Neutron stars in need of holography

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Holography and extreme chromodynamics

w/ Annala, Chesler, Ecker, Henriksson, Hoyos, Järvinen, Loeb, Remes, Rodríguez Fernández, Vuorinen

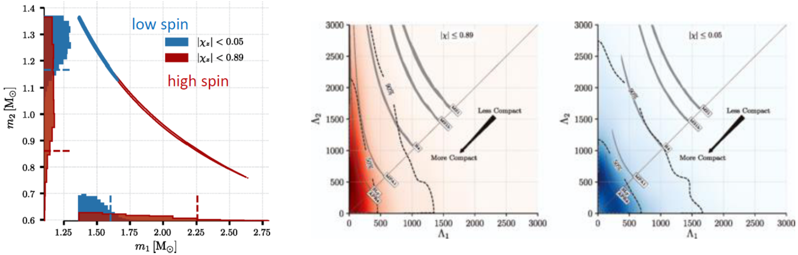
7/3/2018 This year's neutron star collision unlocks cosmic mysteries | Science News

YEAR IN REVIEW ASTRONOMY, PHYSICS, 2017 TOP 10, GRAVITATIONAL WAVES

This year's neutron star collision unlocks cosmic mysteries

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration)
Phys. Rev. Lett. **119**, 161101 – Published 16 October 2017 DOI:https://doi.org/10.1103/PhysRevLett.119.161101



- Nature was kind enough to give us a neutron star collider
- The measurement of squishiness solves for us QCD!
- Well, maybe not quite yet. . .

Simple questions from kindergarden

- Already in the 1930s Landau speculated about the existence of neutron stars (NS): [Phys. Z. Sowjetunion 1, 285 (1932) p.288]

  "...We expect that this must occur when the density of matter becomes so great that atomic nuclei come in contact, forming one gigantic nuclei."

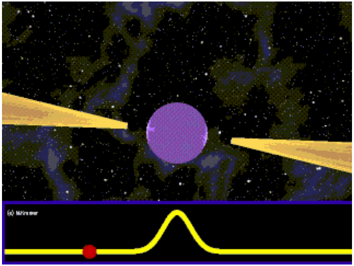
- Yet, my astrophysicist friend posed these questions: [J.Nättilä @Saariselkä'18]
  - How big are neutron stars?
  - What is inside them?
  - How does matter behave under immense pressure?
- I am not going to solve these problems today.

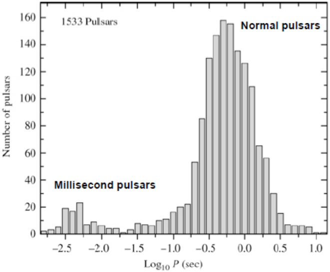
Outline

- Neutron star characteristics
- Anatomy of TOV
- Equation of state
- Holography
  - Proof of principle
  - More realistic models

Most neutron stars are observed as pulsars (highly magnetized rotating star). Observables

- Rotational period $P, \dot{P}$
- Mass ($\lesssim 2M_\odot$), ~~Radius~~ ($9 - 13$km)
- Luminosity
- Temperature $\lesssim keV$
- Magnetic field
- Future: moment of inertia
- From gravitational waves: tidal deformability





- Surface gravitational potential tells how compact the object is

$$2C \equiv \frac{2GM}{c^2 R}$$

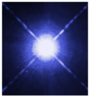
- GR is important macroscopically
- In fact, for any static stable star $2M/R < 8/9$ [Buchdahl'59]

  Anywhere inside $2m(r)/r < 1$ [Hartle'73]

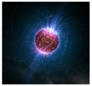
- Coincidentally, limiting case for incompressible star $(P(r) < \infty, \epsilon = \text{const.})$ [Schwarzschild]
- Causality: $2M/R < 0.69$
- For more realistic case, what is the maximum mass?

$\sim 10^{-10}$

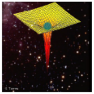
$\sim 10^{-5}$



$\sim 10^{-4} - 10^{-3}$



$\sim 0.2 - 0.4$



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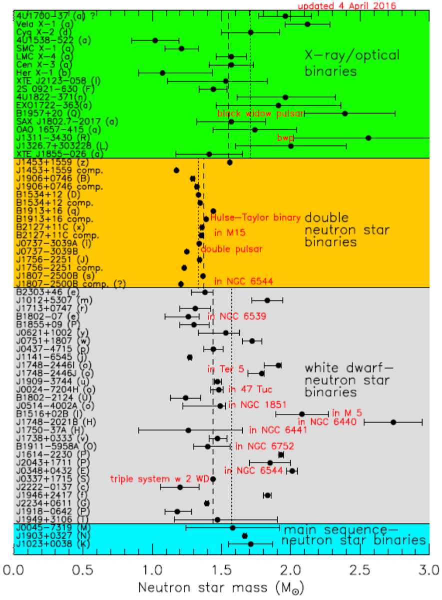
Two accurate Shapiro delay measurements of two solar mass stars

[Demorest et al.'10]

[Antoniadis et al.'13]

$\Rightarrow M_{max} > 2M_{\odot}$



Radius measurements

- Radius very difficult to measure because
  - small $\sim$10 km
  - at least hundreds of light years $\sim 10^{15}$ km away

    $\Rightarrow$ 10^{-14} radians ; angular resolution of Hubble $\sim 10^{-7}$
- A possible way to measure is observing cooling of thermonuclear X-ray bursts from NS-white dwarf binaries where NS accreates matter

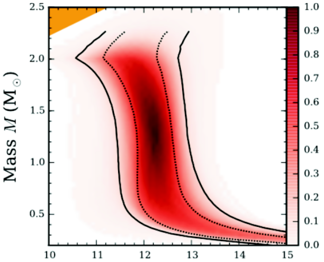
  [state-of-the-art e.g. Nättilä et al.'18]

$\Rightarrow$ Controversial results from the thermal spectrum of 5 quiescent LMXB in globular clusters

[Steiner et al.'14 $R = 12.0 \pm 1.4$ km]

vs.

