Fermions in a spontaneously generated holographic lattice

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Numerical Relativity and Holography

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Outline

1. Introduction
   - Inhomogeneity
   - Coupling
   - Instability
2. Backreaction
   - Expansion
   - Scalar
3. Phase Space
   - Critical Temp.
   - Fermions
4. Summary

Condensed Matter Theory

Semi-Classical explanation

- near the superconducting/normal transition line

Ginzburg-Landau functional

$$F_G = a|\psi|^2 + \gamma|\overrightarrow{\nabla}\psi|^2 + \frac{b}{2}|\psi|^4$$

- a, b, γ are functions of B, T, ψ - "order parameter"

$a = 0$ at the critical temperature
at lower temperatures ψ turns on and F_G is extremized by

$$|\psi|^2 = -a/b$$

Condensed Matter Motivation

FFLO states *[Fulde, Ferrell, and Larkin, Ovchinnikov]*

- near the superconducting/normal transition line

Ginzburg-Landau functional

$$F_G = a|\psi|^2 + \gamma|\vec{\nabla}\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\eta}{2}|\vec{\nabla}^2\psi|^2$$

- a, b, γ, η are functions of B, T

Inhomogeneous Ground State

- finite electron pair momentum
- translation/rotation symmetries broken

$$\psi \sim e^{iqx}$$

- truncation of low-energy IIB string theory *[Arean, Bertolini, Krishnan, Prochazka]*
- holographic unbalanced superconductor *[Bigazzi, Cotrone, Musso, Fokeeva and Seminara]*

Lattice generation

Dynamical generation of a lattice found in

- Q-lattice, *[Donos, Gauntlett]*
- axion type, *[Andrade, Withers]*
- Einstein, Maxwell, Scalar ($F \wedge G$) (Pantelidou talk), *[Donos, Gauntlett, Pantelidou]*

Mechanism

gravity with Λ_{AdS}, scalar field ϕ of mass m and charge $(q, 0)$ coupled to $U(1)$ vector potential A_μ

$$S = \int d^4x\sqrt{-g}\left[\frac{R + 6/L^2}{16\pi G} - \frac{1}{4}F_{AB}F^{AB} + S_\phi + S_{int}\right]$$

AdS/CFT

Gravitational duals related to inhomogeneity have been widely studied

- ▶ allows for calculation of many interesting properties
  - Drude peaks, *[Horowitz, Santos, Tong, ...]*
  - Transport Properties , *[Donos, Gauntlett, Blake, ...]*
  - Momentum relaxation (Kim talk), *[Hartnoll, Hoffman, ...]*
  - Quench (Withers talk) *[Withers]*
  - Metal-Insulator, *[Donos, Hartnoll, Goutraux, ...]*

Fermion spectral functions

- explicit - *[Liu, Schalm, Sun, Zaanen]*
- Q-lattice, *[Ling, Liu, Niu, Wu, Xian]*

Hairy black hole

Asymptotics

near Boundary ($z \to 0$)

$$h \to 1, \quad A_t \sim \mu - \rho z, \quad \Psi \sim \Psi^{\pm} z^{\Delta_{\pm}}, \quad \Delta_{\pm} = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2}$$

- chemical potential, μ and charge density, ρ
- $\langle \mathcal{O}_{\Delta_{\pm}} \rangle = \sqrt{2}\Psi^{\pm}$

Horizon

$z \to 1$

$$A_t \to 0 \ , \quad h \to 0$$

Unbroken phase

Solution with $\phi = 0$

$$
\begin{aligned}
ds^2 &= \frac{1}{z^2}\left[-h(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)}\right], \\
A_t &= \mu(1-z) \\
h(z) &= 1 - \left(1 + \frac{\mu^2}{4}\right) z^3 + \frac{\mu^2}{4} z^4
\end{aligned}
$$

use scaling symmetries: $z_H = 1$, with $z \in [0, 1]$
$\longrightarrow$ AdS boundary at $z \to 0$

Alternative:
self-dual under $\vec{E} \leftrightarrow \vec{B}$

$$A_y = \mathcal{B}x \quad , \qquad A_t = 0$$

Simple Scalar

Scalar Action

$$S_\phi = -\int d^4x\sqrt{-g}\left[g^{AB}(D_A\phi)^* D_B\phi + m^2|\phi|^2\right],\ D_\mu = \partial_\mu - iqA_\mu$$

$$\partial_z^2\phi + \left[\frac{h'}{h} - \frac{2}{z}\right]\partial_z\phi + \frac{1}{h}\nabla_2^2\phi - \frac{1}{h}\left[\frac{m^2}{z^2} - q^2\frac{A_t^2}{h}\right]\phi = 0$$

