

# The holographic Pomeron and low-x physics

Miguel S. Costa

Faculdade de Ciências da Universidade do Porto

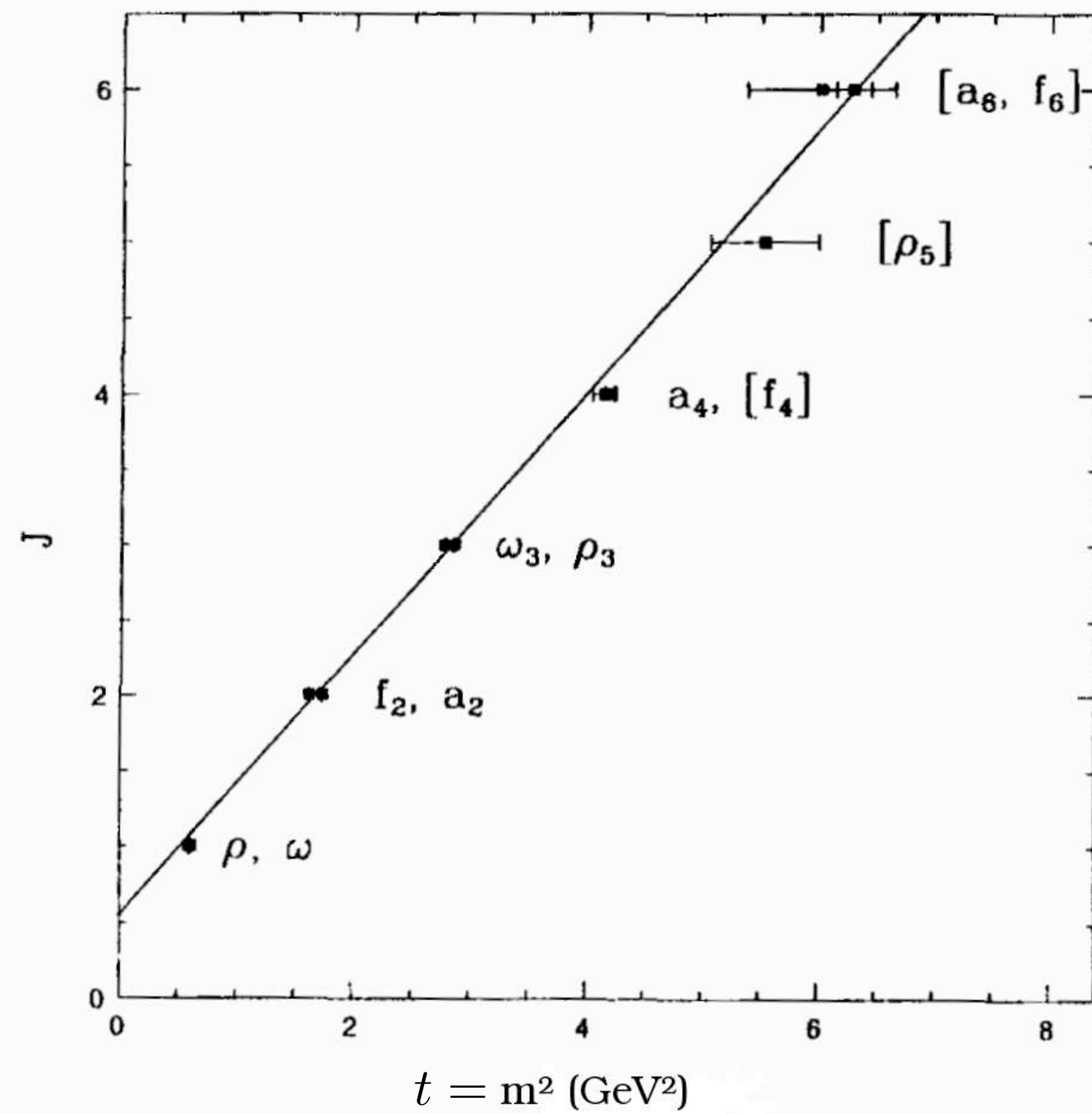
Holography and Extreme Chromodynamics - HoloQuark 2018  
Santiago de Compostela, July 2017

# Regge behaviour in QCD

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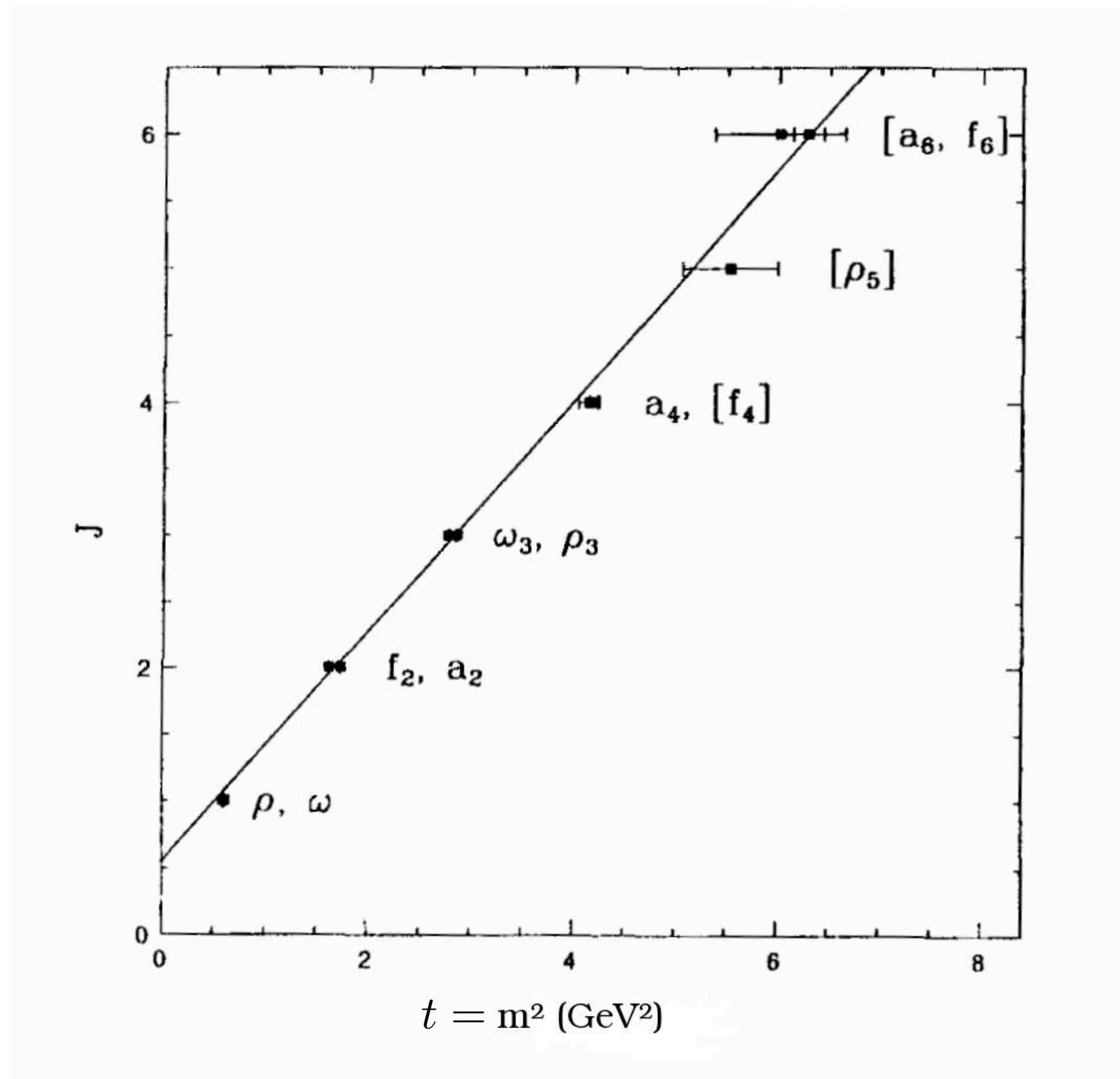
- Hadronic resonances fall in linear trajectories



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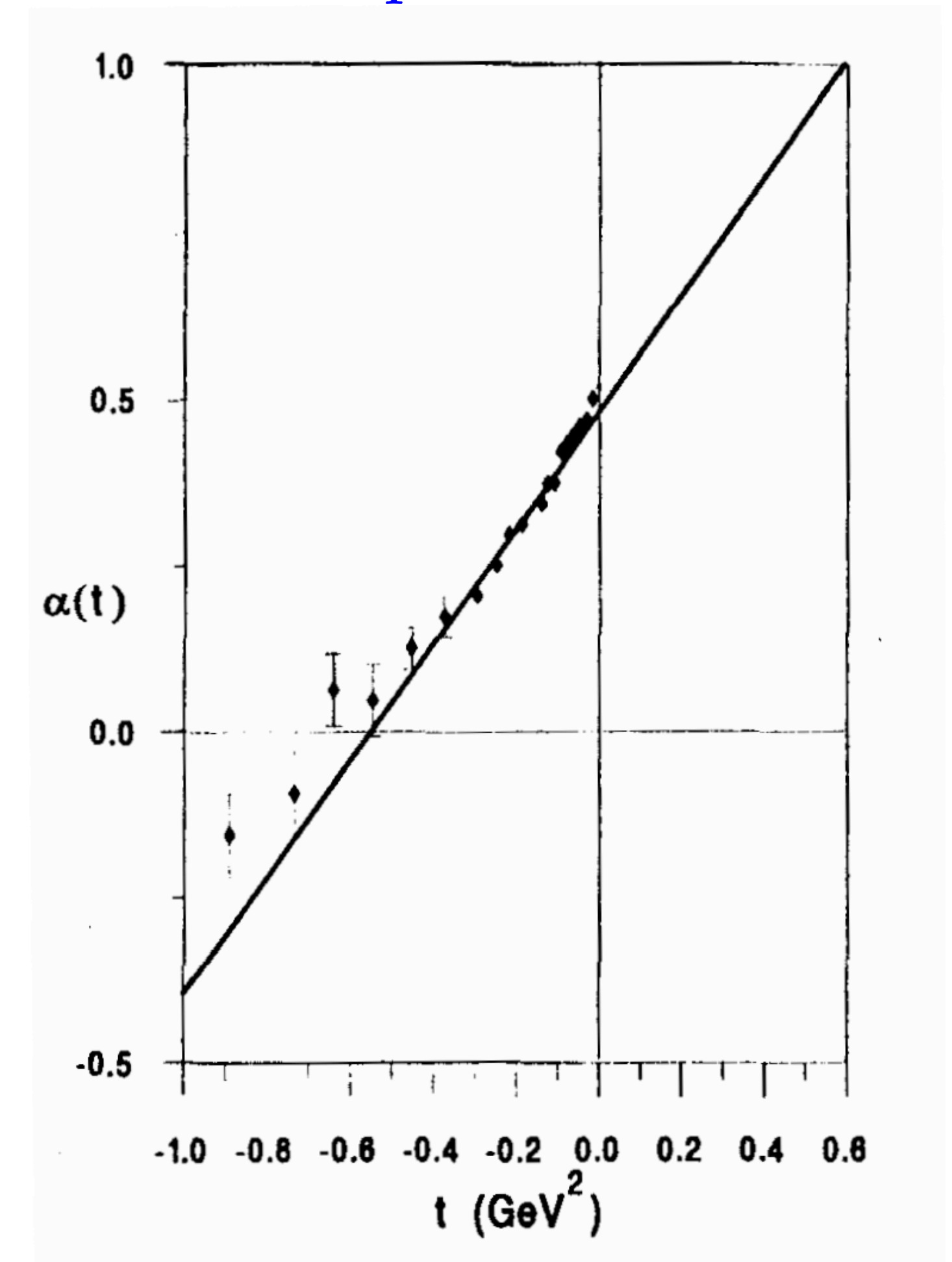
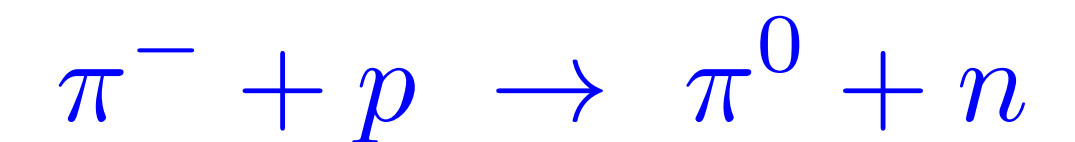
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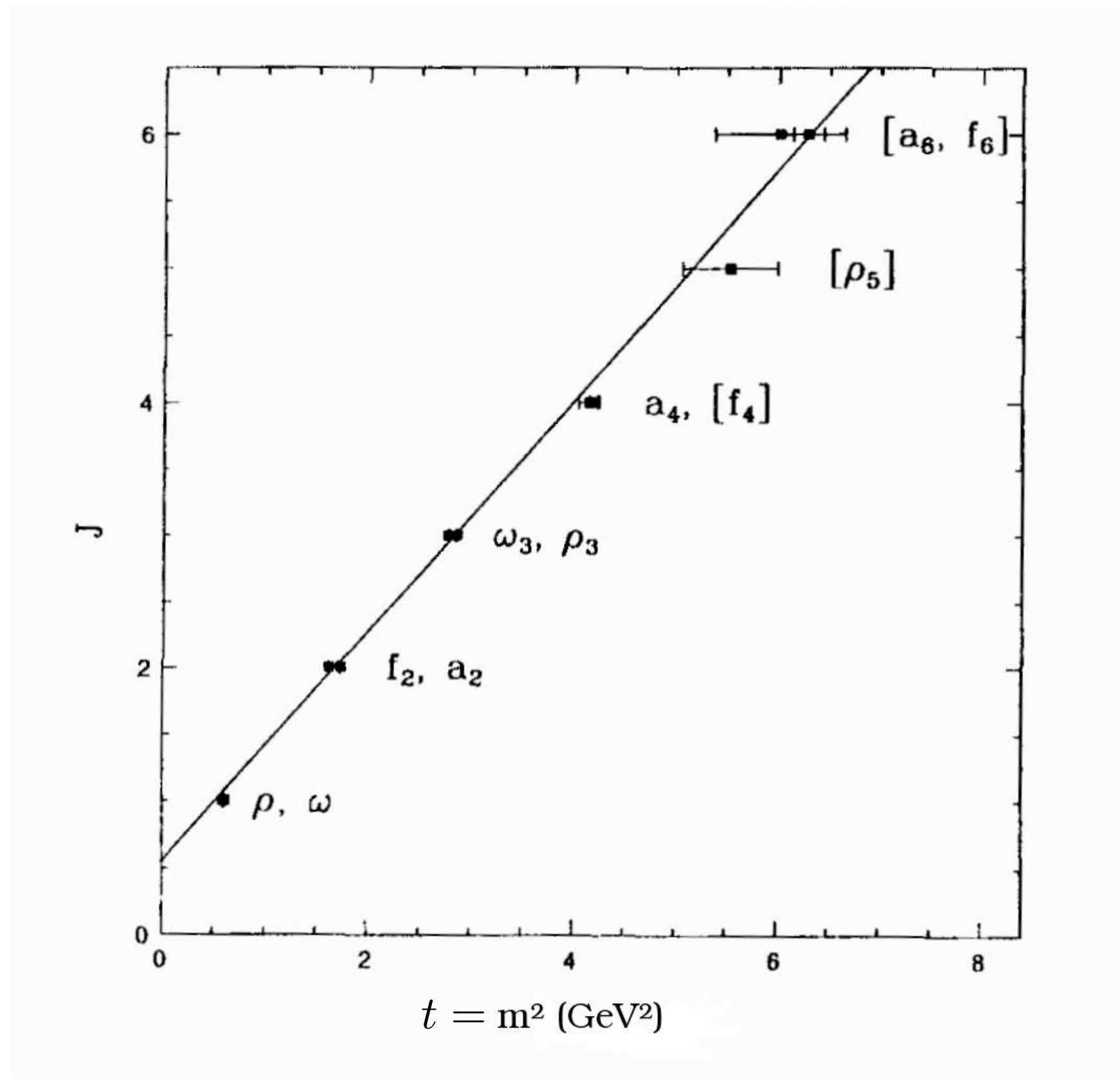
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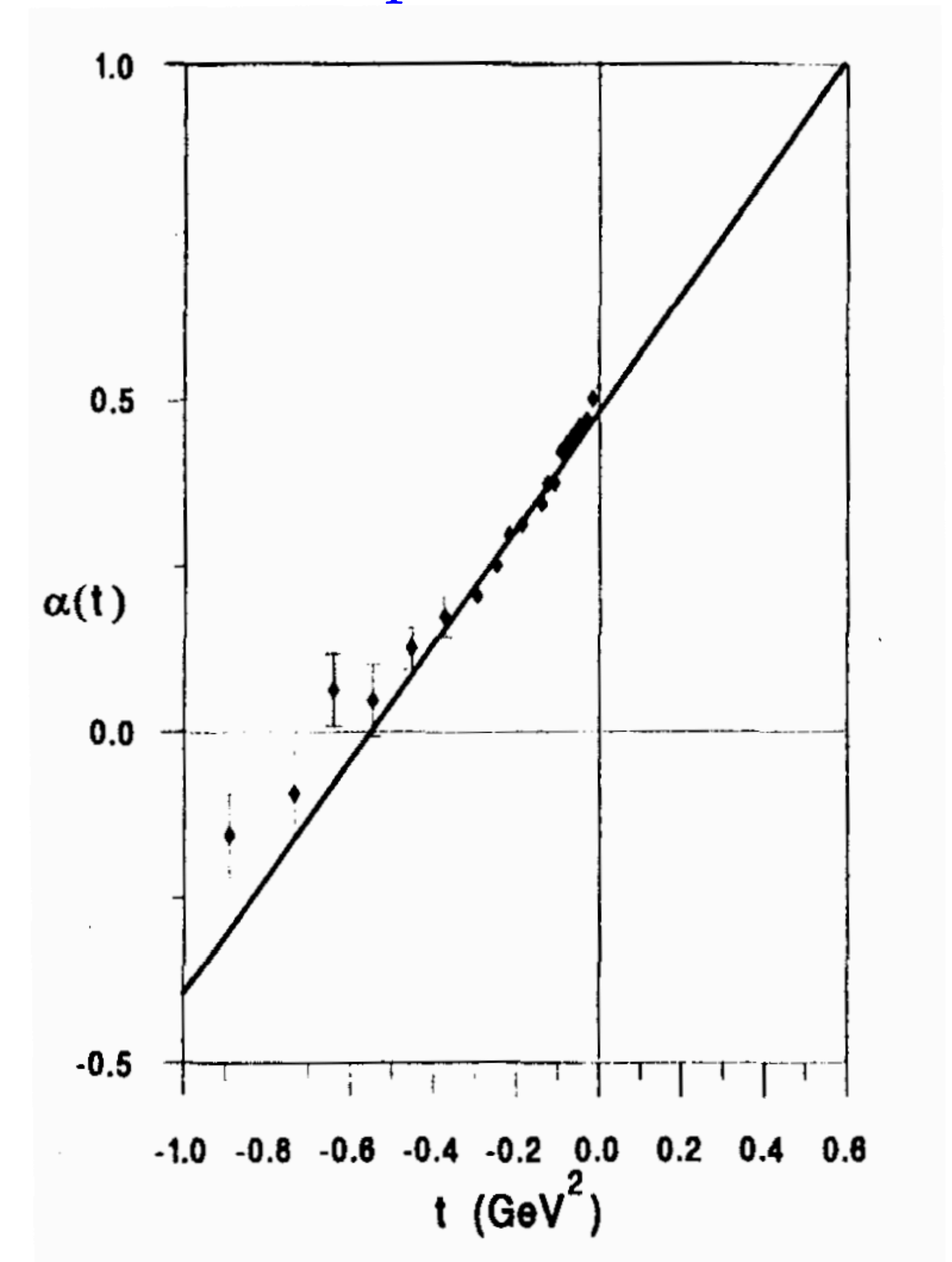
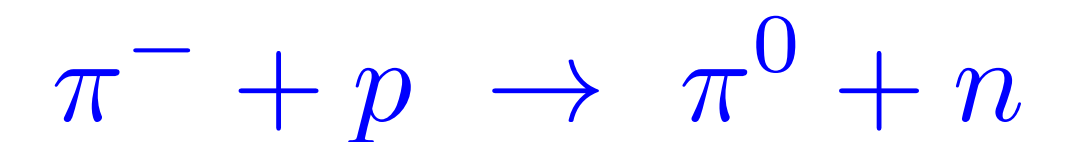
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Total cross section

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## **(Soft) Pomeron trajectory**

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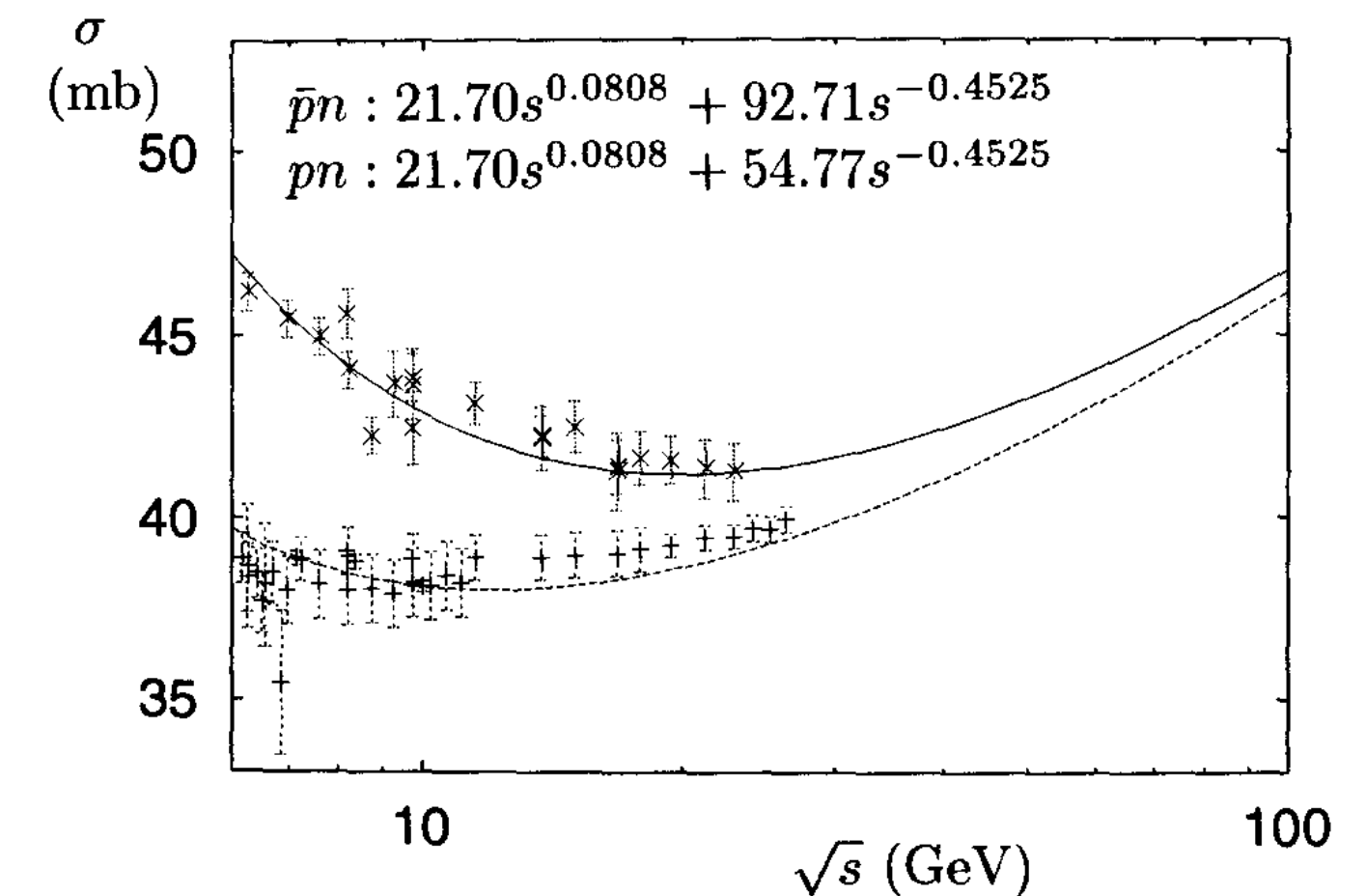
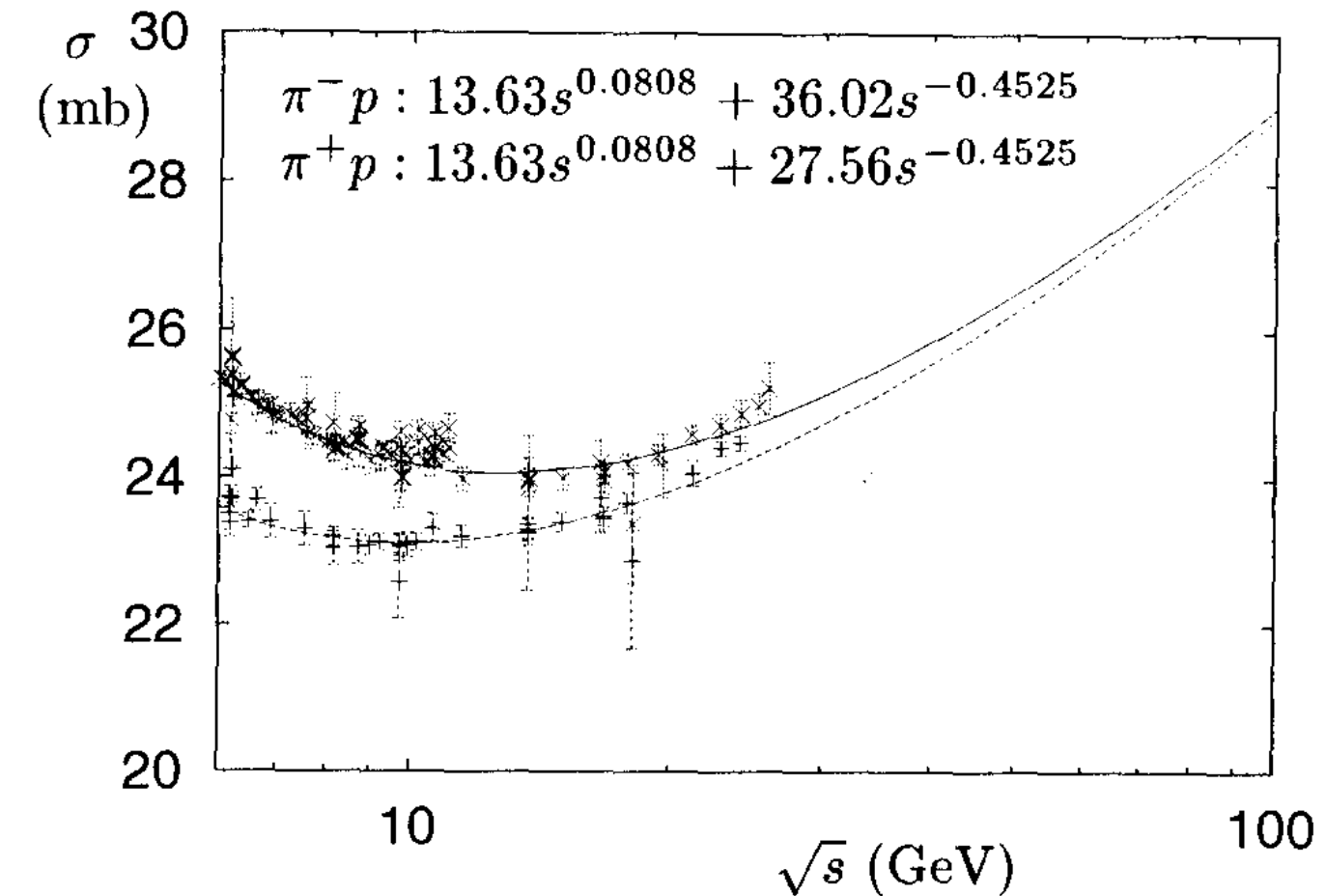
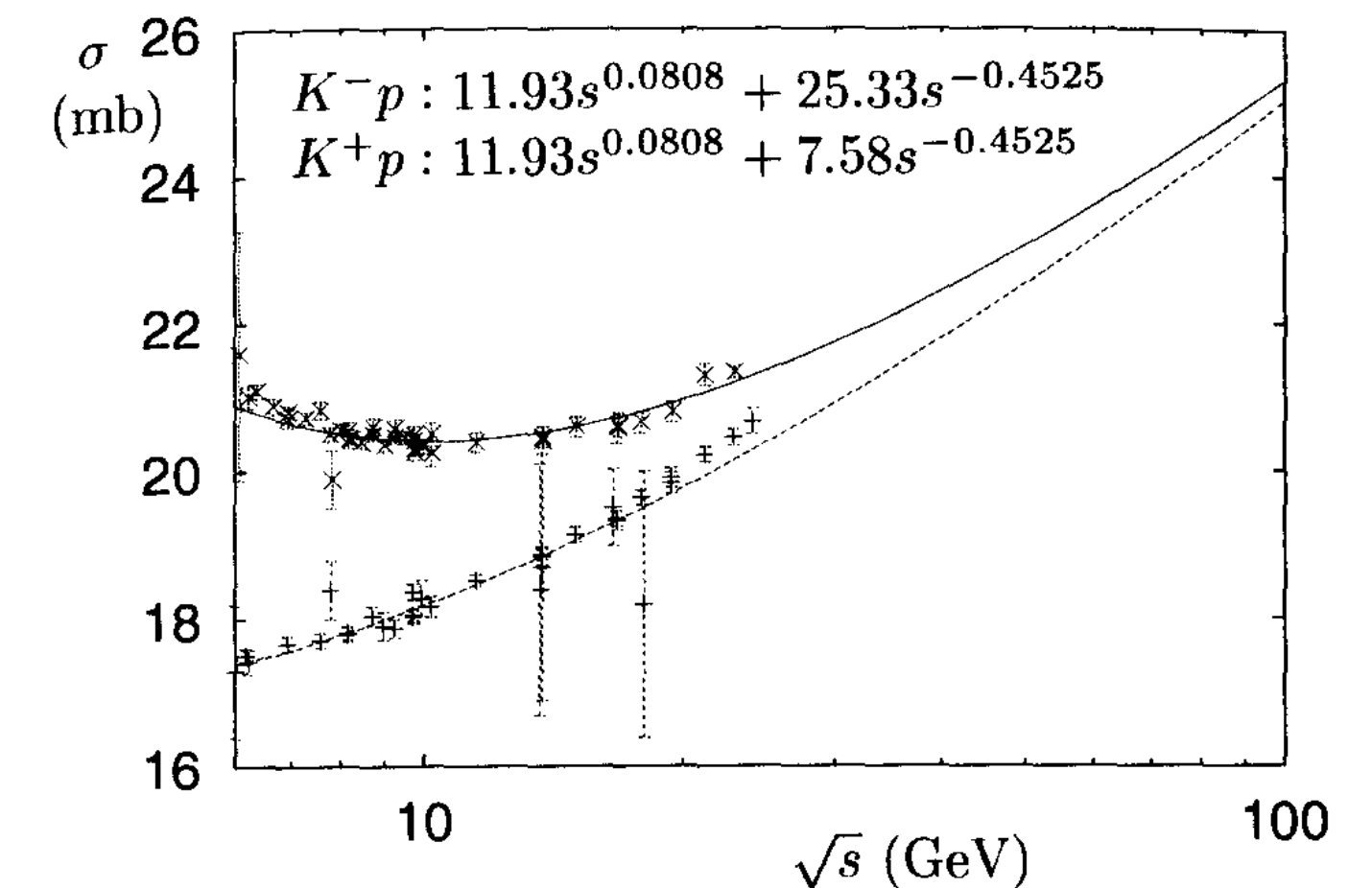
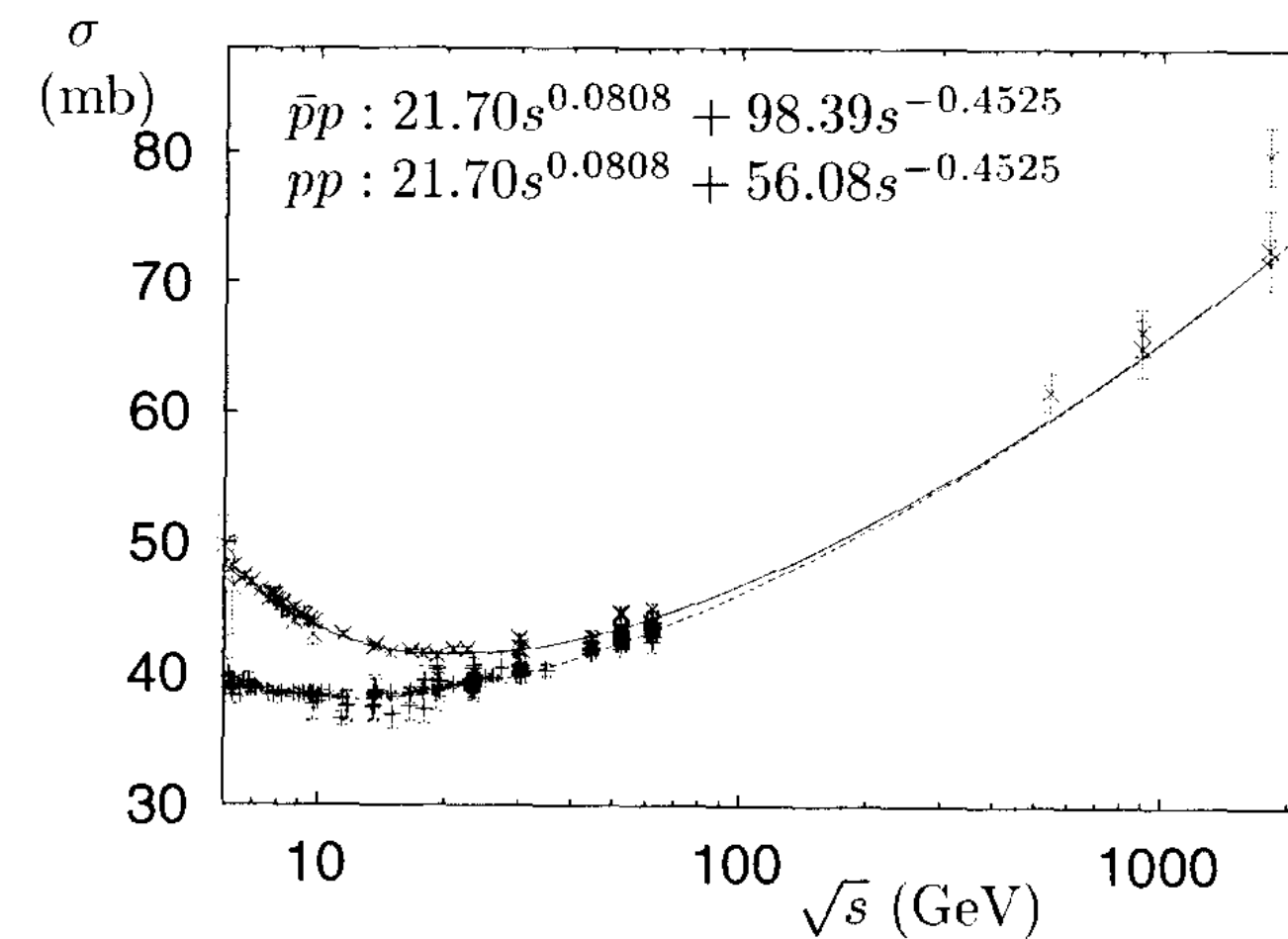
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[Landshoff-Donnachie]