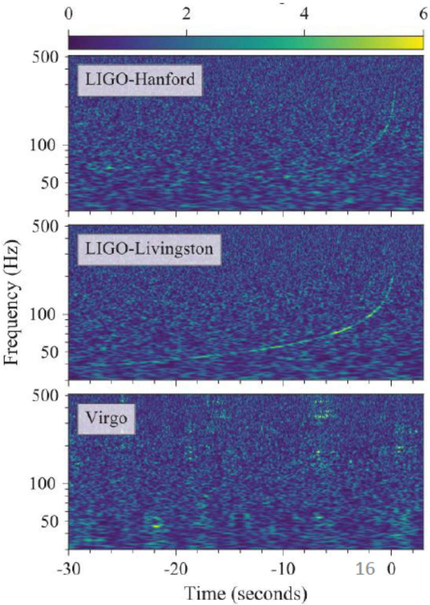
[Guillot et al.'14 R $= 9.4 \pm 1.2$ km]



GW breakthrough of merging neutron stars

Novel features:

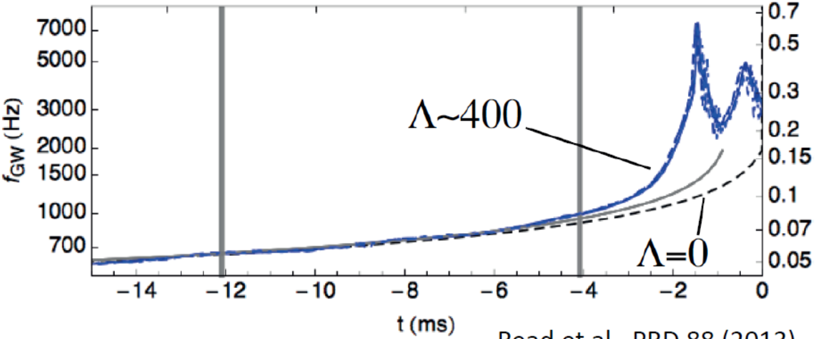
- EM signatures present if no immediate collapse to a BH
- Ringdown pattern, sensitive to EoS, but frequency too high for LIGO
- Tidal deformabilities of the NSs during inspiral provide a good measure of stellar compactness



- Tidal deformability

$$Q_{ij} = -\Lambda E_{ij}$$

- Affects the inspiral phase



Read et al., PRD 88 (2013)

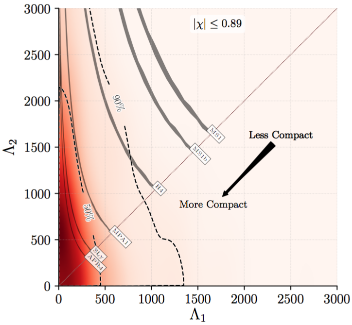
Bounds on tidal deformability

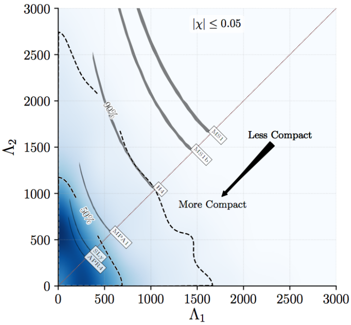
- No detection by LIGO ⇒ upper bound on tidal deformability

$$\Lambda_{1.4M_\odot} < 800 \text{ (low spin prior)}$$

at 90% confidence level

[Abbott et al.'17]





- Updated analysis

[Abbott et al.'18]

$$70 < \Lambda_{1.4M_\odot} < 580 \text{ (low spin prior)}$$

Tolman-Oppenheimer-Volkov

Neutron star structure equations

- GR important → start with Einstein equations ($\Lambda = 0$)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \ , \ \partial_\mu T^\mu_\nu = 0$$

- Non-rotating star with spherical symmetry (TOV equations):

$$ds^2 = -e^{2\nu(r)} dt^2 + \frac{dr^2}{1 - \frac{2Gm(r)}{r}} + r^2 d\Omega_2^2 \ , \ T^0_0 = \epsilon(r) \ , \ T^j_i = P(r)\delta^j_i$$

$$\frac{dP}{dr} = \underbrace{-G \frac{m(r)\epsilon(r)}{r^2}}_{Newtonian} \underbrace{\left(1 + \frac{P(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)}\right)\left(1 - \frac{2GM}{r}\right)^{-1}}_{GR\ \ effects}$$

$$m(r) = 4\pi \int_0^r dr' r'^2 \epsilon(r') \quad \text{mass inside of radius } r$$