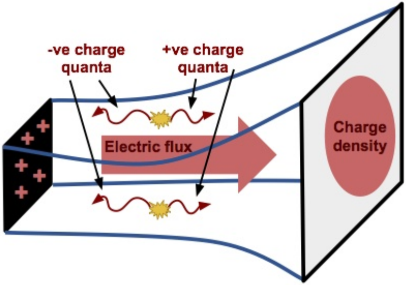
- introduce $\vec{x}$-dependence

$$\nabla_2^2\phi = -k^2\phi \ , \quad \phi \sim \psi(z)\cos kx$$

- fix μ
- $\mu/r_+ \rightarrow$ eigenvalue in scalar equation

$$\frac{T_0}{\mu_0} = \frac{3}{4\pi\mu_0}\left[1 - \frac{\mu_0^2}{12}\right]$$

Instability



picture from Hartnoll

Mechanism for Instability

Extremal black holes near horizon exhibit $AdS_2 \times \mathbb{R}^2$

- effective mass can be below 2D m_{BF}^2

$$m_{eff}^2 < 6 m_{BF,2}^2 = -3/2$$

Instability

Imbalance + Inhomogeneity

$$m_{eff}^2 = m^2 - 2q^2 + k^2$$

Limits placed on maximum imbalance from zero-T instability

$$q_{min}^2 = \frac{3 + 2\Delta(\Delta - 3) + 2k^2}{4}$$

and a limit on k^2

$$k_{max}^2 = 2q^2 - \frac{3}{2} - \Delta(\Delta - 3)$$

Interacting Scalar

Stringy considerations dictate S_ϕ

$$S_{int} = \int d^4x\sqrt{-g}\mathcal{L}_{int}$$

$$\mathcal{L}_{int} = \phi^*\left[\eta\mathcal{G}^{AB}D_AD_B + \eta'\mathcal{H}^{ABCD}D_AD_BD_CD_D + \dots\right]\phi + c.c.$$

- $\mathcal{G}^{AB}$ and $\mathcal{H}^{ABCD}$ may come from Einstein tensor, stress energy tensor, gauge or scalar fields, . . .

Interaction

Can include terms in the Lagrangian

$$|F^{AB}\partial_B\phi|^2$$

$\Rightarrow$ similar to Landau-Ginzburg gradient term

Backreaction

Earlier Coupling

- scalar coupled with Einstein tensor [J.A., E. Papantonopoulous, G. Siopsis]
  - used cosmology with vanishing Λ
    - entry/exit quasi-de Sitter
    - scalar-tensor theory with second order Ψ eqn. *[Sushkov]*

[J.A., E. Papantonopoulous, G. Siopsis, K. Yeter]

$$S_{int} = \int d^4x\sqrt{-g}\left[\eta\mathcal{G}^{AB}(D_A\phi)^* D_B\phi - \eta'|D_A\mathcal{G}^{AB}D_B\phi|^2\right]$$

pick

$$\mathcal{G}_{AB} = T^{EM}_{AB} + g_{AB}\mathcal{L}^{EM} = F_{AC}F_B{}^C - \frac{1}{2}g_{AB}F^2$$

Backreaction

Interaction term contribute linear terms in η, η' to the full system's stress-energy tensor, electromagnetic current, and scalar equation

- Perturbatively solve the EMS equations

$$
\begin{aligned}
ds^2 &= \frac{1}{z^2}\left[-h(z,x)e^{-\alpha(z,x)}dt^2 + \frac{dz^2}{h(z,x)} + e^{\beta(z,x)}dx^2 + e^{-\beta(z,x)}\right] \\
A_t &= A_t(z,x) \\
\phi &= \phi(z,x)
\end{aligned}
$$

Expansion in ξ

$$
\begin{aligned}
h(z,x) &= h_0(z) + \xi^2 h_1(z,x) + \dots\ , \quad \alpha = \xi^2\alpha_1(z,x) + \dots \\
\phi &= \xi\phi_0(z,x) + \xi^3\phi_1(z,x) + \dots\ , \quad \beta = \xi^2\beta_z(z,x) \\
A_t &= A_{t0}(z) + \xi^2 A_{t1}(z,x) + \dots
\end{aligned} \tag{1}
$$

Backreaction

The Hawking temperature is found as

$$\frac{T}{\mu} = -\frac{h'(1)e^{-\alpha(1)}}{4\pi\mu}$$

and chemical potential

$$\mu = A_t(0, x) = \mu_0 + \xi^2\mu_1 + \dots$$

- when r_H is scaled back in, μ is constant

Scalar at (ξ)

$$\partial_z^2\phi + \left[\frac{h'}{h} - \frac{2}{z}\right]\partial_z\phi + \frac{1}{h}\left(1 - \eta\mu^2 z^4 - \eta'\mu^4 z^{10}\nabla_2^2\right)\nabla_2^2\phi - \frac{1}{h}\left[\frac{m^2}{z^2} - q^2\frac{A_t^2}{h}\right]\phi = 0$$

Backreaction

In the limit $k \to \infty$, the k term dominates the scalar equation at the horizon

$$\frac{T}{\mu} = \frac{3}{4\pi}\sqrt{\eta}\left(1 - \frac{1}{12\eta}\right)$$

Large enough $\eta \Rightarrow$ produces a higher transition temperature than the homogeneous $k = 0$.