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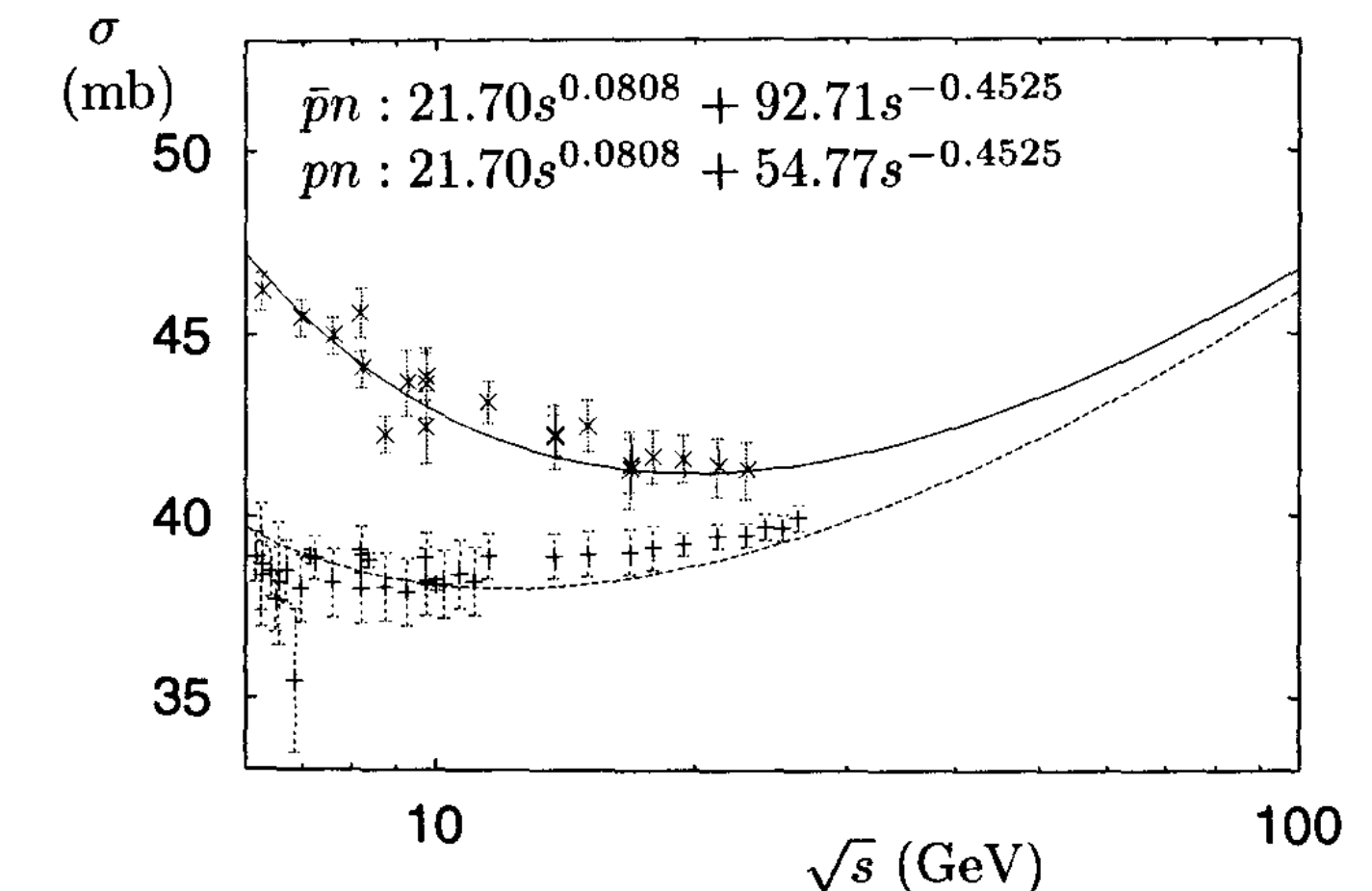
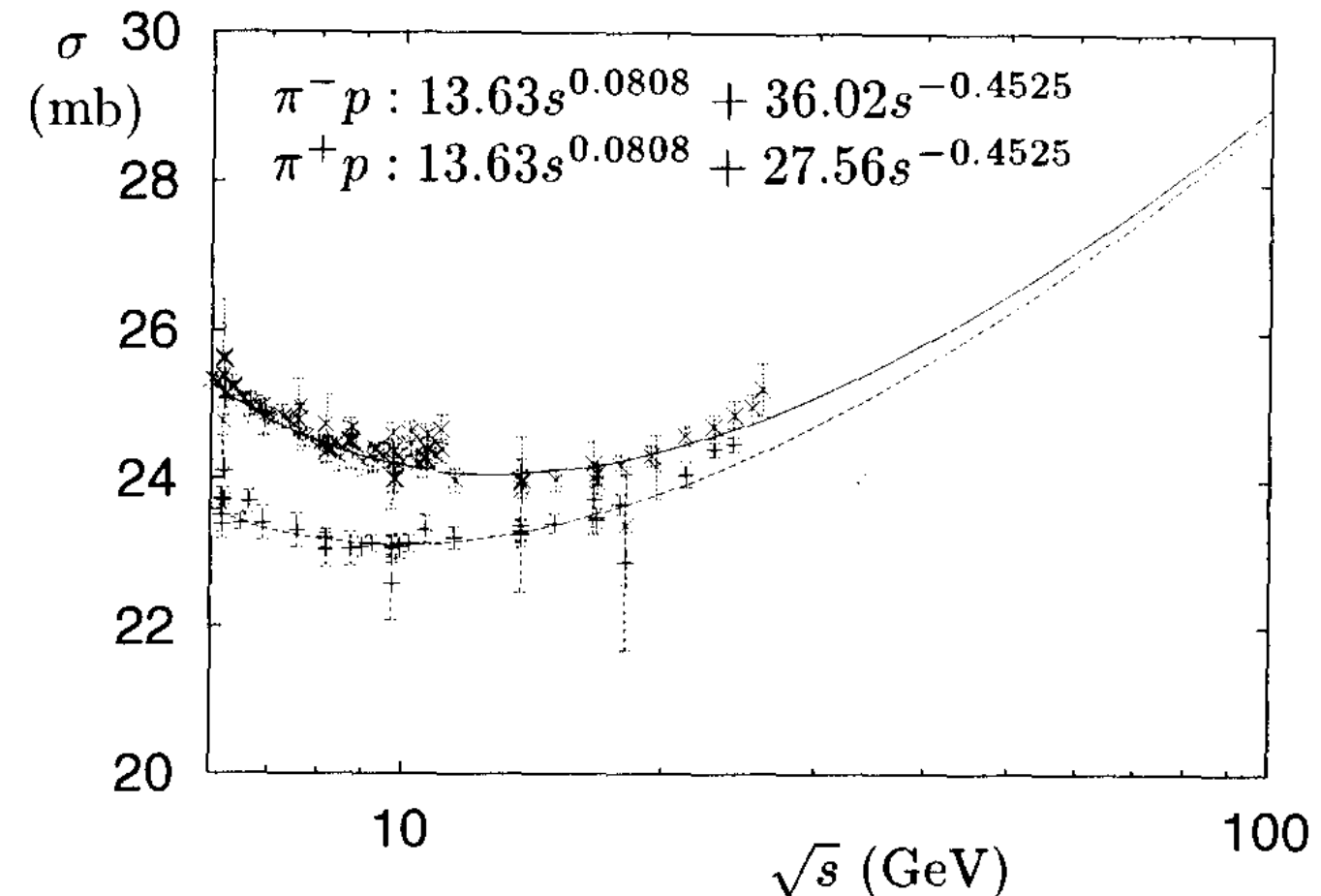
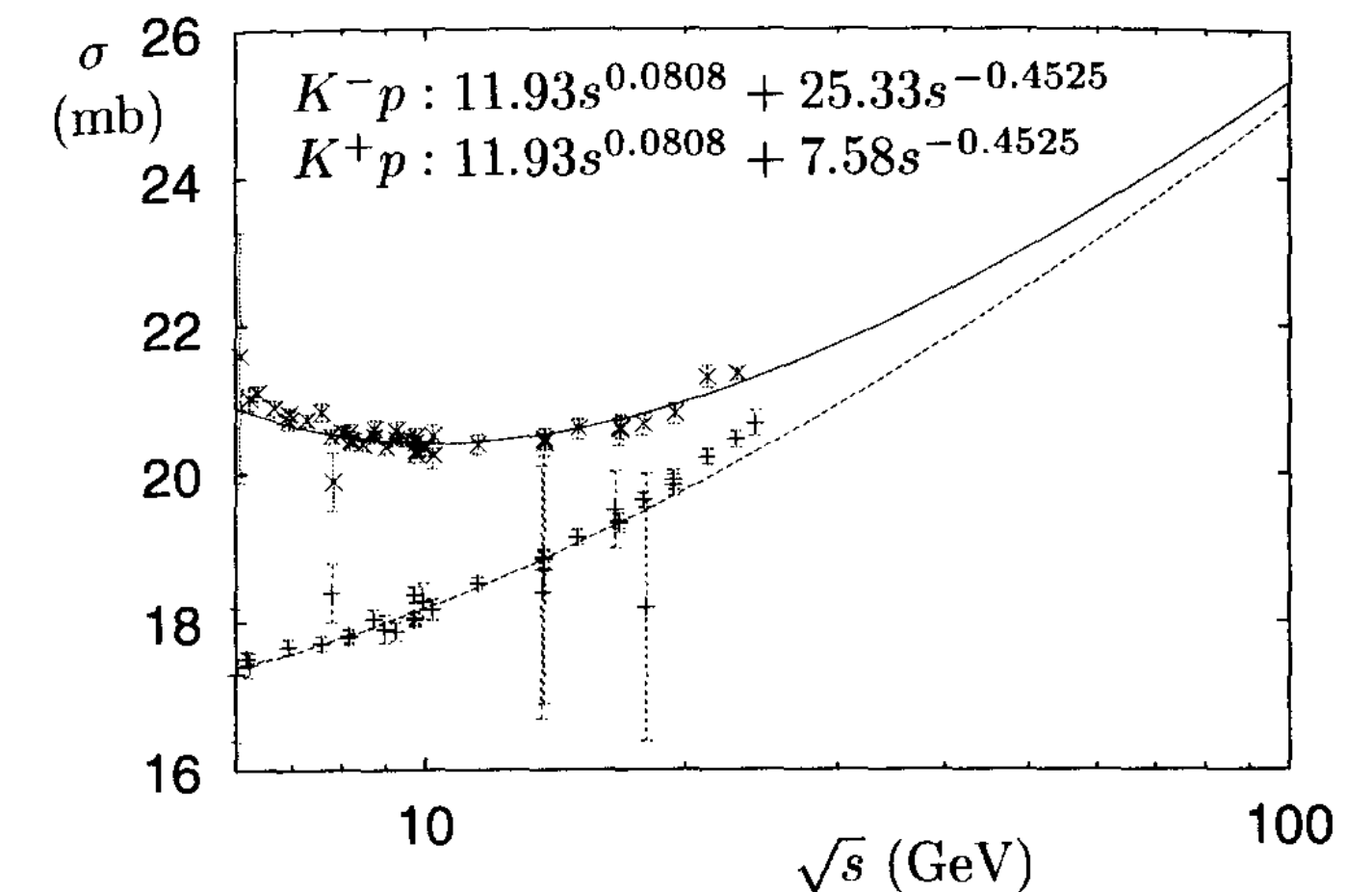
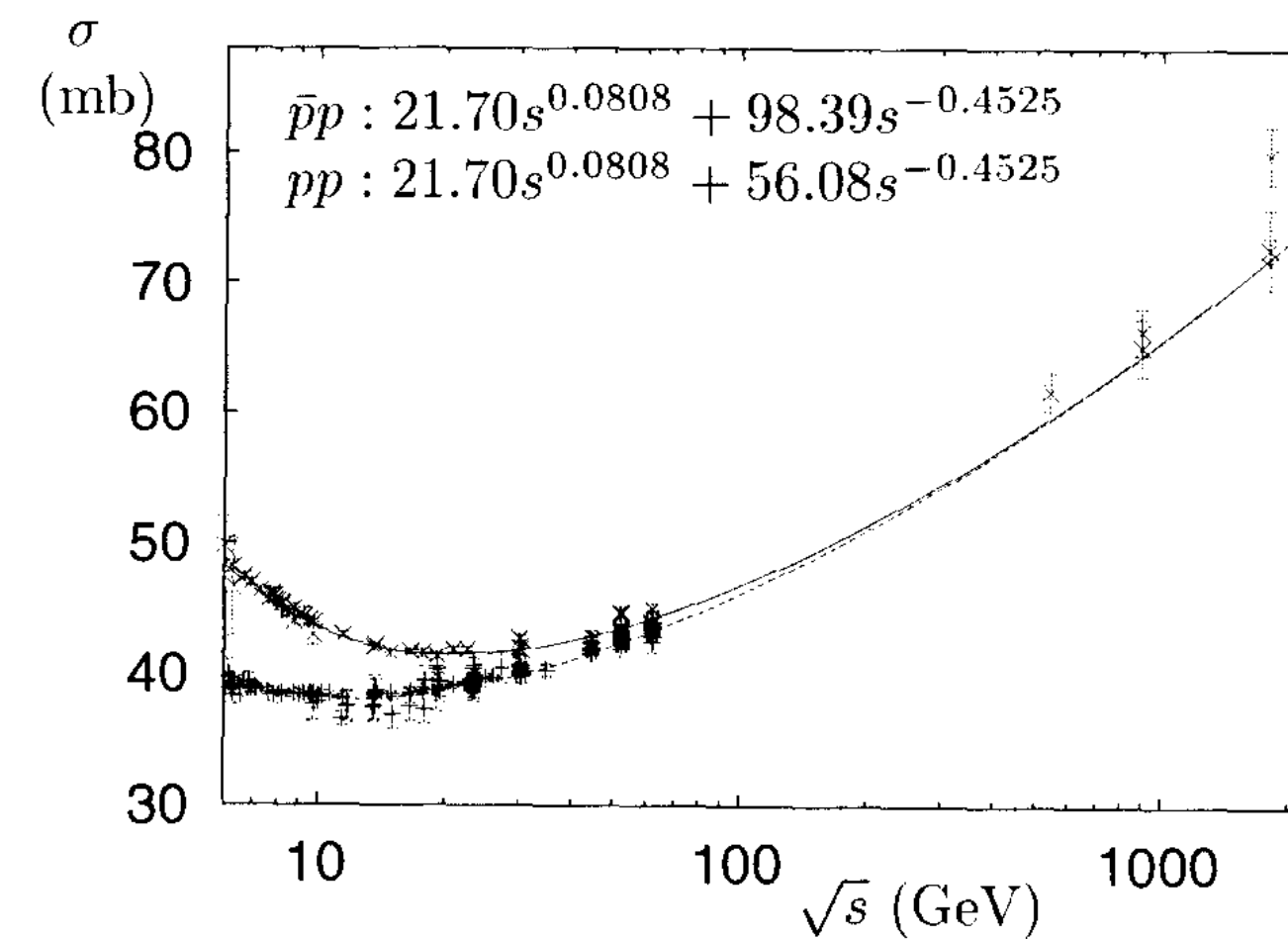
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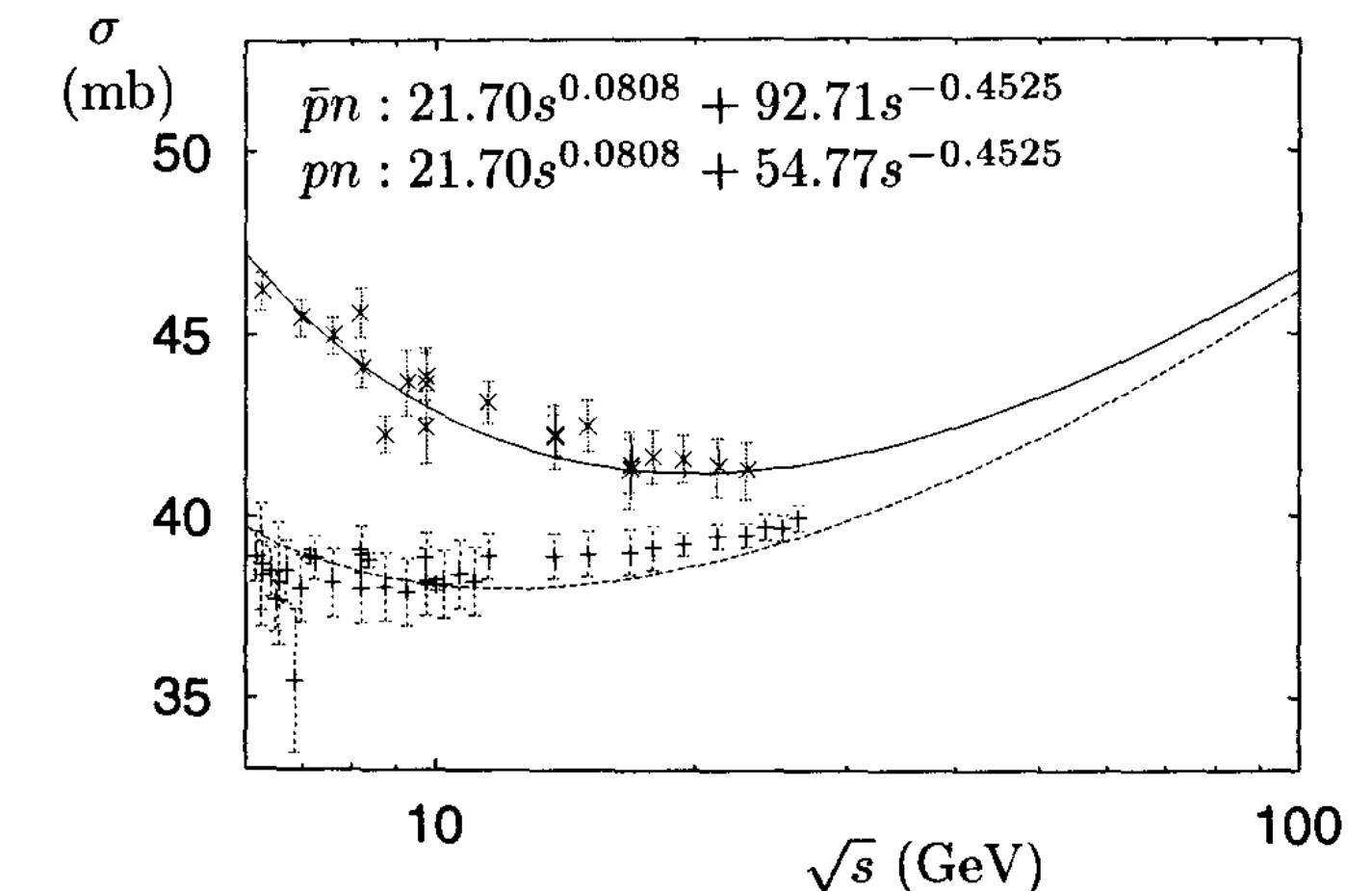
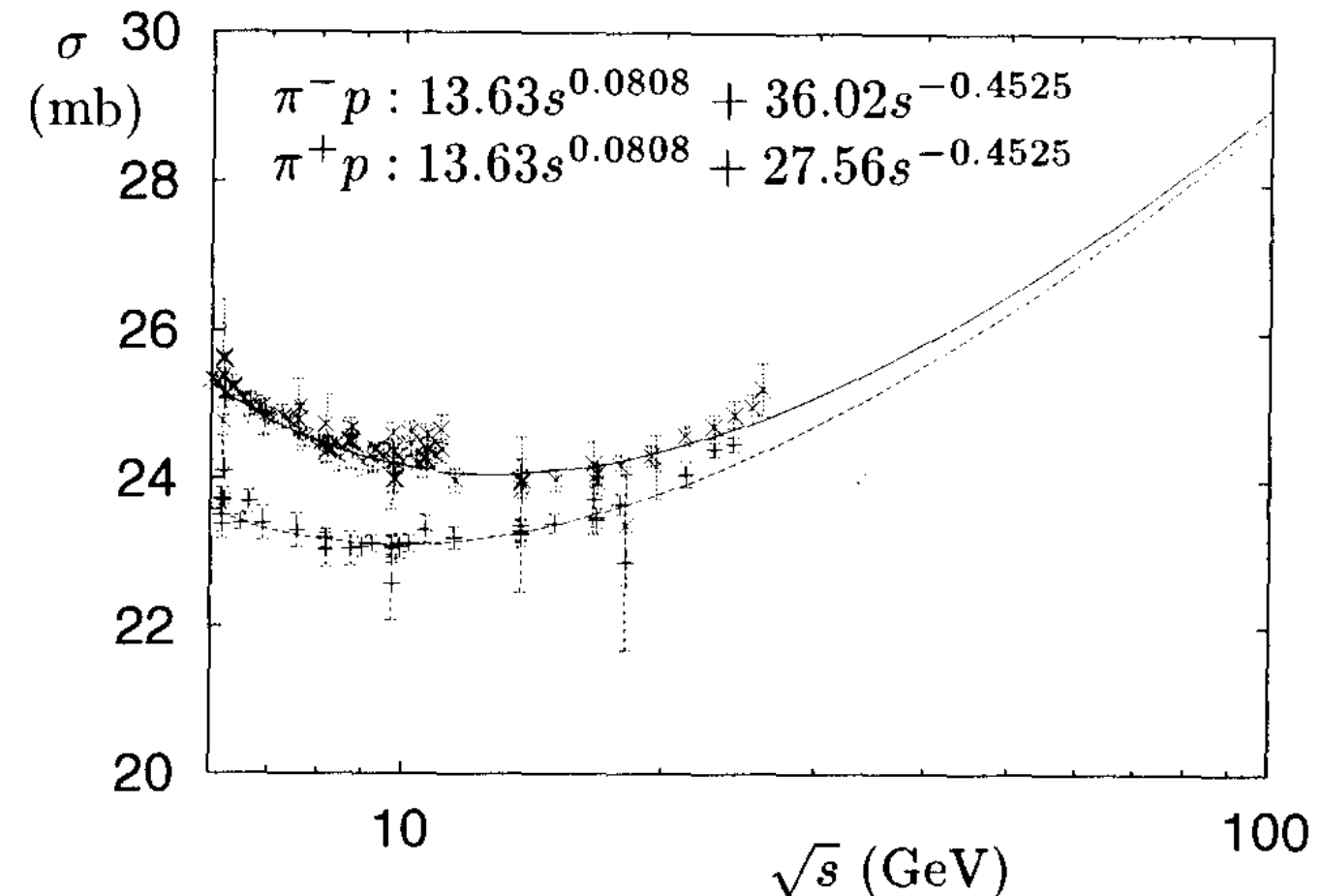
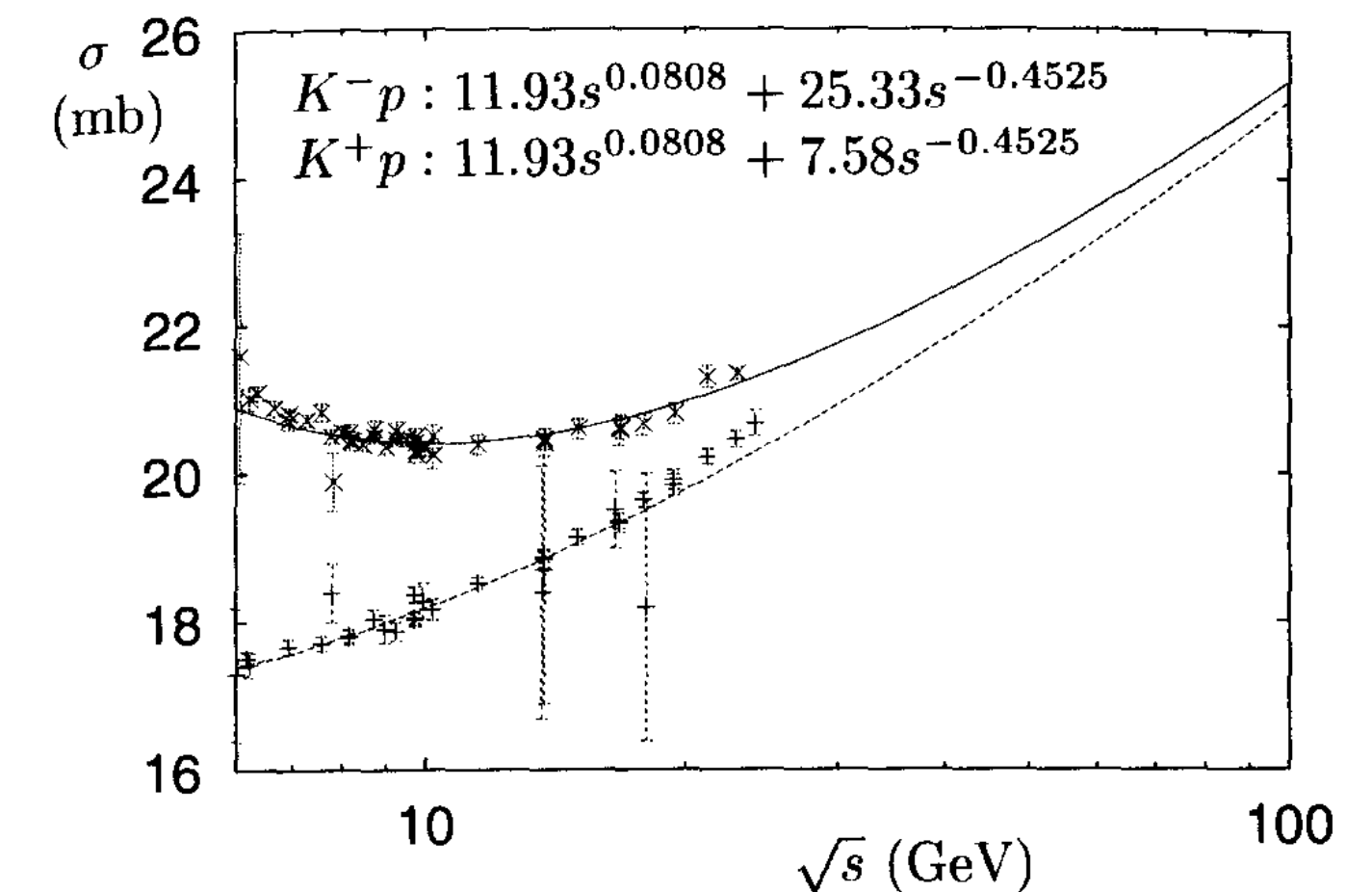
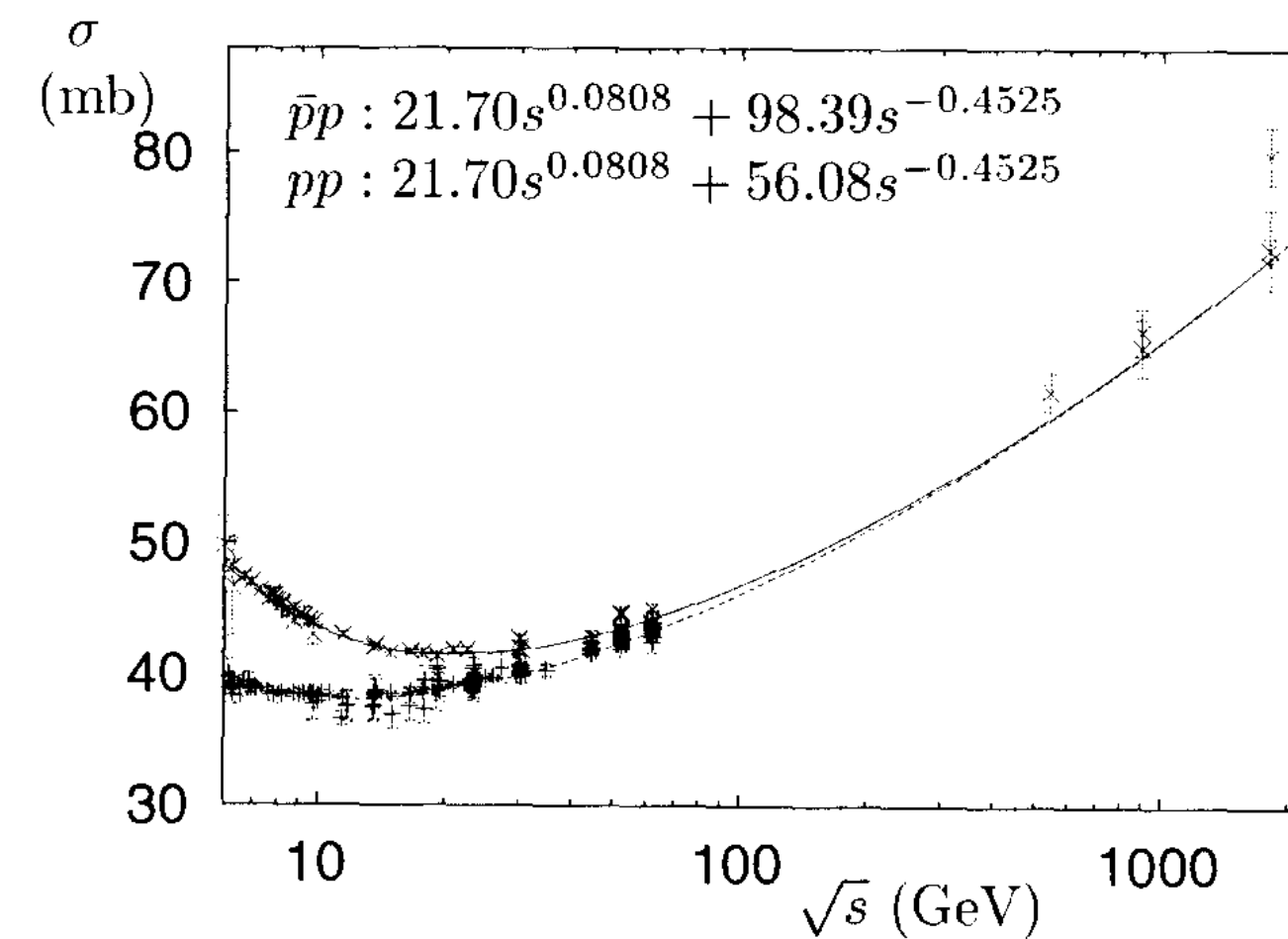
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Evidence from lattice QCD that there are glueballs on this trajectory with  $J \geq 2$ .



