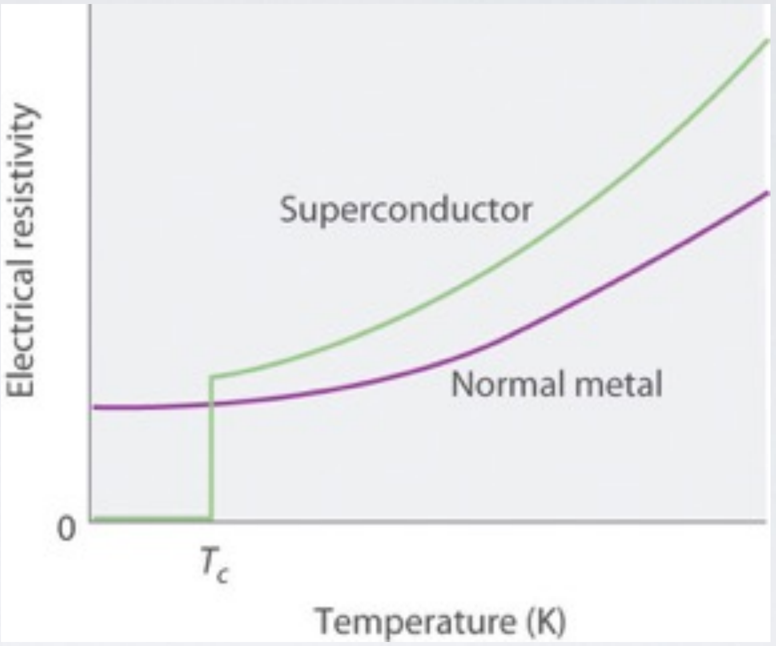
'Correlated Noise'

$$\langle \mu(x)\,\mu(x')\rangle - \mu_0^2 \sim \frac{k_0^{-2\alpha}}{x}\cos(k_0 x) + \ldots \longrightarrow \boxed{\xi \sim \frac{1}{k_0} \sim L}$$

> Review: Holo (s-wave) Superconductor

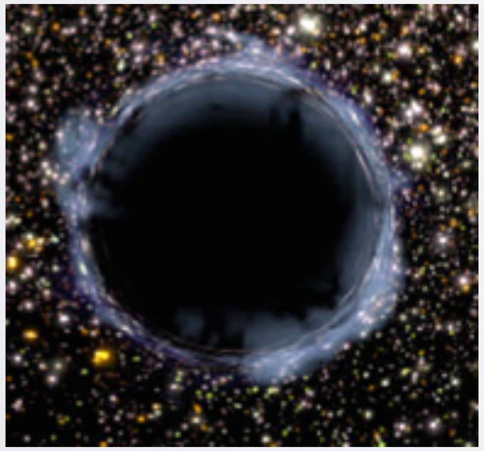
[Hartnoll et al'08]

- **Black Hole gets hair** $\sim$

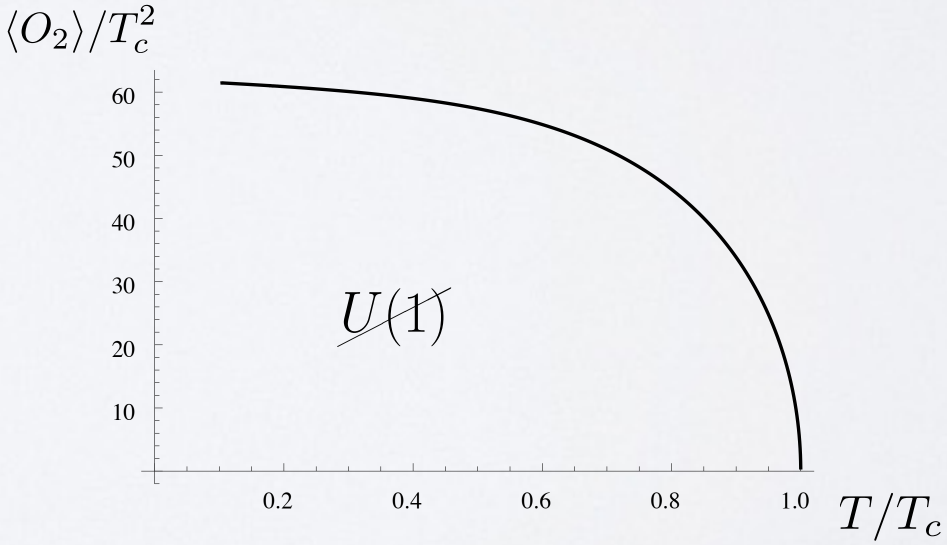


$$\langle \cancel{U(1)} \rangle$$

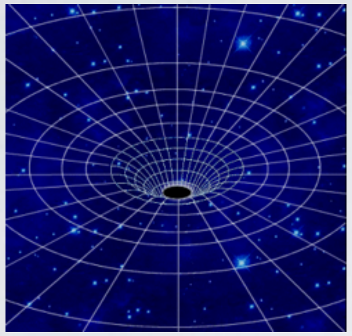
◆ **Charged BH unstable against scalar condensation (Gubser'08)**