Neutron star structure equations

- Solve (numerically) from $r = 0$ to $r = R$ with boundary conditions

$$P(0) = P_c \; , m(0) = 0$$
$$P(R) = 0 \; , \; m(R) = M$$

- Catch: need to know Equation of State $P = P(\epsilon)$
- Rotation breaks spherical symmetry and makes the structure equations "slightly" more complicated
  - deforms the star
  - increase of mass ($\sim 20\%$) due rotation
  - drag of local inertial frames (Lense-Thirring effect)
    - numerical solvers available online [e.g. LORENE]
    - can solve perturbatively [Hartle-Thorne '67-68]

Constancy of chemical potential and temperature

- The metric function $\nu(r)$ is gravitational potential. In vacuum above the star glue it to Schwarzschild:

$$e^{\nu(r)} = \left(1 - \frac{2GM}{r}\right)^{1/2} \; , \; r \geq R$$

- Inside the star

$$\frac{d\nu}{dr} = -\frac{1}{\epsilon(r)}\frac{dP(r)}{dr}\left(1 + \frac{P(r)}{\epsilon(r)}\right)^{-1} \leftrightarrow -d\nu = \frac{dP}{P+\epsilon}$$

- Baryon chemical potential is "constant" at any depth:

$$-d\nu = \frac{dP}{\mu n} = \frac{d\mu}{\mu}$$

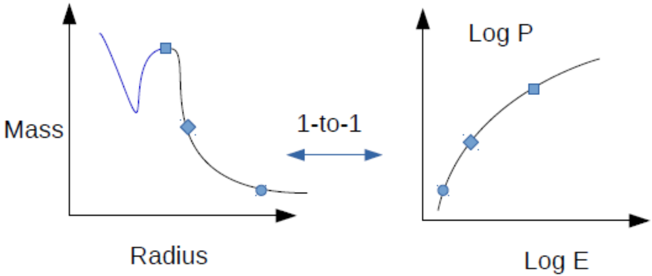
- For idealized cold NS with iron surface ($P_{Fe^{56}} = 0 = \mu n - \epsilon$)

$$\mu(r)e^{\nu(r)} = const. = \mu_{Fe}\left(1 - \frac{2GM}{R}\right)^{1/2}, \mu_{Fe} \approx \frac{m(Fe)}{56} \sim 930 MeV$$

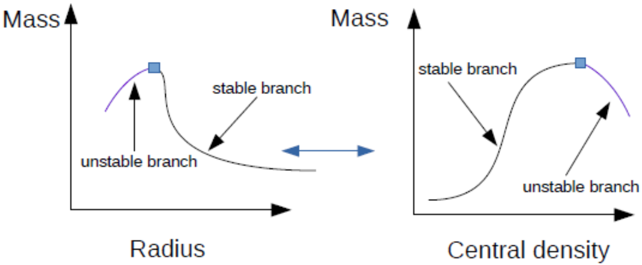
- For thermal equilibrium: $T(r)e^{\nu(r)}$=const.

[Zel'dovich-Novikov'71]

Solutions of TOV and stability



- Sols of TOV represent static equilibrium configurations
  - Charge neutrality
  - Beta equilibrium
- Given central density find a star
- Equation of state is observable!



- Stability is required, necessary:

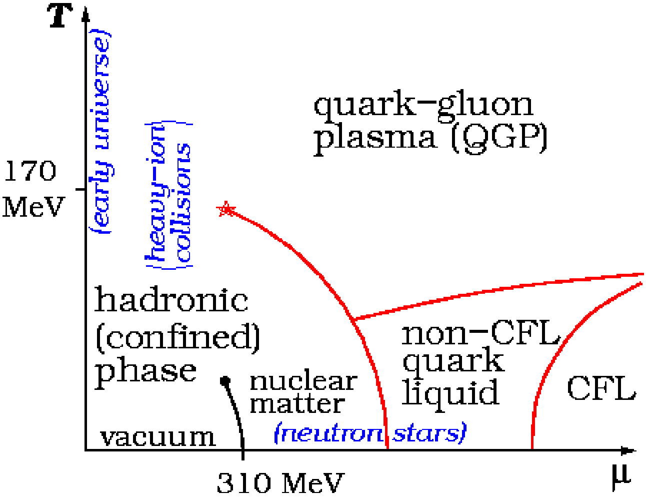
$$\frac{dM}{d\epsilon_c} > 0 \leftrightarrow \frac{dM}{dR} < 0$$

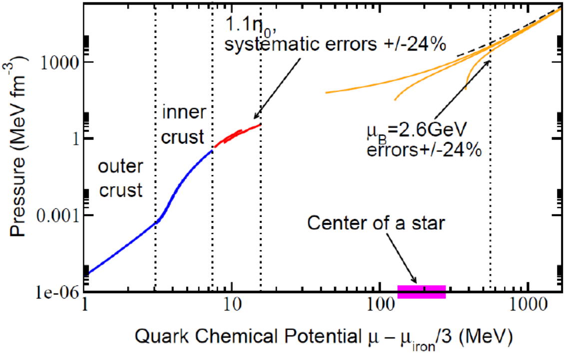
- Sufficient: stable wrt small radial perturbations and convection