# Regge theory

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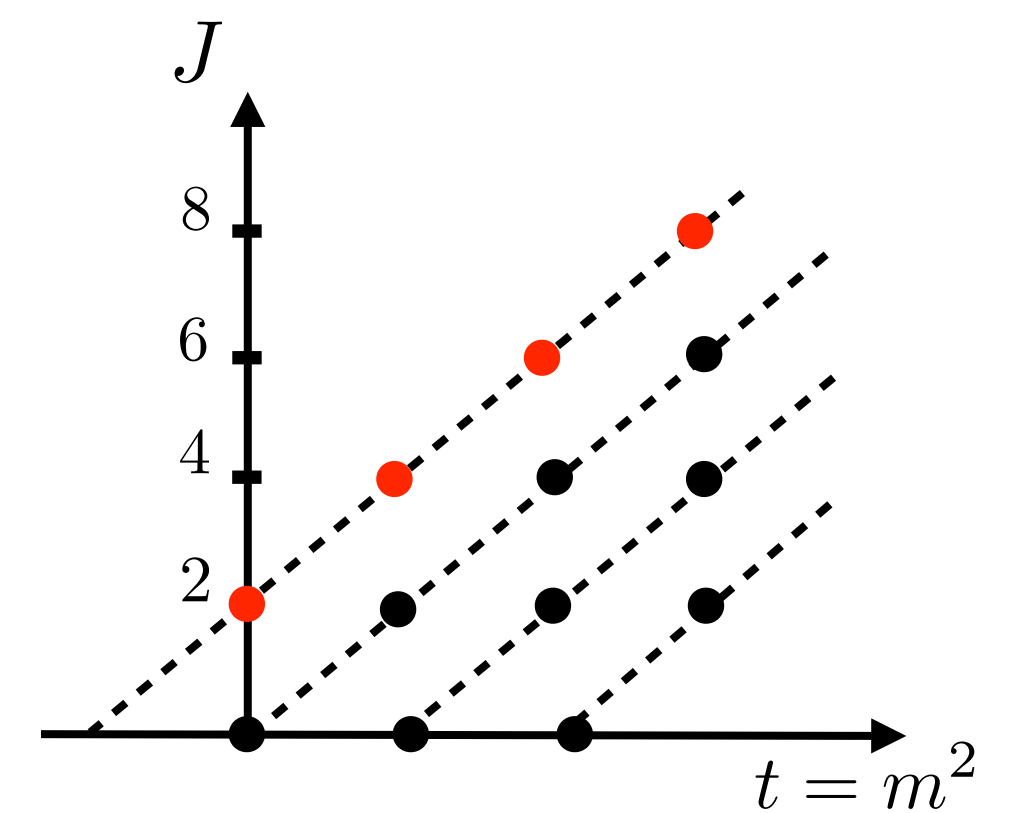
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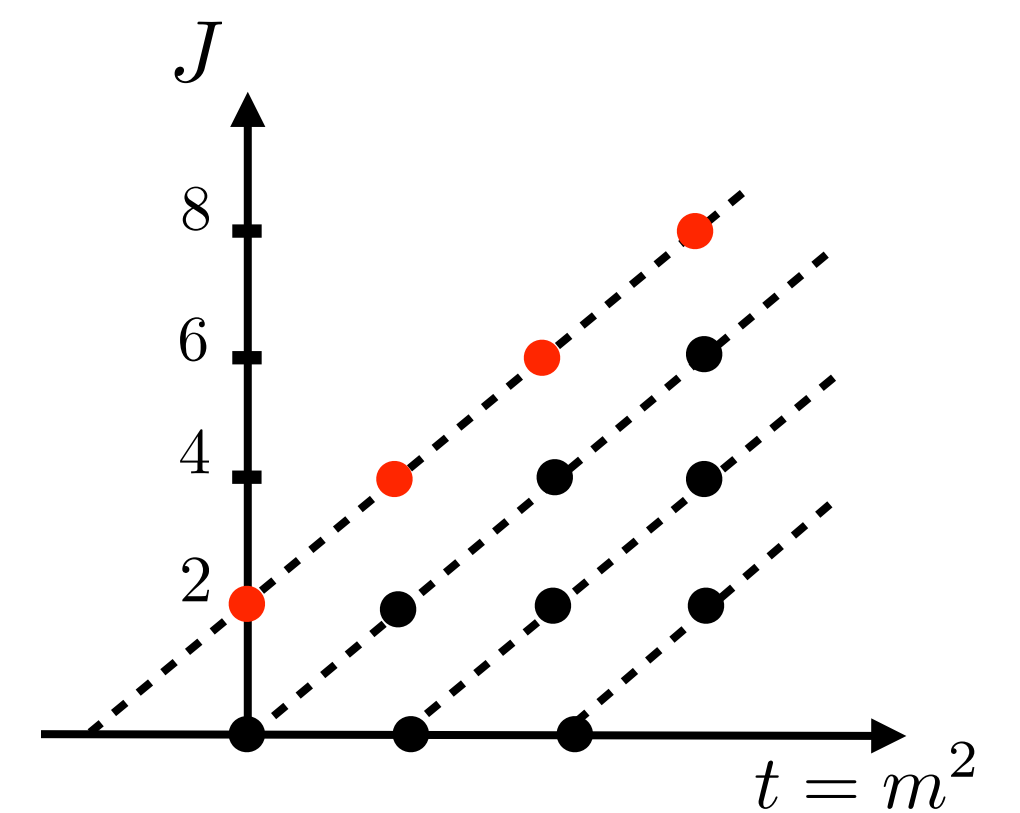
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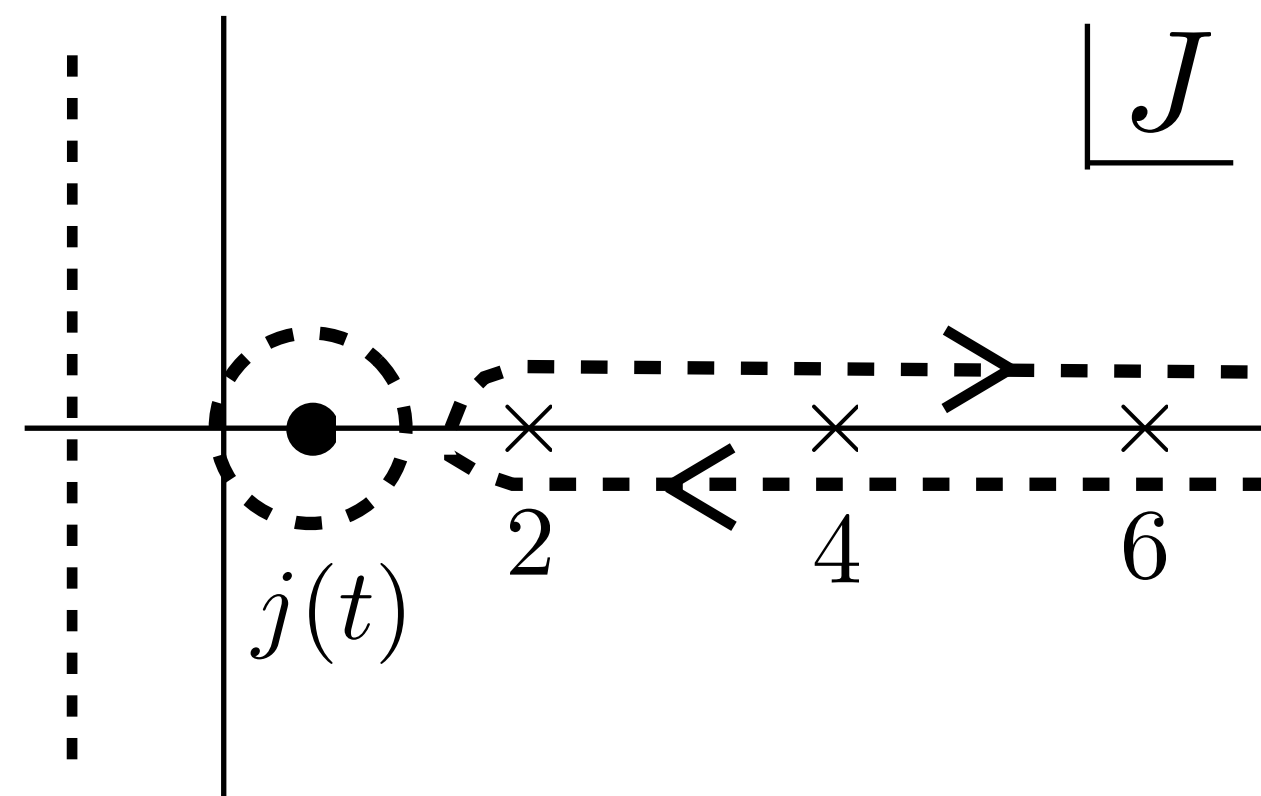
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- Pick leading pole

$$a_J(t) \approx - \frac{j'(t) r(j(t))}{J - j(t)}$$

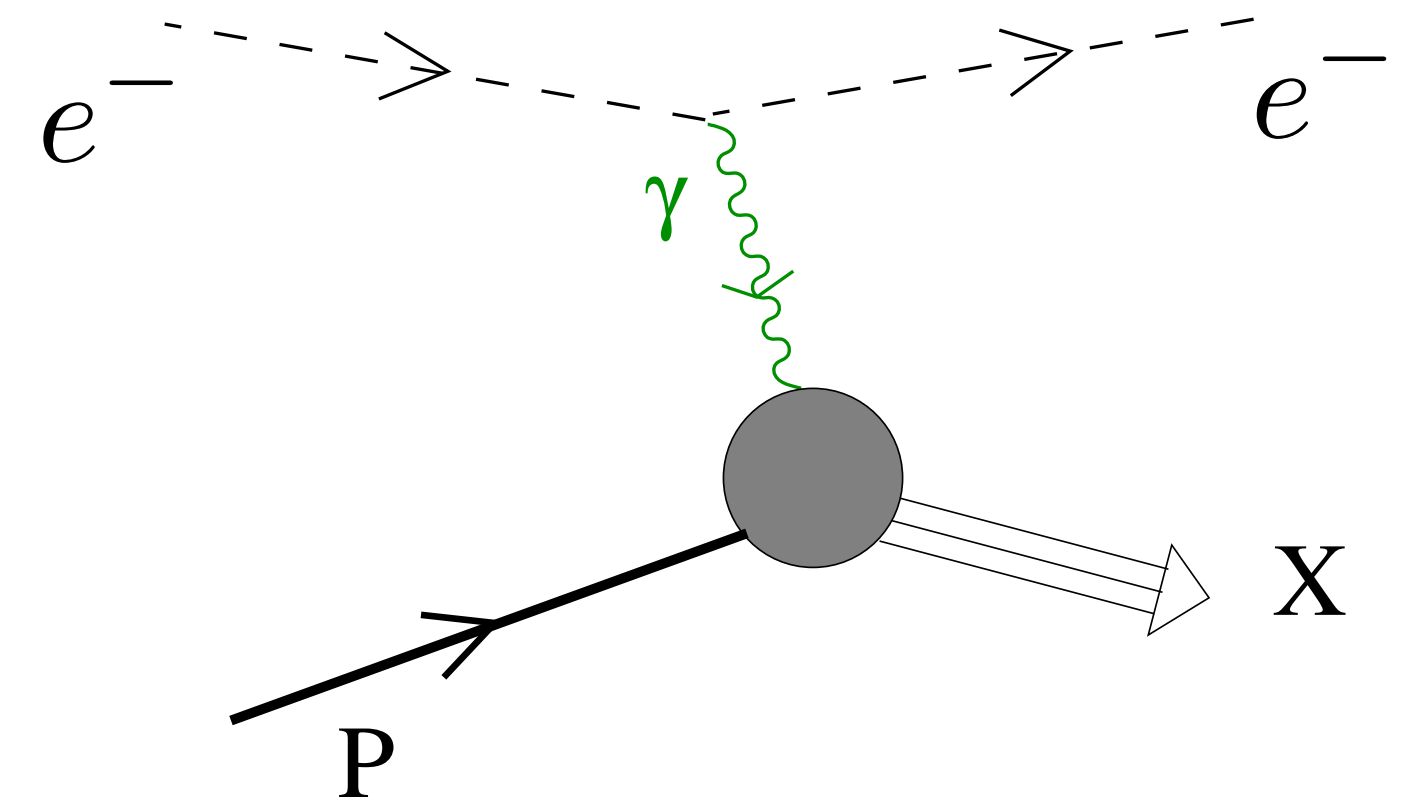


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## Deep Inelastic Scattering (DIS)

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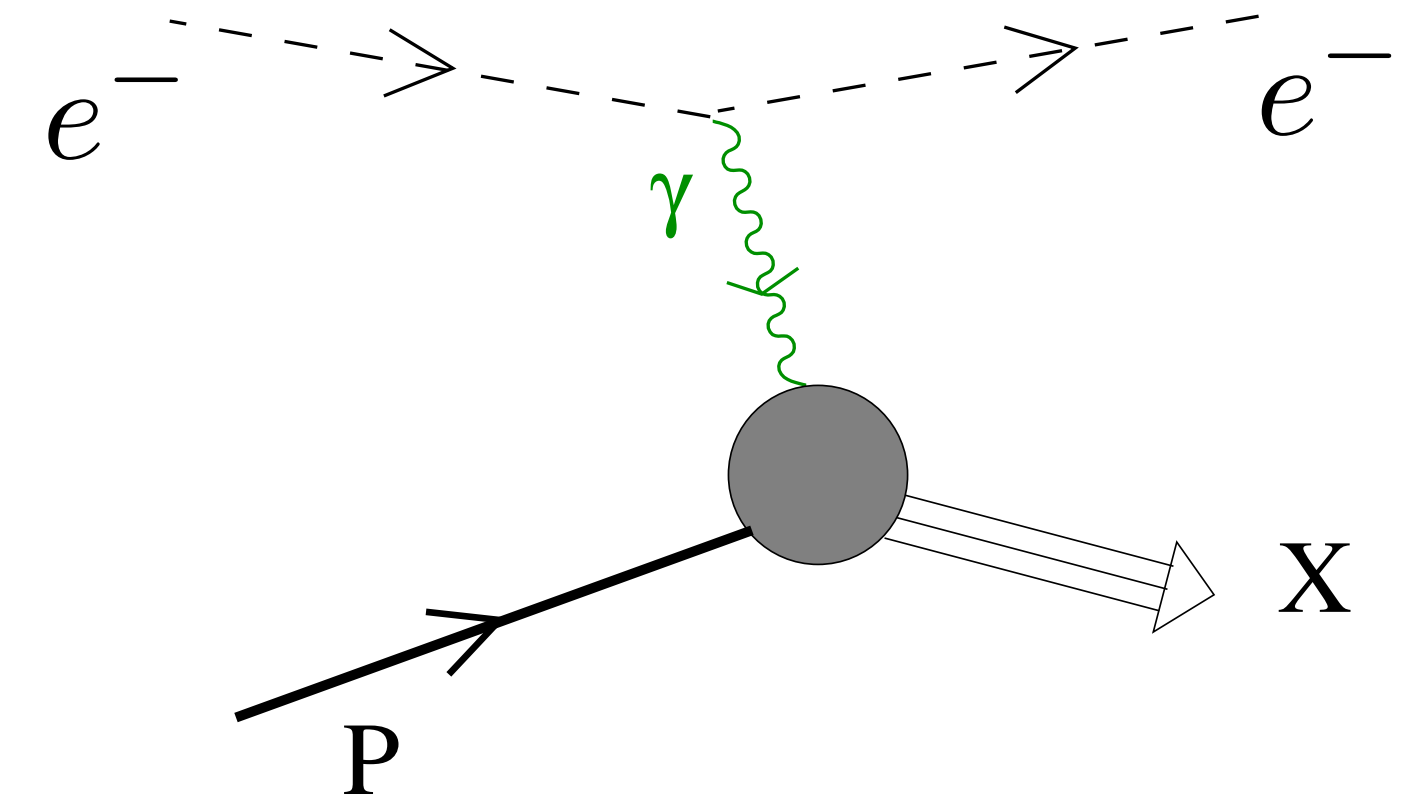
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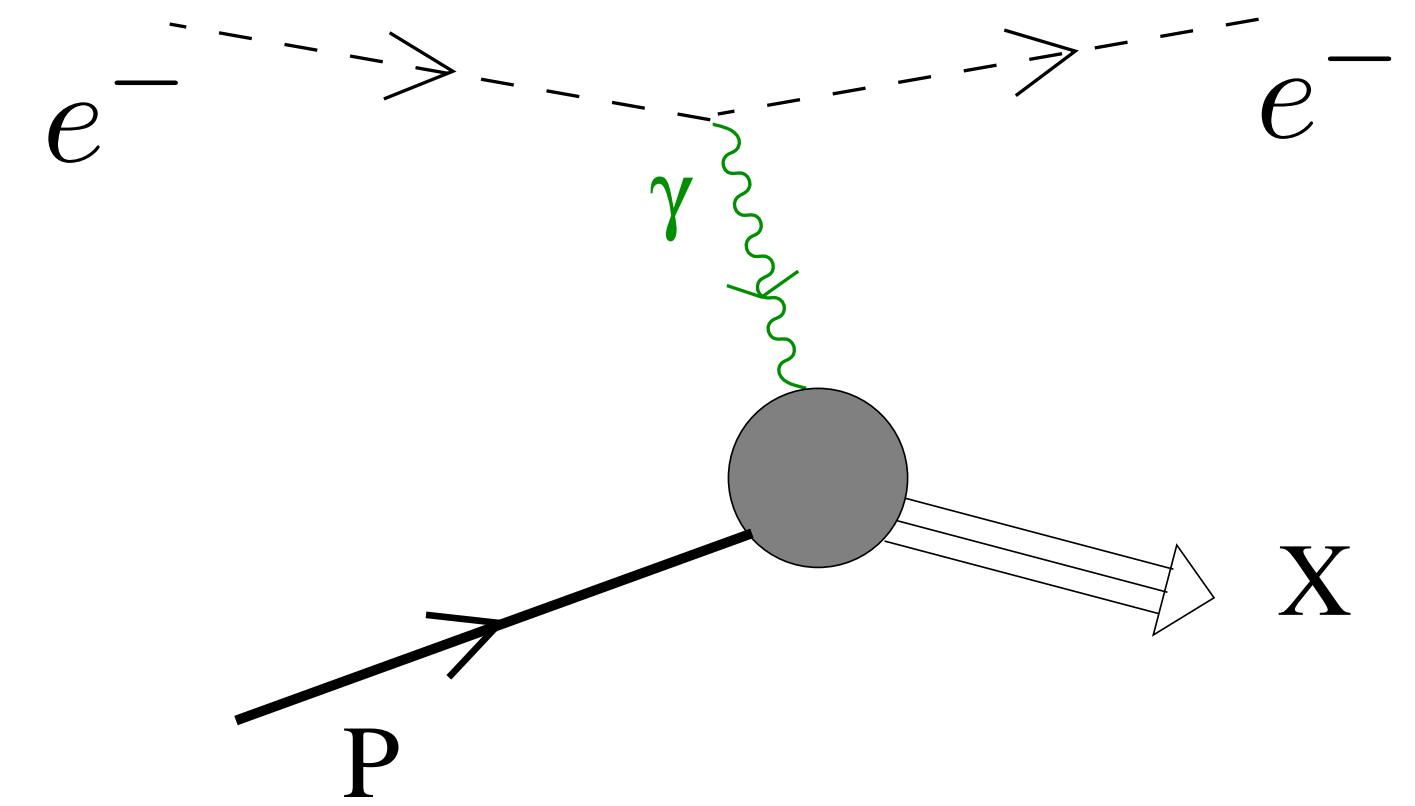
- Optical theorem

$$\sum_X \left| \left| \begin{array}{c} \gamma \\ \nearrow \\ \text{P} \end{array} \right. \right. \left. \left. \begin{array}{c} \longrightarrow \\ \text{X} \end{array} \right| \right|^2 = \text{Im} (t=0) \begin{array}{c} \gamma \\ \nearrow \\ \text{P} \end{array} \begin{array}{c} \searrow \\ \text{P} \end{array} \begin{array}{c} \gamma \end{array}$$

The equation relates the sum of squared amplitudes for all possible final states  $X$  to the imaginary part of the forward scattering amplitude  $T$  at zero momentum transfer ( $t=0$ ). The left side shows a diagram where a proton ( $P$ ) and a virtual photon ( $\gamma$ ) interact at a grey vertex to produce a final state  $X$ . The right side shows a diagram where a proton ( $P$ ) and a virtual photon ( $\gamma$ ) interact at a grey vertex to produce another proton ( $P$ ) and a virtual photon ( $\gamma$ ).

