Semi-holographic approaches to quark-gluon plasma physics

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with: C. Ecker, A. Kurkela, A. Mukhopadhyay, F. Preis, A. Soloviev, S. Stricker

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QGP: Strongly and weakly coupled

Quark-gluon plasma/liquid:

- Much evidence for strongly coupled physics (η/s close to $\hbar/4\pi$)
- Weak-coupling physics present simultaneously:
  - Thermodynamics well described by resummed perturbation theory above $\sim 3T_c$
  - Quark susceptibilities at $\mu \approx 0$
  - Hard components of high-p_T jets

Theoretical challenge: How to combine strongly and weakly coupled sectors of a theory when their descriptions are radically different?

Semi-holographic models

Semi-holography:

dynamical boundary theory (weakly self-coupled in the UV)
coupled to a strongly coupled conformal sector with gravity dual

oxymoron coined by Faulkner & Polchinski, JHEP 1106 (2011) 012 [arXiv:1001.5049]
in study of holographic non-Fermi-liquid models

further developed for NFLs in:
A. Mukhopadhyay, G. Policastro, PRL 111 (2013) 221602

B. Douçot, C. Ecker, A. Mukhopadhyay, G. Policastro, Phys.Rev. D96 (2017) 106011

- only part of the d.o.f.'s described by holography
- more flexible model-building

Semi-holographic model for heavy-ion collisions

Aim: hybrid strong/weak coupling model of quark-gluon plasma formation
(QCD: strongly coupled in IR, weakly coupled in UV)

(different) example:
J. Casalderrey-Solana et al., JHEP 1410 (2014) 19 and JHEP 1603 (2016) 053
J. Brewer et al., JHEP 1802 (2018) 015 → talk by Wilke Van Der Schee

Idea of semi-holographic model by
E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003 [arXiv:1410.6448]:

combine pQCD (Color-Glass-Condensate) description of initial stage of HIC through overoccupied gluons with AdS/CFT description of thermalization

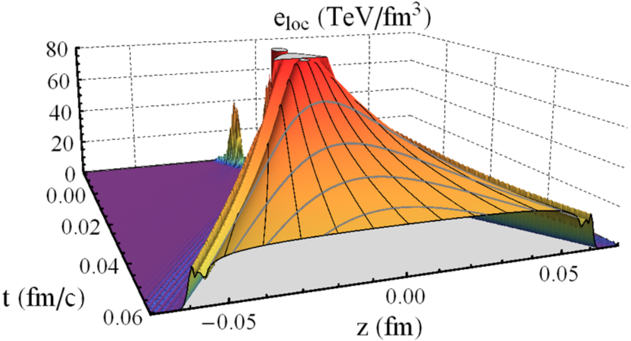
modified and extended in
A. Mukhopadhyay, F. Preis, A.R., S. Stricker, JHEP 1605 (2016) 141 [arXiv:1512.06445]

- s.t. $\exists$ conserved local energy-momentum tensor for combined system
- verified in (too) simple test case, recently in:
  C. Ecker, A. Mukhopadhyay, F. Preis, A.R., A. Soloviev, arXiv:1806.01850
  cf. talk by Christian Ecker

Gravity dual of heavy-ion collisions

pioneered and developed in particular by P. Chesler & L. Yaffe [JHEP 1407 (2014) 086]

attempt towards quantitative analysis along these lines:
Wilke van der Schee, Björn Schenke, PRC92 (2015) 064907 [1507.08195]



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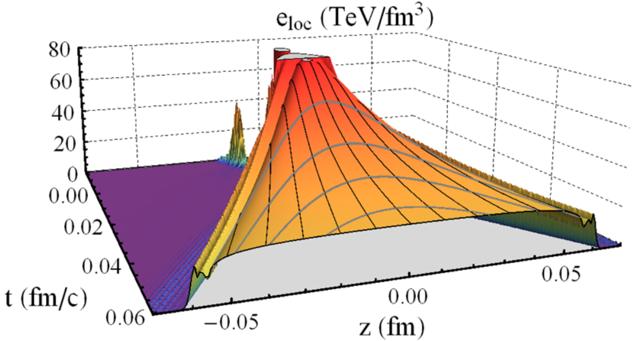


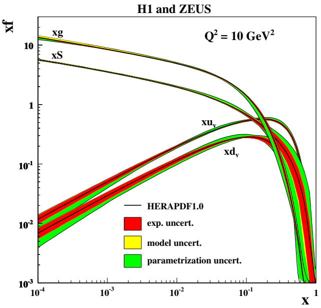
had to scale down energy density by a factor of 20 (6) for the top LHC (RHIC) energies

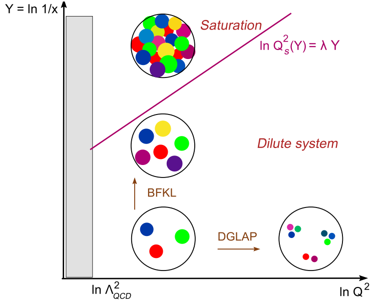
calls for input from pQCD in earliest stage?

pQCD and Color-Glass-Condensate framework

developed by Larry McLerran and collaborators

gluon distribution $xG(x, Q^2)$ in a proton rises very fast with decreasing longitudinal momentum fraction x at large, fixed Q^2





HIC: high gluon density $\sim \alpha_s^{-1}$ at "semi-hard" scale Q_s ($\sim$ few GeV)

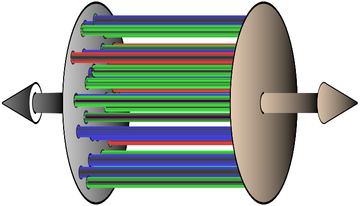
weak coupling $\alpha_s(Q_s) \ll 1$ but highly nonlinear because of large occupation numbers

description in terms of classical YM fields as long as gluon density nonperturbatively high

Color-Glass-Condensate evolution of HIC at LO

effective degrees of freedom in this framework:

1. color sources ρ at large x (frozen on the natural time scales of the strong interactions and distributed randomly from event to event)
2. gauge fields A^μ at small x
   (saturated gluons with large occupation numbers $\sim 1/\alpha_s$, with typical momenta peaked about $k_\perp Q_s$)

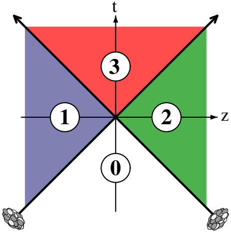


glasma: non-equilibrium matter, with high occupation numbers $\sim 1/\alpha_s$

initially longitudinal chromo-electric and chromo-magnetic fields that are screened at distances $1/Q_s$ in the transverse plane of the collision

