Universality and transient effects in N=4 SYM plasma

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Abstract

Numerical simulations of expanding plasma based on the AdS/CFT correspondence as well as kinetic theory and hydrodynamic models strongly suggest that some observables exhibit universal behaviour even when the system is not close to local equilibrium. This leading behaviour is expected to be corrected by transient, exponentially decaying contributions which carry information about the initial state. Focusing on late times, when the system is already in the hydrodynamic regime, we analyse numerical solutions describing expanding plasma of strongly coupled N=4 supersymmetric Yang-Mills theory and identify these transient effects. In the process we offer additional evidence supporting the recent identification of the Borel sum of the hydrodynamic gradient expansion with the far-from-equilibrium attractor in this system.

Introduction

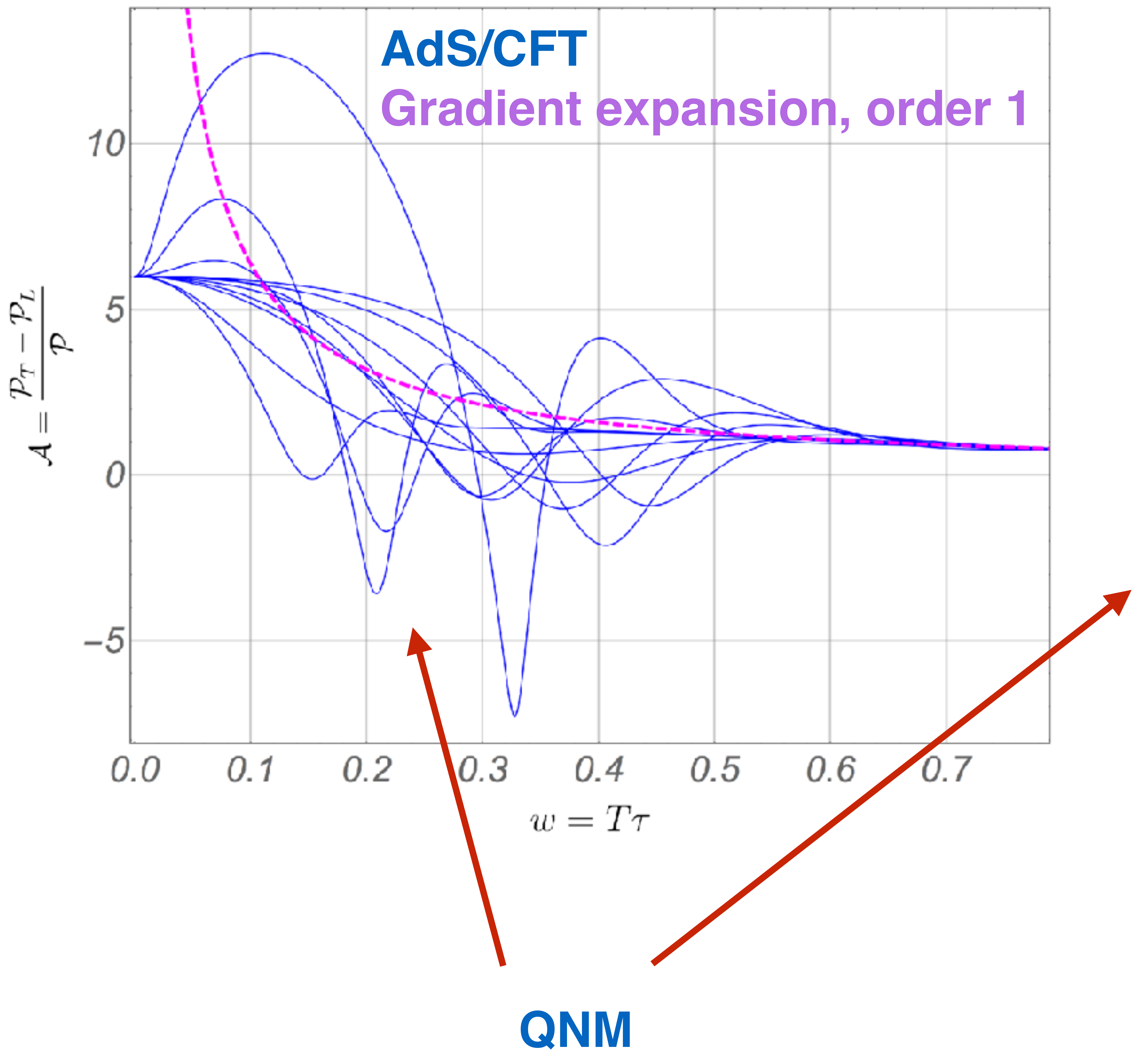
- **Physical motivation**: dynamics of quark-gluon plasma
- **Main tool**: AdS/CFT description of Bjorken flow (in N=4 SYM, not QCD)
- **Important developments of more general validity:**
  - Domain of applicability of hydrodynamics and the role of non-hydrodynamic modes
  - Far-from-equilibrium attractor behaviour
  - Universal observables
  - Transseries which capture the dissipation of initial state information

Hydrodynamization

- Complex initial states approach a universal regime through the decay of transients, leaving long-lived excitations which can be modelled by hydrodynamics

- It is very convenient to use **universal observables**, whose behaviour at late times is independent of initial conditions, up to exponentially suppressed terms

- In N=4 SYM the transient effects correspond to black brane QNM and exhibit damped oscillatory behaviour, which can be seen in numerical solutions also **at late times.**



Hydrodynamics and the gradient expansion

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(\mathcal{E})(g^{\mu\nu} + u^\mu u^\nu) + \Pi^{\mu\nu}$$
$$\nabla_\alpha T^{\alpha\beta} = 0$$

Navier-Stokes theory

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$

needs a **"UV-completion"** to cure its acausality (MIS, BRSSS, aHYDRO, DTH, …)

- This introduces non-hydrodynamic modes which act as a **regulator** to ensure v<1.
- **No unique way** to do this
- Domain of applicability of hydrodynamics: **regulator independence**

Standard approach: **MIS**, **BRSSS** (implies purely decaying nonhydro modes)

$$\left(\tau_\pi \mathscr{D} + 1\right) \Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \ldots$$

HJSW: an alternative designed to **model the leading QNM of SYM**

$$\left((\frac{1}{T}\mathscr{D})^2 + 2\Omega_I \frac{1}{T}\mathscr{D} + |\Omega|^2 \right) \Pi^{\mu\nu} = -\eta |\Omega|^2 \sigma^{\mu\nu} - c_\sigma \frac{1}{T}\mathscr{D}\left(\eta\sigma^{\mu\nu}\right) + \ldots$$

The choice of relaxation equations determines the non-hydrodynamic sector.

The gradient expansion: matching hydrodynamics to a microscopic theory

$$\Pi^{\mu\nu} = \boxed{-\eta\sigma^{\mu\nu}} + \tau_\pi \mathscr{D}(\eta\sigma^{\mu\nu}) + \ldots$$

Bonus: divergent gradient expansions carry information about the non-hydro sector.

Bjorken flow

Very symmetric dynamics: effective numerics as well as analytical methods.