$\sim$



> Review: Holo (s-wave) Superconductor

★  Black Hole in AdS₄ $\sim \boxed{2+1 \text{ CFT } (T \neq 0)}$

★ U(1) Gauge field $A_\mu \sim J_\mu \rightarrow$ **Chemical Potential**

★ Charged scalar condenses ⟶ **Black Hole gets hair**

$\psi \sim O$

$\langle O \rangle \neq 0 \longrightarrow \cancel{U(1)}$

> Review: Holo (s-wave) Superconductor

> Setup:

Maxwell-Einstein + scalar $\mathcal{S} = \int dx^4 \sqrt{-g}\left[\frac{R}{\mathcal{K}} + \frac{6}{\mathcal{K}\,L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - i\,q\,A_\mu)\Psi|^2 - m^2\,|\Psi|^2\right]$

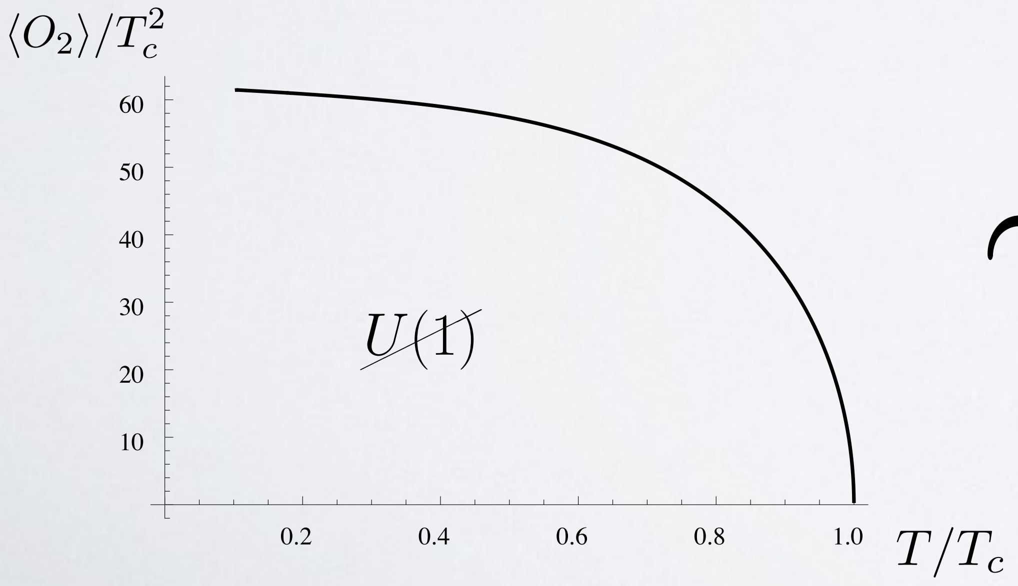
> Looking for superconducting solutions:

Ansatz ⟶

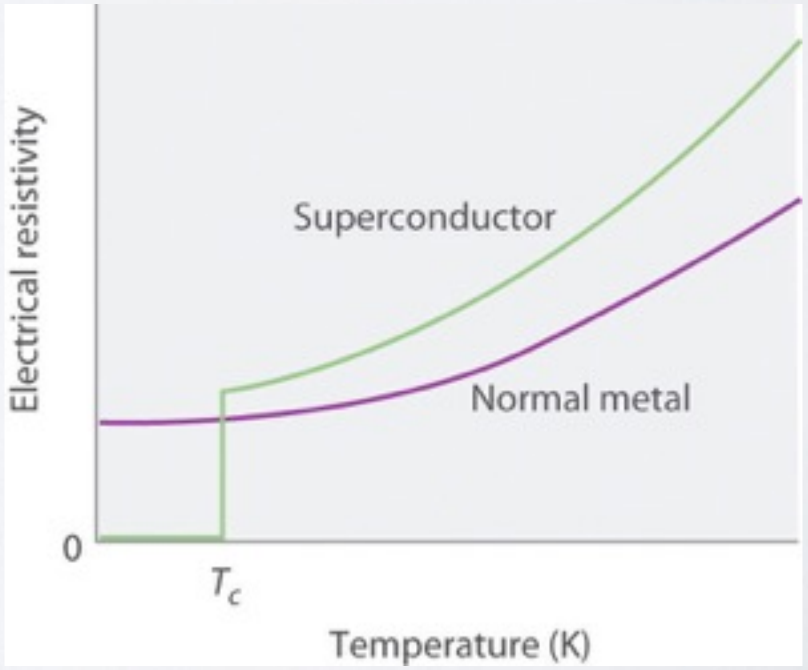
$$\mathcal{S} = \int dx^4 \sqrt{-g}\left[\frac{R}{\mathcal{K}} + \frac{6}{\mathcal{K}\,L^2} \boxed{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - i\,q\,A_\mu)\Psi|^2 - m^2\,|\Psi|^2}\right]$$