[Kovetz'67,Schutz'70,Detweiler-Ipser'73]

Equation of State

QCD phase diagram

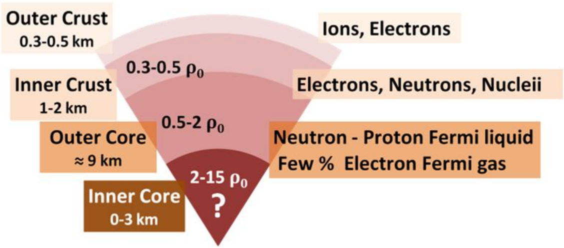




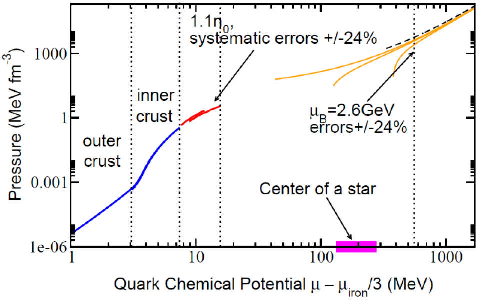
[Kurkela-Fraga-Schaffner-Bielich-Vuorinen '14]

- Low density pretty well-understood
- In order to reach and pass n_s, need to treat neutron interactions systematically: Chiral Effective Theory [NNNLO Tews et al.'13,Hebeler et al.'13]
- High density pQCD at 3 loops for unpaired $m_q \neq 0$ [Kurkela-Romatschke-Vuorinen'09]

Nuclear matter EoS

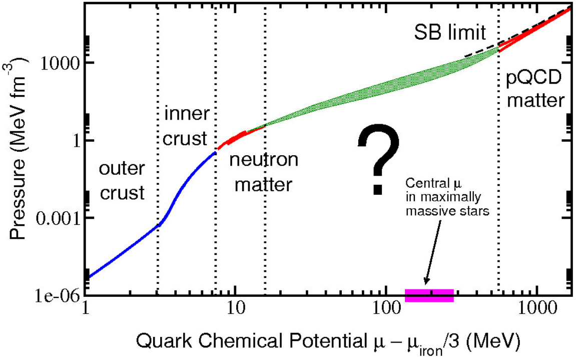


- CET
  $n \leq n_s \sim 0.16\mathrm{fm}^{-3}$:
  $n \sim 5n_s$
- pQCD $\mu_q > 1\mathrm{GeV}$:
  $\mu_q < 500\mathrm{MeV}$
- Lattice $\mu_q \ll T$:
  $T \sim 0.1\mathrm{MeV}$



Connecting the extremes

- Traditionally two methods:
  - Pheno models, eg. MIT bag model, NJL
  - Parametrize ignorance by a controlled interpolation: polytropes
- Realistic EoS needs to
  - satisfy causality $dP/d\epsilon \leq 1$ & stability $dP/d\epsilon > 0$
  - conform with $M_{max} > 2M_{\odot}$ & $\Lambda_{1.4M_{\odot}} = 70...580$ (slow-spin prior) $\Rightarrow$ likely stiff somewhere, $\boxed{\frac{dP}{d\epsilon} > \frac{1}{\sqrt{3}}}$. [Bedaque-Steiner'14]



- Interpolation w/ piecewise polytropic EoSs

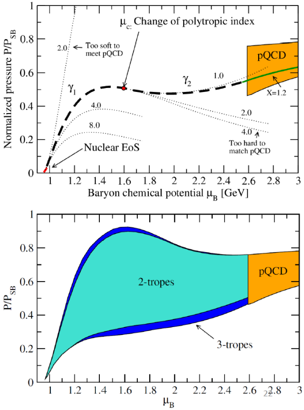
$$p_i(n) = \kappa_i n^{\gamma_i}$$

varying all relevant parameters

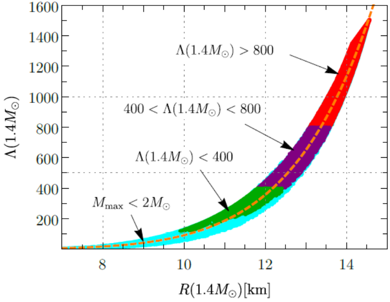
Require:

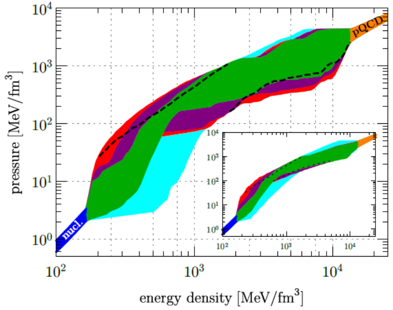
1. Smooth matching to nuclear and quark matter EoS
2. Continuity of p, n, possibly allow 1st order phase trans.
3. Subluminality
4. $M_{max} > 2M_{\odot}$
5. LIGO bound

[Annala-Gorda-Kurkela-Vuorinen'17]



Tidal deformability as radius measurement?