The energy-momentum tensor

$$\langle T^{\mu\nu}\rangle = \mathrm{diag}(\mathscr{E}, \mathscr{P}_L, \mathscr{P}_T, \mathscr{P}_T)^{\mu\nu}$$

$$\mathscr{P}_L = -\mathscr{E} - \tau\dot{\mathscr{E}}\ , \quad \mathscr{P}_T = \mathscr{E} + \frac{1}{2}\tau\dot{\mathscr{E}}$$

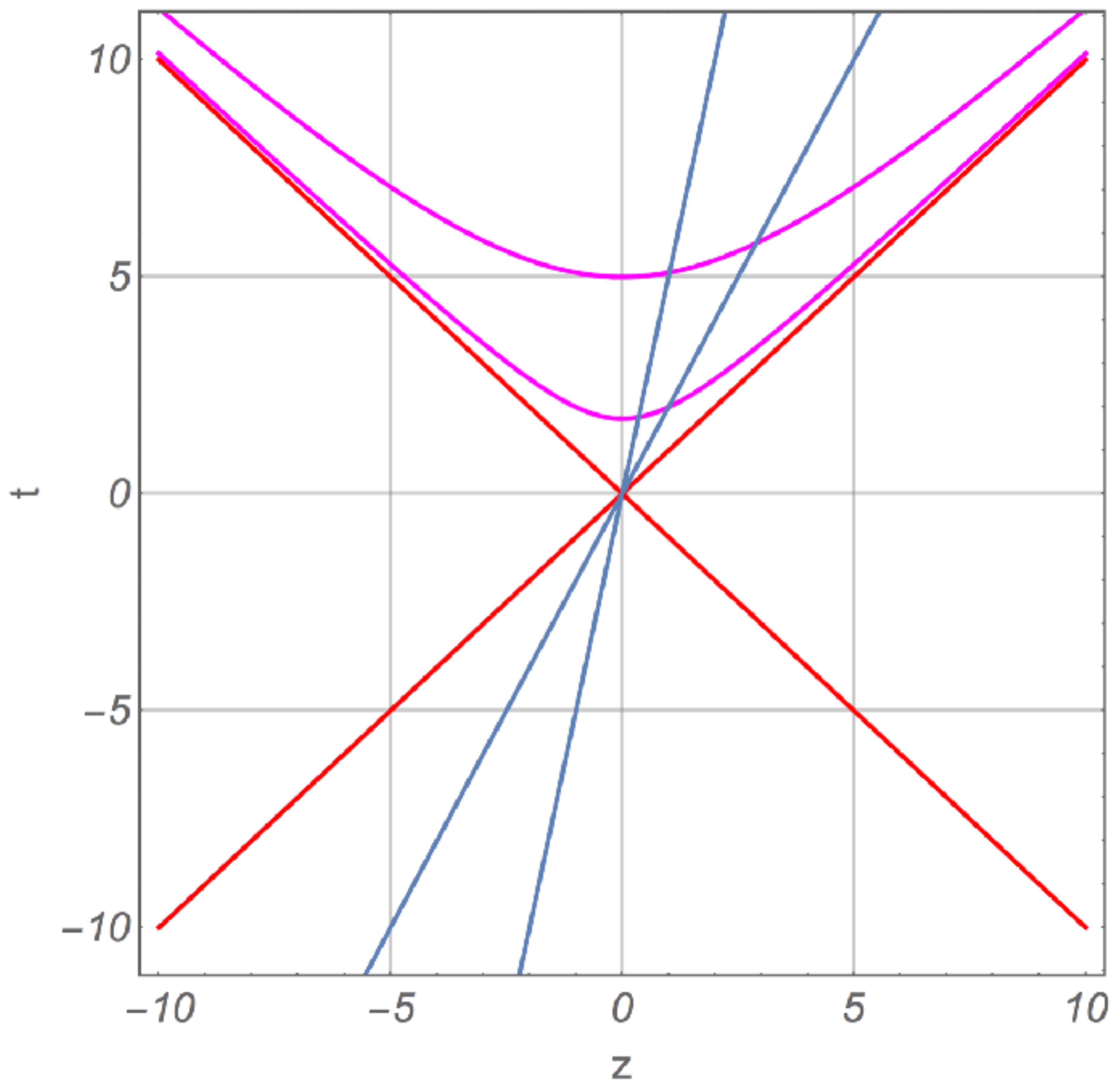
with $\mathscr{E} = \mathscr{E}(\tau)$

Effective temperature defined by $\mathscr{E} \sim T^4$

Dimensionless **pressure anisotropy**:

$$\mathscr{A} \equiv \frac{\mathscr{P}_T - \mathscr{P}_L}{\mathscr{P}}$$

$$t = \tau \cosh\lambda\ , \quad z = \tau \sinh\lambda$$



Universality in Bjorken flow

Late proper-time expansion

$$T(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}}\left(1 + \sum_{n=1}^{\infty}\frac{t_n}{(\Lambda\tau)^{2n/3}}\right)$$

Gradient expansion of the pressure anisotropy

$$\mathscr{A}(w) = \sum_{n=1}^{\infty}\frac{a_n^{(0)}}{w^n} = 8\frac{\eta}{s}\frac{1}{w} + \ldots$$

where $w \equiv \tau T \sim \tau/\tau_\pi$ is proper time in units of the relaxation time.

- This (asymptotic) solution is **universal** - independent of initial conditions
- Leading coefficient can be scaled away: $\tilde{w} \equiv \frac{w}{4\pi\eta/s} \implies \mathscr{A}(\tilde{w}) = \frac{2}{\pi\tilde{w}} + \ldots$
- The emergence of Navier-Stokes at late times is **manifest**

Transseries

$$\mathcal{A} = \Phi_0(w) + \sum_{m>0} \sigma_m w^{-\beta_m} e^{-A_m w} \Phi_m(w)$$

$$\Phi_m(w) = \sum_{n \geq 0} \frac{a_n^{(m)}}{w^n}$$

- The hydrodynamic sector (m=0) is **universal** - has no memory of initial conditions
- Nonhydro sectors correspond to **nonhydrodynamic modes** and their coupling
- All universal coefficients can be recovered from the hydro ones - **resurgence**
- The **transseries parameters** contain information about the initial state
- The transseries captures the **dissipation of initial state information**