Probe Limit

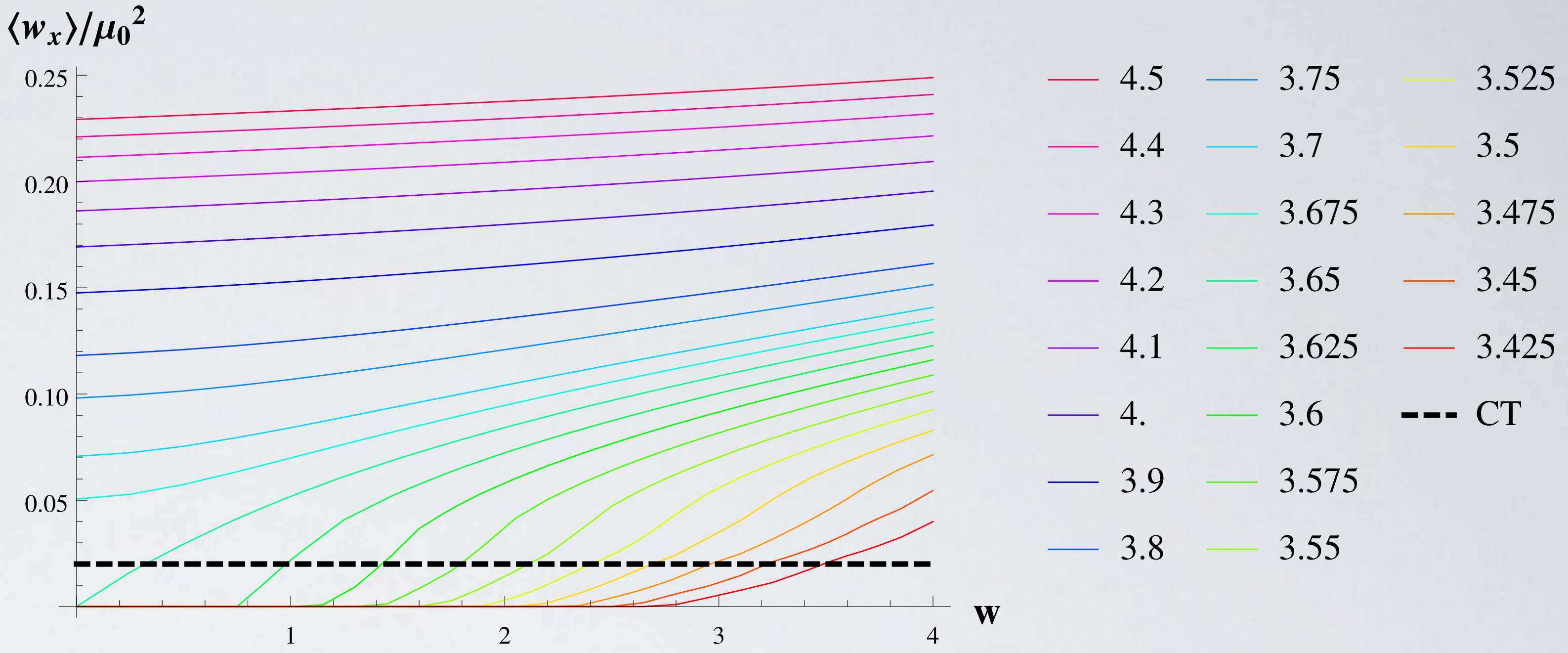
⬇

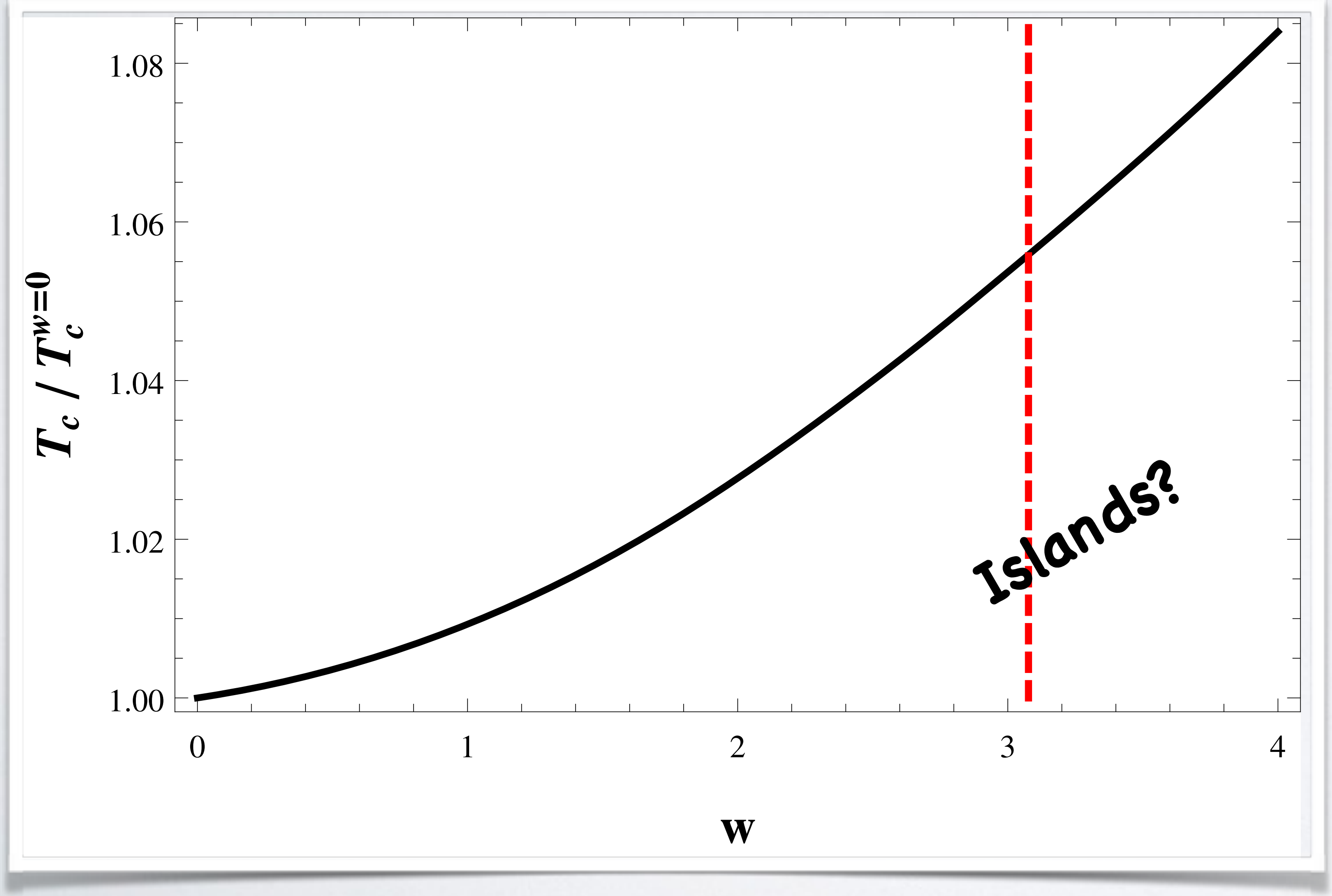


$\sim$



> Results: ‘phase diagram’