Color-Glass-Condensate evolution of HIC at LO

- colliding nuclei as shock waves with frozen color distribution



classical YM field equations

$$D_\mu F^{\mu\nu}(x) = \delta^{\nu+}\rho_{(1)}(x^-, \mathbf{x}_\perp) + \delta^{\nu-}\rho_{(2)}(x^+, \mathbf{x}_\perp)$$

in Schwinger gauge $A^\tau = (x^+ A^- + x^- A^+)/\tau = 0$
with ρ from random distribution (varying event-by-event)

outside the forward light-cone (3):
(causally disconnected from the collision)
pure-gauge configurations

$$A^+ = A^- = 0$$
$$A^i(x) = \theta(-x^+)\theta(x^-)A^i_{(1)}(\mathbf{x}_\perp) + \theta(-x^-)\theta(x^+)A^i_{(2)}(\mathbf{x}_\perp)$$
$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{\mathrm{i}}{g}\, U_{(1,2)}(\mathbf{x}_\perp)\partial_i U^\dagger_{(1,2)}(\mathbf{x}_\perp)$$
$$U_{(1,2)}(\mathbf{x}_\perp) = \mathrm{P}\,\exp\left(-\mathrm{i}g\int \mathrm{d}x^\mp \frac{1}{\nabla^2_\perp}\rho_{(1,2)}(x^\mp, \mathbf{x}_\perp)\right)$$

inside forward light-cone:
numerical solution with initial conditions at $\tau = 0$:

$$A^i = A^i_{(1)} + A^1_{(2)}, \qquad A^\eta = \frac{\mathrm{i}g}{2}\left[A^i_{(1)}, A^i_{(2)}\right], \qquad \partial_\tau A^i = \partial_\tau A^\eta = 0$$

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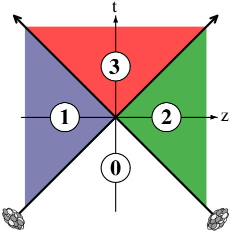

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• **Aim of semi-holographic model:** include bottom-up thermalization from relatively soft gluons with higher α_s and their backreaction when they build up thermal bath

Semi-holographic glasma evolution

[E. Iancu, A. Mukhopadhyay, JHEP 1506 (2015) 003]

[A. Mukhopadhyay, F. Preis, AR, S. Stricker, JHEP 1605 (2016) 141]

UV-theory=classical Yang-Mills theory for overoccupied gluon modes with $k \sim Q_s$
IR-CFT=effective theory of strongly coupled soft gluon modes $k \ll Q_s$,
modelled by N=4 SYM gravity dual marginally deformed by:
❶ boundary metric $g^{(\mathrm{b})}_{\mu\nu}$, ❷ dilaton $\phi^{(\mathrm{b})}$, and ❸ axion $\chi^{(\mathrm{b})}$ which are functions of A_μ

$$S = S_{\mathrm{YM}}[A] + W_{\mathrm{CFT}}\left[g^{(\mathrm{b})}_{\mu\nu}[A], \phi^{(\mathrm{b})}[A], \chi^{(\mathrm{b})}[A]\right]$$

W_{CFT}: generating functional of the IR-CFT (on-shell action of its gravity dual)
minimalistic coupling through gauge-invariant dimension-4 operators

① IR-CFT energy-momentum tensor $\frac{1}{2\sqrt{-g^{(\mathrm{b})}}}\frac{\delta W_{\mathrm{CFT}}}{\delta g^{(\mathrm{b})}_{\mu\nu}} = \mathcal{T}^{\mu\nu}$ coupled to energy-momentum tensor $t_{\mu\nu}$ of YM (glasma) fields through

$$g^{(\mathrm{b})}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}, \quad t_{\mu\nu}(x) = \frac{1}{N_c}\mathrm{Tr}\Big(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\Big);$$

② $\phi^{(\mathrm{b})} = \frac{\beta}{Q_s^4}h, \quad h(x) = \frac{1}{4N_c}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})$; ③ $\chi^{(\mathrm{b})} = \frac{\alpha}{Q_s^4}a, \quad a(x) = \frac{1}{4N_c}\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$

α, β, γ dimensionless and $O(1/N_c^2)$

Semi-holographic glasma evolution

IR-CFT: marginally deformed AdS/CFT

in Fefferman-Graham coordinates:

$$
\begin{aligned}
\chi(z,x) &= \frac{\alpha}{Q_s^4} a(x) + \cdots + z^4 \frac{4\pi G_5}{l^3} \mathcal{A}(x) + \mathcal{O}(z^6), \\
\phi(z,x) &= \frac{\beta}{Q_s^4} h(x) + \cdots + z^4 \frac{4\pi G_5}{l^3} \mathcal{H}(x) + \mathcal{O}(z^6), \\
G_{rr}(z,x) &= \frac{l^2}{z^2}, \\
G_{r\mu}(z,x) &= 0, \\
G_{\mu\nu}(z,x) &= \frac{l^2}{z^2} \Big(\underbrace{\eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)}_{g^{(b)}_{\mu\nu} = g_{(0)\mu\nu}} + \cdots + z^4 \big(\underbrace{\frac{4\pi G_5}{l^3}}_{2\pi^2/N_c^2} \mathcal{T}_{\mu\nu}(x) + P_{\mu\nu}(x) \big) \\
&\qquad + \mathcal{O}(z^4 \ln z) \Big),
\end{aligned}
$$

with $P_{\mu\nu} = \frac{1}{8} g_{(0)\mu\nu} \left(\left(\mathrm{Tr}\, g_{(2)}\right)^2 - \mathrm{Tr}\, g_{(2)}^2 \right) + \frac{1}{2} (g_{(2)}^2)_{\mu\nu} - \frac{1}{4} g_{(2)\mu\nu} \mathrm{Tr}\, g_{(2)}$

[de Haro, Solodukhin, Skenderis, CMP 217 (2001) 595]