Example: Bjorken flow in BRSSS hydrodynamics

Evolution equation for the pressure anisotropy in Bjorken flow

$$C_{\tau_\pi}\left(1+\frac{\mathcal{A}}{12}\right)\mathcal{A}' + \left(\frac{C_{\tau_\pi}}{3w}+\frac{C_{\lambda_1}}{8C_\eta}\right)\mathcal{A}^2 = \frac{3}{2}\left(\frac{8C_\eta}{w}-\mathcal{A}\right)$$

in terms of dimensionless transport coefficients

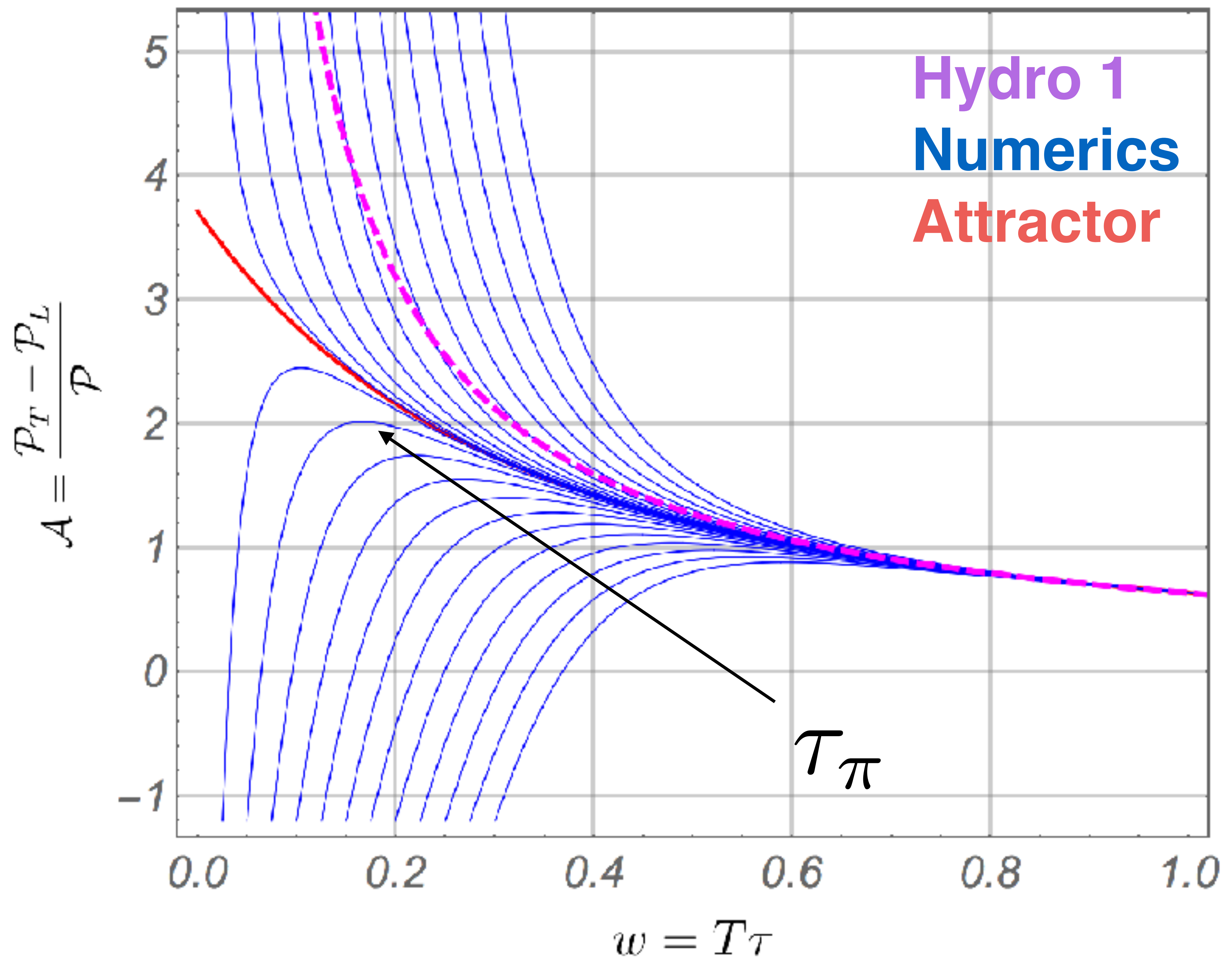
$$C_{\tau_\pi} = T\tau_\pi, \quad C_\eta = \eta/s, \quad C_{\lambda_1} = T\lambda_1/\eta$$

Gradient expansion solution

$$\mathcal{A} = \boxed{\frac{8C_\eta}{w}} + \frac{16C_\eta(C_{\tau_\pi}-C_{\lambda_1})}{3w^2} + \ldots$$

Transseries

$$\mathcal{A} = \sum_{n>0}\frac{a_n^{(0)}}{w^n} + \left(\sigma\, e^{-\frac{3}{2C_{\tau_\pi}}w}\, w^{\frac{C_\eta-2C_{\lambda_1}}{C_{\tau_\pi}}}\right)\sum_{n>0}\frac{a_n^{(1)}}{w^n} + \ldots$$