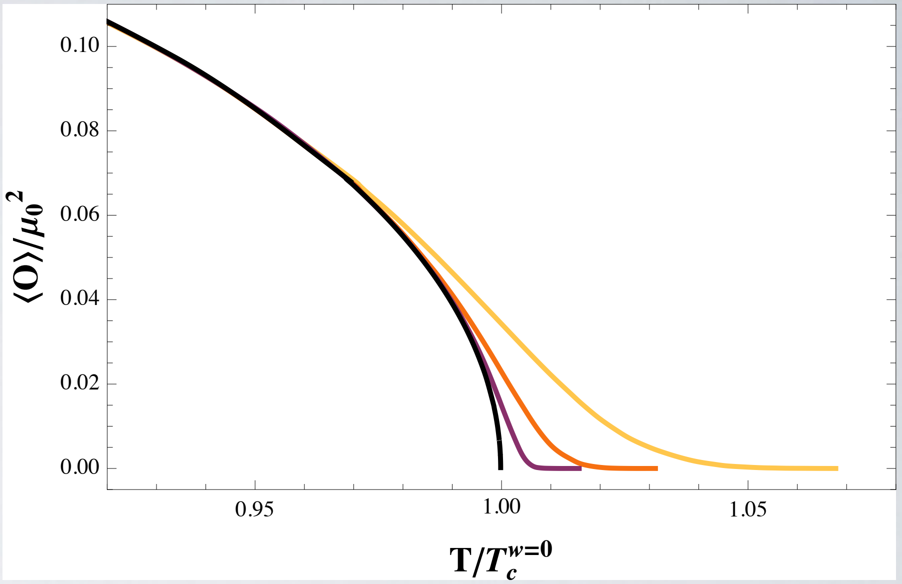


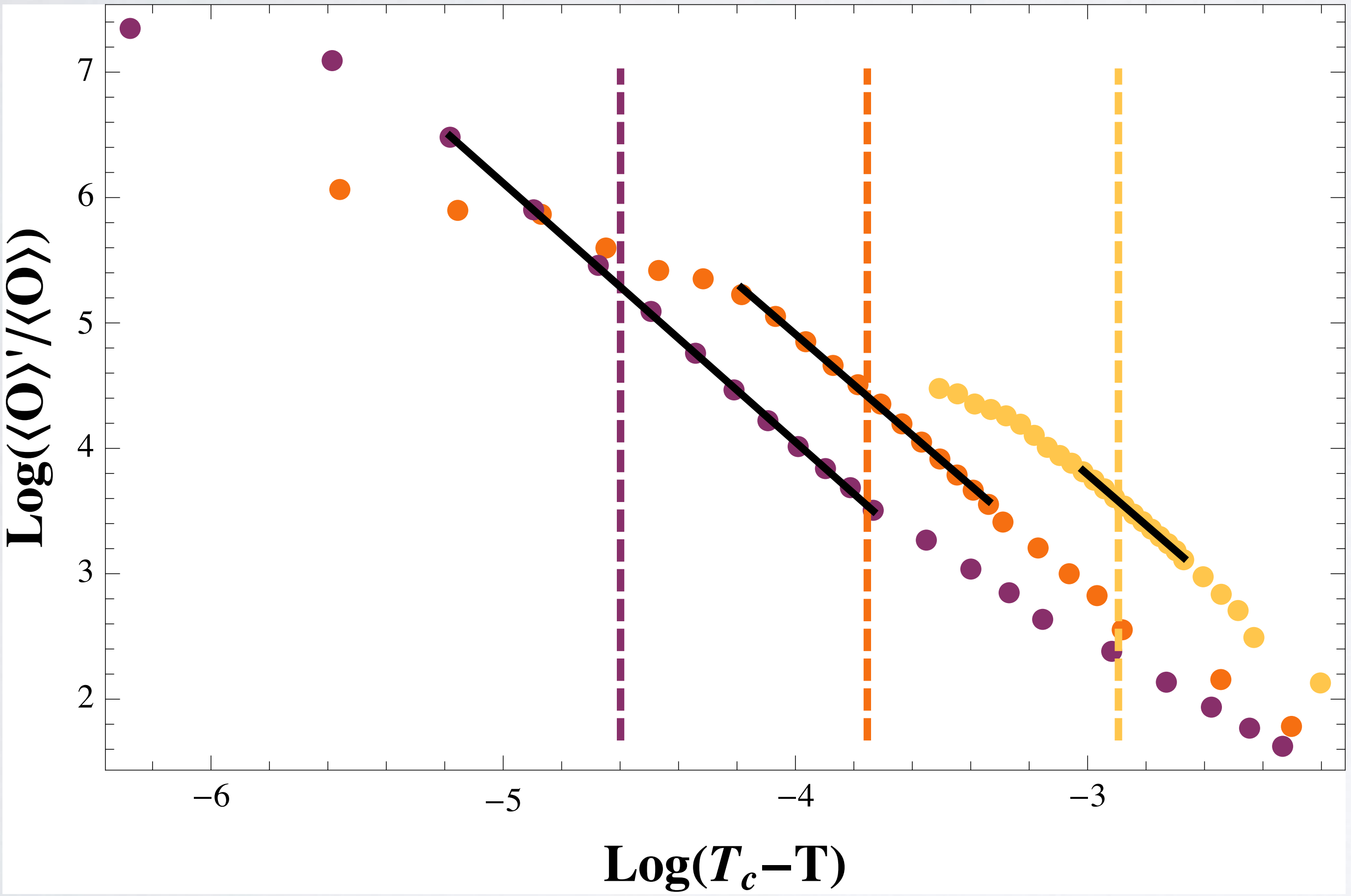


P-WAVE SUPERCONDUCTOR
[1407.7526]