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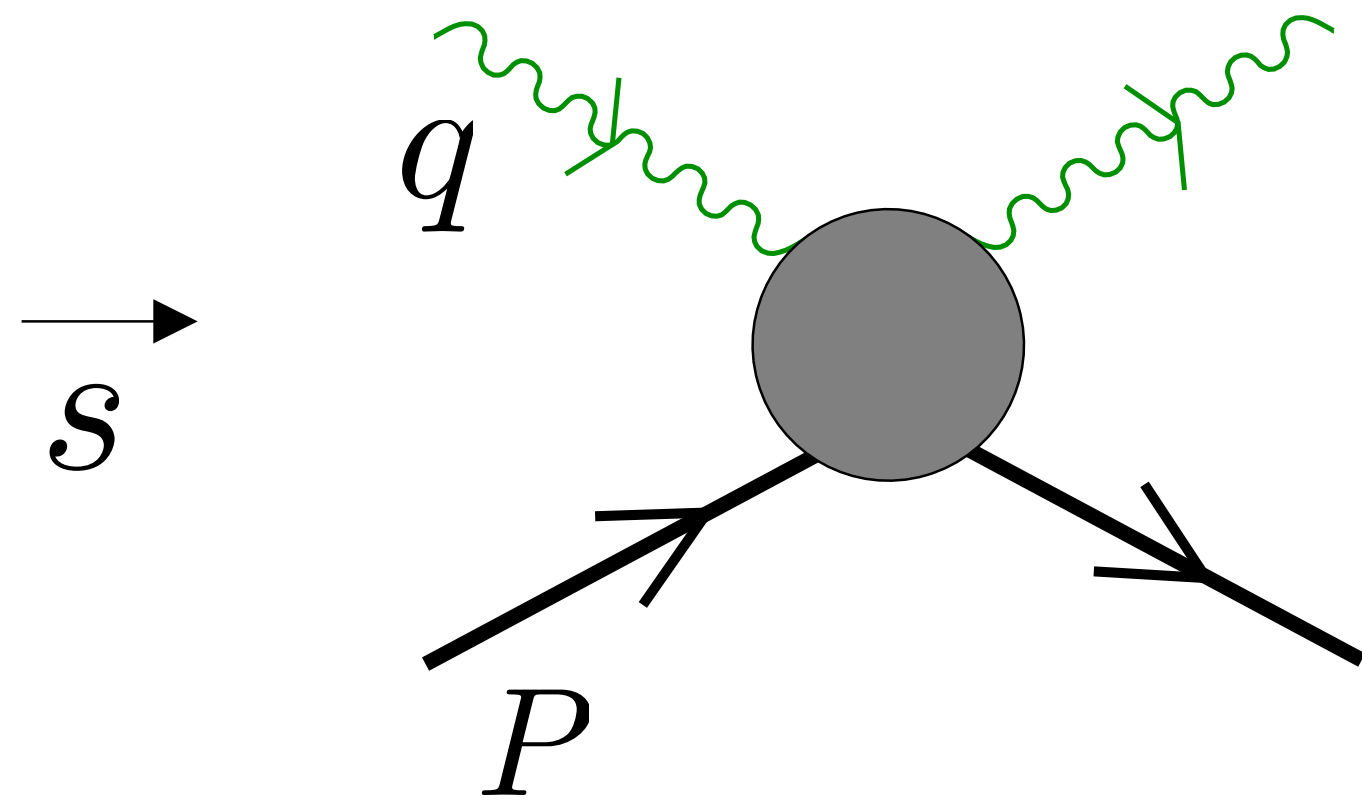
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The equation relates the squared magnitude of the forward scattering amplitude (left side) to the imaginary part of the total cross-section at zero momentum transfer (right side). The left side shows a proton (P) and a photon (γ) entering a vertex, with a final state X. The right side shows a proton (P) entering a vertex from the left and another proton (P) exiting to the right, with two photons (γ) entering from above.

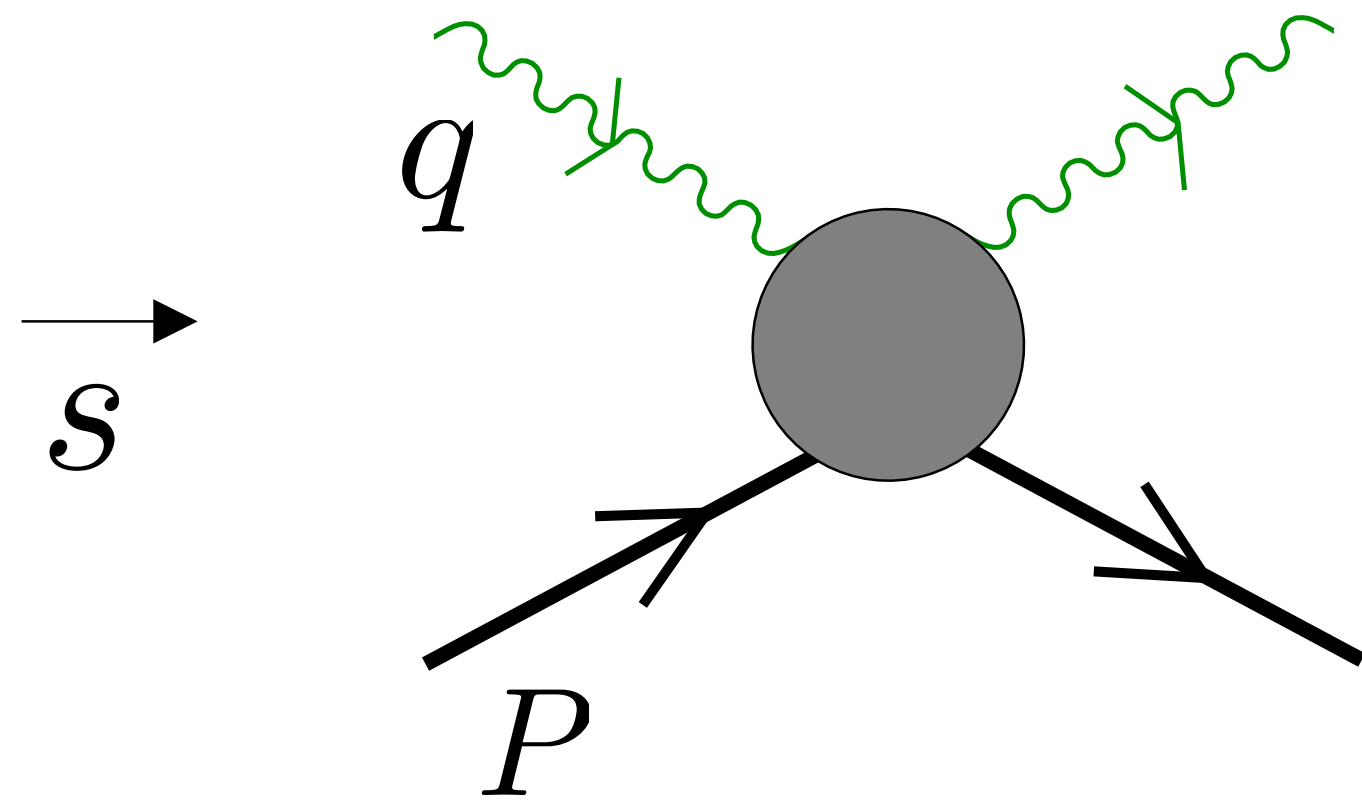
- Hadronic tensor

$$W^{ab}(x, Q, t) = i \int d^4y e^{iq \cdot y} \langle P | T \{ j^a(y) j^b(0) \} | P' \rangle$$



$$s = -(q + P)^2$$

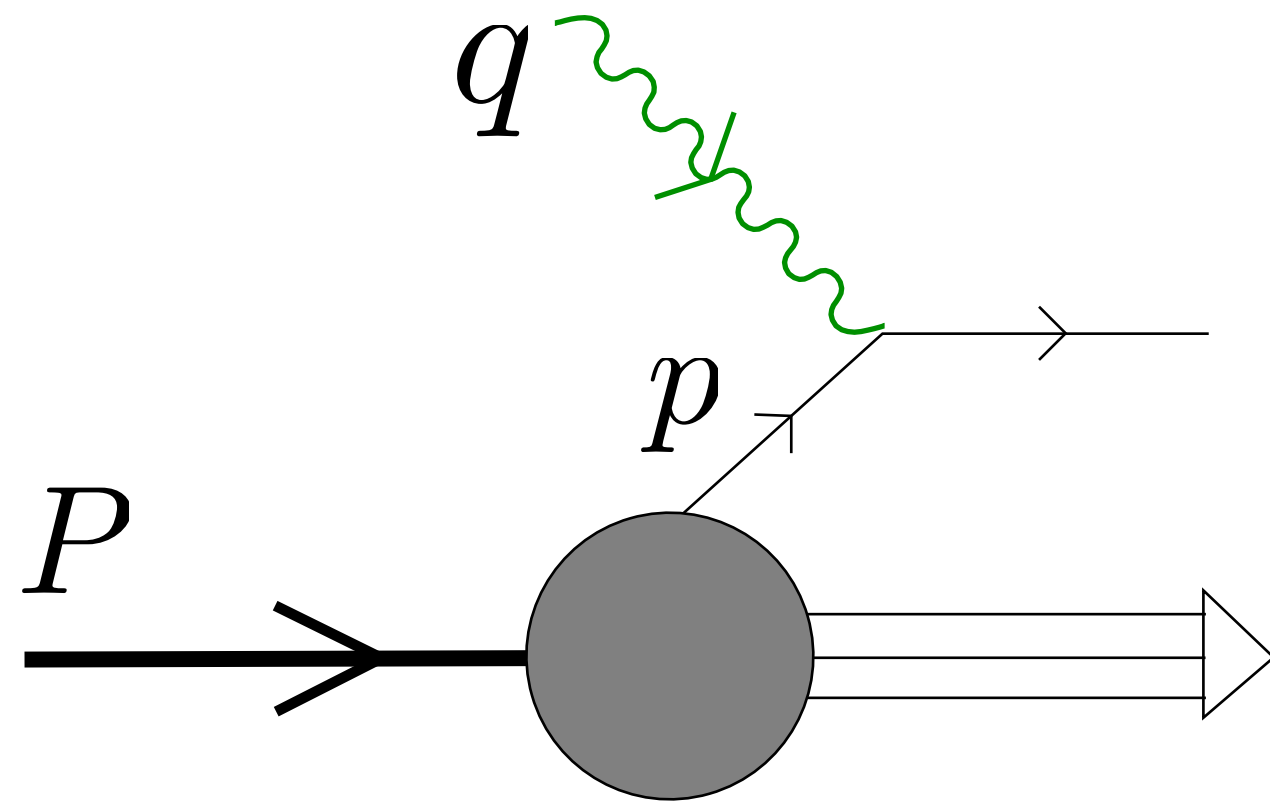
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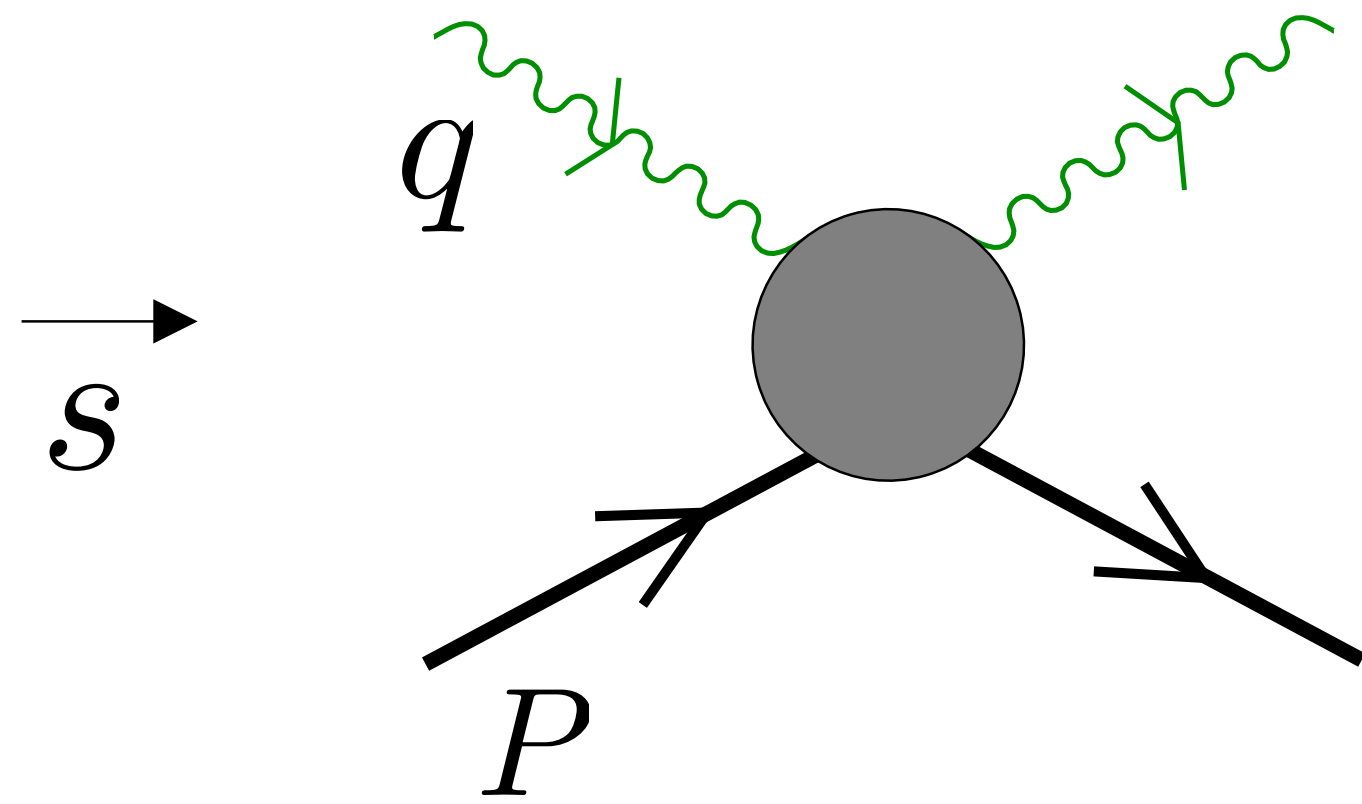
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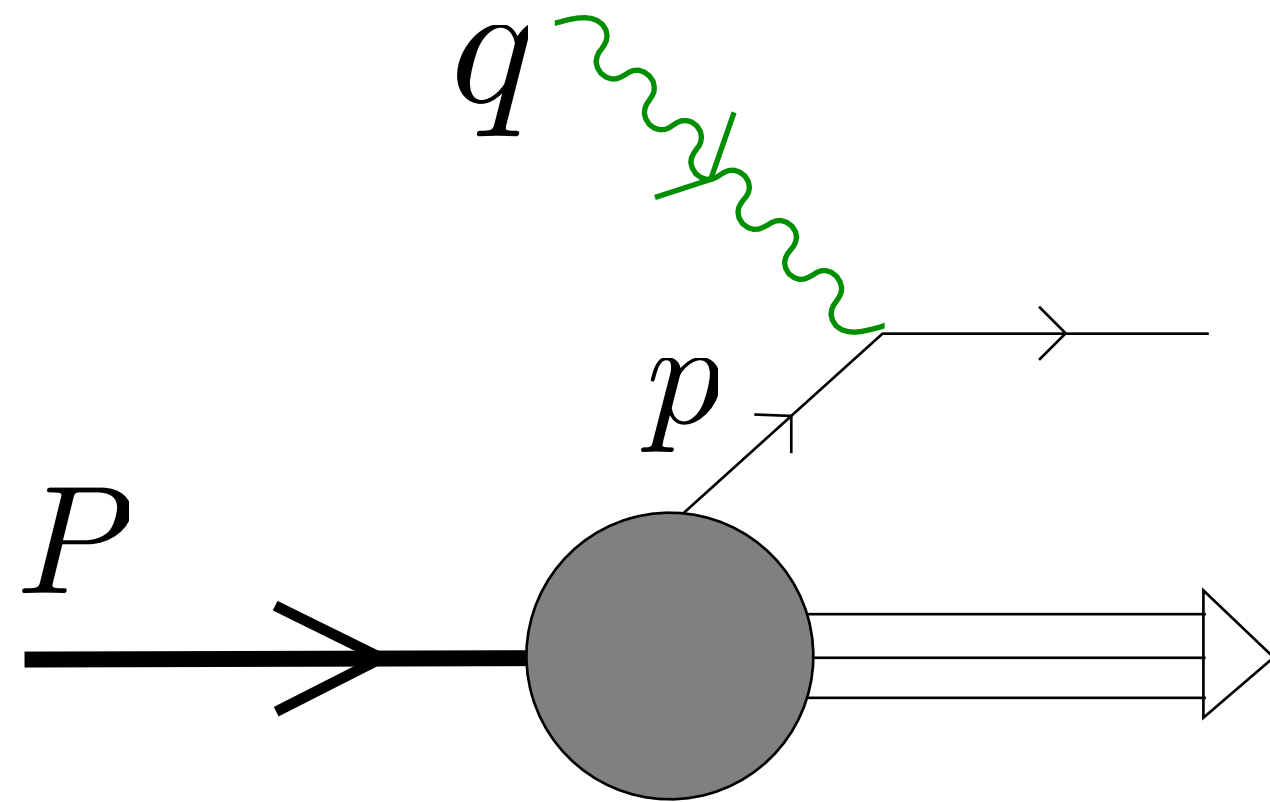
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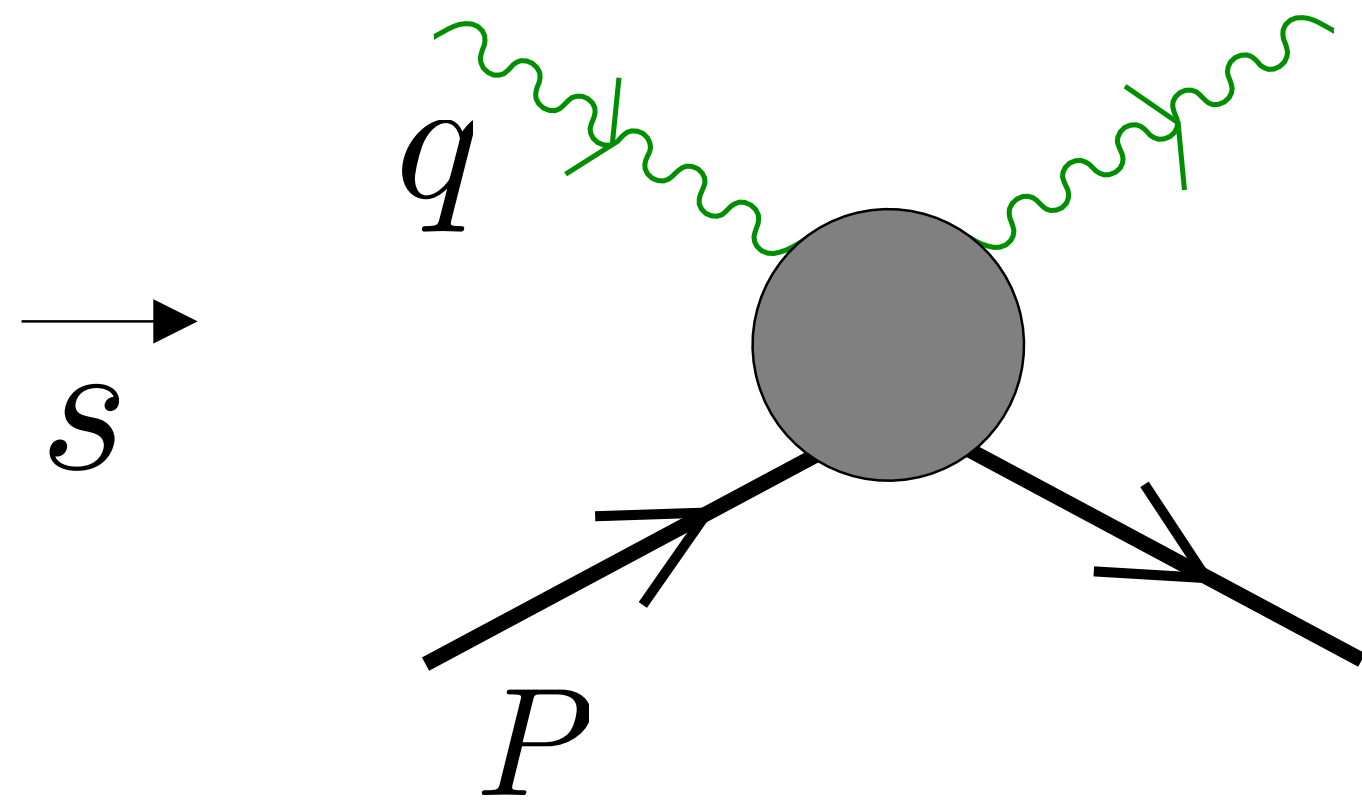
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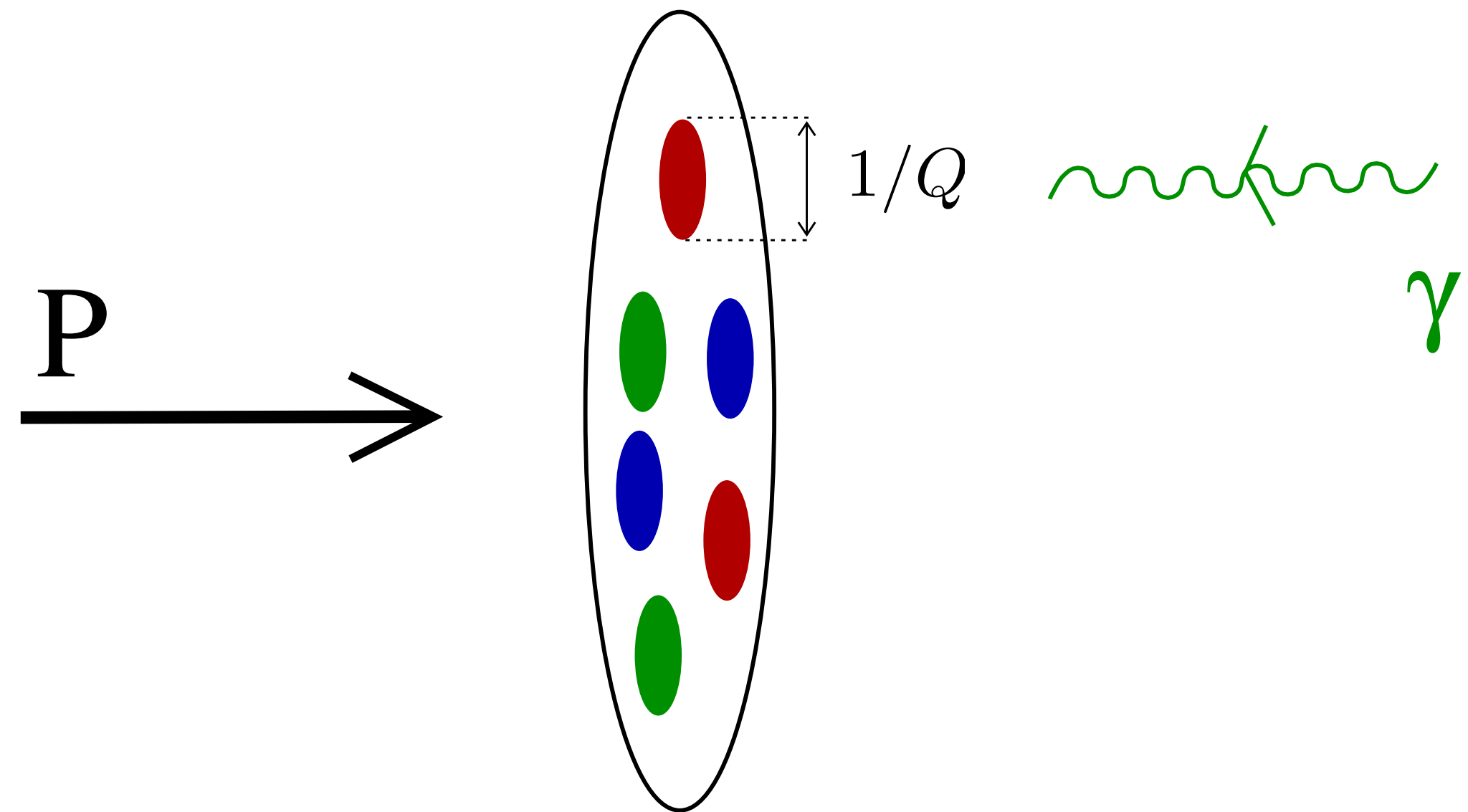
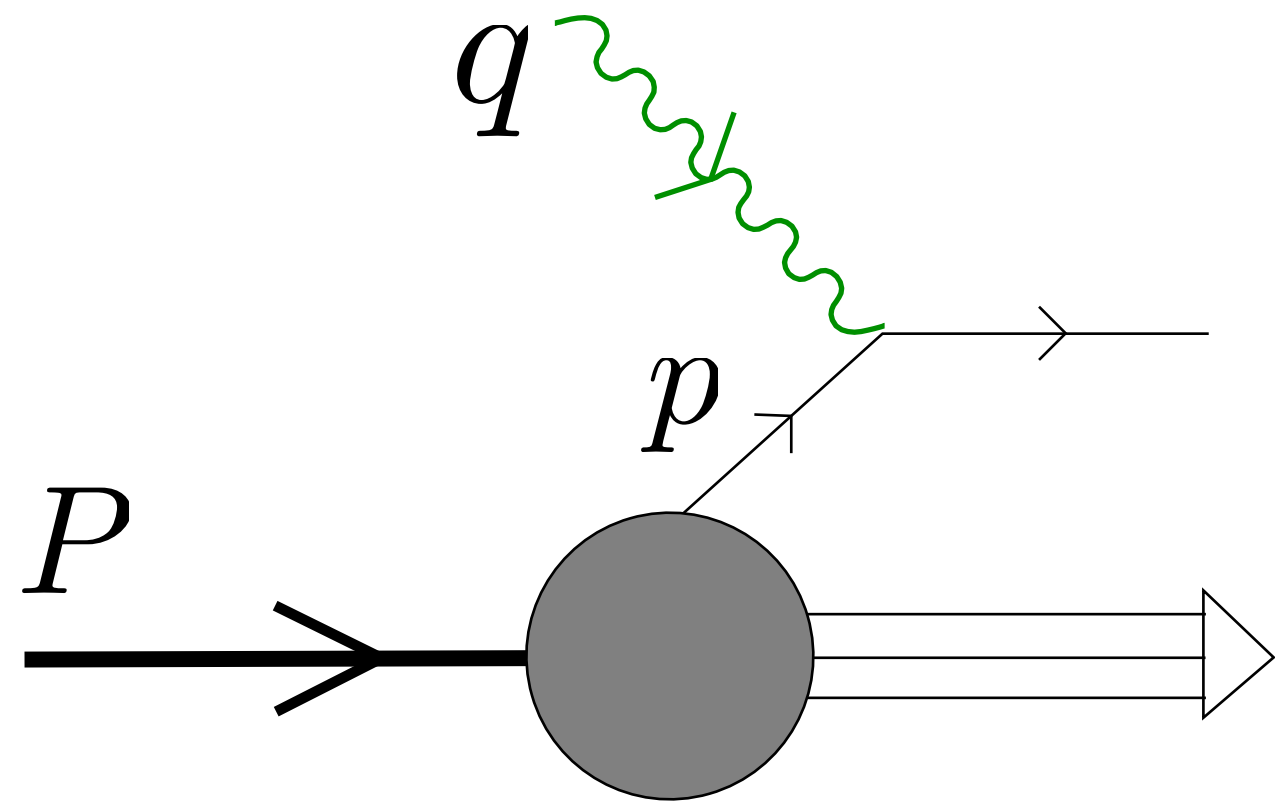