Borel summation

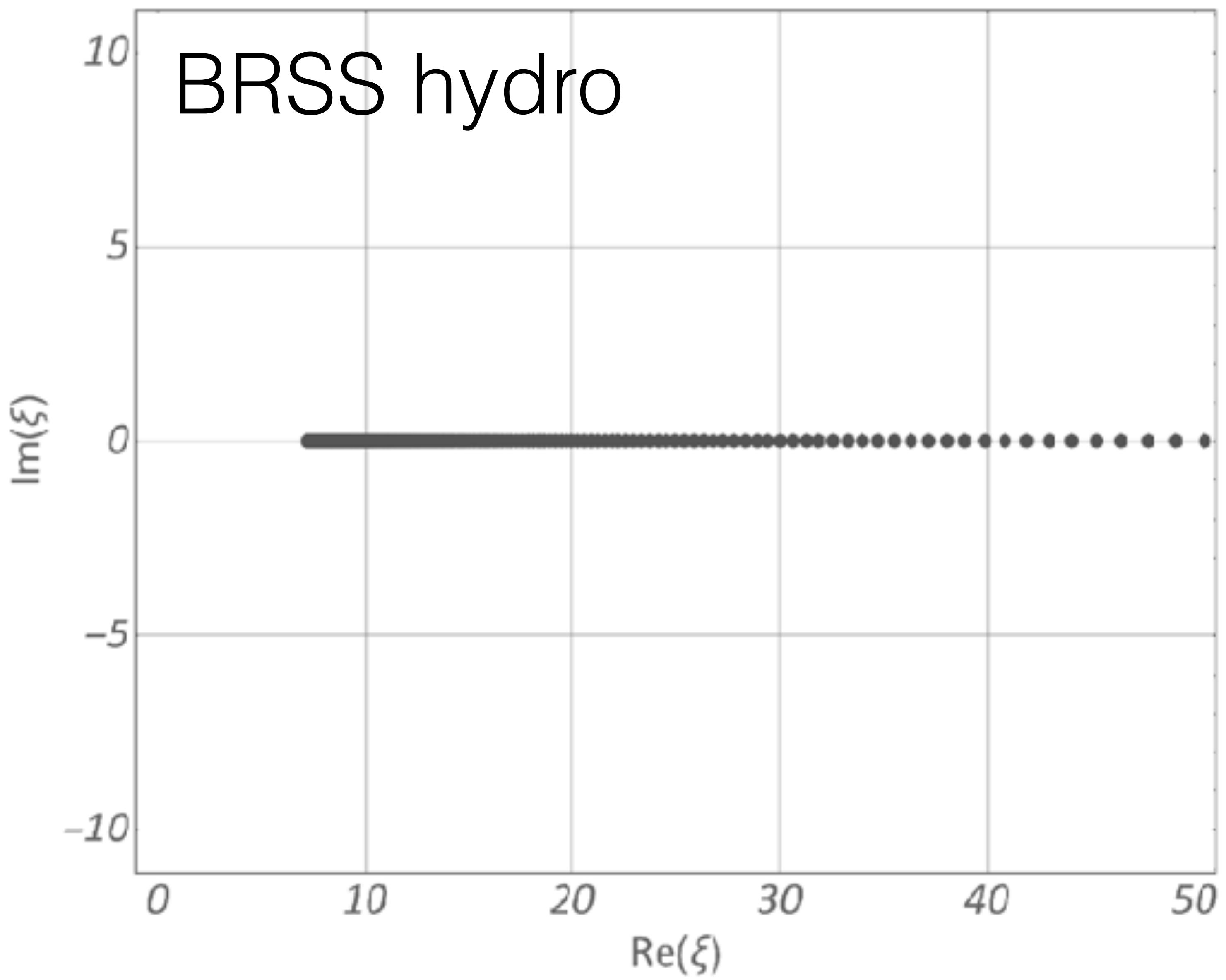
- Borel transform

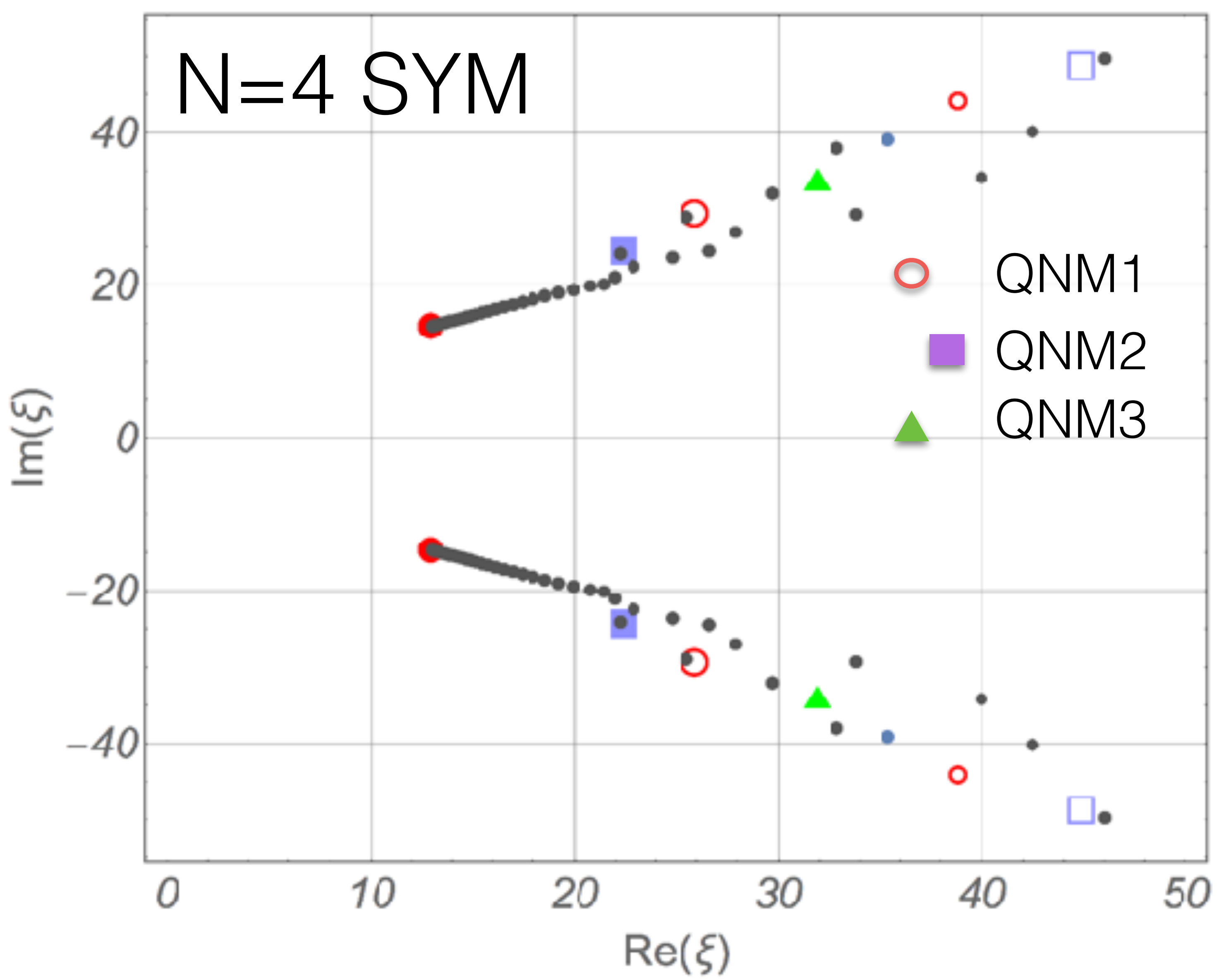
$$\mathscr{B}[\Phi_0](\xi) = \sum_{n\geq 0} \frac{a_{n+1}^{(0)}}{\Gamma[n+1]}\xi^n$$