Semi-holographic glasma evolution

Modified YM (glasma) field equations

$$\frac{\delta S}{\delta A_\mu(x)} = \frac{\delta S_{\rm YM}}{\delta A_\mu(x)} + \int d^4y \left(\frac{\delta W_{\rm CFT}}{\delta g^{(\rm b)}_{\alpha\beta}(y)} \frac{\delta g^{(\rm b)}_{\alpha\beta}(y)}{\delta A_\mu(x)} + \frac{\delta W_{\rm CFT}}{\delta \phi^{(\rm b)}(y)} \frac{\delta \phi^{(\rm b)}(y)}{\delta A_\mu(x)} + \frac{\delta W_{\rm CFT}}{\delta \chi^{(\rm b)}(y)} \frac{\delta \chi^{(\rm b)}(y)}{\delta A_\mu(x)} \right)$$

gives

$$D_\mu F^{\mu\nu} = \frac{\gamma}{Q_s^4} D_\mu \left(\hat{\mathcal{T}}^{\mu\alpha} F_\alpha{}^\nu - \hat{\mathcal{T}}^{\nu\alpha} F_\alpha{}^\mu - \frac{1}{2} \hat{\mathcal{T}}^\alpha_\alpha F^{\mu\nu} \right) + \frac{\beta}{Q_s^4} D_\mu \left(\hat{\mathcal{H}} F^{\mu\nu} \right) + \frac{\alpha}{Q_s^4} \left(\partial_\mu \hat{\mathcal{A}} \right) \tilde{F}^{\mu\nu}$$

with $\hat{\mathcal{T}}^{\alpha\beta} = \frac{\delta W_{\rm CFT}}{\delta g^{(\rm b)}_{\alpha\beta}} = \sqrt{-g^{(b)}}\mathcal{T}^{\alpha\beta}, \quad \hat{\mathcal{H}} = \frac{\delta W_{\rm CFT}}{\delta \phi^{(\rm b)}} = \sqrt{-g^{(b)}}\mathcal{H}, \quad \hat{\mathcal{A}} = \frac{\delta W_{\rm CFT}}{\delta \chi^{(\rm b)}} = \sqrt{-g^{(b)}}\mathcal{A}$

Total energy-momentum tensor of combined system

IR-CFT, like glasma EFT, interpreted as living in Minkowski space

covariant conservation equation for energy-momentum tensor

$$\nabla_\mu \mathcal{T}^{\mu\nu}(x) = -\frac{\beta}{Q_s^4}\,\mathcal{H}(x)\nabla^\nu h(x),$$

with *effective metric* $g^{(\mathrm{b})}_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu}(x)$

$\rightarrow$ nonconservation in Minkowski space, with driving forces derived from UV $t_{\mu\nu}[A]$

$$\partial_\mu \mathcal{T}^{\mu\nu} = -\frac{\beta}{Q_s^4}\,\mathcal{H}\, g^{\mu\nu}_{(\mathrm{b})}[t]\,\partial_\mu h - \mathcal{T}^{\alpha\nu}\Gamma^\gamma_{\alpha\gamma}[t] - \mathcal{T}^{\alpha\beta}\Gamma^\nu_{\alpha\beta}[t]$$

with $\Gamma^\mu_{\nu\rho}[t] = \frac{\gamma}{2Q_s^4}\left(\partial_\nu t^\mu{}_\rho + \partial_\rho t^\mu{}_\nu - \partial^\mu t_{\nu\rho}\right) + \mathcal{O}\left(t^2\right)$

Total conserved energy-momentum tensor in Minkowski space ($\partial_\mu T^{\mu\nu} = 0$):

$$T^{\mu\nu} = t^{\mu\nu} + \mathcal{T}^{\mu\nu} + \text{hard-soft interaction terms}$$

(but $\mathcal{T}^{\mu\nu}$ not purely soft, contains also some hard-soft pieces through $g^{\mu\nu}_{(\mathrm{b})}[t]$)

Total energy-momentum tensor of combined system

Temporarily replacing Minkowski metric $\eta_{\mu\nu}$ by $g^{\rm YM}_{\mu\nu}$:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g^{\rm YM}}}\left[\frac{\delta S_{\rm YM}}{\delta g^{\rm YM}_{\mu\nu}(x)} + \int d^4y \left(\frac{\delta W_{\rm CFT}}{\delta g^{(b)}_{\alpha\beta}(y)}\frac{\delta g^{(b)}_{\alpha\beta}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)} + \frac{\delta W_{\rm CFT}}{\delta \phi^{(b)}(y)}\frac{\delta \phi^{(b)}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)} + \frac{\delta W_{\rm CFT}}{\delta \chi^{(b)}(y)}\frac{\delta \chi^{(b)}(y)}{\delta g^{\rm YM}_{\mu\nu}(x)}\right)\right]$$

At $g^{\rm YM}_{\mu\nu} = \eta_{\mu\nu}$, this gives

$$T^{\mu\nu} = t^{\mu\nu} + \hat{\mathcal{T}}^{\mu\nu} - \frac{\gamma}{Q_s^4 N_c}\hat{\mathcal{T}}^{\alpha\beta}\left[{\rm Tr}(F_\alpha{}^\mu F_\beta{}^\nu) - \frac{1}{2}\eta_{\alpha\beta}{\rm Tr}(F^{\mu\rho}F^\nu{}_\rho) + \frac{1}{4}\delta^\mu_{(\alpha}\delta^\nu_{\beta)}{\rm Tr}(F^2)\right] - \frac{\beta}{Q_s^4 N_c}\hat{\mathcal{H}}\,{\rm Tr}(F^{\mu\alpha}F^\nu{}_\alpha) - \frac{\alpha}{Q_s^4}\eta^{\mu\nu}\hat{\mathcal{A}}\,a$$

Satisfies $\boxed{\partial_\mu T^{\mu\nu} = 0}$ w.r.t. flat Minkowski space

Simple (nonexpanding) test case

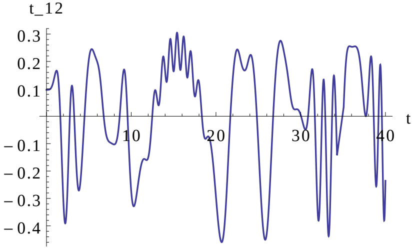
First test with dimensionally reduced (spatially homogeneous) YM fields $A^a_\mu(t)$ which already have nontrivial (in general chaotic) dynamics

SU(2) gauge fields, $a = 1, 2, 3$, using temporal gauge $A^a_0 = 0$, $g = 1$

$$D^\mu F_{\mu j} = 0 \Rightarrow \ddot{A}^a_j - A^a_i A^b_i A^b_j + A^a_j A^b_i A^b_i = 0,$$

Gauss law
$D^\mu F^d_{\mu 0} = 0 \Rightarrow \epsilon^{dea} A^{ei} \dot{A}^a_i = 0$, satisfied by initial conditions $A(t_0) = 0$ or $\dot{A}(t_0) = 0$

General case: 9 degrees of freedom with chaotic dynamics
conserved, traceless energy-momentum tensor with $\varepsilon = const.$, $t^{0i} = 0$,
but otherwise wildly fluctuating:



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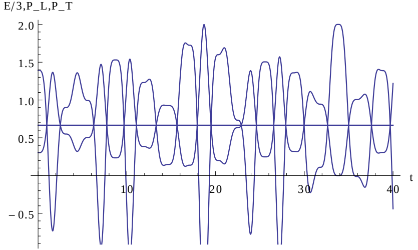
Color-spin-locked: $A^a_i \propto \delta^a_i$, 3 degrees of freedom
$\rightarrow$ diagonal but anisotropic traceless energy-momentum tensor
e.g., only one direction singled out:

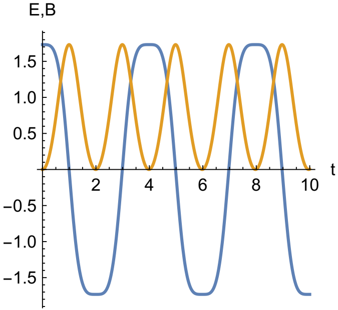


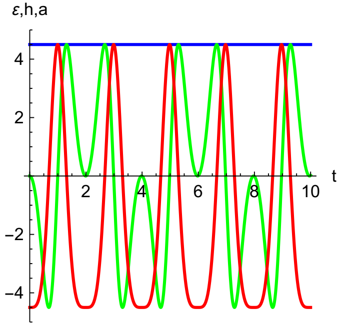
Simplest test case

$\exists$ a nontrivial solution with homogeneous isotropic energy-momentum tensor ($p = \varepsilon/3$) by homogeneous and isotropic color-spin locked oscillations $A_0^a = 0, \quad A_i^a = \delta_i^a f(t)$

$$f(t) = C\,\mathrm{sn}(C(t - t_0)| - 1) \qquad \text{(Jacobi elliptic function sn)}$$

$$E_i^a = \delta_i^a f', \quad B_i^a = \delta_i^a f^2 \qquad \varepsilon = const.,\ h = -\tfrac{1}{2}(\mathbf{E}^a \cdot \mathbf{E}^a - \mathbf{B}^a \cdot \mathbf{B}^a),\ a = -\mathbf{E}^a \cdot \mathbf{B}^a$$





Simplest test case – gravitational solution

Switching off $\alpha = 0 = \beta$ (otherwise also nontrivial sources for dilaton and axion!)

IR-CFT $\hat{\mathcal{T}}^{\mu\nu} = \mathrm{diag}(\hat{\mathcal{E}}, \hat{\mathcal{P}}, \hat{\mathcal{P}}, \hat{\mathcal{P}})$ to be determined by gravitational problem with

$$g^{(\mathrm{b})}_{\mu\nu} = \eta_{\mu\nu} + \frac{\gamma}{Q_s^4} t_{\mu\nu} = \mathrm{diag}\left(-1 + \frac{3\gamma}{Q_s^4} p(t), 1 + \frac{\gamma}{Q_s^4} p(t), 1 + \frac{\gamma}{Q_s^4} p(t), 1 + \frac{\gamma}{Q_s^4} p(t)\right)$$

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Analytic result:

$$\hat{\mathcal{E}} := \hat{\mathcal{T}}^{tt} = \frac{N_c^2}{2\pi^2}\left(\frac{3c}{4r_{(0)}(t)^2 v'_{(0)}(t)} + \frac{3r'_{(0)}(t)^4}{16 r_{(0)}(t)^6 v'_{(0)}(t)^5}\right),$$

$$\hat{\mathcal{P}} := \hat{\mathcal{T}}^{xx} = \hat{\mathcal{T}}^{yy} = \hat{\mathcal{T}}^{zz} = \frac{N_c^2}{2\pi^2}\left\{\frac{c v'_{(0)}(t)}{4 r_{(0)}(t)^2} + \frac{r'_{(0)}(t)^2\left[4 r_{(0)}(t) r'_{(0)}(t) v''_{(0)}(t) + r_{(0)}(t)\left(5 r'_{(0)}(t)^2 - 4 r_{(0)}(t) r''_{(0)}(t)\right)\right]}{16 r_{(0)}(t)^6 v'_{(0)}(t)^4}\right\},$$

with

$$r_{(0)}(t) = \sqrt{1 + (\gamma/Q_s^4)p(t)}\,, \qquad v'_{(0)}(t) = \sqrt{\frac{1 - (\gamma/Q_s^4)3p(t)}{1 + (\gamma/Q_s^4)p(t)}}$$

because of isotropy and homogeneity, gravity solution locally diffeomorphic to AdS-Schwarzschild with integration constant c corresponding to mass of black hole

Convergence of iterations

Coupled glasma equation of test case is 4th order nonlinear ODE
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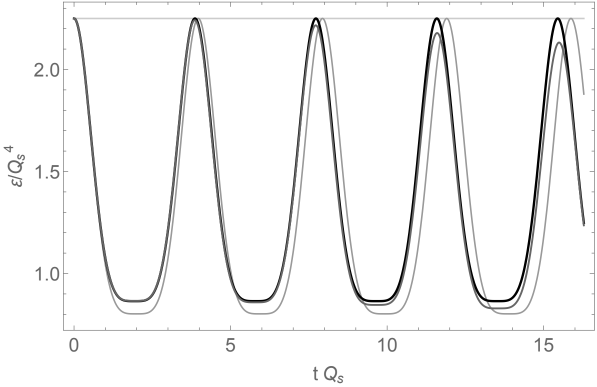
Coupled glasma equation of test case is 4th order nonlinear ODE
— no reasonable solutions found directly —
but iterative solution converges very quickly:



oscillating YM energy (but constant total energy)
UV not able to give off energy to IR permanently because of isotropy and homogeneity:
gravity dual does not have propagating degrees of freedom!

First tests with thermalization/black hole formation

Turning on coupling β (glueball operator $\leftrightarrow$ bulk dilation)
allows black hole formation (dilaton excitations falling behind horizon)
also in homogeneous and isotropic situation

needs numerical GR code in AdS space

until recently we had prohibitive numerical difficulties in finding convergent iteration

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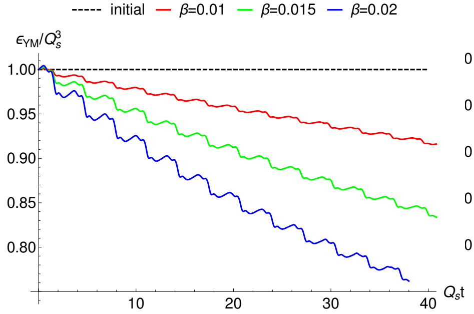
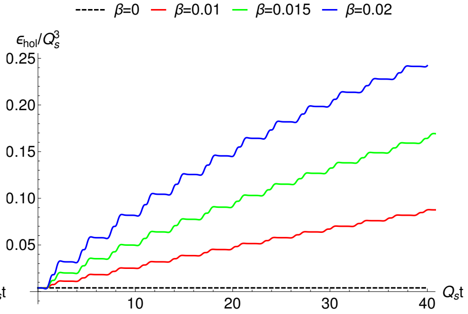
[C. Ecker, A. Mukhopadhyay, F. Preis, AR, A. Soloviev, arXiv:1806.01850]
success in slightly simpler AdS_4/QFT_3 duality with analogous setup and only β turned on
$\rightarrow$ talk by Christian Ecker