- Assuming no 1st order phase transition in the outer crust:
  [Annala-Gorda-Kurkela-Vuorinen'17]

$$\Lambda \;\leftrightarrow\; R$$

- If transition, non-monotonic

[1711.06244]

Holography

Use holography to connect the dots

- Strongly coupled $\mathcal{N} = 4$ good proxy for heavy ion physics
  [see list by Mateos' talk]
- However, to mimic cold and dense QCD:
  - need finite density of fundamental flavors $N_f \neq 0$, while vanilla $\mathcal{N} = 4$ only adjoints
  - $N_c = 3$ is actually very important for baryon structure, color superconductivity etc.
  - need ~~SUSY~~ and ~~conformality~~ and impose confinement
  - need different (bare) masses for the quarks
- No holographic dual to even come close to meet these criteria

The idea is

Compute EoS using AdS/CFT $\Rightarrow$ use TOV equations to build the star in flat space

Choosing your model

- Top-down (correct calculation, wrong theory)
  - Compact stars in AdS_5
    [de Boer-Papadodimas-Verlinde'09]
  - Quark stars in Sakai-Sugimoto and D4-D6
    [Burikham-Hirunsirisawat-Pinkanjanarod'10]
    [Kim-Lee-Shin-Wan'11&'14]
    [Ghoroku-Kubo-Tachibana-Toyoda'13]
  - Add quenched flavors to $\mathcal{N} = 4 \rightarrow$ D3-D7 models
    [1603.02943,1711.06244]
  - Stiff phases from consistent truncations; no NS yet
    [$U(1)_R$:1609.03480,1707.00521]
    [$U(1)_B$:work in progress]
  - Increasing number of papers on cold holographic EoS at finite μ, not stiff
    [e.g. Noronha et al.]
- Bottom-up (less correct calculation, less wrong theory)
  - V-QCD
    [works in progress]
  - Family of models with stiff phases
    [1609.03480,1707.00521]

Proof of principle

Proof of principle

- Start with the simplest model
- $\mathcal{N} = 2$ SQCD theory: $\mathcal{N} = 4$ $SU(N_c)$ SYM + N_f hypers in fundamental
- Gravity dual: probe D7-branes in $AdS_5 \times S^5$
  [Karch-Katz'02]
- Focus on black hole embeddings at finite density and $T \ll \mu_q$
  [Mateos et al.'07]
- Identify as the quarkyonic phase in large-N_c QCD
  [McLerran-Pisarski'07]

~~Proof of principle~~ Fun digression

- Equation of state is analytic

[Karch-O'Bannon'07]

$$p = \kappa^2(\mu_q^2 - m_0^2)^2 + \mathcal{O}(\mu^3 T, T^4)$$

$$n = \frac{\partial p}{\partial \mu} \;\rightarrow\; \epsilon = \mu n - p = 3p + 4\kappa^2 m_0^2 \sqrt{p}$$

- TOV is known to be soluble analytically e.g. for

[Buchdahl'67, Lattimer lecture notes, textbooks]

$$\epsilon = -\sqrt{5}p + 12\sqrt{p_* p}$$

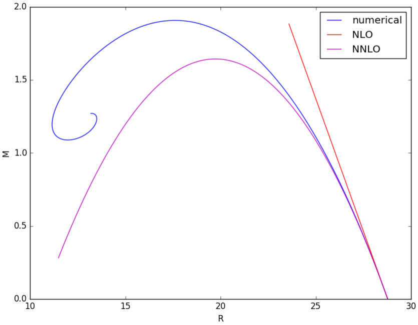
$$\text{and}$$

$$\epsilon \propto p^{1/\gamma} \;\rightarrow\; R \sim M^{\frac{\gamma-2}{3\gamma-4}}$$

- Here at low p we have $\gamma = 2$ and can set up perturbation in terms of

$$\varepsilon \equiv \frac{\mu_c - m_0}{m_0} \ll 1 \leftrightarrow \text{parametrically} \;\; \varepsilon \sim C = \frac{GM}{R} \ll 1$$

~~Proof of principle~~ Fun digression



- Scaling symmetry of TOV: $p \to a^2 p, \epsilon \to a^2 \epsilon, r \to r/a, m \to m/a$
- Find

$$\boxed{c_0 M = R^{(0)} - R - \frac{c_1}{c_0 R^{(0)}} (R - R^{(0)})^2 + \ldots}$$

- For any m_0, good approx upto $C \approx 0.116$

Proof of principle

- Equation of state

[Karch-O'Bannon'07]

$$p = \kappa^2(\mu_q^2 - m_0^2)^2 + \mathcal{O}(\mu_q^3 T, T^4)$$

$$n_q = \frac{\partial p}{\partial \mu_q} \;\rightarrow\; \epsilon = \mu_q n_q - p = 3p + 4\kappa^2 m_0^2 \sqrt{p}$$

$$\kappa^2 = \# \frac{N_c N_f}{\lambda_{YM}}$$

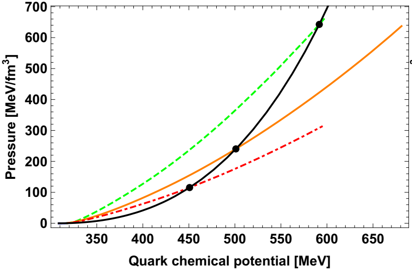
- Extrapolate to pQCD $\mu_q \to \infty$: $N_c = 3 = N_f$, $\lambda_{YM} \approx 10.74$
- Maintain charge neutrality & β-equilibrium:

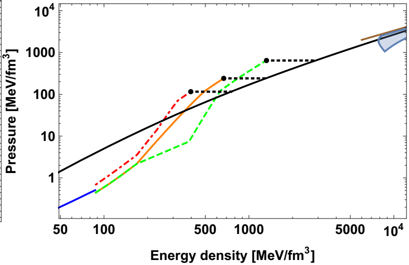
$$\mu_e = 0 \;,\; \mu_u = \mu_d = \mu_s \equiv \mu_q$$

- Point of zero pressure as for Fe^{56} in vacuum:

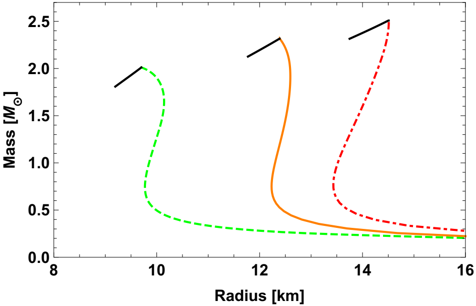
$$\boxed{m_0 \approx 310 MeV}$$

Matching to state-of-the-art EoSs from CET





- Strong 1st order transitions at phenomenologically reasonable densities $2.4 - 6.9 n_s$, can support $2M_\odot$
- No quark matter cores



Generalize, strange matter hypothesis

- Quarks in atomic nuclei are confined within nucleons:

$$\frac{E_{u,d}}{A} > \frac{E(Fe^{56})}{56} \sim 930 MeV$$

- Strange matter hypothesis: three-flavor quark matter absolutely stable in vacuum ($p = 0$):
  [Bodmer'71,Terazawa'79,Witten'84]

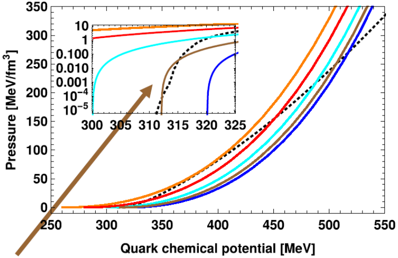
$$\frac{E_{SQM}}{A} = \frac{\epsilon}{n_B} < \frac{E(Fe^{56})}{56}$$

- Point of zero pressure

m_0 free parameter

- Other parameters as before

Equations of state, part dos



Two crossings

$\mu_0 < \mu_N$ quark matter

$\mu_0 \lesssim \mu_N$

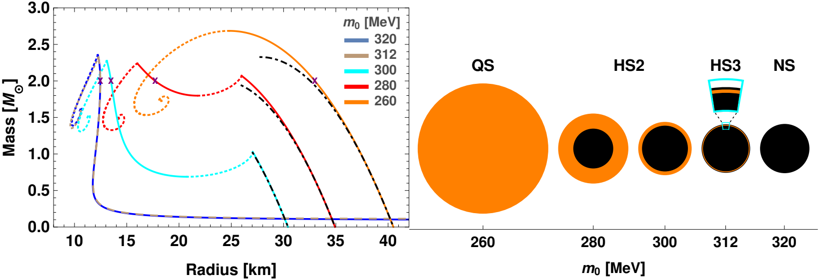
$\mu_0 \lesssim \mu_N$

$\mu_0 \gtrsim \mu_N$

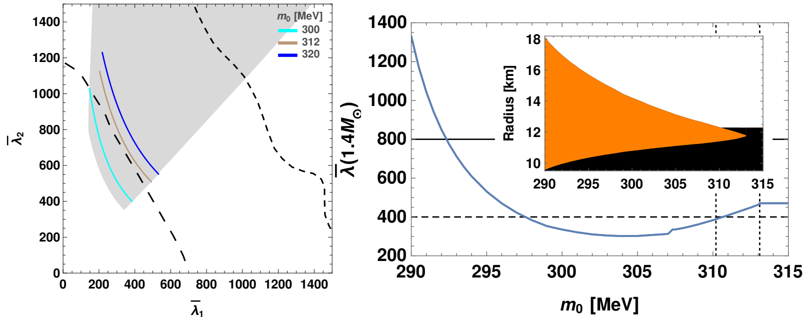
both phases

$\mu_0 > \mu_N$ nuclear matter

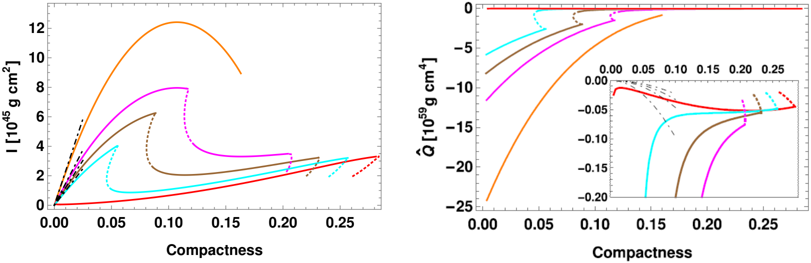
- Dashed curve "intermediate" HLPS
- Can have 1st order phase transition both at low and high density



- QS & HS2: three-flavor QM absolutely stable
- HS2 & HS3 also found in some other pheno models
  [Alford-Braby-Paris-Reddy'04]



- Tidal deformabilities fit GW observations
- Hydrid stars are actually "better fit" than neutron stars
- Heat up, numerics, $\eta(T, \mu_q)$ is on the correct ballbark for HS3
  [Mateos-Myers-Thomson'06+μ vs. Caballero-Postnikov-Horowitz-Prakash'08]
- How then can we distinguish our hydrid stars from NS?