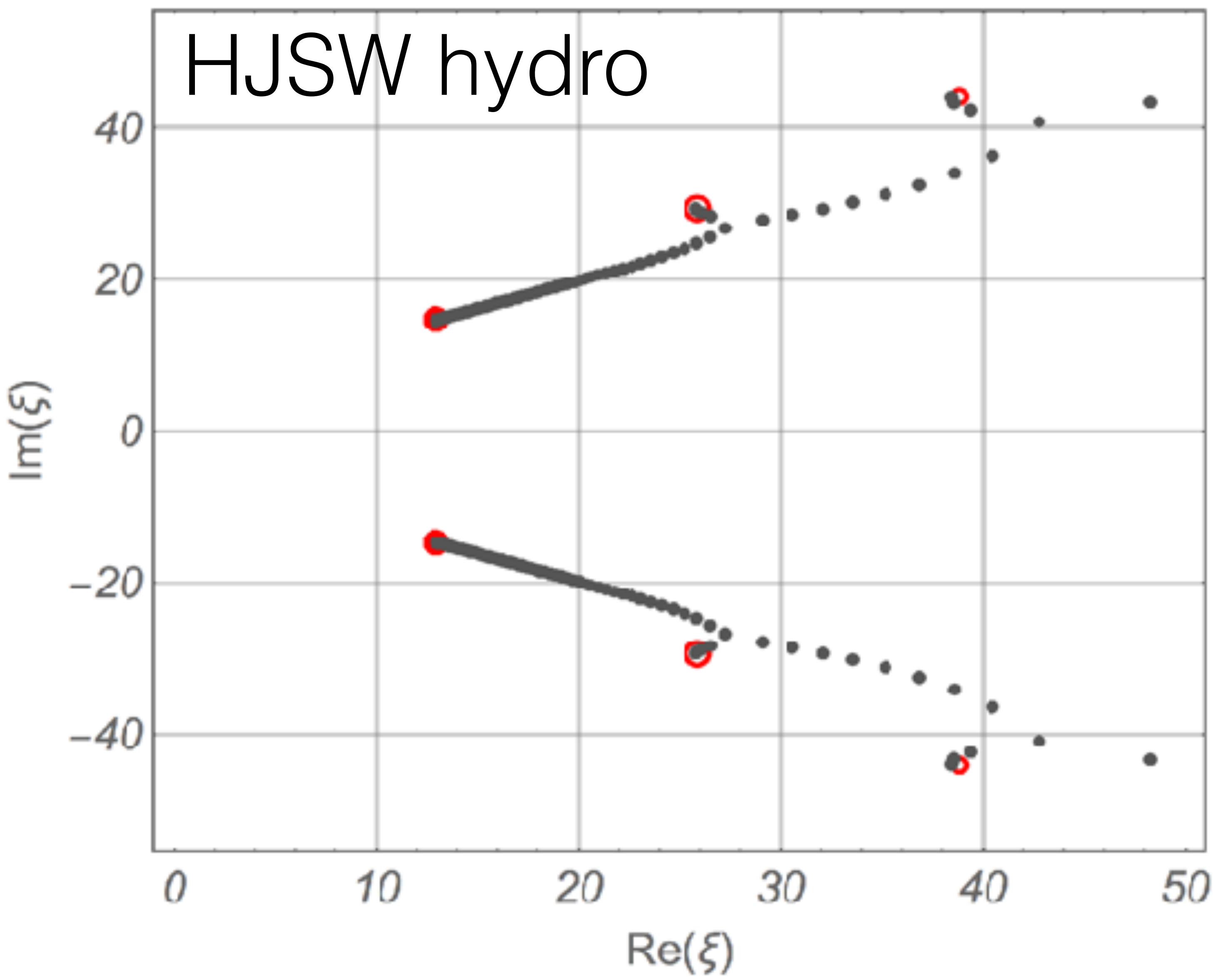
- Borel sum

$$\mathcal{S}\Phi_0(w) = \int_C d\xi\, e^{-w\xi}\, \tilde{\mathscr{B}}[\Phi_0](\xi)$$

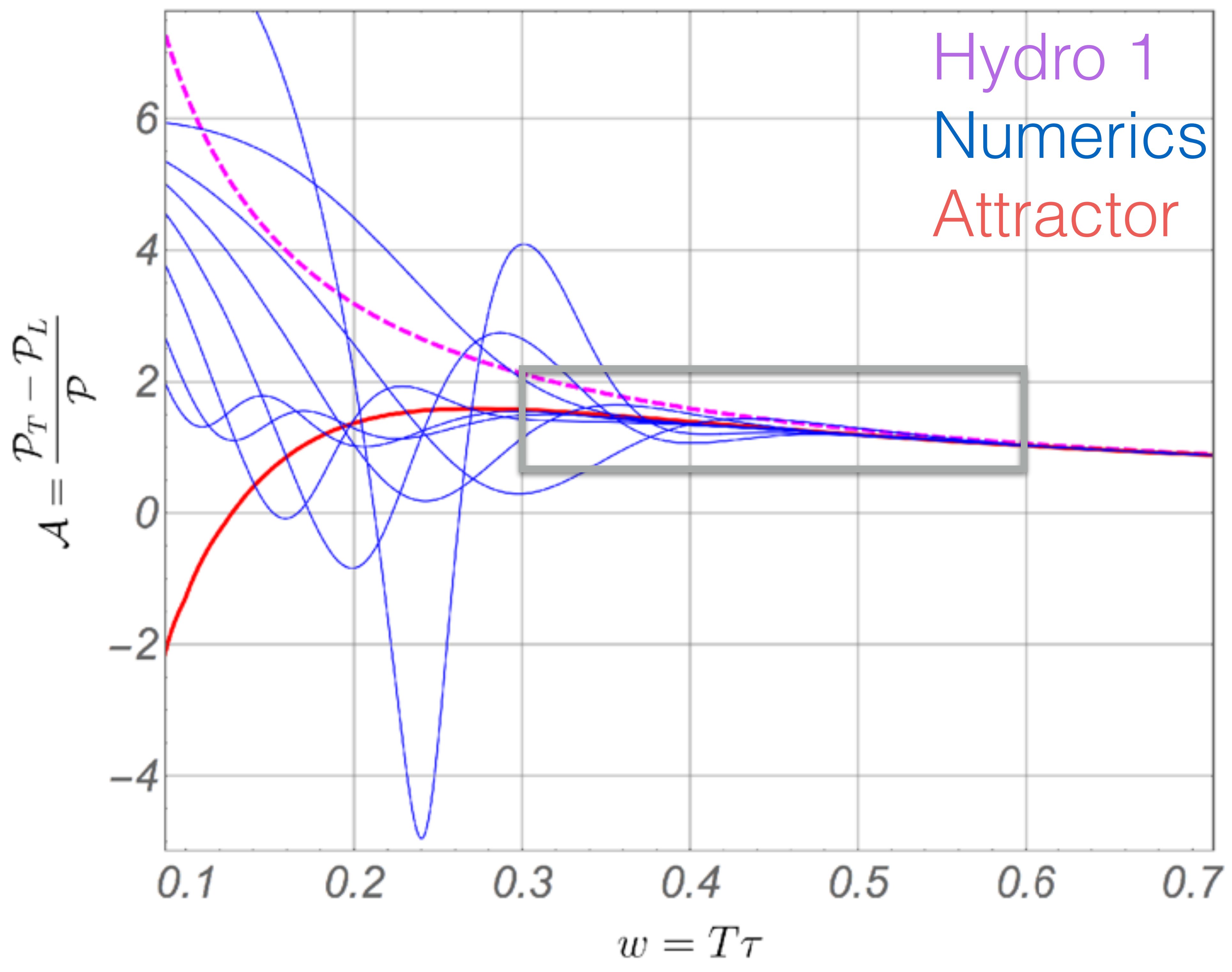
Analytic continuation (Pade)