> Results: Phase transition @ finite disorder

> Average of the condensate vs Temperature…





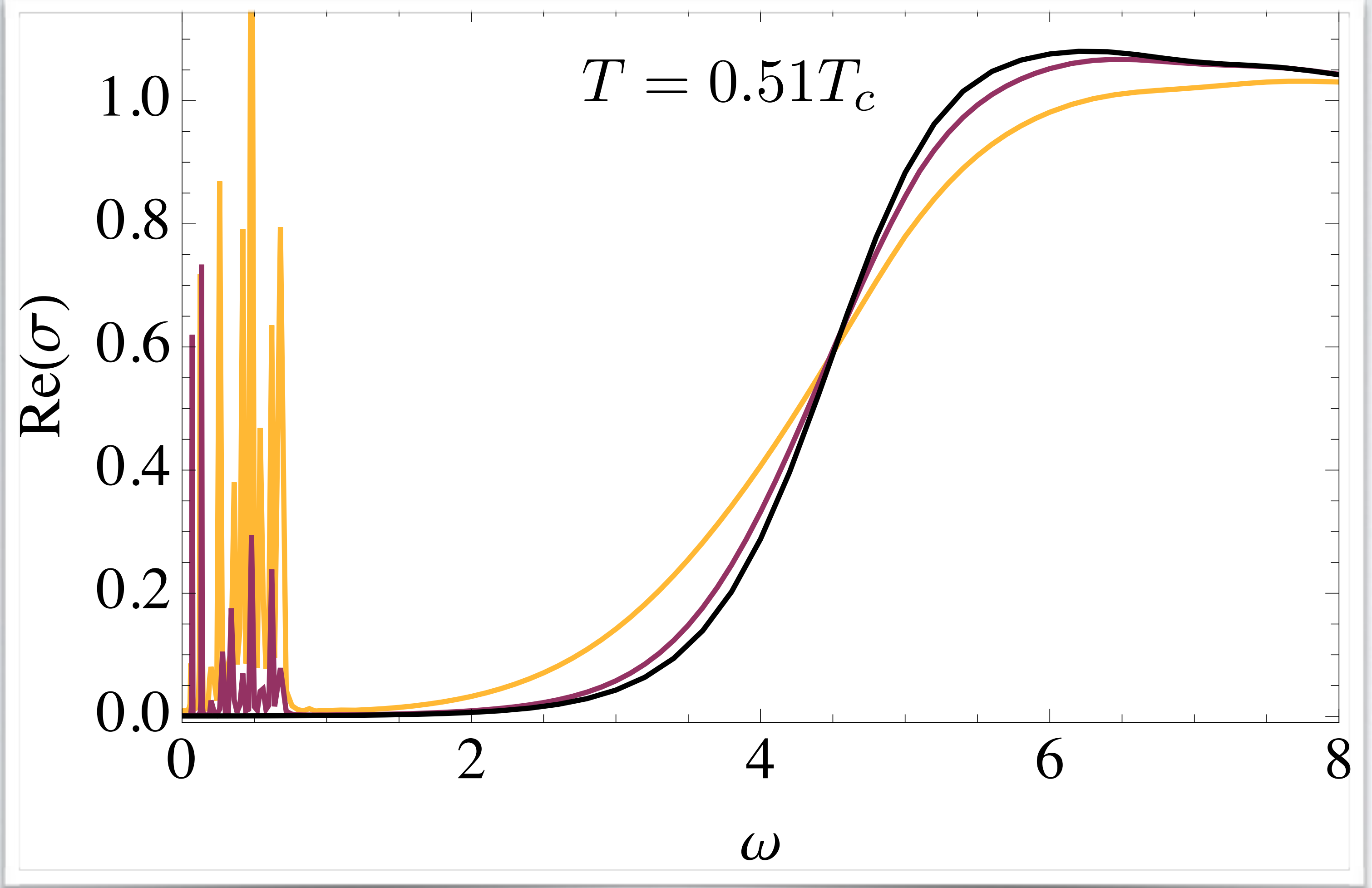
$$\langle \mathcal{O} \rangle \sim \exp\left(-A|T-T_c|^{-\nu}\right)$$