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- Bjorken  $x$

- Transverse resolution  $1/Q$



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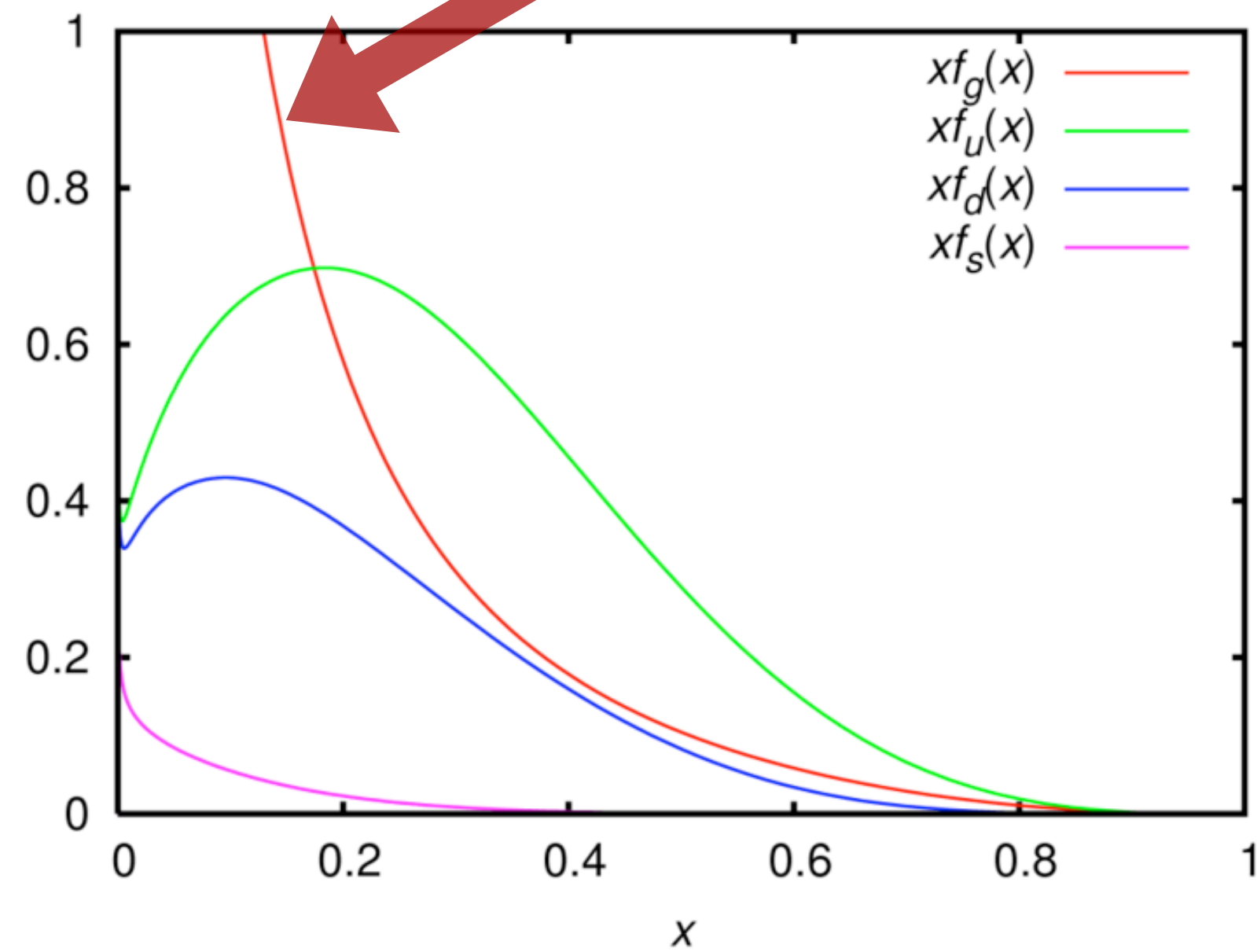
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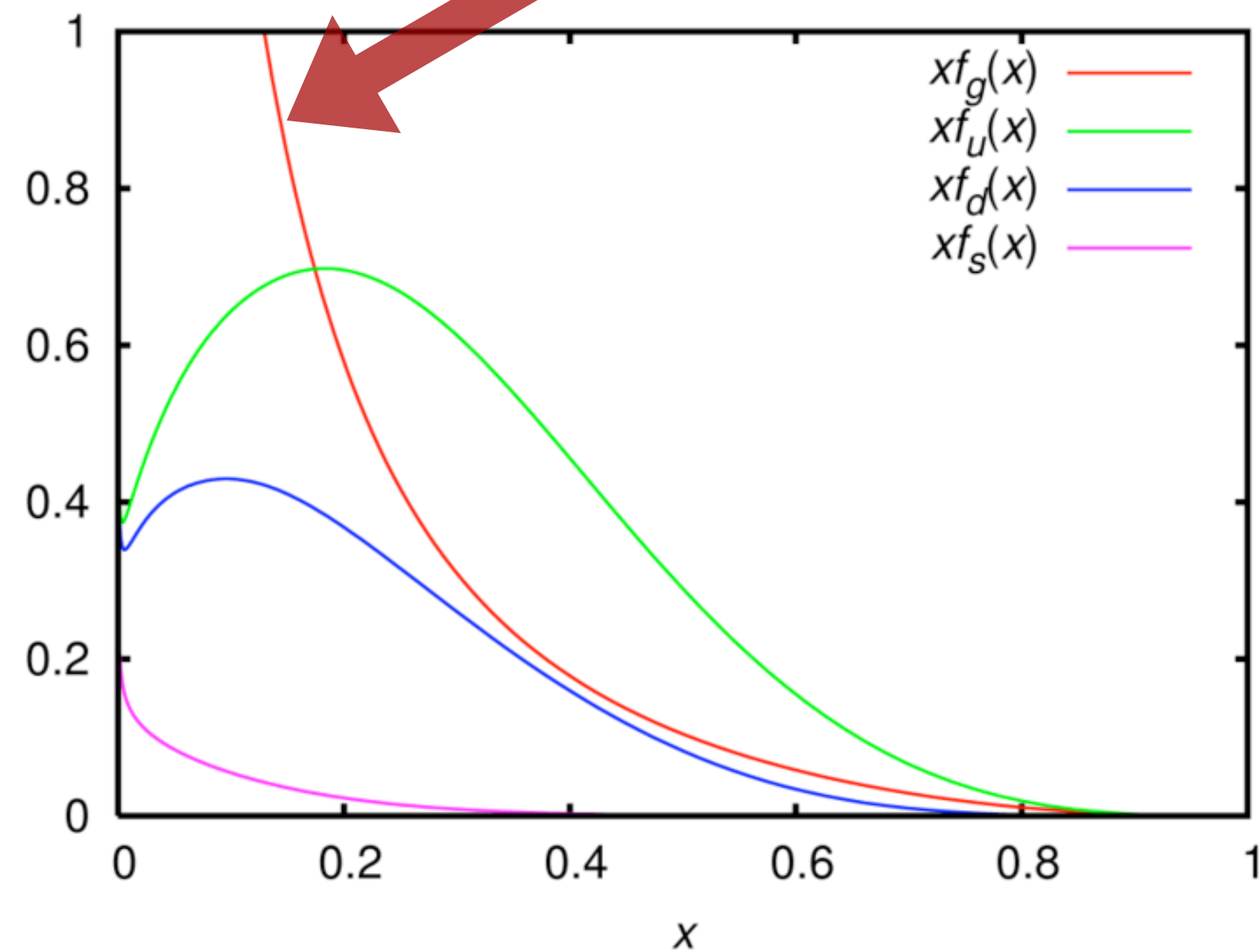
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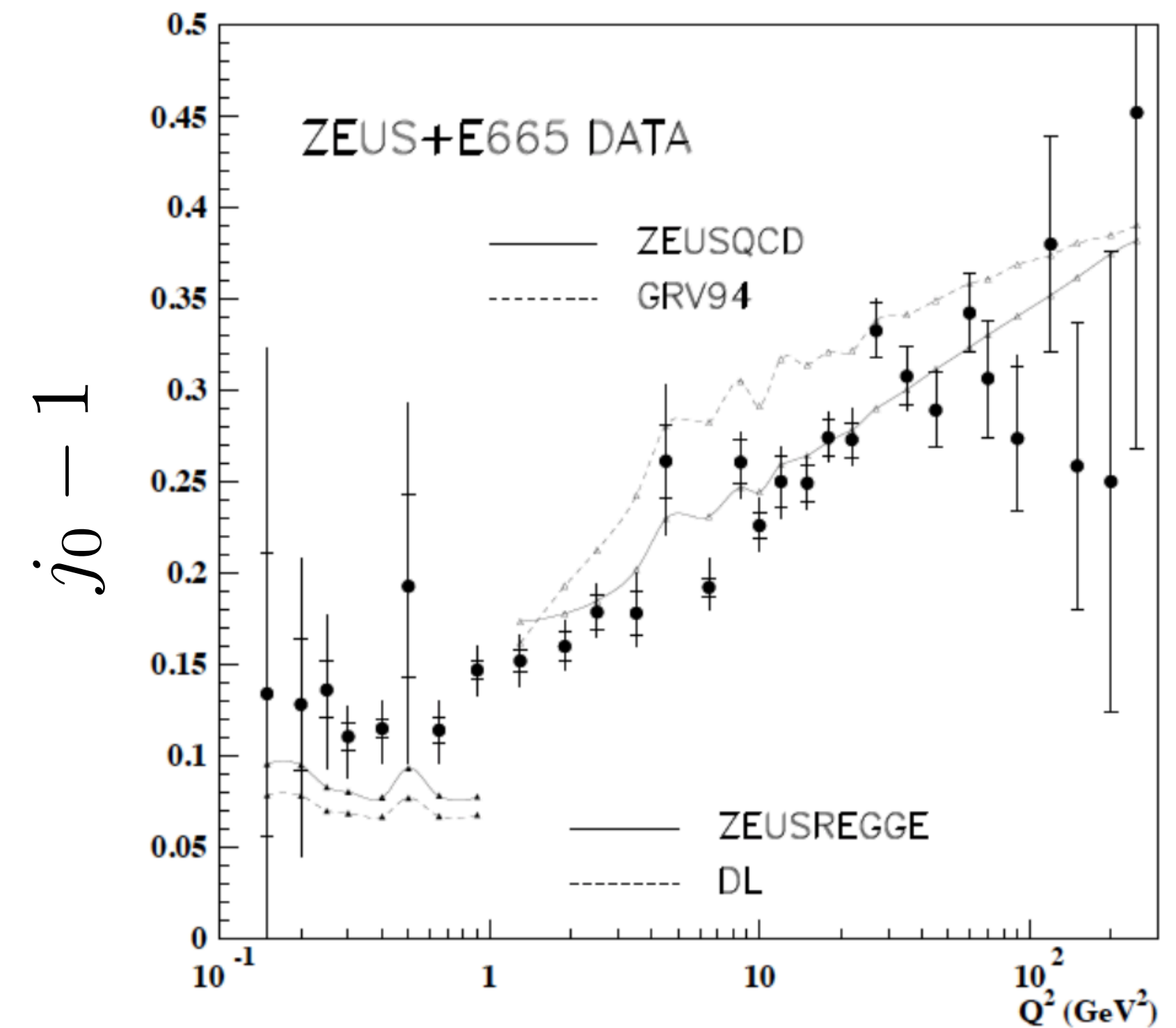
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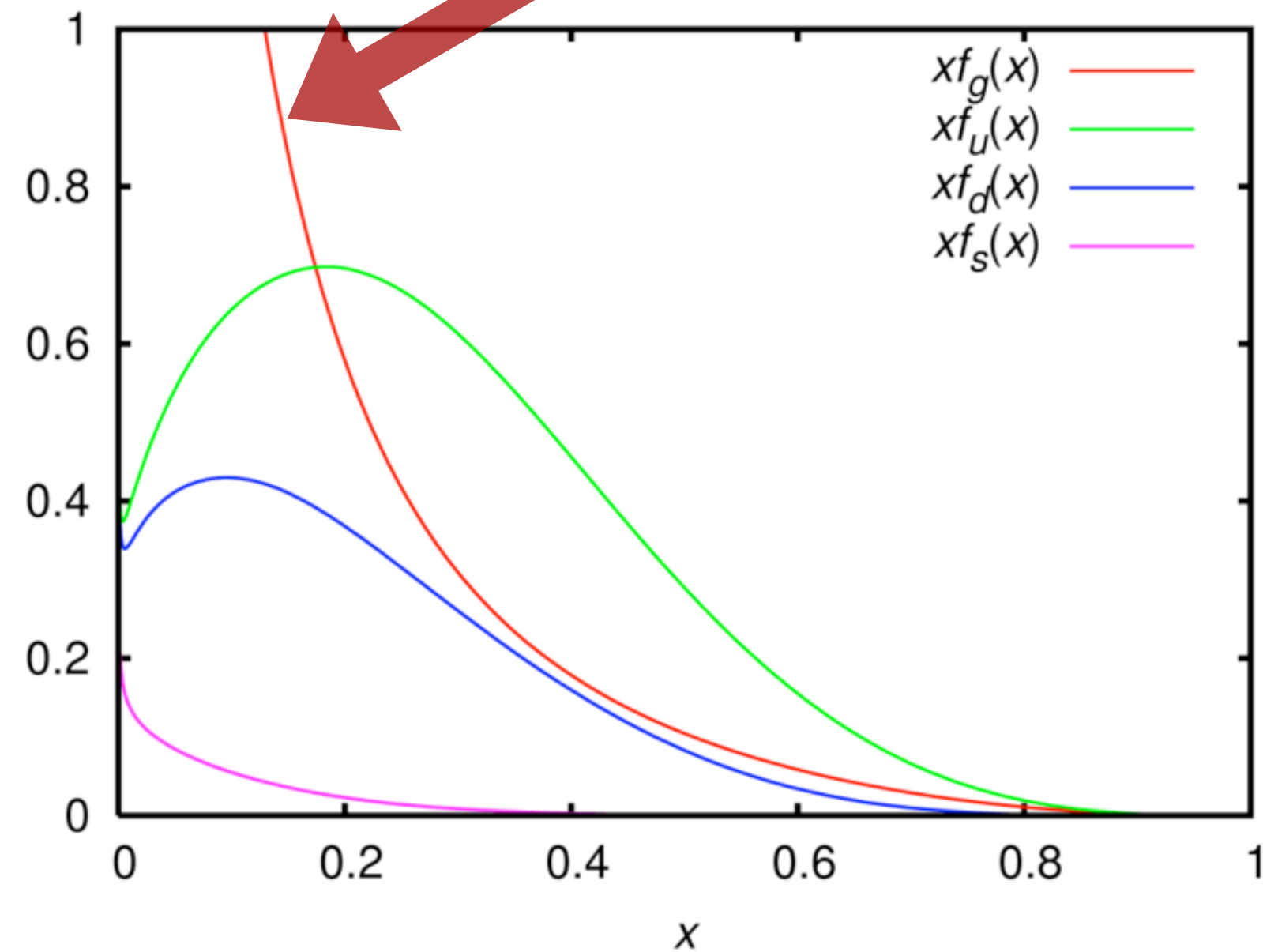
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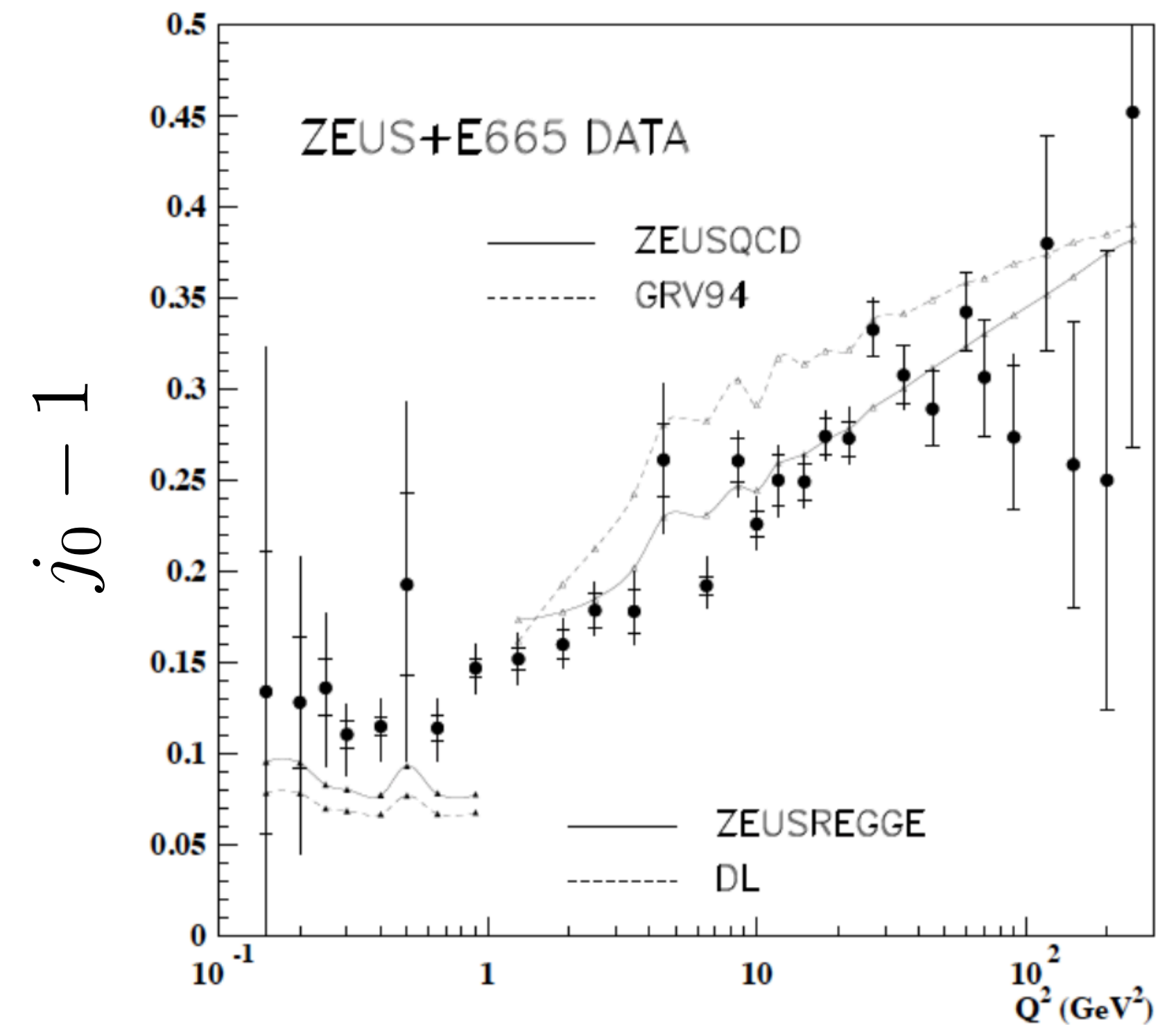
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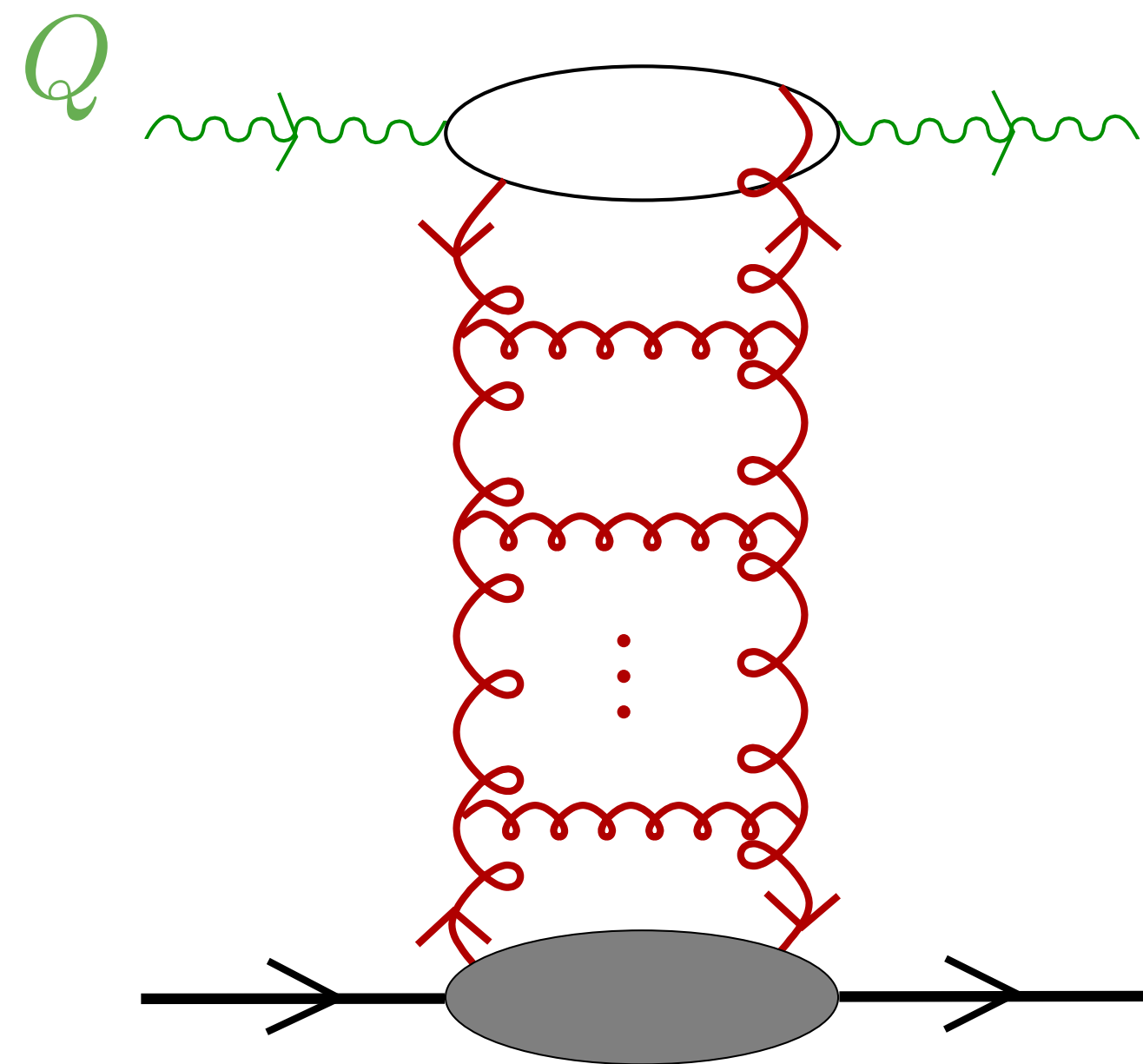
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**Is it the same Regge trajectory? One or two pomerons (soft and hard)?**