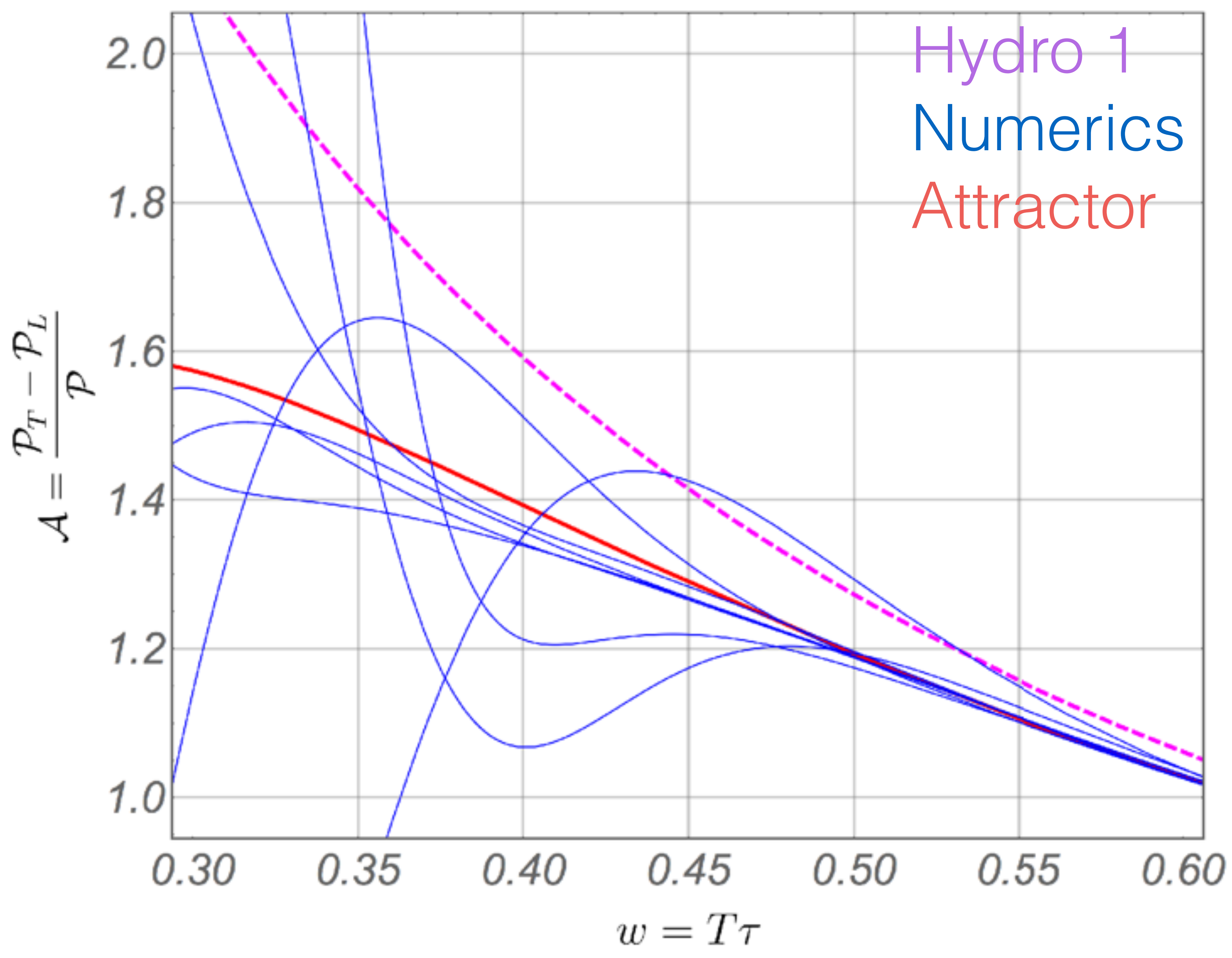






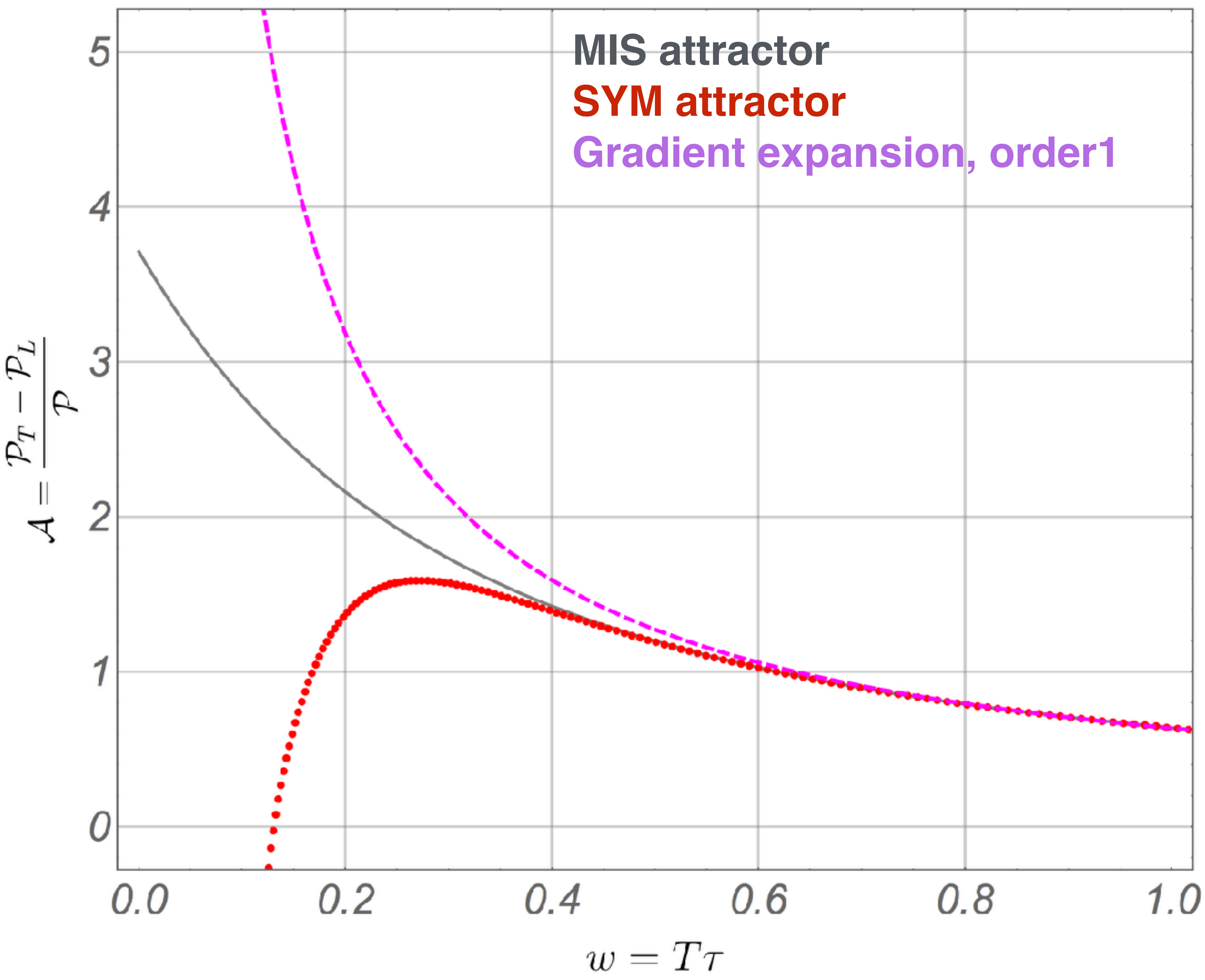
Attractor in N=4 SYM plasma





- Borel sum of the hydro series using the symmetric contour acts as an attractor
- Some solutions reach the attractor for w<0.5
- The difference between the attractor and hydro-1 is parameter dependent
- Behaviour for small w hard to get this way

Comparing attractors



Seeing the transients in SYM plasma

The QNM can be seen directly in numerical solutions of Bjorken flow at late times.