with

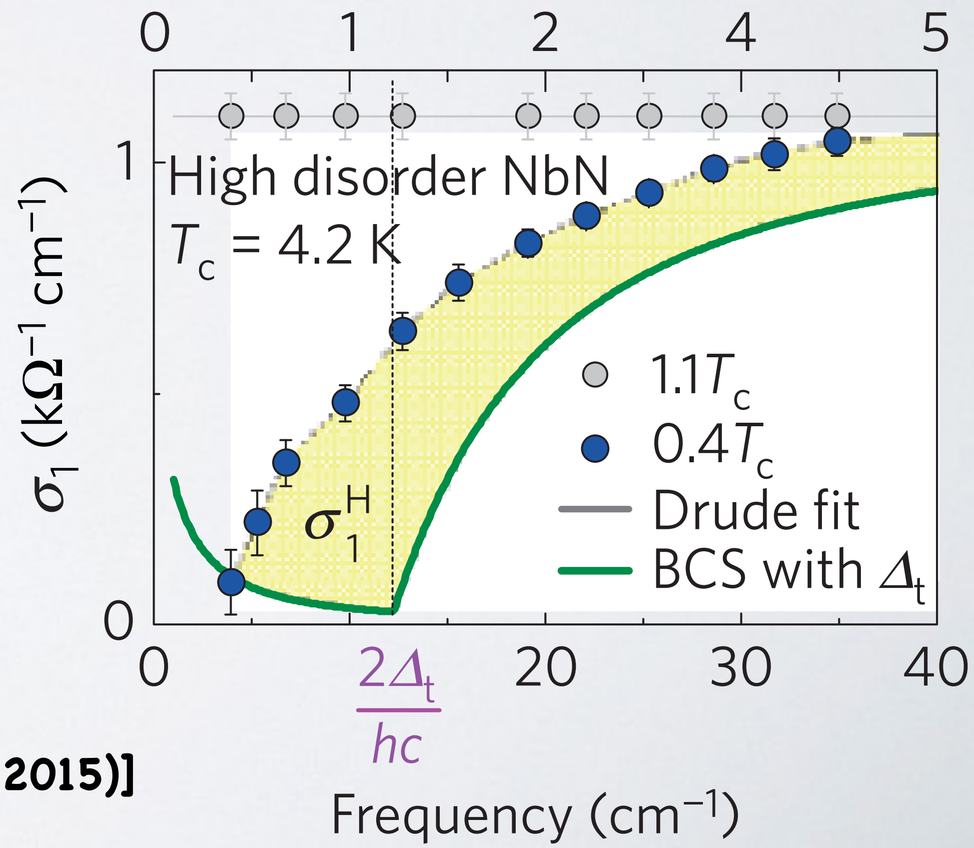
$$\nu = 1.03 \pm 0.02$$

[See Griffiths' phases… T. Vojta, PRL'03]

> σ_AC: Large disorder & Higgs mode(?)