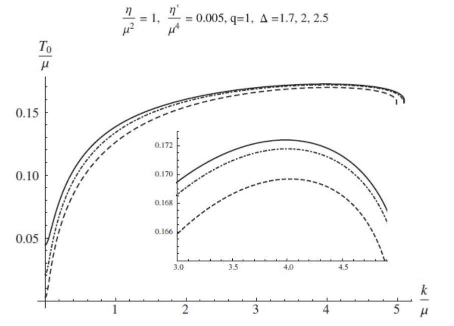
k_c^2

η' creates a limit new k^2_{max}

- η' a cutoff to compete with this effect and select a preferred finite

Inhomogeneous Solution, order ξ

Transition Temperature



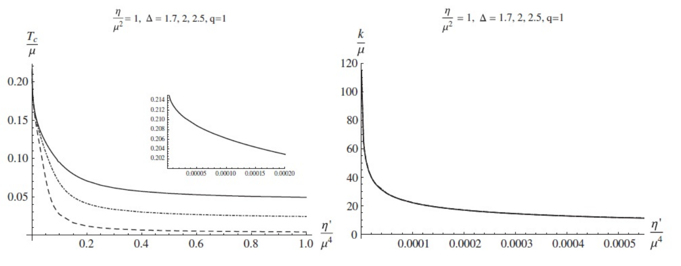
$$\Delta = 1.7, 2, 2.5$$
$$\frac{\eta}{\mu^2} = 1.$$
$$\frac{\eta'}{\mu^4} = 0.005$$

- inhomogeneous solutions possess higher transition temperature than homogenous solution
- in CFT, dominant terms possess modulated order parameter

$$\langle \mathcal{O} \rangle \sim \cos kx$$

Inhomogeneous Solution

Critical Temperature, $\Delta = 1.7, 2., 2.5$, $\frac{\eta}{\mu^2} = 1.$, $\frac{\eta'}{\mu^4} = 0.005$



$\Rightarrow \eta'$ sets UV cutoff and selects the lattice size

Below T_c

Expansion

At each order in ξ only a finite number of modes

- $\mathcal{O}(1)$ - 0
- $\mathcal{O}(\xi)$ - k
- $\mathcal{O}(\xi^2)$ - $0, 2k$
- $\mathcal{O}(\xi^3)$ - $k, 3k$

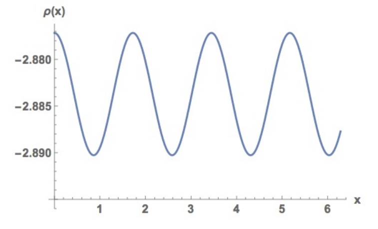
$$\frac{T}{T_c} \approx 1 - \xi^2 \left(\alpha_{10}(1) + \frac{\mu_1}{\mu_0} - \frac{h'_{10}(1)}{3 - \mu_0^2/4} \right) \;, \quad \frac{\langle \mathcal{O} \rangle}{T_c} \sim \sqrt{1 - \frac{T}{T_c}}$$

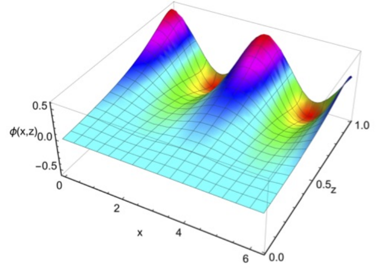
$$\frac{\rho}{\mu^2} = -\frac{\partial_z A_t(0,x)}{[A_t(0,x)]^2} \approx \frac{\rho_0}{\mu_0^2} + \xi^2 \frac{\rho_1(x)}{\mu_0^2} \;, \quad \rho_1 \sim \cos 2kx$$

Inhomogeneous Solution

The charge density is spatially inhomogeneous in presence of lattice and spatially homogeneous chemical potential

Charge and Scalar



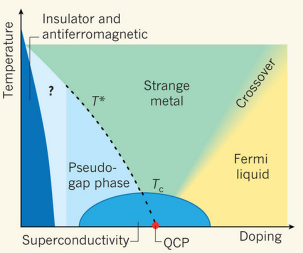


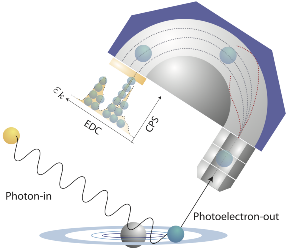
$\Delta = 1$, $q = 0$, $\xi = .1$, $\eta\mu_0^2 = .41$, $\eta'/\mu_0^4 = 0.005$