Next steps

Work in progress:

- AdS_5
- evolution of single flux tubes
- boost-invariant expansion
- inclusion of fluctuations (2PI action)

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Remainder of talk:

- Democratic extension of metric coupling
  S. Banerjee, N. Gaddam and A. Mukhopadhyay, Illustrated study of the semiholographic nonperturbative framework, Phys. Rev. D95 (2017) 066017, [1701.01229]
- Consequences for assumed thermal equilibrium and hydrodynamics

Hybrid thermodynamics & hydrodynamics

[A. Kurkela, A. Mukhopadhyay, F. Preis, AR, A. Soloviev, arXiv:1805.05213]

Late-time hydrodynamic behavior (assuming thermalization and
including only universally present tensorial couplings):
each sector described by dynamics with own *effective metric*, which encodes interactions with the other sector which deforms it through its energy-momentum tensor

("democratic coupling" necessary when diluted glasma enters quantum regime
– permits second tensorial coupling γ')

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \gamma\,\eta_{\mu\alpha}\tilde{t}^{\alpha\beta}\sqrt{-\tilde{g}}\eta_{\beta\nu} + \gamma'\,\eta_{\mu\nu}\eta_{\alpha\beta}\tilde{t}^{\alpha\beta}\sqrt{-\tilde{g}}, \\ \tilde{g}_{\mu\nu} &= \eta_{\mu\nu} + \gamma\,\eta_{\mu\alpha}t^{\alpha\beta}\sqrt{-g}\eta_{\beta\nu} + \gamma'\,\eta_{\mu\nu}\eta_{\alpha\beta}t^{\alpha\beta}\sqrt{-g}, \end{aligned} \quad (1)$$

$\eta_{\mu\nu}$ physical metric of total system (NB: all in same topological space!)

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$\eta_{\mu\nu}$ physical metric of total system (NB: all in same topological space!)

Total energy-momentum conserved w.r.t. Minkowski background: $\boxed{\partial_\mu T^{\mu\nu} = 0}$

$$\begin{aligned} T^{\mu\nu} &= \frac{1}{2}(t^{\mu}{}_{\nu}\sqrt{-g} + \tilde{t}^{\mu}{}_{\nu}\sqrt{-\tilde{g}})\eta^{\rho\nu} \\ &+\frac{1}{2}\eta^{\mu\rho}(t_{\rho}{}^{\nu}\sqrt{-g} + \tilde{t}_{\rho}{}^{\nu}\sqrt{-\tilde{g}}) \\ &-\frac{1}{2}\left[\gamma\,(t^{\rho\alpha}\sqrt{-g})\eta_{\alpha\beta}(\tilde{t}^{\beta\sigma}\sqrt{-\tilde{g}})\eta_{\sigma\rho} + \gamma'\,(t^{\alpha\beta}\sqrt{-g})\eta_{\alpha\beta}(\tilde{t}^{\sigma\rho}\sqrt{-\tilde{g}})\eta_{\sigma\rho}\right] \end{aligned}$$

Hybrid thermodynamics

Equilibrium solutions require $\boxed{\gamma > 0}$ for causality,

$$g_{\mu\nu} = \mathrm{diag}(-a^2, b^2, b^2, b^2), \quad \tilde{g}_{\mu\nu} = \mathrm{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{b}^2)$$

have effective lightcone speeds $v^{l.c.} = a/b$, $v^{l.c.} = \tilde{a}/\tilde{b} < 1$

NB: total system has Minkowski metric with unit lightcone speed!

(cp. thermal masses: they reduce propagation speed, but less strictly)

$$t^{\mu\nu} = (\epsilon_1(T_1) + P_1(T_1))u^\mu u^\nu + P_1(T_1)g^{\mu\nu}, \quad \text{with} \quad u^\mu = (1/a, 0, 0, 0)$$
$$\tilde{t}^{\mu\nu} = (\epsilon_2(T_2) + P_2(T_2))\tilde{u}^\mu \tilde{u}^\nu + P_2(T_2)\tilde{g}^{\mu\nu}, \quad \text{with} \quad \tilde{u}^\mu = (1/\tilde{a}, 0, 0, 0)$$

(1) $\Rightarrow b^2 - a^2 = \gamma \left(\frac{\epsilon_2(T_2)}{\tilde{a}^2} + \frac{P_2(T_2)}{\tilde{b}^2} \right) \tilde{a}\tilde{b}^3 \geq 0$

$\tilde{b}^2 - \tilde{a}^2 = \gamma \left(\frac{\epsilon_1(T_1)}{a^2} + \frac{P_1(T_1)}{b^2} \right) ab^3 \geq 0$ if $\gamma > 0$

Temperatures of the subsystems given by

$T_1^{-1} = \int_0^\beta \sqrt{-g_{00}}\, d\tau = a\beta = a\mathcal{T}^{-1}$,

$T_2^{-1} = \tilde{a}\mathcal{T}^{-1}$,

with $\mathcal{T}$ the temperature of the total system with Minkowski metric

Moreover: (1) $\Rightarrow \mathcal{T}\mathcal{S} = T_1 s_1(T_1) ab^3 + T_2 s_2(T_2)\tilde{a}\tilde{b}^3 = \mathcal{T}\left[s_1(T_1)b^3 + s_2(T_2)\tilde{b}^3 \right]$

thermodynamically consistent

Hybrid thermodynamics

Equilibrium solutions   require $\boxed{\gamma > 0}$ for causality, $\boxed{r \equiv -\gamma'/\gamma > 1}$ for UV-completeness

$$g_{\mu\nu} = \mathrm{diag}(-a^2, b^2, b^2, b^2), \quad \tilde{g}_{\mu\nu} = \mathrm{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{b}^2)$$

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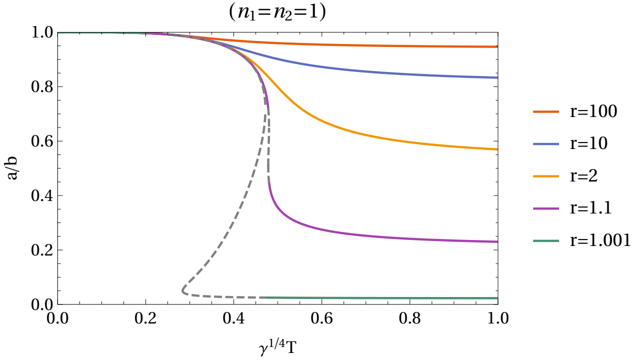
for example: coupling *two identical conformal systems* with $P_i = n_i T^4$:

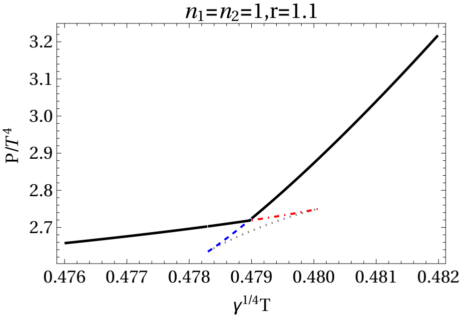


multivalued solutions below $r_c = 1.1145\ldots$: **first-order phase transition**

Hybrid thermodynamics

mutual effective metric coupling of two identical conformal systems

- $1 < r \equiv -\gamma'/\gamma < r_c = 1.1145$: first-order phase transition



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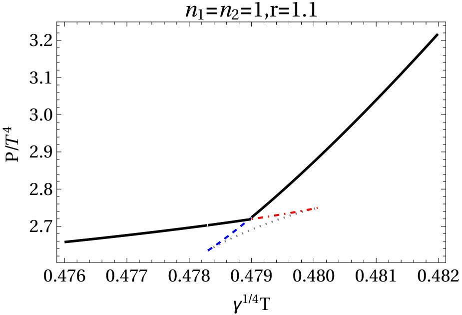
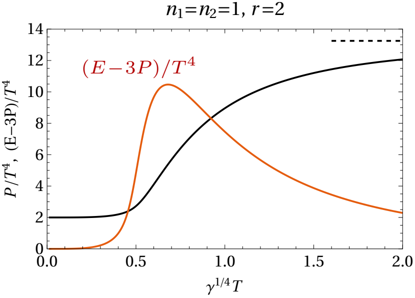


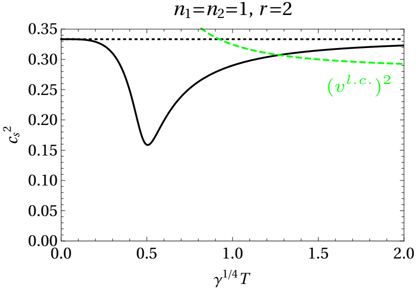
- 2nd order phase transition at critical end point $r = r_c = 1.1145\ldots$
  with critical exponent $\alpha = 2/3$ (specific heat $C_V \sim |T - T_c|^{-\alpha}$)
  (larger than Ising and polymers,
  but close to deconfinement matrix model of Pisarski and Skokov with $\alpha = 0.6$)

Hybrid thermodynamics

mutual effective metric coupling of two identical conformal systems

- $r \equiv -\gamma'/\gamma > r_c = 1.1145$: crossover region





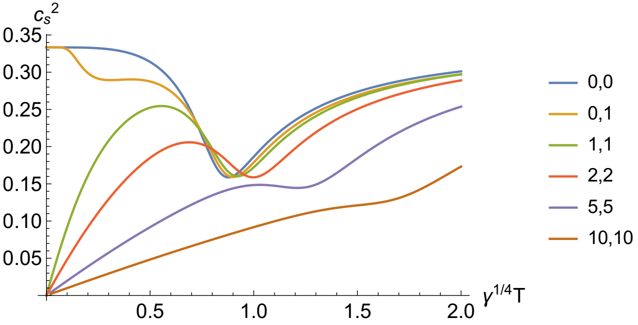
Increase of density of d.o.f.'s from spatial rescaling

Conformal behavior also at $\gamma^{1/4}\mathcal{T} \to \infty$!

NB: acoustic mode from coherent fluctuations involving both sectors

Massive subsystems

Speed of sound (squared) for two systems
with one or both replaced by a gas of free massive bosons (crossover region, $r = 2$)



(Values given in the plot legend refer to the two masses in units of $\gamma^{-1/4}$)

Hybrid hydrodynamics

Coupling individual hydrodynamic descriptions of two sectors

- $\nabla_\mu t^\mu{}_\nu = 0$, $\tilde{\nabla}_\mu \tilde{t}^\mu{}_\nu = 0$ with mutual metric coupling
- gradient expansion of energy-momentum tensors $t^\mu{}_\nu$, $\tilde{t}^\mu{}_\nu$, $T^{\mu\nu}$
  with given transport coefficients $\eta_{1,2}$ (shear viscosity), $\zeta_{1,2}$ (bulk viscosity)

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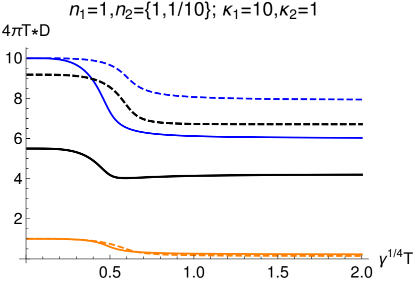
E.g. ❶ $\eta_1/s \equiv \kappa_1/4\pi = 10/4\pi$ (weakly coupled), ❷ $\eta_2/s = 1/4\pi$ (strongly coupled)



D: shear diffusion ($TD \equiv \eta/s$)

full lines: equal pressure contributions;

dashed lines: weakly coupled system dominates pressure

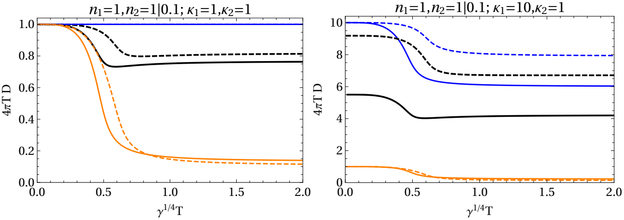
blue, orange: 2 eigenmodes in two-fluid system (fluctuation equations)

black: overall conserved momentum tensor (Kubo formula)

- overall shear viscosity interpolates those of subsystems, decreases with mutual interaction strength γ

Hybrid hydrodynamics - Shear sector

Comparing equal and unequal $\kappa_{1,2} = 4\pi\eta_{1,2}/s$:



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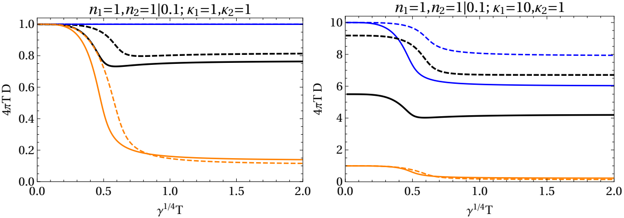
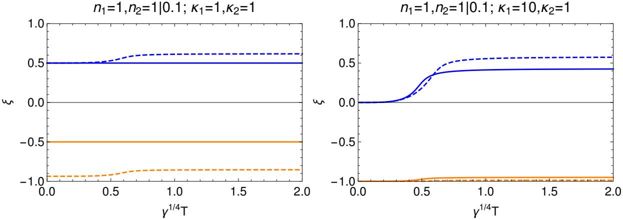


- relation between velocity fields: $\xi \equiv \frac{2}{\pi}\arctan(\tilde{\nu}/\nu)$



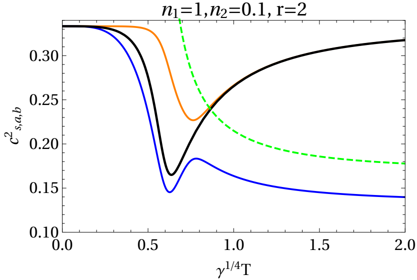
$\xi = 0$ or $\xi = \pm 1$: mode is carried only by subsystem 1 or 2, respectively;

$\xi = 0.5$ or $\xi = -0.5$: exactly equal amplitudes with equal or opposite phase

Hybrid hydrodynamics - Sound sector

❶ $\eta_1/s \equiv \kappa_1/4\pi = 10/4\pi$ (weakly coupled), ❷ $\eta_2/s = 1/4\pi$ (strongly coupled)

Sound velocities — Sound attenuations ($\gamma_{a,b,s} = \Gamma_{a,b,s} k^2$)



(here $\zeta = 0$)

blue, orange: 2 eigenmodes in two-fluid system (fluctuation equations)

parallel and anti-parallel velocity fields in subsystems with phase change at crossover

black: from overall conserved momentum tensor ($c_s^2 = dP/dE$, Γ_s from Kubo formula)

dashed green: lightcone speed limit $\tilde{v}$

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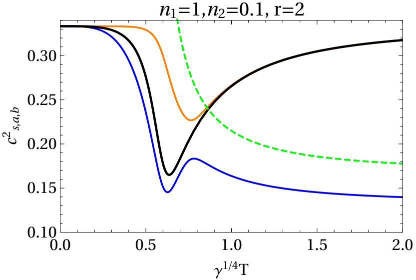

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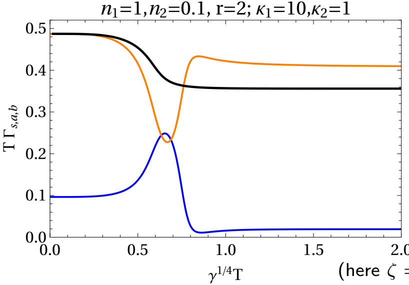

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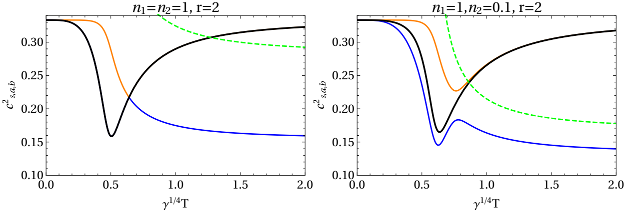
Since $\gamma_{a,b,s} = \Gamma_{a,b,s} k^2 \to 0$ for $k \to 0$:

no equilibration in homogeneous & isotropic limit!

$\Rightarrow$ needs other couplings (e.g., dilaton↔Lagrangian density)
as also found in the case of semi-holographic toy model with classical YM equations

Hybrid hydrodynamics - Sound sector

Comparing equal and unequal densities:



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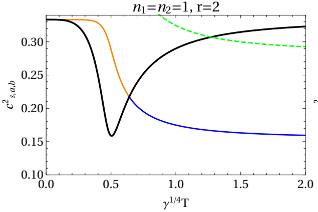

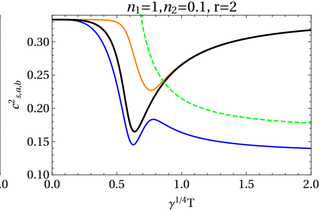

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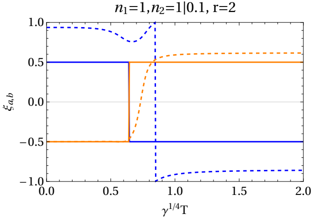

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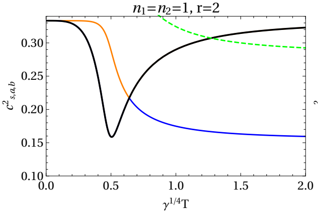
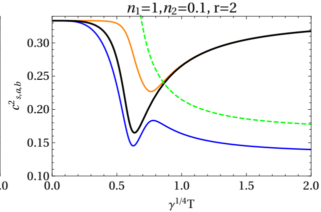
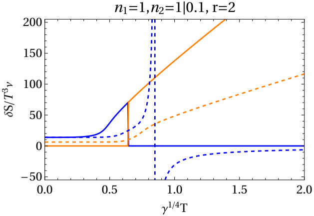




- entropy content $\delta S/(\mathcal{T}^3 v)$ (dashed=unequal systems)



entropy predominantly in equal-phase mode; other mode more quasiparticle-like

Coupling of hydro model and kinetic theory model

❶ Kinetic theory model of massless particles in RTA with $\tau(T_1) = \frac{5\kappa_1}{4\pi T_1} = \frac{5\eta_1}{T_1 s_1}$

❷ Strongly coupled hydro model (embedded in IS with extremely small relaxation time)

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Anderson-Wittig equation:

$$\left(\partial_t + \frac{p^i}{p^0}\partial_i\right)\delta f - \delta\Gamma^i_{\beta\gamma}\frac{p^\beta p^\gamma}{p^0}\frac{\partial}{\partial p^i}f_0 = -\frac{a}{\tau}(\delta f - \delta f^{eq}), \quad a = \sqrt{-g_{00}}$$

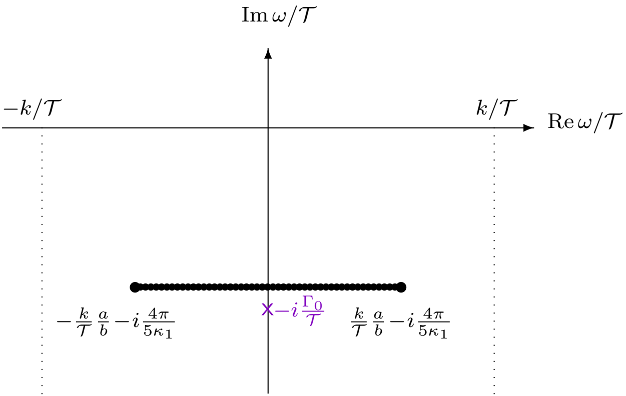
besides modes in coupled system that agree with bi-hydro case:

$\exists$ infinitely many fluctuations with $\delta t^{\mu\nu} \propto \int \frac{\mathrm{d}^3 p}{p_0} p^\mu p^\nu \delta f(\mathbf{x},\mathbf{p},t) = 0$ which decouple from strongly coupled sector $\rightarrow$

$$\omega = \frac{a}{b}\frac{\mathbf{k}\cdot\mathbf{p}}{p} - i\frac{a}{\tau(T_1)} = \frac{a}{b}k\cos\theta - i\frac{1}{\tau(\mathcal{T})}$$

Coupling of hydro model and kinetic theory model

Analytic structure of the response function in the kinetic sector:



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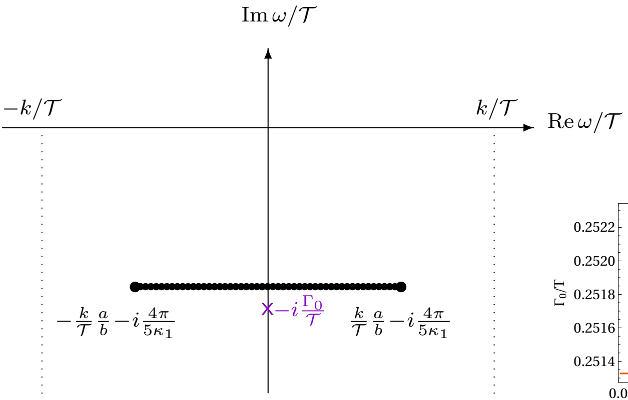

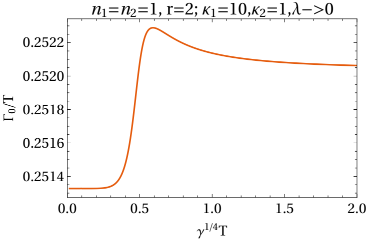

— Interactions between subsystems reduce lightcone speed to a/b

— Pure relaxational mode at $k = 0$ moved from branch cut down into second Riemann sheet

Conclusions

Pure gauge/gravity thermalization treats infinite coupling limit

Aim: hybrid $_{\text{qualitative}}$ description with less strongly coupled UV sector

- Semi-holographic framework of Iancu and Mukhopadhyay proposes to combine LO glasma evolution with thermalization of soft degrees of freedom in AdS/CFT
- New scheme has conserved total energy-momentum tensor (in Minkowski space)
  formal proof + numerical verification in simple test case
- Toy model now also with actual black hole formation $\leftrightarrow$ thermalization!
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  formal proof + numerical verification in simple test case
- Toy model now also with actual black hole formation $\leftrightarrow$ thermalization!
  (talk of Christian Ecker)
- Late-time behavior analysed within simplest effective bi-metric model
  - Interesting phase structure
  - Interesting bi-hydrodynamics
  - Complete equilibration needs interactions beyond ultralocal mutual effective metric couplings (scalars, fluctuations, ...)

Action formulation of democratic metric coupling

When subsystems can be described by action principle, total action for democratic metric coupling is given by

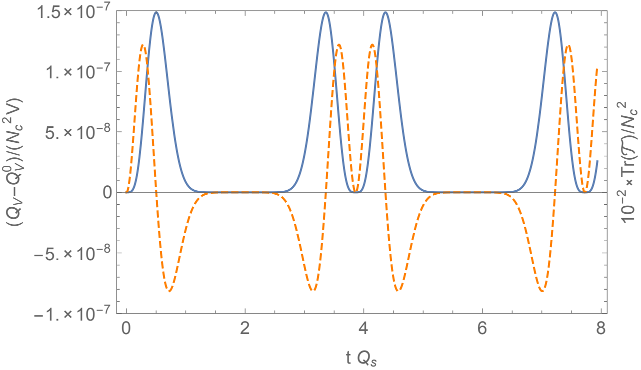
$$\begin{aligned} S[\phi, \tilde{\phi}, g_{\mu\nu}, \tilde{g}_{\mu\nu}, g^{(\mathrm{B})}_{\mu\nu}] \quad = \quad & \int \mathrm{d}^d x \sqrt{-g}\mathcal{L}_1[\phi, g_{\mu\nu}] + \int \mathrm{d}^d x \sqrt{-\tilde{g}}\mathcal{L}_2[\tilde{\phi}, \tilde{g}_{\mu\nu}] \\ & + \frac{1}{2\gamma} \int \mathrm{d}^d x \sqrt{-g^{(\mathrm{B})}} \left(g_{\mu\alpha} - g^{(\mathrm{B})}_{\mu\alpha}\right) g^{(\mathrm{B})\,\alpha\beta} \left(\tilde{g}_{\beta\nu} - g^{(\mathrm{B})}_{\beta\nu}\right) g^{(\mathrm{B})\,\nu\mu} \\ & + \frac{1}{2\gamma} \frac{\gamma'}{d\gamma' - \gamma} \int \mathrm{d}^d x \sqrt{-g^{(\mathrm{B})}} \left(g_{\mu\nu} g^{(\mathrm{B})\,\mu\nu} - d\right) \left(\tilde{g}_{\alpha\beta} g^{(\mathrm{B})\,\alpha\beta} - d\right). \end{aligned}$$

$g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ appear as auxiliary fields and the interaction terms of the two subsystems represented by the last two lines of the above action merely implement the algebraic relations between the effective metrics and the subsystem energy-momentum tensors.
If we vary with respect to ϕ and $\tilde{\phi}$ first, we evidently obtain the two subsystem dynamical equations in the respective effective metrics, since the last two lines are independent of ϕ and $\tilde{\phi}$.
Individual subsystem actions diffeomorphism invariant $\rightarrow \nabla_\mu t^\mu_\nu = 0$ and $\tilde{\nabla}_\mu \tilde{t}^\mu_\nu = 0$ on-shell.

Energy exchanges but no thermalization

Isotropic and homogeneous toy model with only tensor coupling:

entropy (area of the black hole) is conserved,
canonical charge of the black hole changes only according to trace anomaly:



blue: canonical charge (thermal energy) returns to same value at stationary points (at different extrema of $\varepsilon^{\mathrm{YM}}$)
orange: $\mathrm{Tr}\,\mathcal{T}$