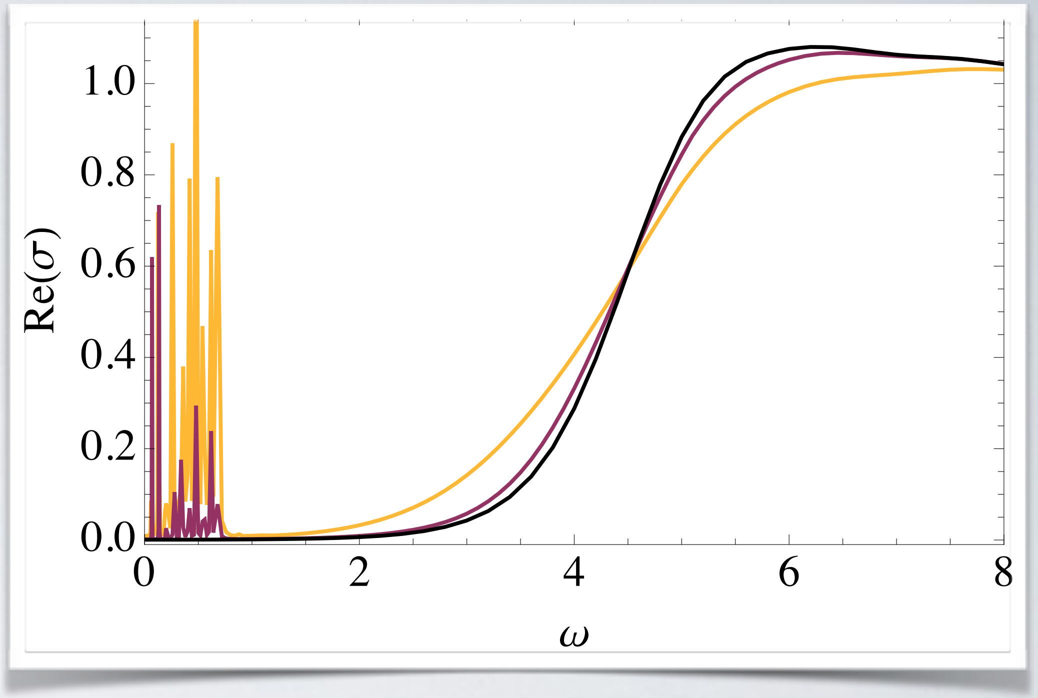


Disorder suppresses the gap

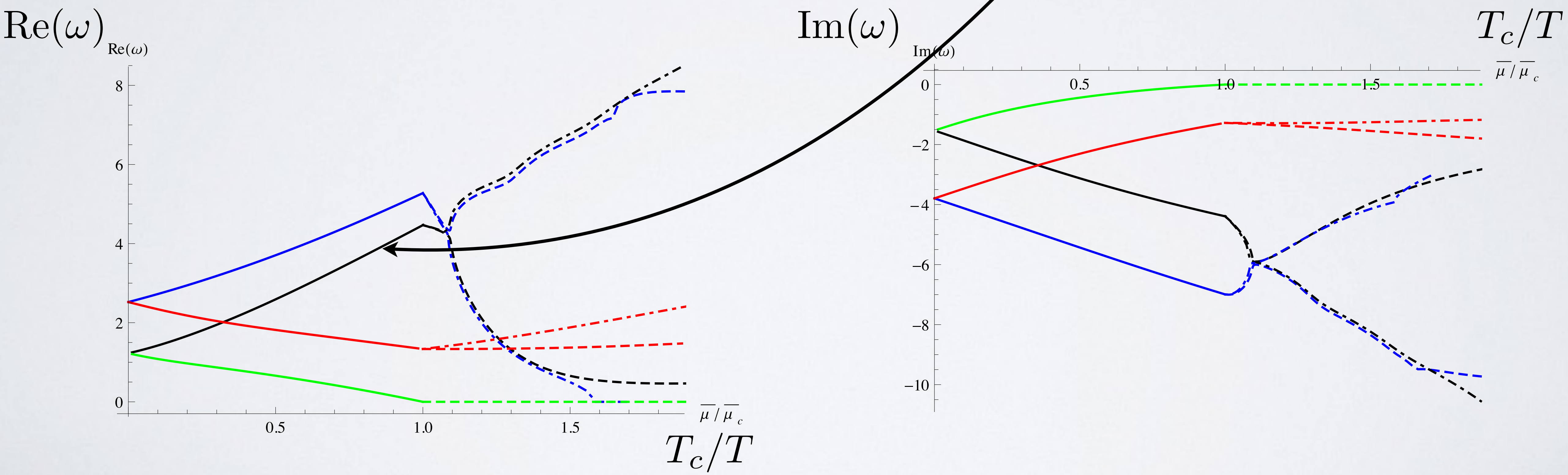


[Sherman et al, Nature Phys 11, 188–192 (2015)]

> σ_AC: Large disorder & Higgs mode(?)



The Goldstone QNM has a massive partner...



> Background: Charge Density

- Charge Density vs Noise Strength ⇒ Study ***ρ*(μ)** clean system

- Homogeneous (massless) case:

$$\boxed{\begin{aligned} &\rho \approx \sqrt{2}\mu\,; && (\rho \ll 1) \\ &\rho \approx 0.291\mu^2\,; && (\rho \gg 1) \end{aligned}}$$

$$\left[\langle \mathrm{noise}(x)\rangle = \frac{1}{L}\int_0^L dx\,\mathrm{noise}(x) = 0\right]$$

- Assume

$$\mu = \mu_0\,(1 + w\,\mathrm{noise}(x)) \longrightarrow \boxed{\begin{aligned} &\langle\rho\rangle \neq \langle\rho\rangle(w)\,; && (\rho \ll 1) \\ &\boxed{\langle\rho\rangle \propto w^2}; && (\rho \gg 1) \end{aligned}}$$

Charge Density Enhancement

> Conductivity. 'Weak Noise' (~ ignoring gradients along x)

- Homogeneous limit:

$$\xrightarrow[a^{(2)} = \frac{\sqrt{2}\rho}{\sqrt{4+\rho^2}}]{} \quad \sigma_{\rm DC} = \left\langle \frac{1}{\frac{1}{L}\int_0^{L_x} dx \, \frac{2}{\sqrt{4+\rho^2}}} \right\rangle_{\rm noise}$$

- Noise strength expansion:

$$\mu = \mu_0 \left(1 + \frac{w}{25} n(x)\right) \to \rho(n(x)) \quad \textbf{in} \quad \sigma_{\rm DC} = \left\langle \frac{1}{\frac{1}{L}\int_0^{L_x} dx \, \frac{2}{\sqrt{4+\rho(x)^2}}} \right\rangle_{\rm noise}$$