Fermion phases

Fermions

- (aspects of) pseudogap
- insulating antiferromagnet
- strange metal
- Fermi liquid





Fermionic phases

Spectral Function

Scattering light illuminates constituents

- extract Green's function

$$G = \frac{Z}{\omega - \nu_F(k - k_F) + \Sigma(\omega, k)}$$

  - calculate the self-energy Σ

$$S_{\text{fermion}} = i \int d^4x \sqrt{-g}\bar{\Psi}\left[\not{D} - m_f\right]\Psi \ ,$$

Fermion

Bloch Expansion

$$\Psi_{\alpha s} = \sum_{l=0,\pm1,\pm2,\ldots} \psi^l_{\alpha s}(z) e^{2ilkx}$$

Expand ψ in terms of parameter ξ

$$\psi^l_{\alpha s} = \psi^{0,l}_{\alpha s} + \xi^2 \psi^{1,l}_{\alpha s} + \xi^4 \psi^{2,l}_{\alpha s} + \ldots$$

G_R comes from the $z \to 0$ behavior

$$\psi^l_\alpha(z) \approx A^l_\alpha \, z^{-m_f} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + B^l_\alpha \, z^{m_f} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$G_R = BA^{-1}$$

Leading Order

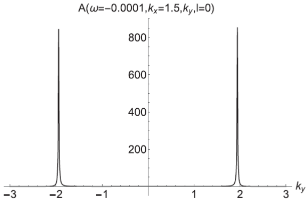
Poles

- at leading order, the Dirac equation

$$\partial_z \psi^{0,l}_{\alpha s} - i\frac{q_f \mu_0 (1-z)+\omega}{h(z)}\sigma^2 \psi^{0,l}_{\alpha s} + \frac{k_x + 2kl}{\sqrt{h(z)}}\sigma^3 \psi^{0,l}_{\alpha s} - \frac{k_y}{\sqrt{h(z)}}\sigma^1 \psi^{0,l}_{-\alpha\ 3-s} = 0$$

and spectral function

$$A^{0,l}(\omega, k_x, k_y) = \Im\left[\frac{\psi^{0,l}_{+1}(\epsilon)}{\psi^{0,l}_{+2}(\epsilon)} + \frac{\psi^{0,l}_{-1}(\epsilon)}{\psi^{0,l}_{-2}(\epsilon)}\right]$$



► Same behavior at 1^{st} order

2^{nd} Order

- four nearest modes to l are excited
- l, $l \pm 1$ and $l \pm 2$

Near the Fermi surface,

$$G_R \sim \frac{A^{0,l} B^{0,l}}{\left(A^{0,l} + \xi^4 A^{2,l}\right)^2 - \xi^4 A^{1,l-1} A^{1,l+1}}$$

The type of (non)-Fermi fluid is selected by

$$\nu_{k_l} = \frac{\sqrt{2}}{\mu_0} \sqrt{k_l^2 - \frac{q_f^2 \mu_0^2}{6}}$$

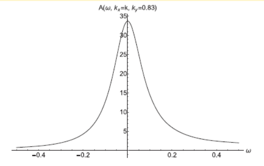
Which also determines the expansion of G_R and size of pseudogap Δ

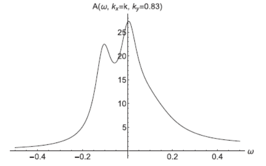
Pseudogap

$\nu_{k_l} < 1/2 \rightarrow$ non-Fermi liquid

$$\Delta \sim \xi^{1/2\nu_{k_l}}$$

small gap, broad peaks

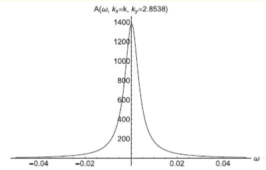


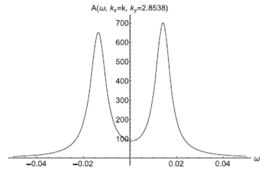


$\nu_{k_l} > 1/2 \rightarrow$ Fermi liquid

$$\Delta \sim \xi^2$$

broad gap, tight peaks





$\nu_{k_l} = 1/2 \rightarrow$ marginal Fermi liquid

Summary

- superconducting, strongly-coupled matter
- lattice mechanism
  - lattice structure
  - modulated charge density
- fermion pseudogap creation

work to be done

$\Rightarrow$ effect on other types of states, insulators

$\Rightarrow$ transport coefficients

$\Rightarrow$ general features of other dynamical lattices?