The form of the leading trans-series correction follows from

- HJSW hydro: by solving ODEs
- direct AdS/CFT calculation: formal solution including QNM contributions

$$\mathscr{A}(w) \sim \mathscr{A}_H(w) + e^{-\frac{3}{2}\Omega_I w} w^{\beta_R} \left[\Phi_+(w) \cos\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) + \Phi_-(w) \sin\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) \right]$$

$$\Phi_\pm(w) = C_\pm \left(1 + \sum_{n>0} \frac{a_n^{(\pm)}}{w^n}\right) \approx C_\pm$$

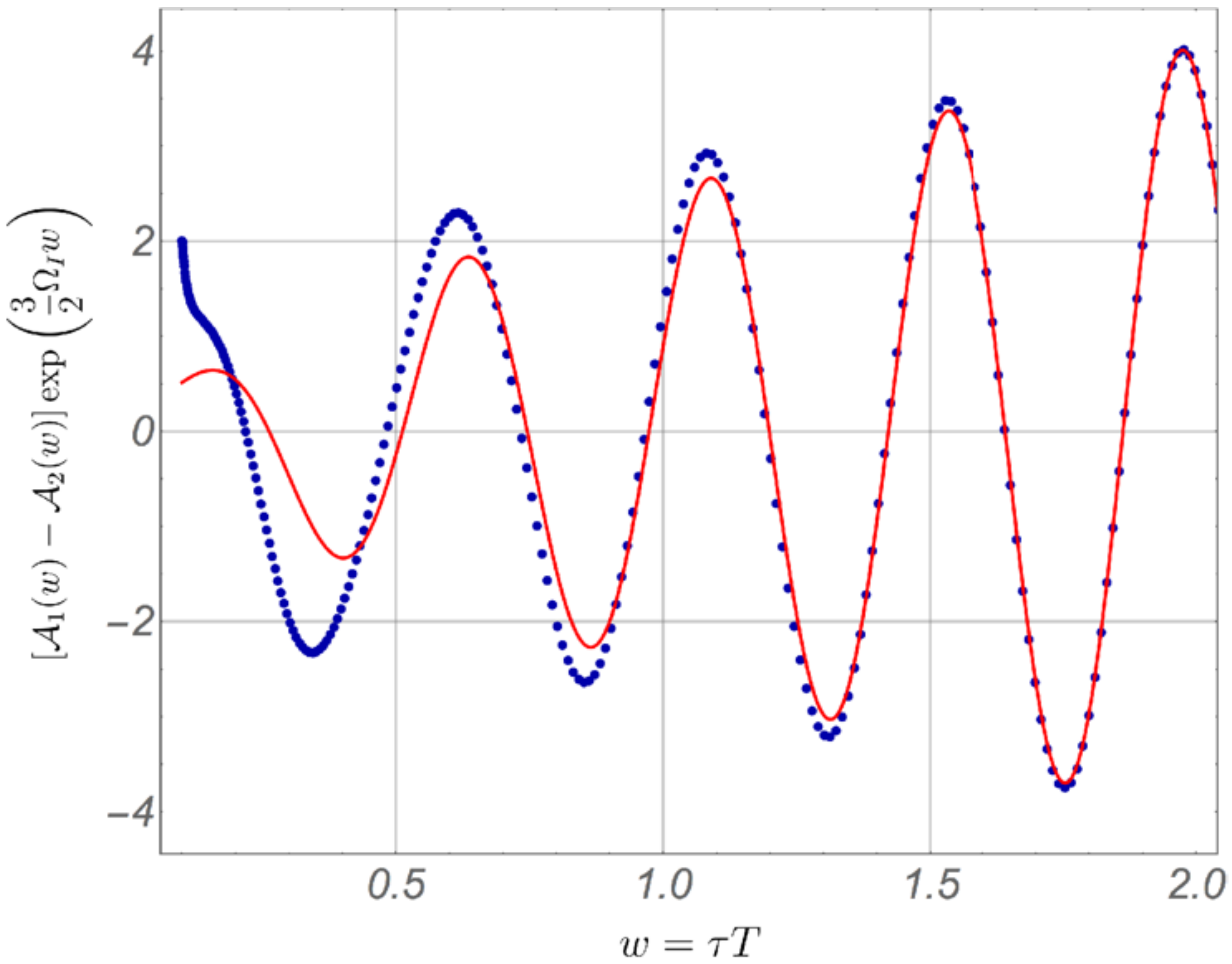
But we **do not really know the universal part** $\mathscr{A}_H$.

To see that the transient, damped oscillations can be resolved with the existing numerical methods we can consider **pairs of solutions**; their difference will not involve the universal, hydrodynamic part:

$$\mathscr{A}_1(w) - \mathscr{A}_2(w) \sim e^{-\frac{3}{2}\Omega_I w} w^{\beta_R} \left[C_{12}^{(+)} \cos\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) + C_{12}^{(-)} \sin\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right)\right]$$

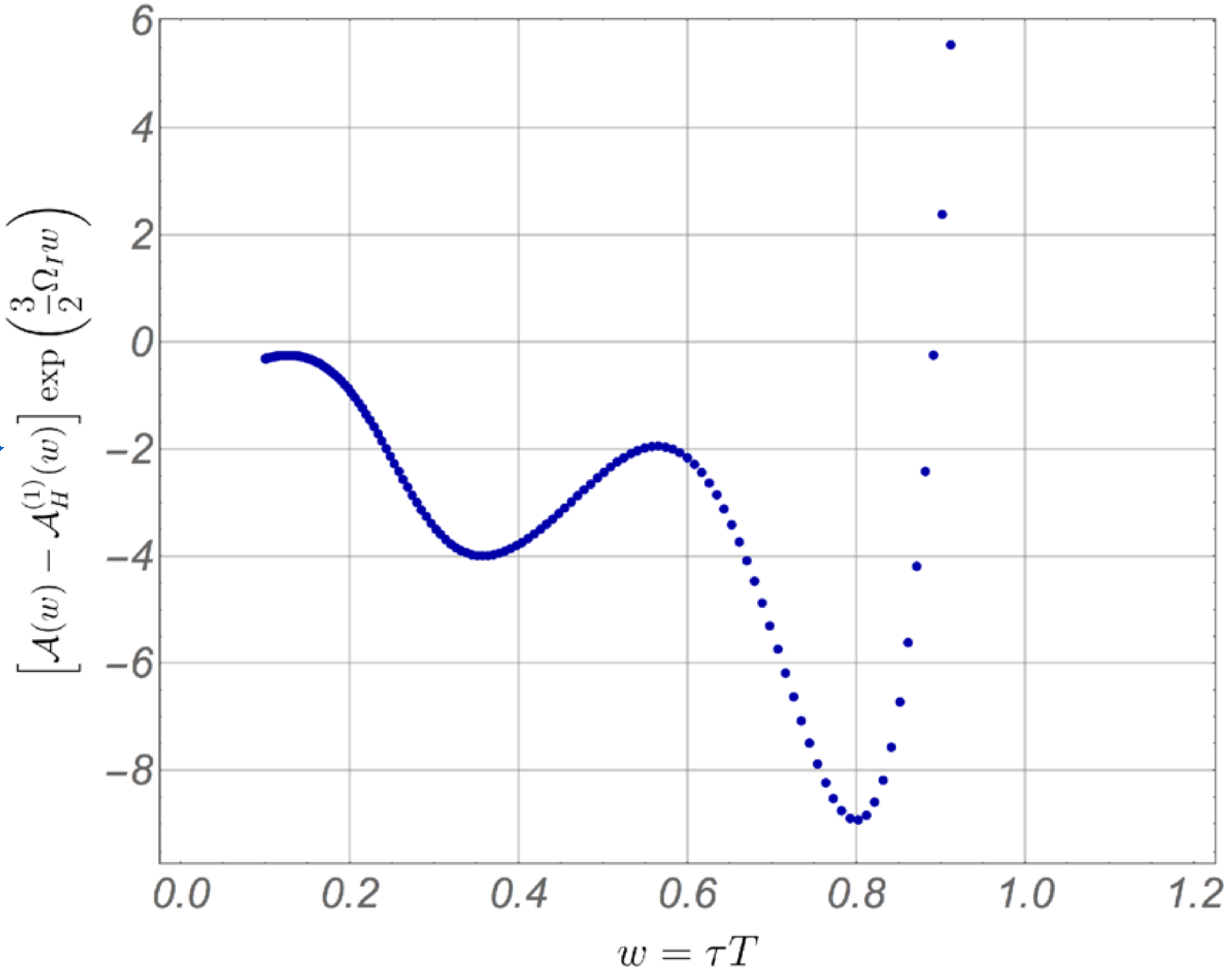
Here all the parameters are fixed apart from the **two amplitudes**, which reflect the initial conditions and differ from one pair of solution to another.