# Hard Pomeron [BFKL - Balitsky, Fadin, Kuraev & Lipatov]

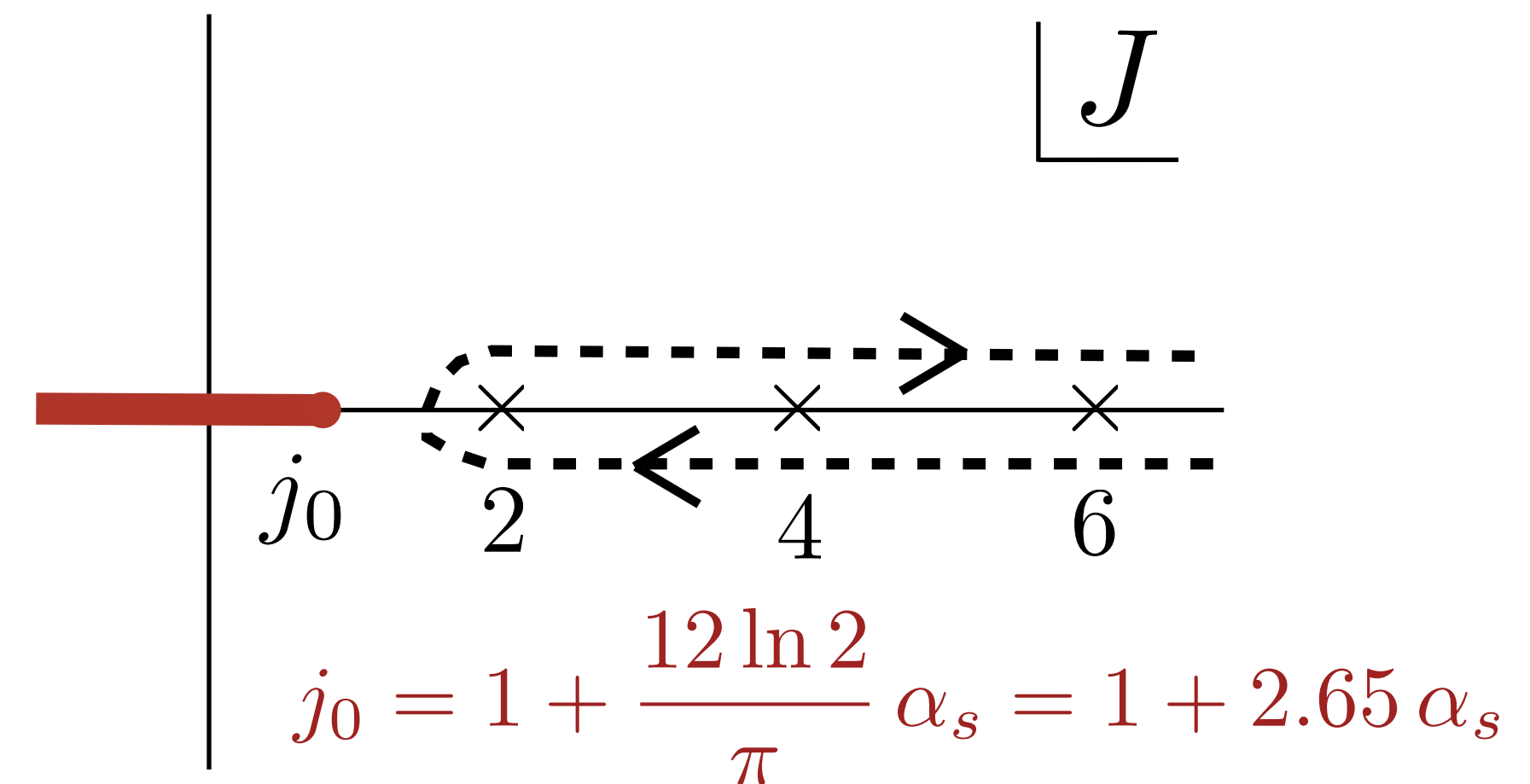


Two gluon exchange with ladder interactions

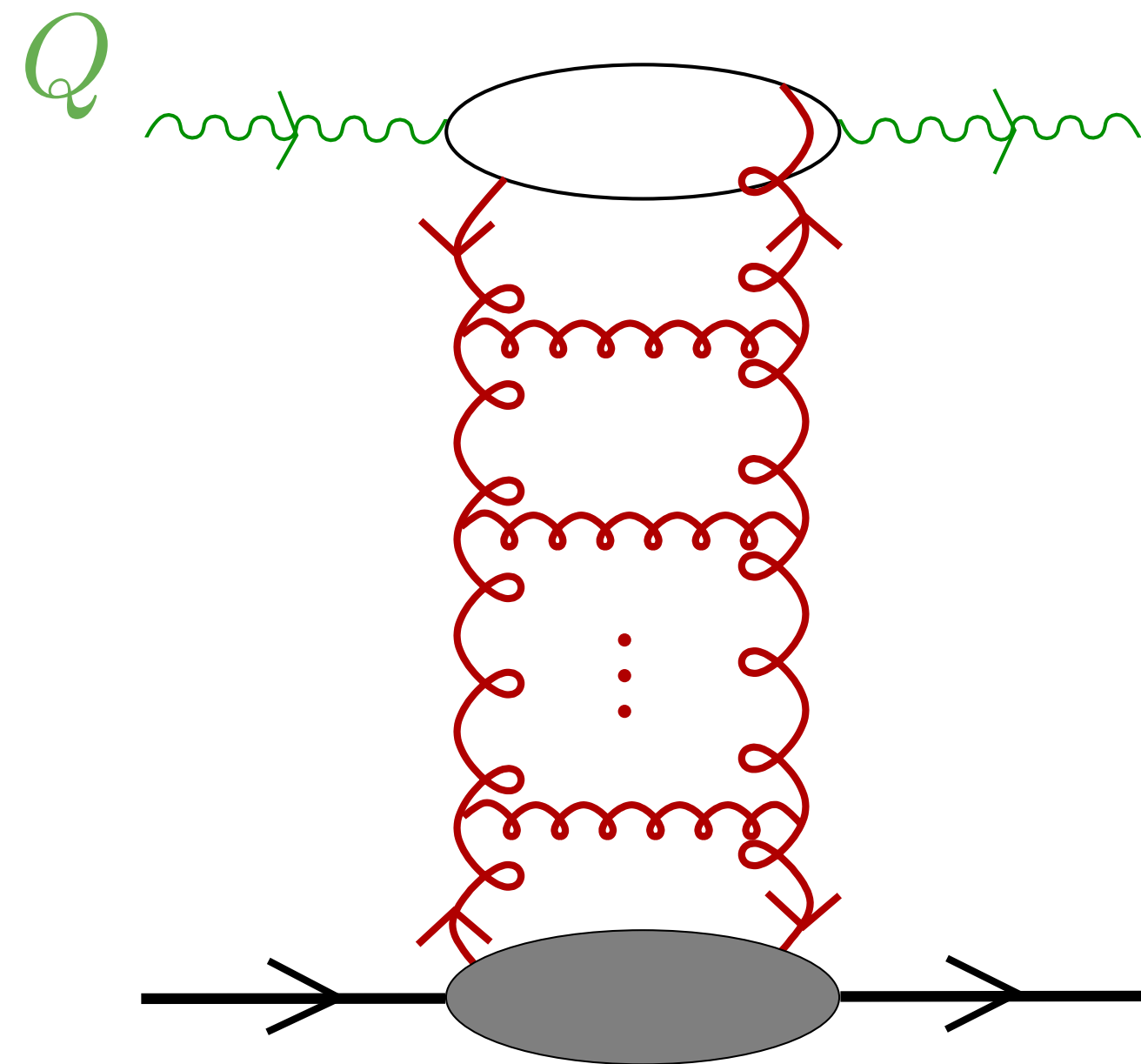
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Valid for hard probes  $Q \gg \Lambda_{QCD}$  (conformal limit)

Hard pomeron is a cut in J-plane



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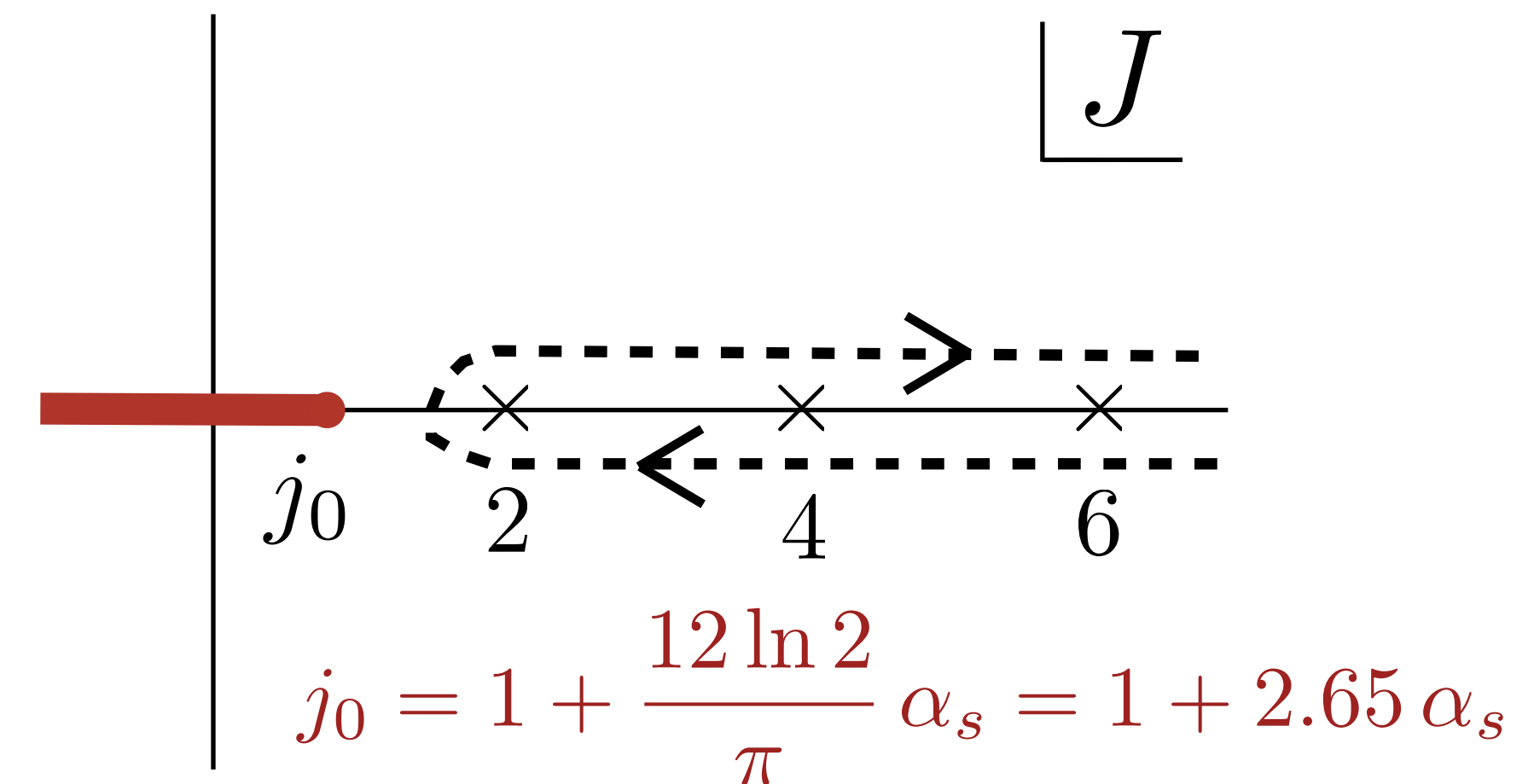


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- Breaking conformal symmetry, explains well DIS data outside the confining region  $Q > \Lambda_{QCD}$  [Kowalski, Lipatov, Ross, Watt 10]

- Strong rise in  $1/x$  , violating Froissart bound

$$\sigma \lesssim m_{\pi} (\ln s)^2$$

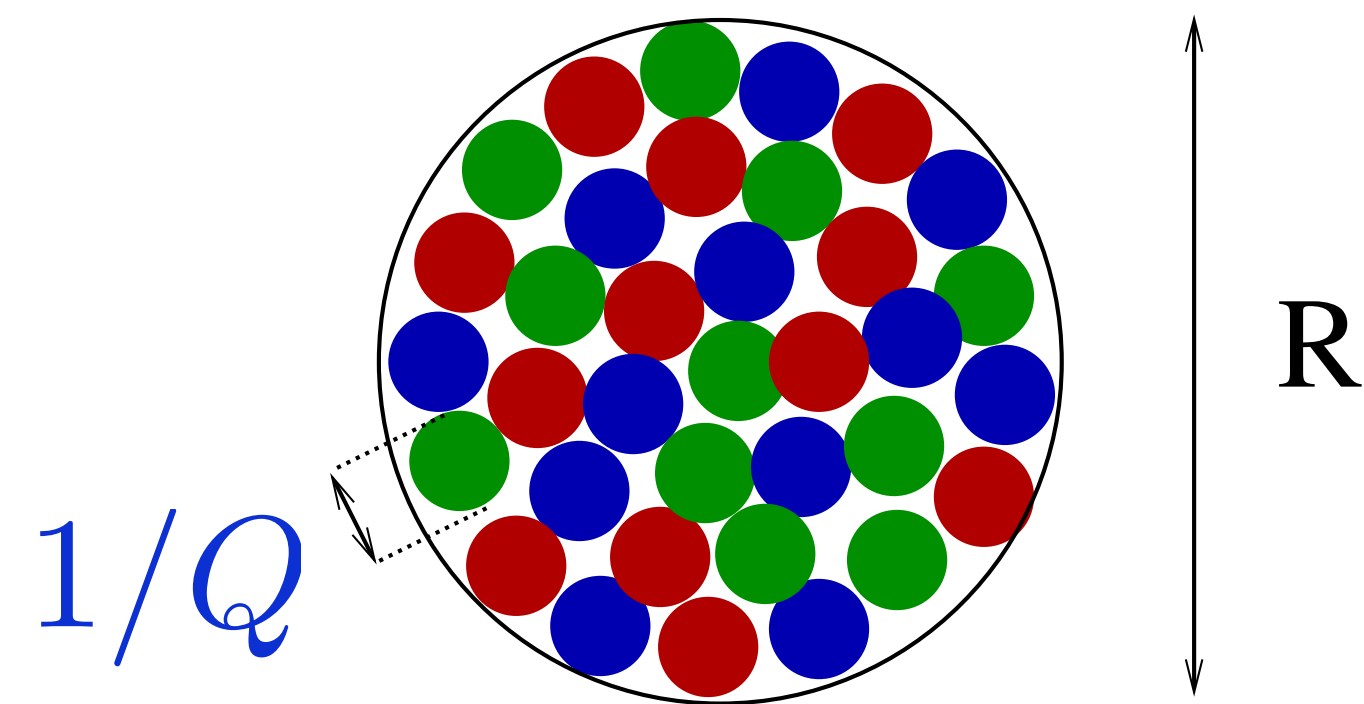
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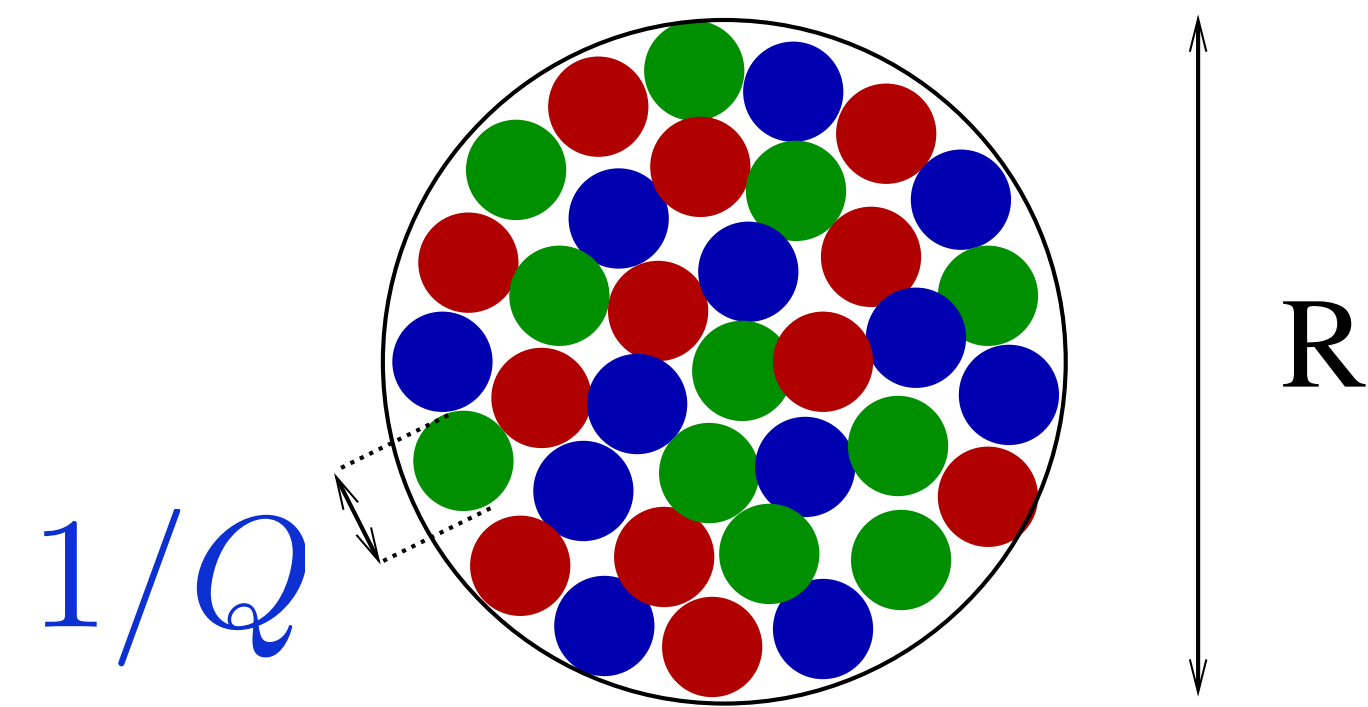


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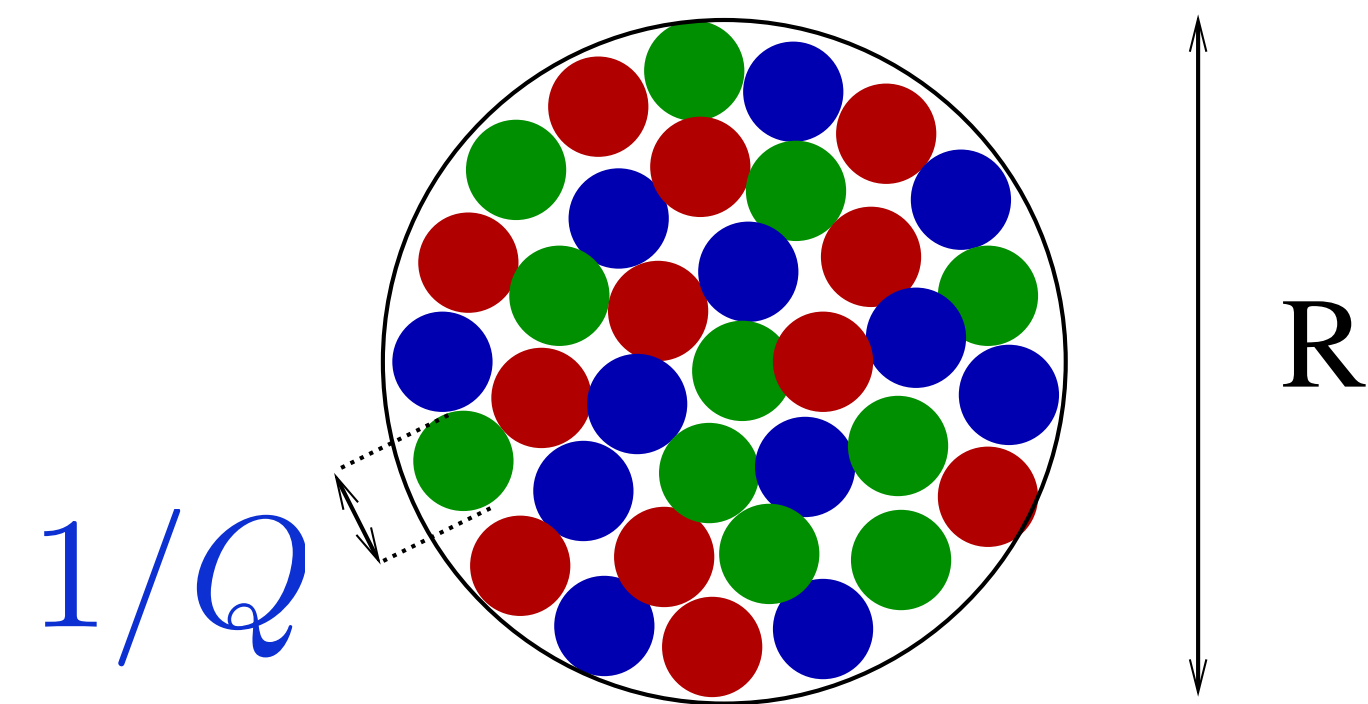
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**Graviton Regge trajectory dual to pomeron trajectory**

[Brower, Polchinski,  
Strassler, Tan 06]

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- Construct Regge theory for CFTs, or for the dual gravity (string theory). For the example of N=4 SYM results are valid at any coupling.
- Once dual description of pomeron well understood, can apply to low  $x$  physics in QCD starting from holographic QCD description, including confining region.

## Regge Kinematics in CFTs [Cornalba, MSC, Penedones, Schiappa 06]

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- Consider correlator with EMG current  $j^a = \bar{\psi}\gamma^a\psi$  and scalar operator  $\mathcal{O}$

$$A^{ab}(y_i) = \langle j^a(y_1) \mathcal{O}(y_2) j^b(y_3) \mathcal{O}(y_4) \rangle$$

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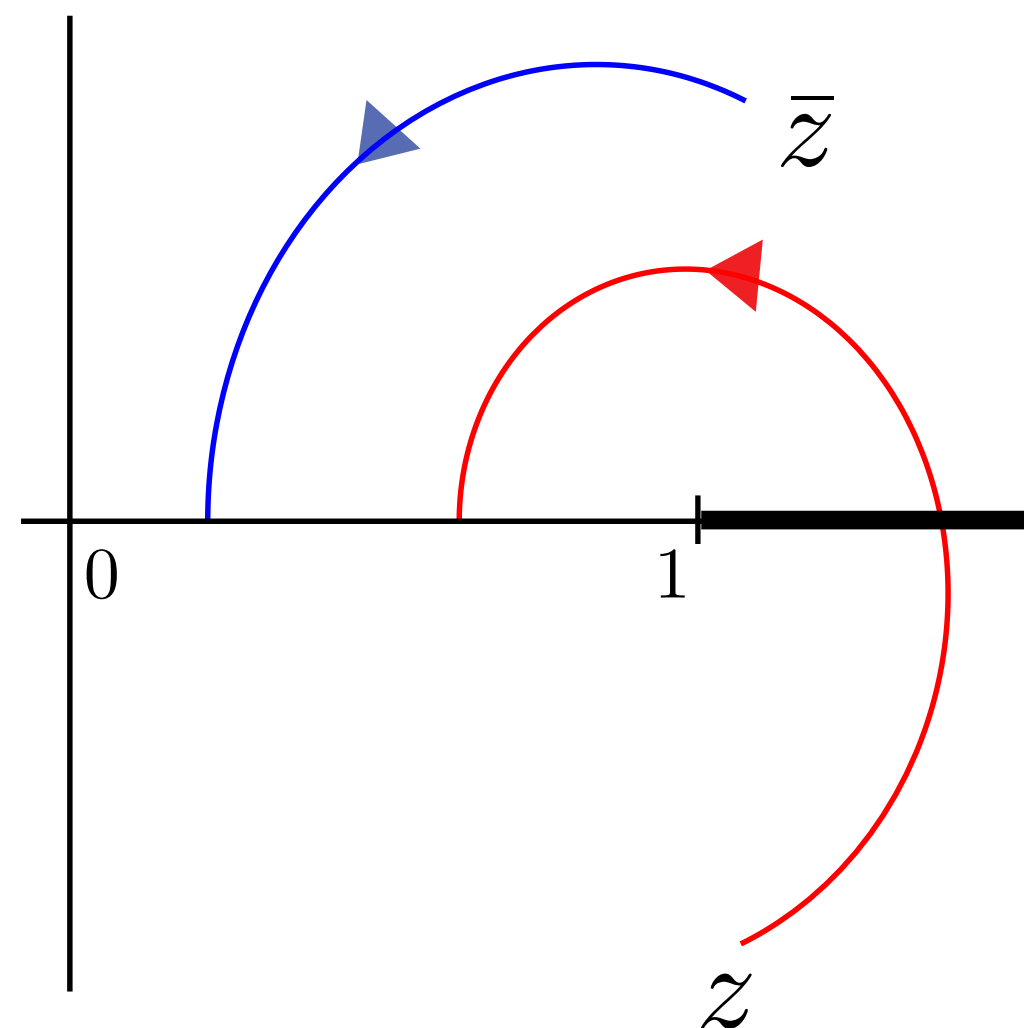
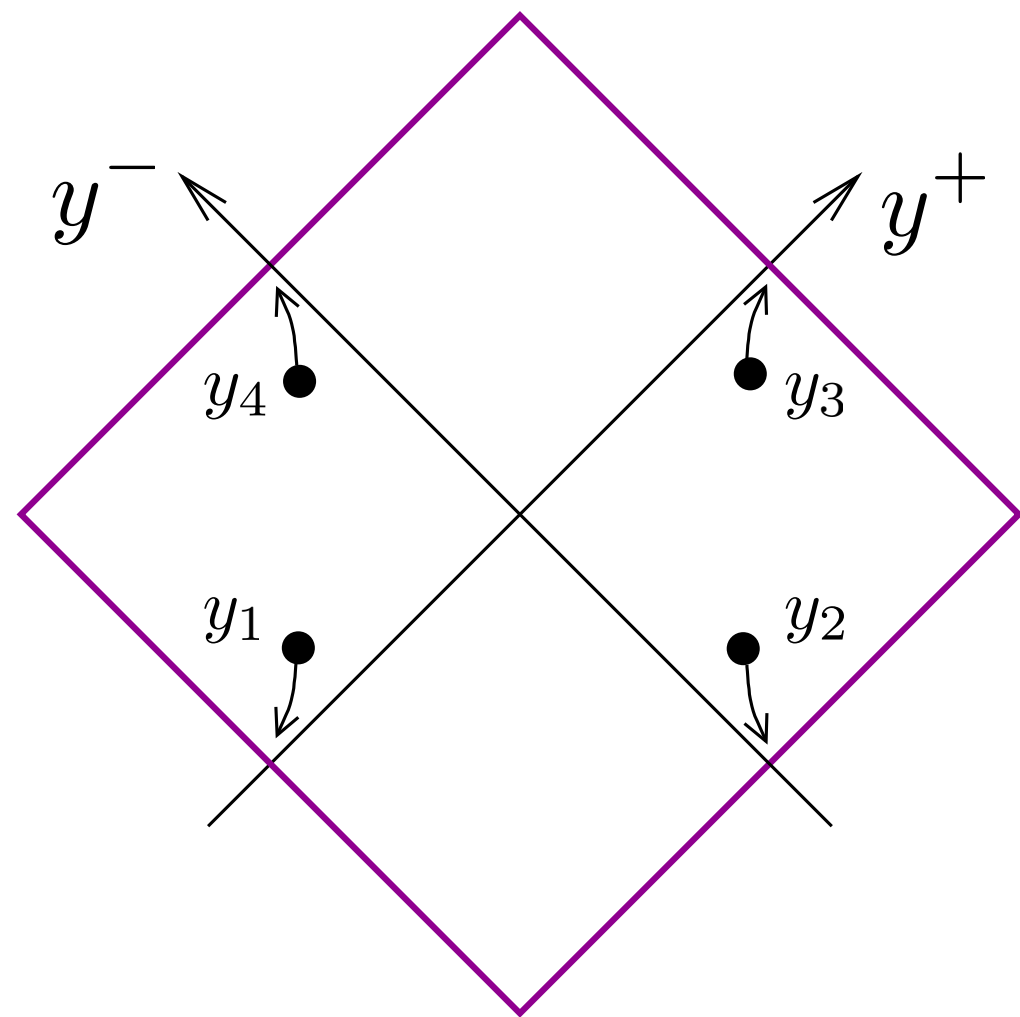
$$A^{\cancel{\phi}}(y_i) = \langle j^{\cancel{\times}}(y_1) \mathcal{O}(y_2) j^{\cancel{\times}}(y_3) \mathcal{O}(y_4) \rangle = \frac{A(z, \bar{z})}{(y_{13})^{2\xi} (y_{24})^{2\Delta}} \quad \begin{aligned} z\bar{z} &= \frac{y_{13}y_{24}}{y_{12}y_{34}} \\ (1-z)(1-\bar{z}) &= \frac{y_{14}y_{23}}{y_{12}y_{34}} \end{aligned}$$

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- Regge kinematics is Lorentzian. Analytically continue from Euclidean theory ( $\bar{z} = z^*$ ) to  $z, \bar{z}$  on real axis.