expand in powers of w → $\sigma_{\rm DC}(w)$

$\rho \approx \sqrt{2}\mu \,; \qquad (\rho \ll 1)$

$\rho \approx 0.291\mu^2 \,; \qquad (\rho \gg 1)$

> Conductivity. ‘Strong Noise’ [pseudo-analytics]

– (Pseudo-analytic) DC Conductivity at all orders in w –

- $$\sigma_{\mathrm{DC}} = \left\langle \frac{1}{\frac{1}{L}\int_0^{L_x} dx\, \frac{2}{\sqrt{4+\rho(x)^2}}} \right\rangle_{\mathrm{noise}}$$
- $$\mu(x) = \frac{\rho(x)}{\sqrt{2}}\, {}_1F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{\rho(x)^2}{4}\right)$$

~ Averaging over homogeneous(x) systems

➡ **Invert (numerically) for** $\rho(\mu(x)) \longrightarrow$ **Integrate** $\longrightarrow \sigma_{\mathrm{DC}}(\mu_0, w)$

- Large charge (small w) limit

Linear behavior!

$$\boxed{\sigma_{\mathrm{DC}} \approx \frac{1 - 3\left(w^2/25^2\right)\,\Delta + \left(9\Delta^2/25^4 - 5\Delta_4/25^4\right) w^4}{1 + \left(w^2/25^2\right)\Delta}\,\frac{\langle\rho\rangle}{2}}$$

$$\Delta = \left\langle \frac{1}{L}\int dx\, n(x)^2 \right\rangle_{\mathrm{noise}}$$

$$\Delta_4 = \left\langle \frac{1}{L}\int dx\, n(x)^4 \right\rangle_{\mathrm{noise}}$$

we’ll see that $\sigma_{\mathrm{DC}} \propto \langle\rho\rangle$ **true at all orders in w**