The two amplitudes appearing in the formula above can then be **fitted to the numerical solution**.



Alternatively, instead of looking at differences of solutions, we can try to subtract the universal part ("hydro to all orders") by estimating it using the gradient expansion.

Can we just remove the universal part by subtracting first or second order of the gradient expansion?

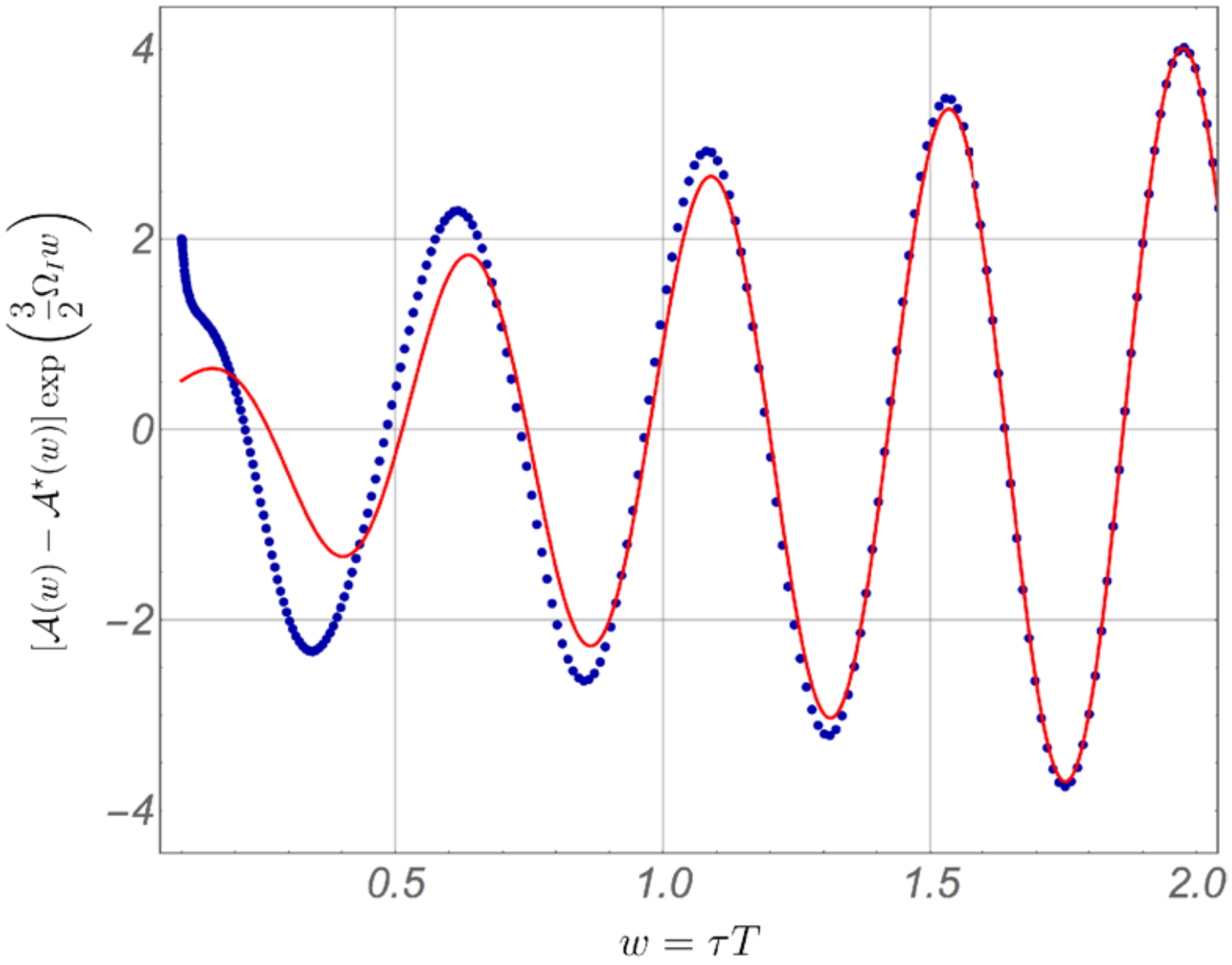


First order of the gradient expansion

$$\mathcal{A}_H^{(1)}(w) = 8\,(\eta/s)\,\frac{1}{w}$$

Instead, we can try to use the estimate of “all-order hydro” provided by the attractor calculated as the **Borel sum** of the truncated gradient expansion:

$$\mathscr{A}(w) - \mathscr{A}^{\star}(w) \sim e^{-\frac{3}{2}\Omega_I w} w^{\beta_R} \left[C^{(+)} \cos\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) + C^{(-)} \sin\left(\frac{3}{2}\Omega_R w - \beta_I \log(w)\right) \right]$$



Summary

- The emergence of hydrodynamic behaviour is governed by the decay of non-hydrodynamic modes rather than local equilibration.
- Universal observables which exhibit attractor behaviour provide a clean way to study the decay of transients.
- The symmetric Borel sum provides a reasonable approximation to the attractor for a useful range of w.
- The transseries expresses the “dissipation” of initial state information
- The least-damped QNM can be seen directly in the late time behaviour