- Regge limit

$$z, \bar{z} \rightarrow 0 \quad \text{with} \quad \frac{z}{\bar{z}} \quad \text{fixed}$$

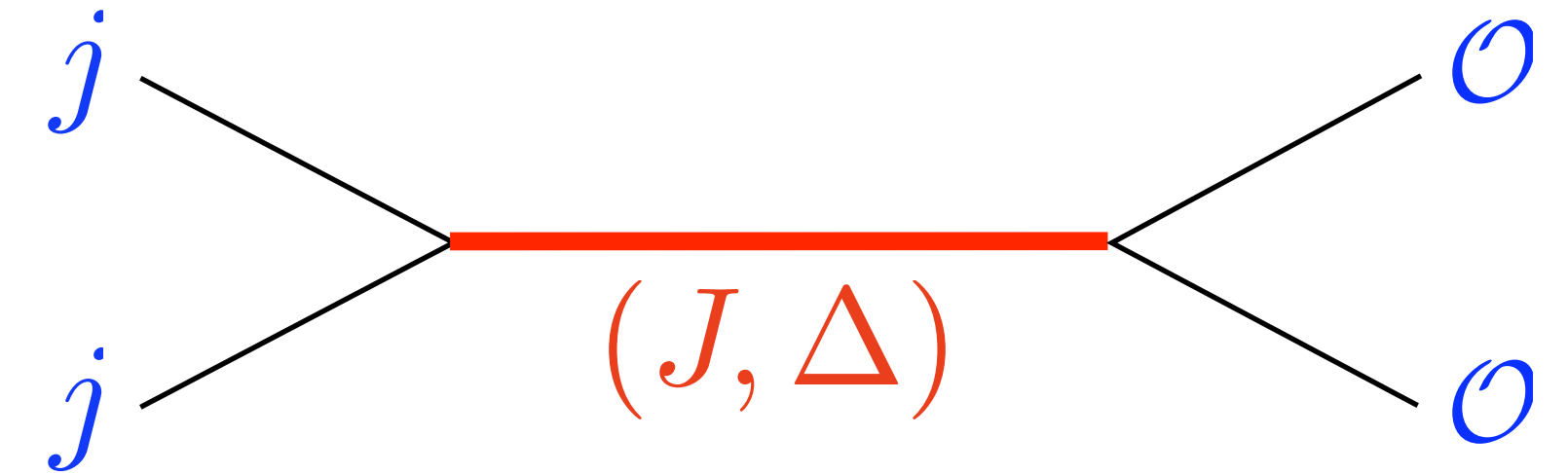
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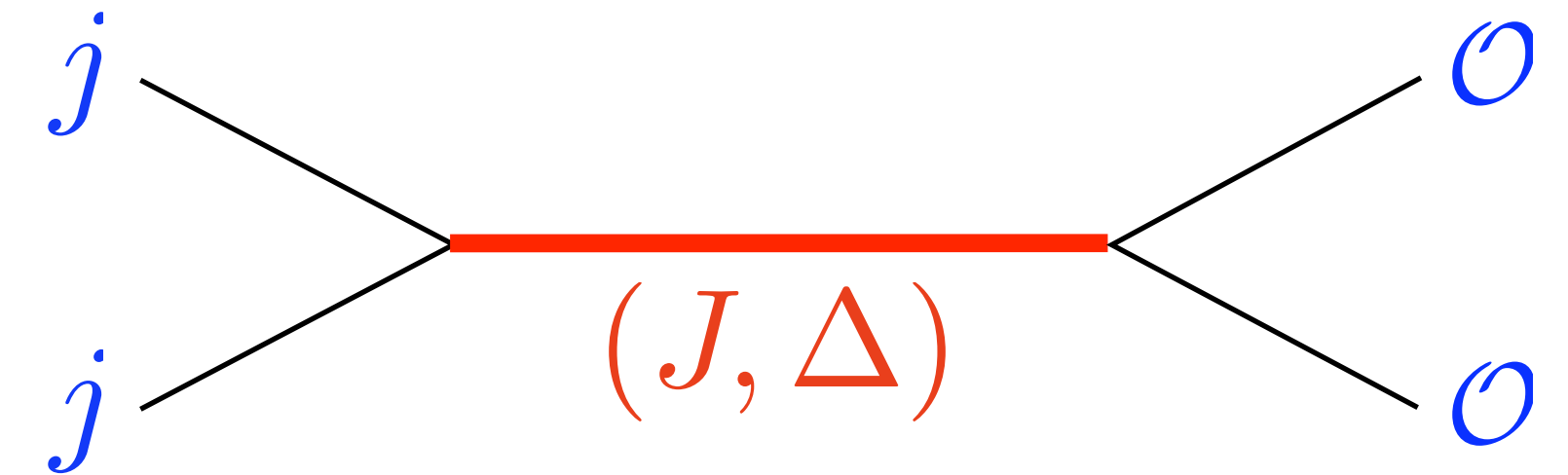


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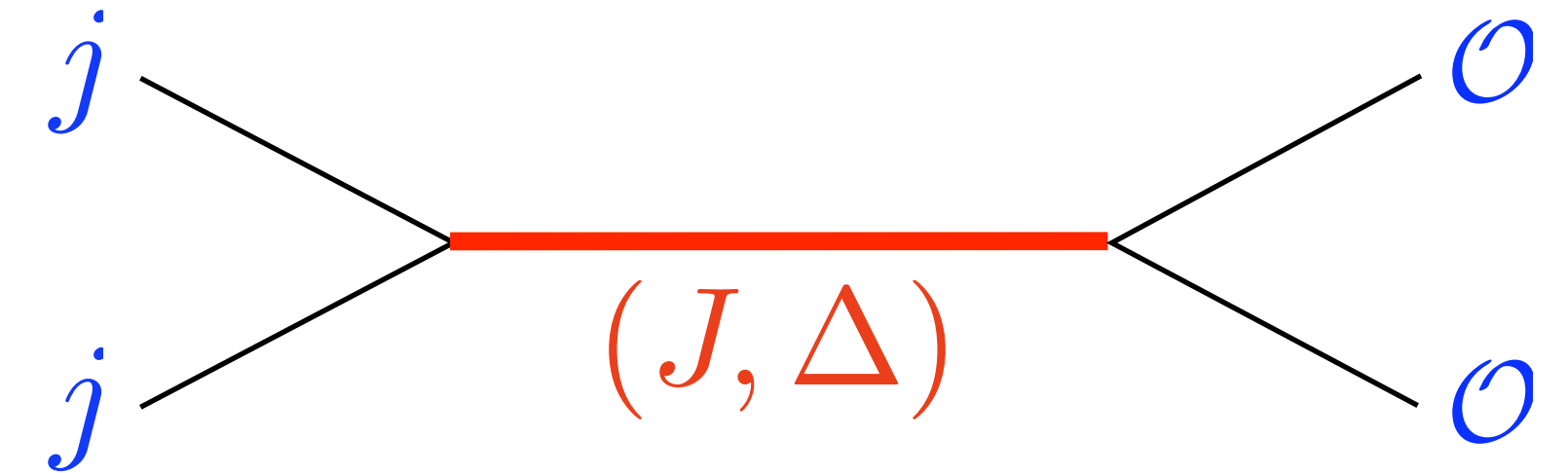


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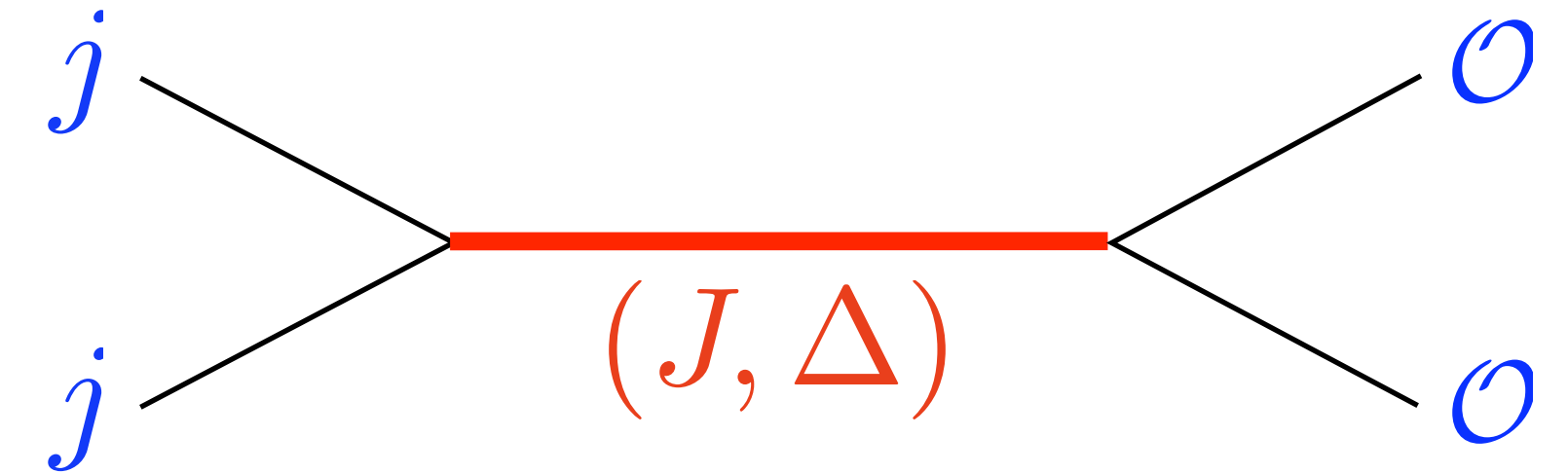
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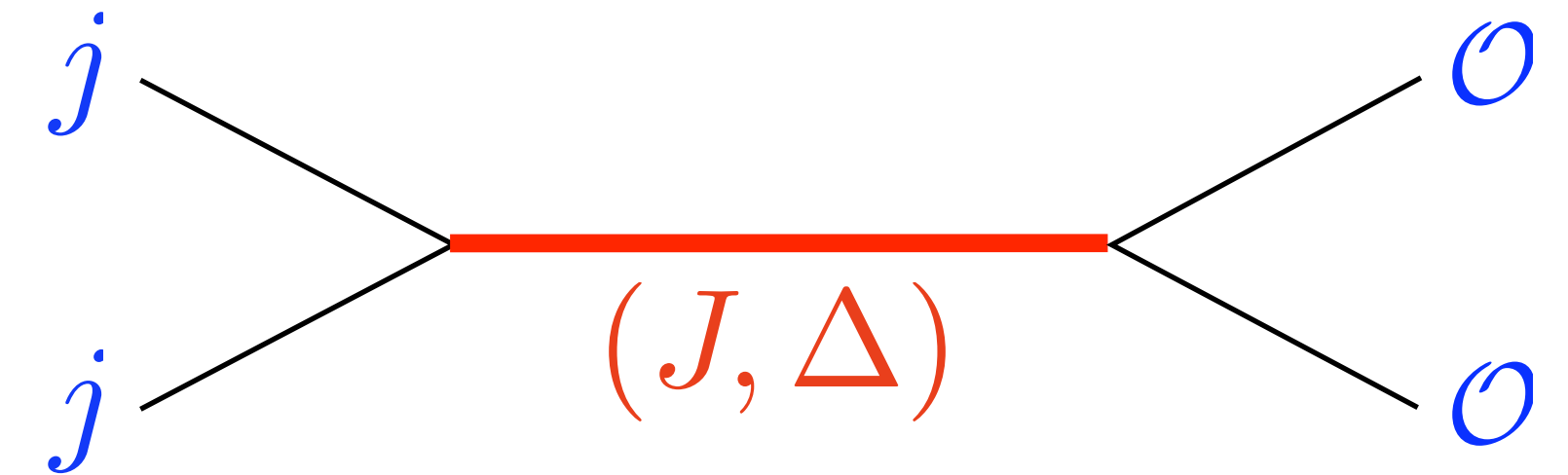
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$$\sim \sum_J \sigma^{1-J} \int d\nu \alpha_J(\nu) \Omega_{i\nu}(\rho)$$

$$\sigma^2 = z\bar{z}, \quad e^{2\rho} = \frac{z}{\bar{z}}$$

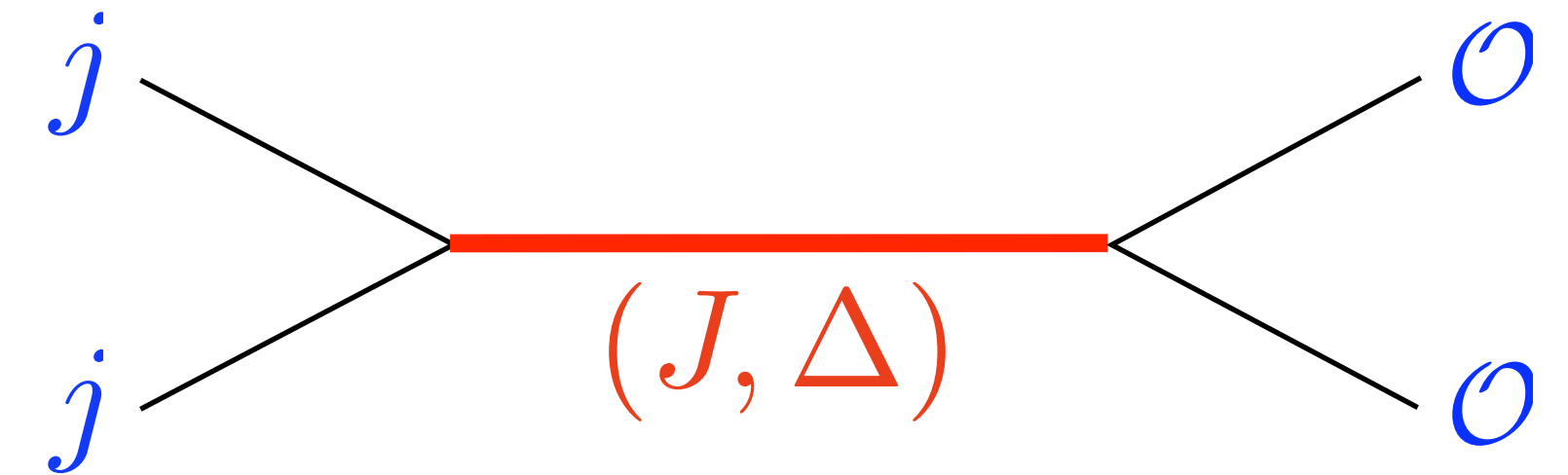
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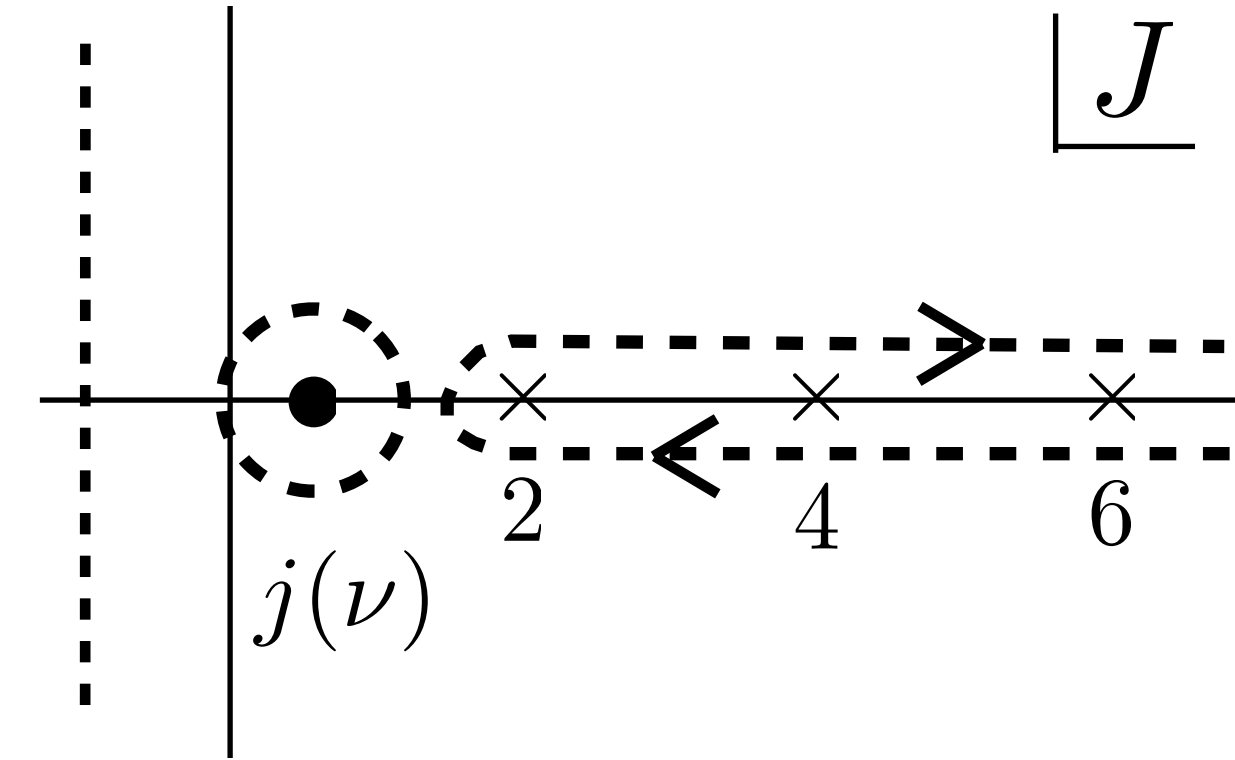
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**Regge limit**  $\sigma \rightarrow 0$ ,  $\rho$  fixed

$$\sim \sum_J \sigma^{1-J} \int d\nu \alpha_J(\nu) \Omega_{i\nu}(\rho) \longrightarrow \text{Harmonic function on } \mathbb{H}_{d-1} \left( \nabla^2 \Omega_{i\nu} = -(\nu^2 + h - 1) \Omega_{i\nu} \right)$$

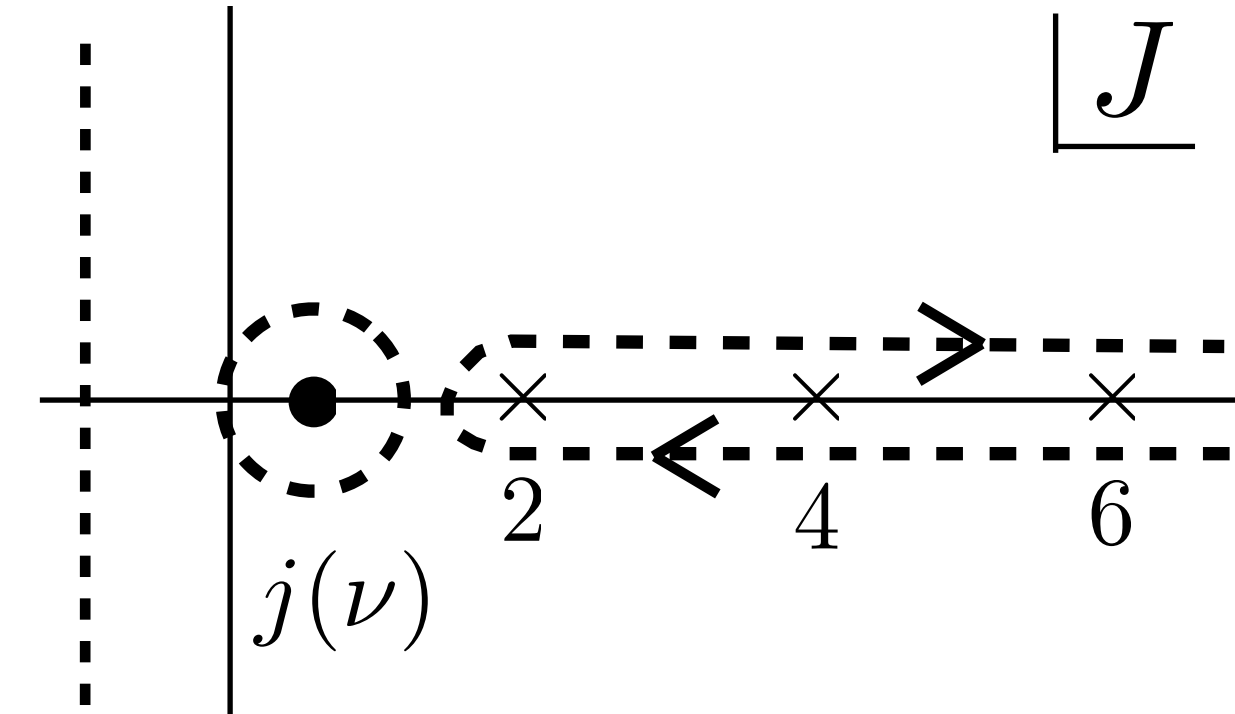
- Sum over spin using Sommerfeld-Watson transform

$$\mathcal{A}(\sigma, \rho) = \sigma \int d\nu \int_C \frac{dJ}{2\pi i} \frac{\pi}{2 \sin(\pi J)} \left( \sigma^{-J} + (-\sigma)^{-J} \right) \alpha_J(\nu) \Omega_{i\nu}(\rho)$$



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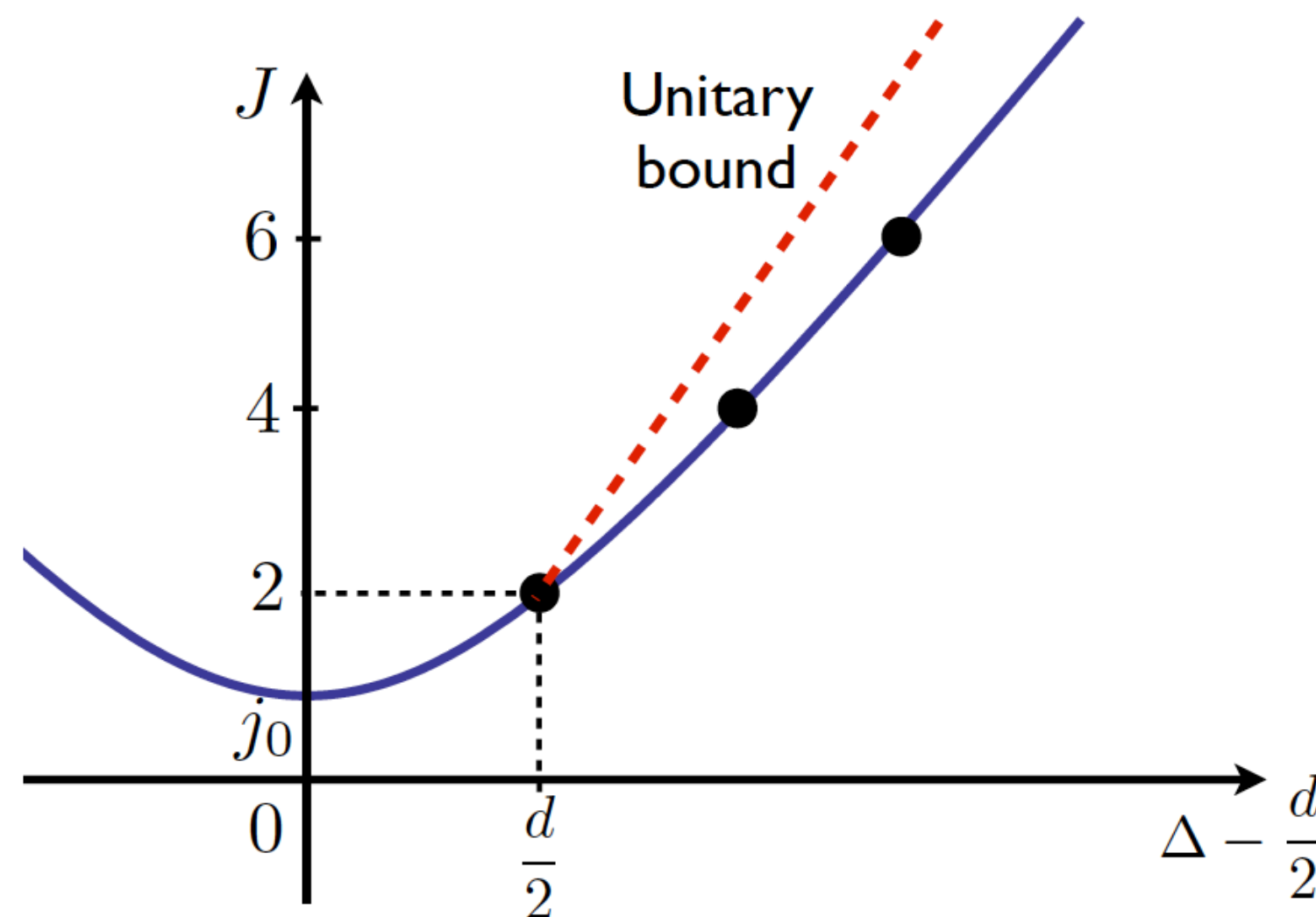
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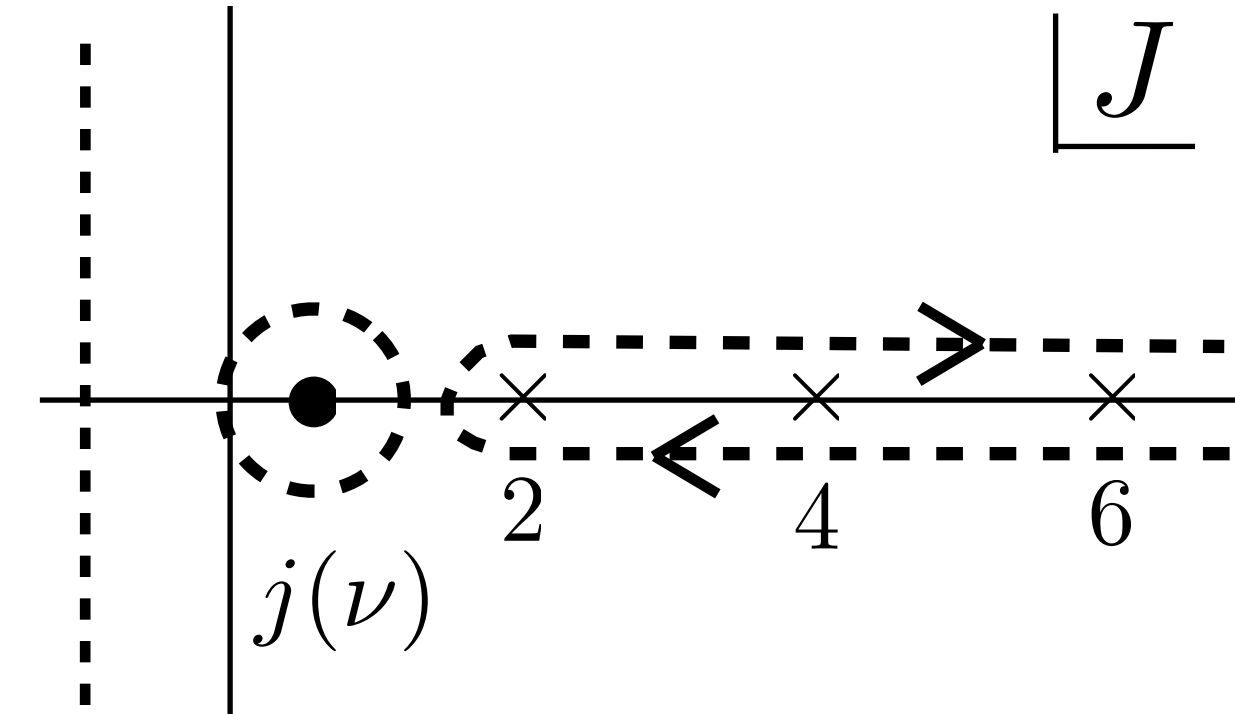
**Regge pole for  $J = j(\nu)$  such that**

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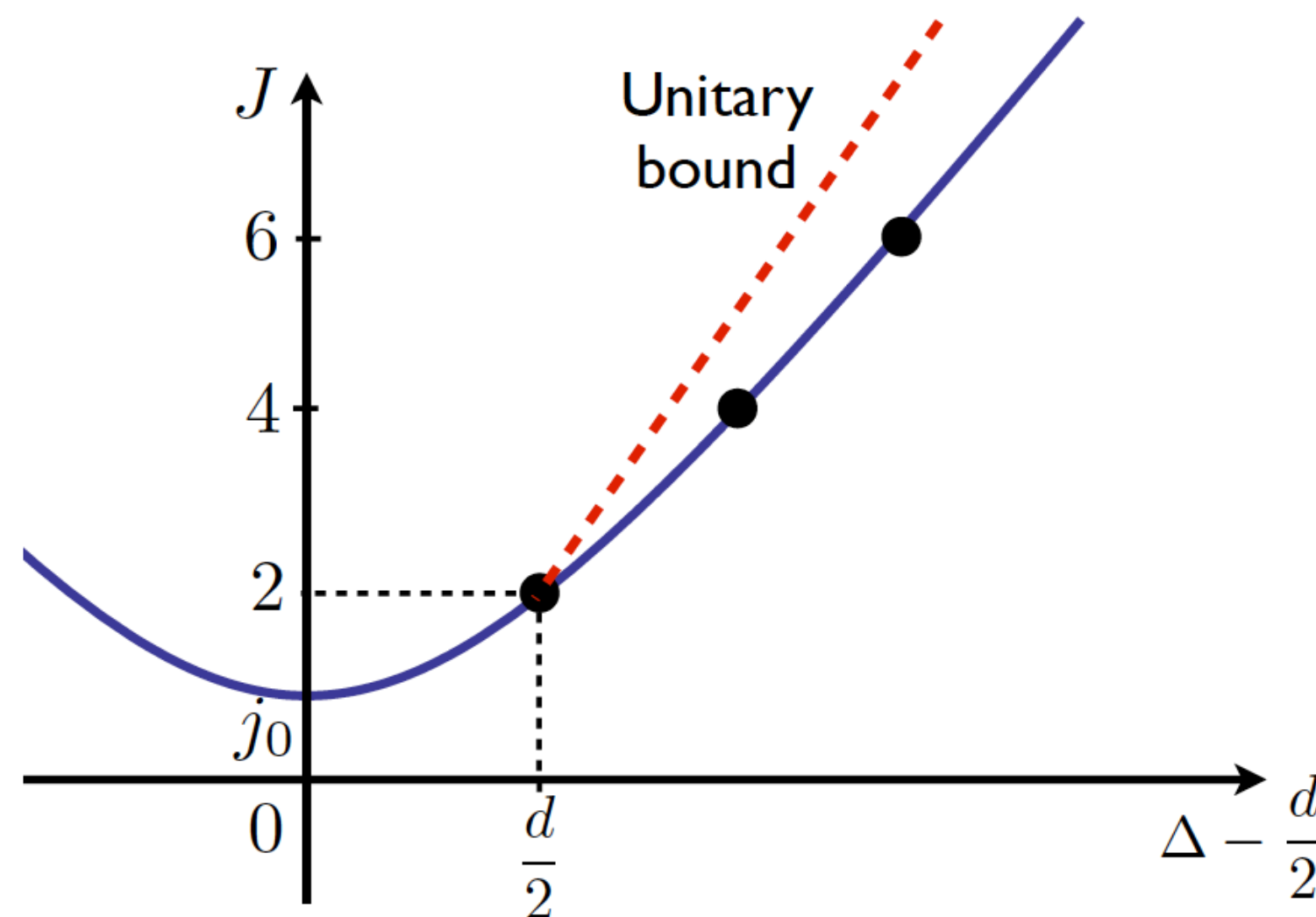
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# N=4 Super Yang Mills