- Consider stars with small angular velocity
- Compute moment of inertia and quadrupole moment of mass distribution [Glendenning's book,Hartle-Thorne'68,Raithel-Özel-Psaltis'16]
- Analytics for QM tails

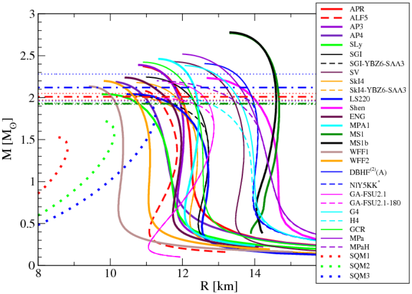
$$\bar{I} \approx 0.261 C^{-2}$$

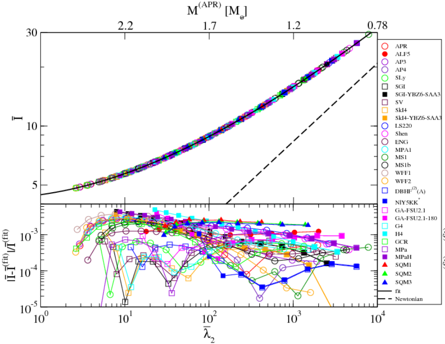
$$\bar{Q} \approx -30.35 C$$

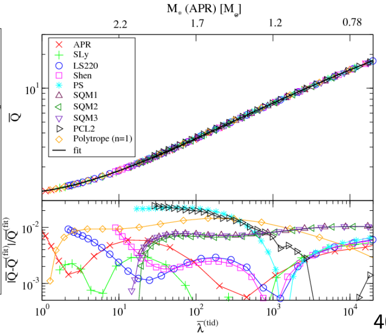
I-Love-Q relations

- Take a plethora of EoS
- Assume no 1st order phase transition in the crust
- They all obey universal relations to within ∼ %1

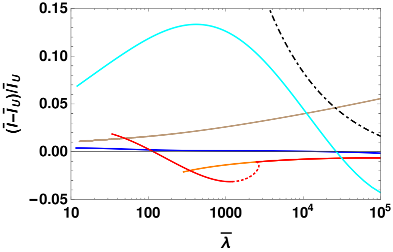
[Yagi-Yunes'13]

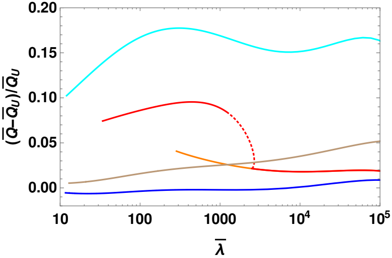




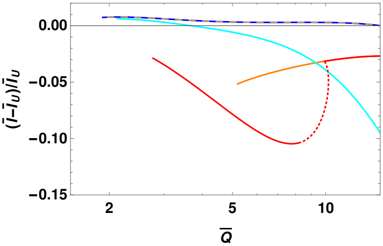


Violation of I-Love-Q relations





- Hybrid stars can violate universal relations up to $\sim 15\%$
- Conjecture: violation if strong 1st order phase transition in the crust



More realistic model: V-QCD

V-QCD as a proxy for QCD

- For longer intro, see Matti Järvinen's talk
- Bottom-up holographic theory to mimic QCD as closely as possible

[Järvinen-Kiritsis'11]

- Three potentials to be fitted against available lattice QCD data at $\mu = 0$
- Extrapolate from there to finite μ

V-QCD more details

Two bulk scalars $\lambda = e^{\phi} \leftrightarrow g^2 N_c$, $\tau \leftrightarrow \bar{q}q$

- Model physics in chirally symmetric phase ($m_q = 0$), set $\tau = 0$:

$$S_{V-QCD} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

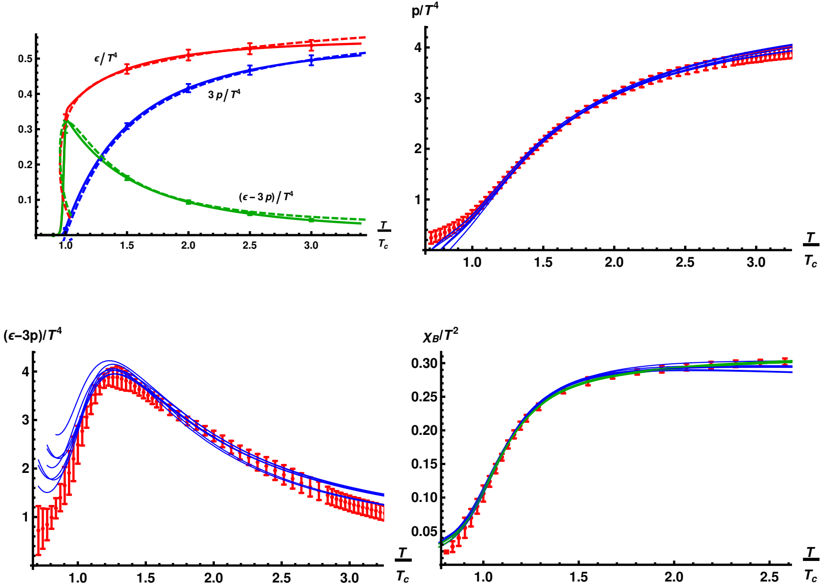
$$- N_f N_c M^3 \int d^5x V_{f0}(\lambda) \sqrt{- \det(g_{ab} + w(\lambda) F_{ab})}$$

$$F_{rt} = \Phi'(r) \quad , \quad \Phi(\infty) = \mu_q$$

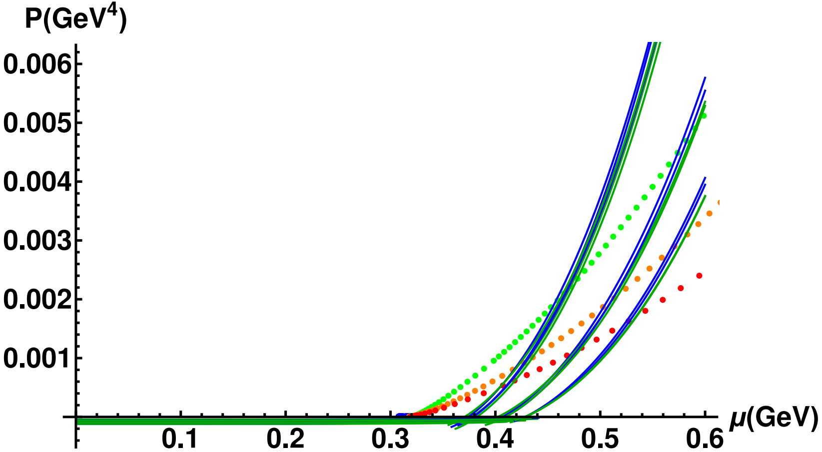
- Functions V_g , V_{f0} , w and two parameters: M and the dynamical energy scale Λ to be determined
- Use both qualitative features (e.g. confinement, asymptotic freedom) and fit to lattice/experimental data

[Järvinen et al. work in progress]

Fitting to full QCD data at $\mu = 0$

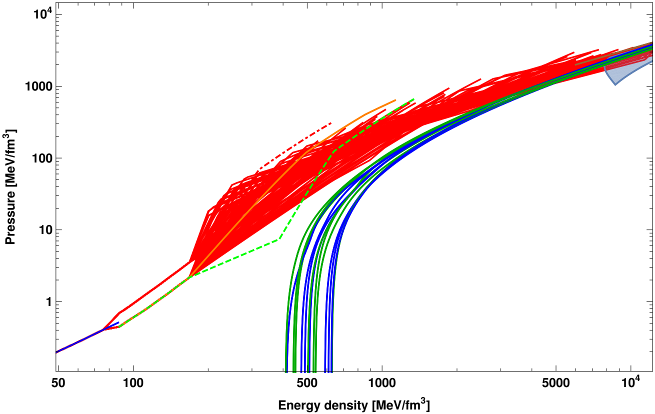


Pressures for $T = 0$

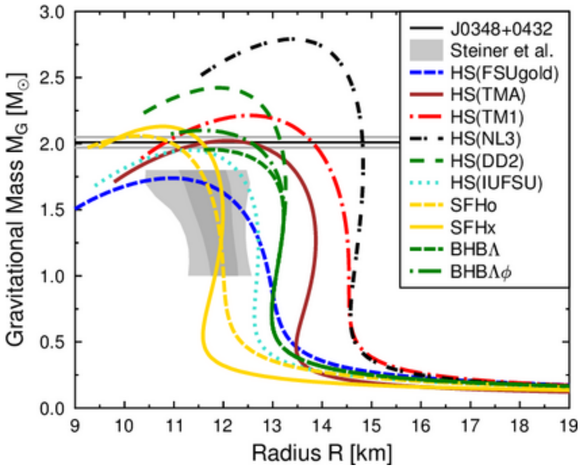


- Pressures automatically at the correct place

Matching with polytropes

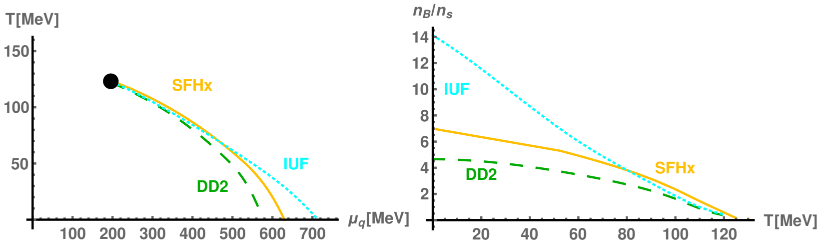


- Low density use tritropes assuming no transition and no lower bound on M
- Conjecture: Strong 1st order phase transition is generic!
  - $\Rightarrow$ Upper bound on M



- Only few EoS (DD2,SFHx,IUF) available from nuclear side at $T \neq 0$ that survive LIGO/Virgo

[https://astro.physik.unibas.ch/people/matthias-hempel/equations-of-state.html]



- Strong 1st order phase transition at $T = 0$ as in D3-D7
- Critical point at the same ballbark for all EoS

Thank you!