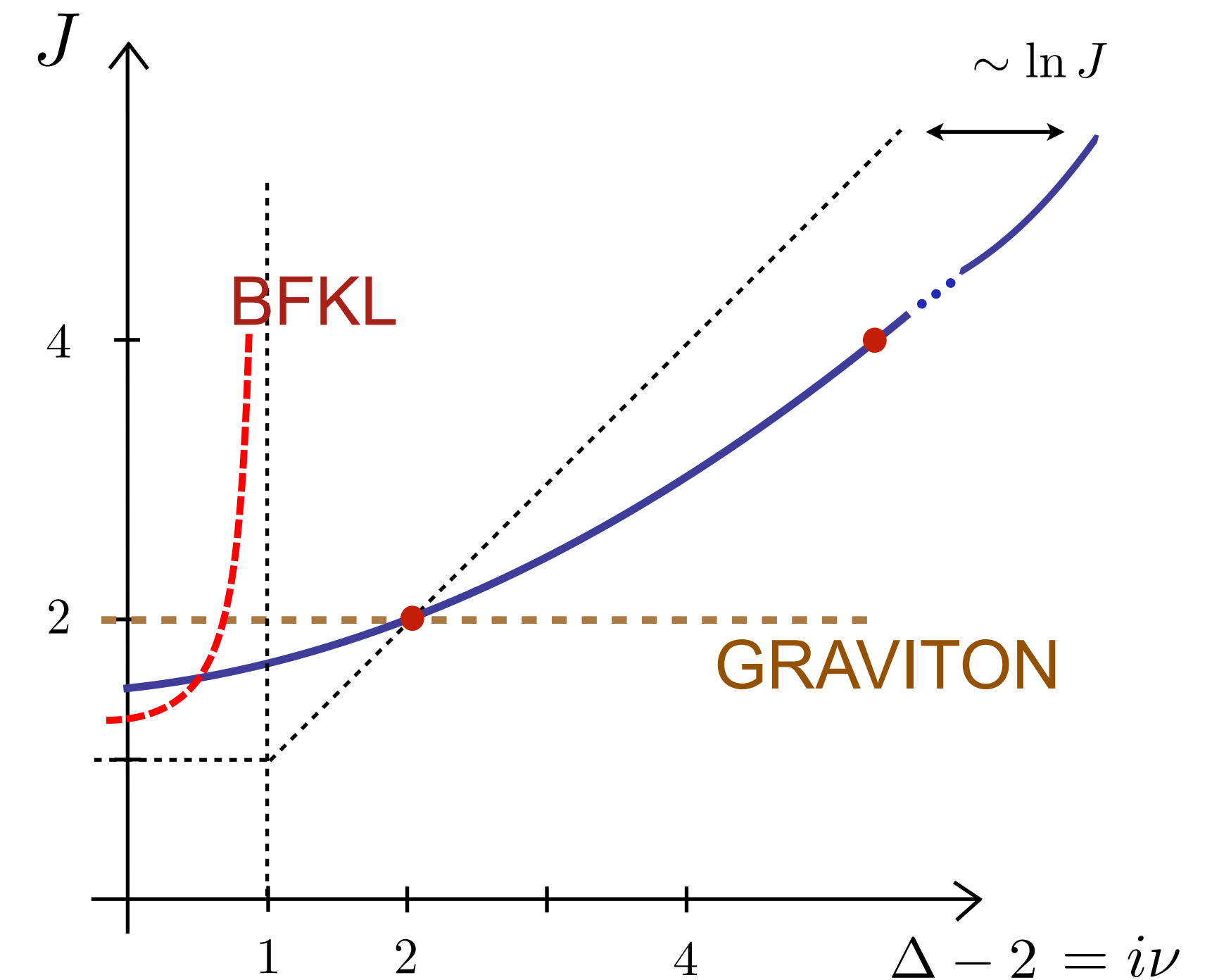
- Correlation functions that exchange vacuum quantum numbers are dominated in Regge limit by exchange of pomeron/graviton Regge trajectory (twist 2)

$$\mathcal{A}(\sigma, \rho) = \int d\nu \alpha(\nu, \lambda) \sigma^{1-j(\nu, \lambda)} \Omega_{i\nu}(\rho)$$

$$J = j(\nu, \lambda) = j_0(\lambda) - \mathcal{D}(\lambda) \nu^2 + \dots$$

$$\mathcal{O}_J \sim \text{Tr} (F_{\alpha\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J}{}^\alpha)$$

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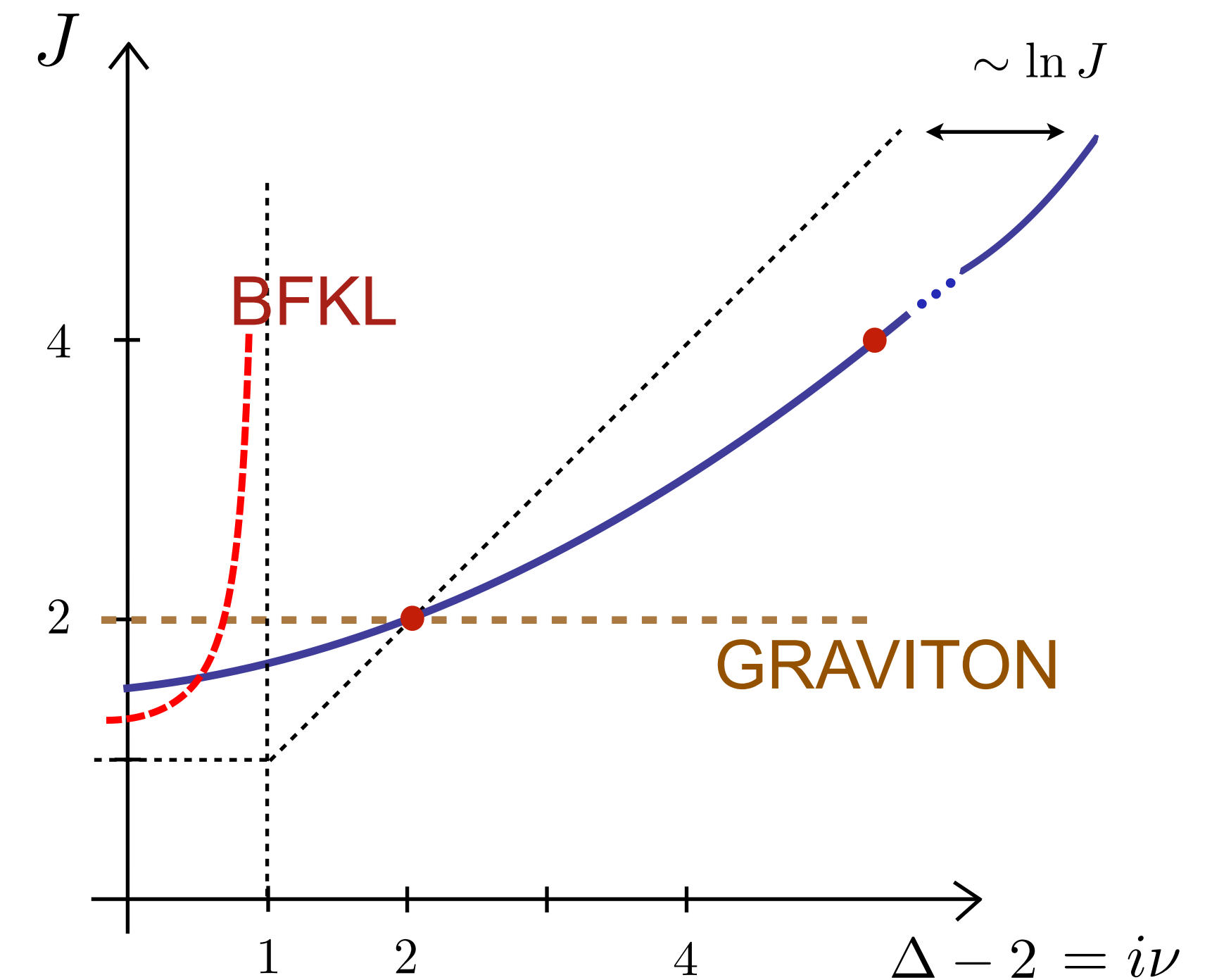
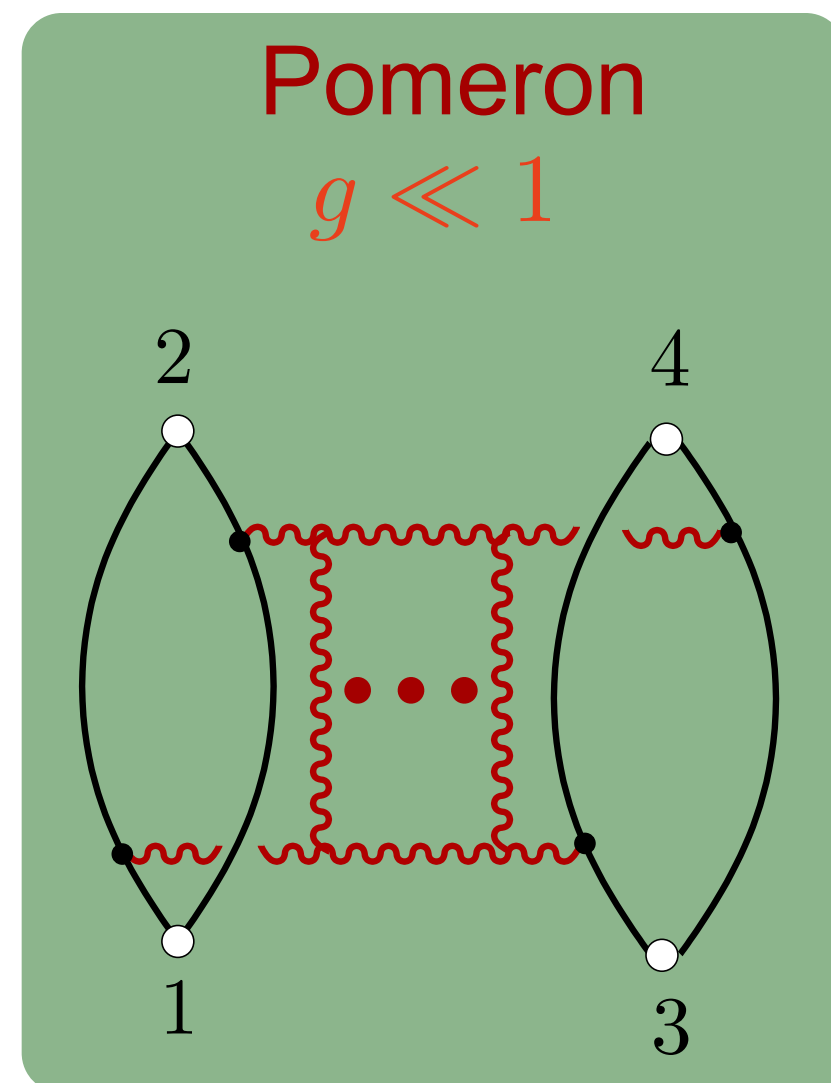
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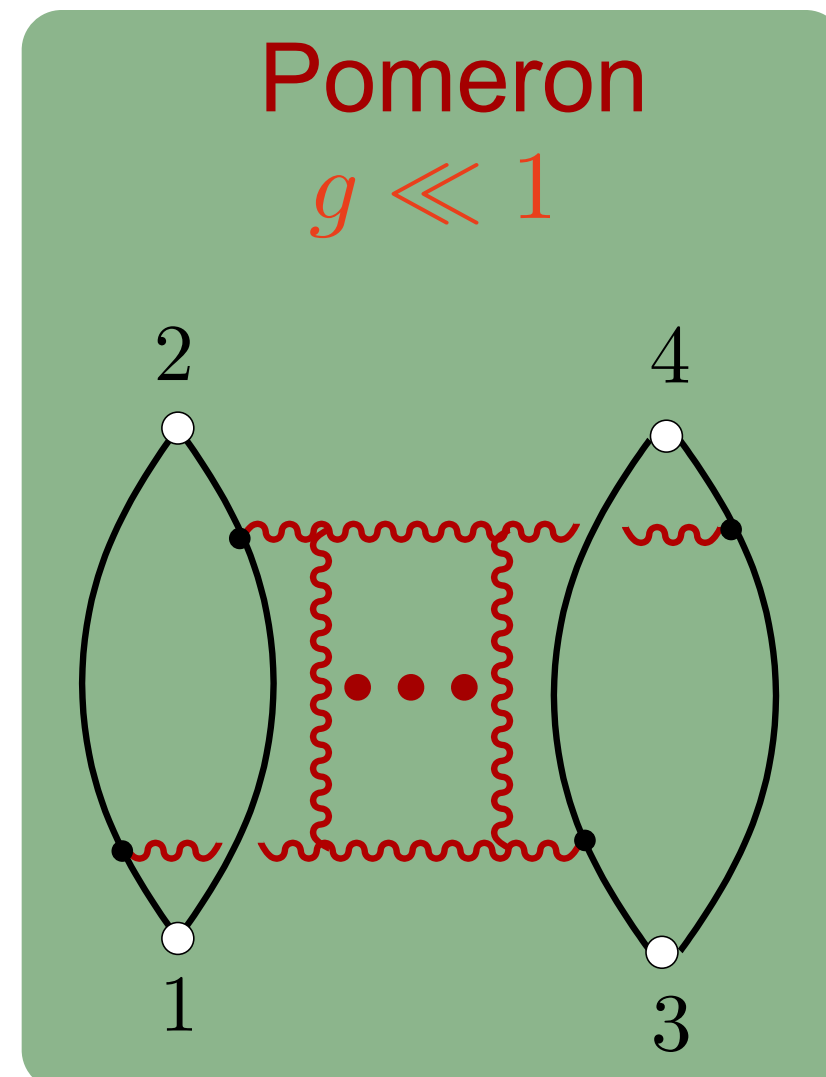
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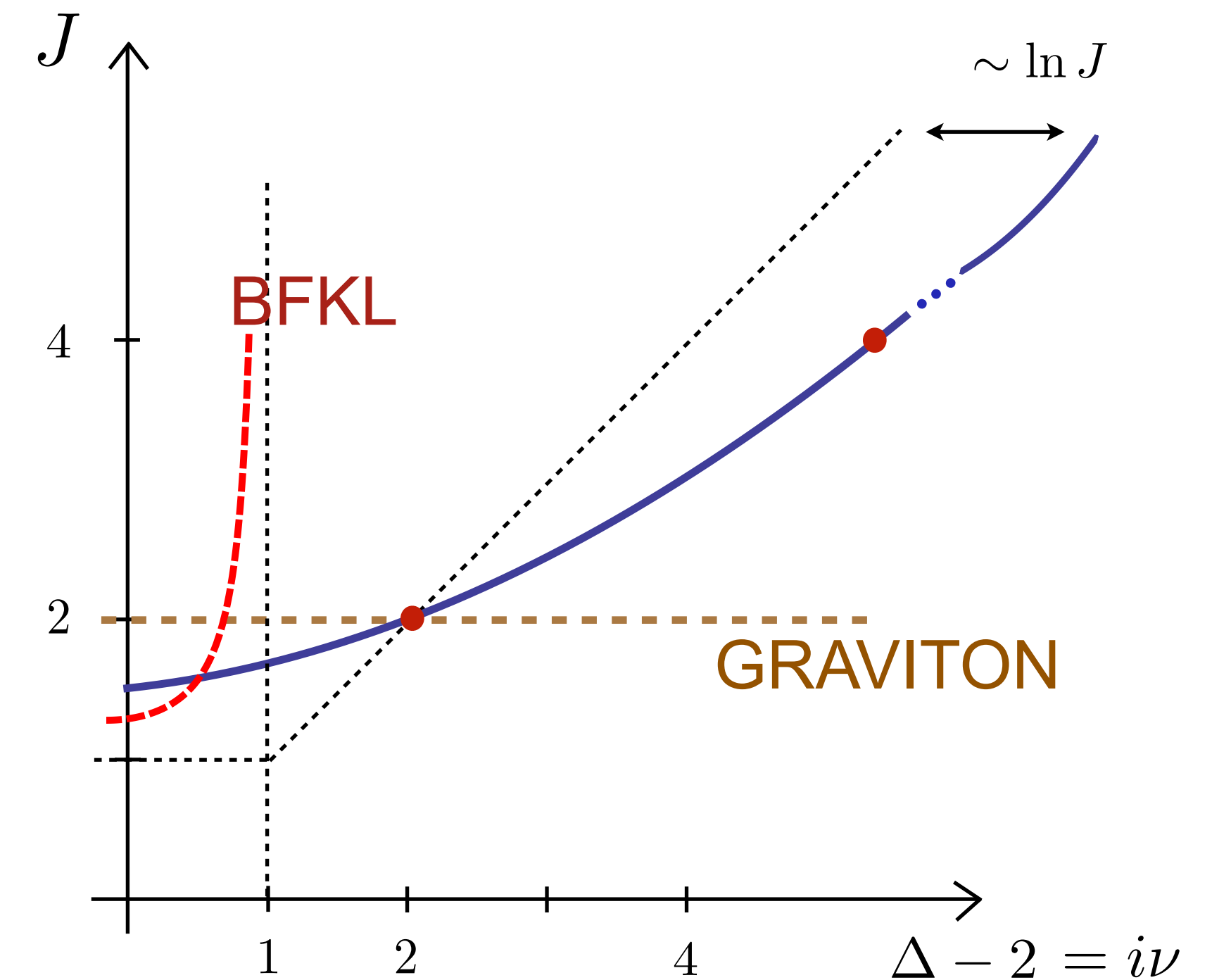
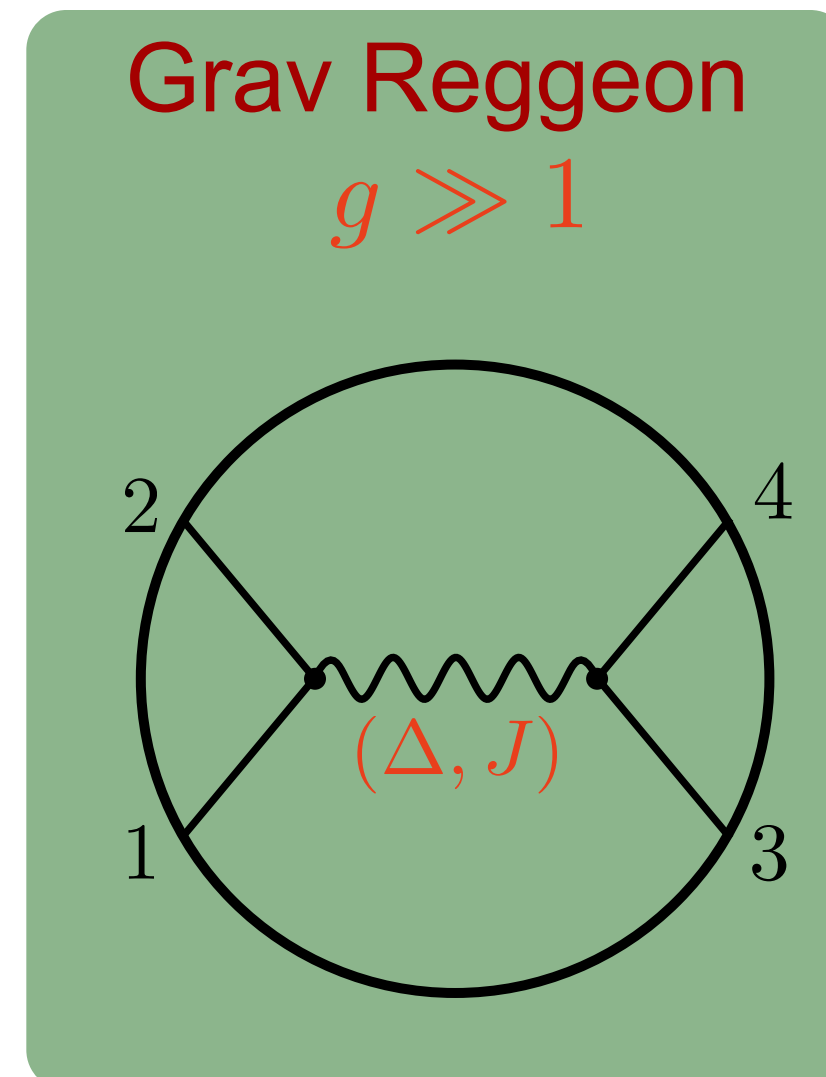
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# Graviton/Pomeron Regge trajectory at strong coupling [BPST 06]

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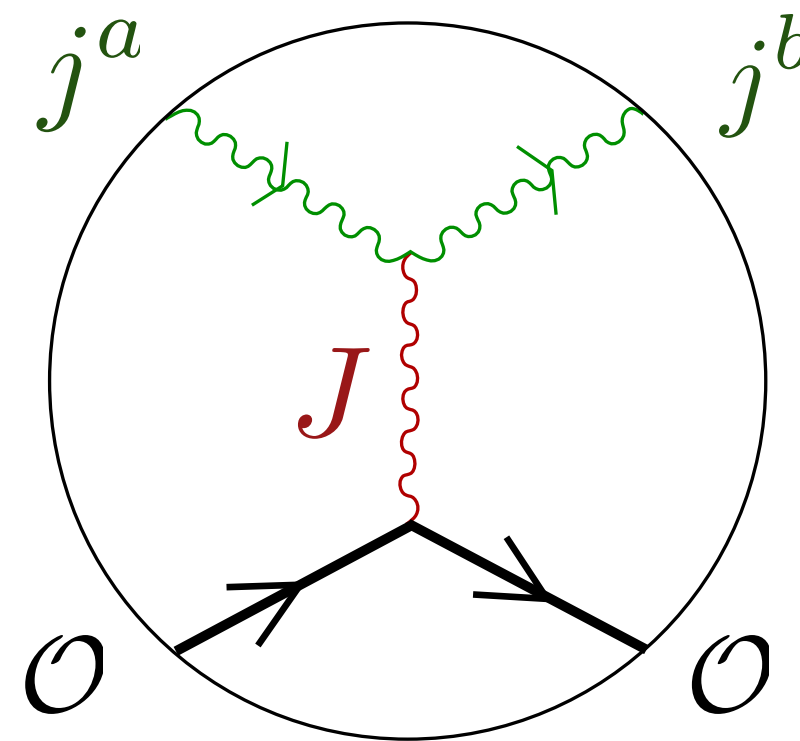
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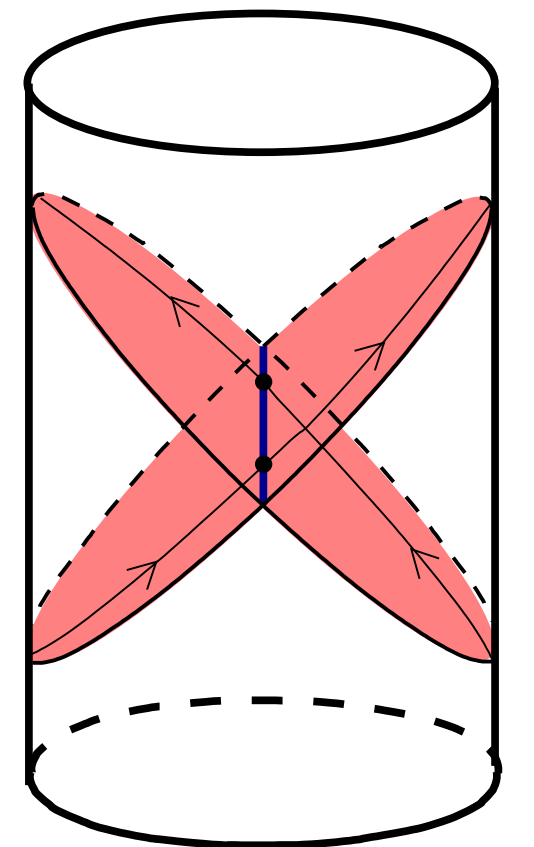
Exchange of spin J field in AdS  
(symmetric, traceless and transverse)

$$(D^2 - m^2) h_{a_1 \dots a_J} = 0$$

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AdS scattering  
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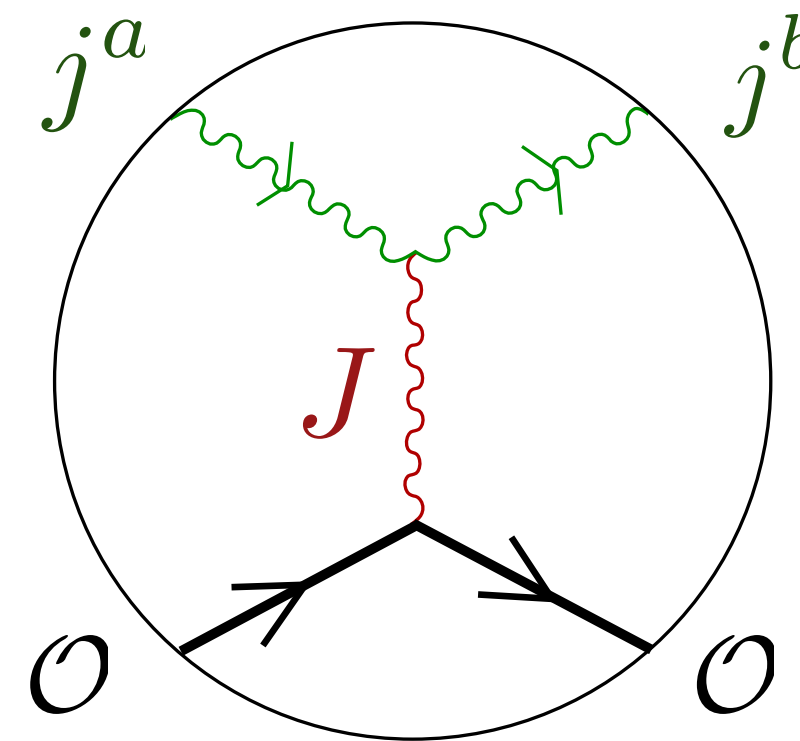
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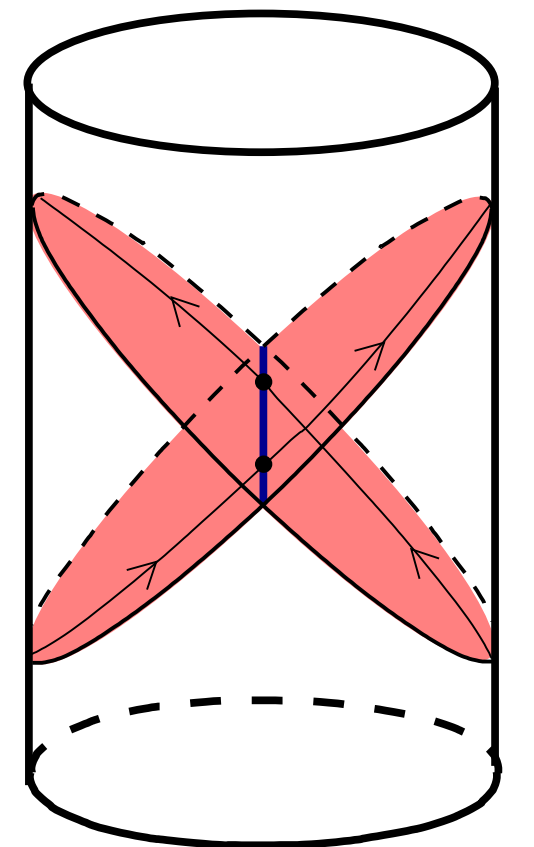
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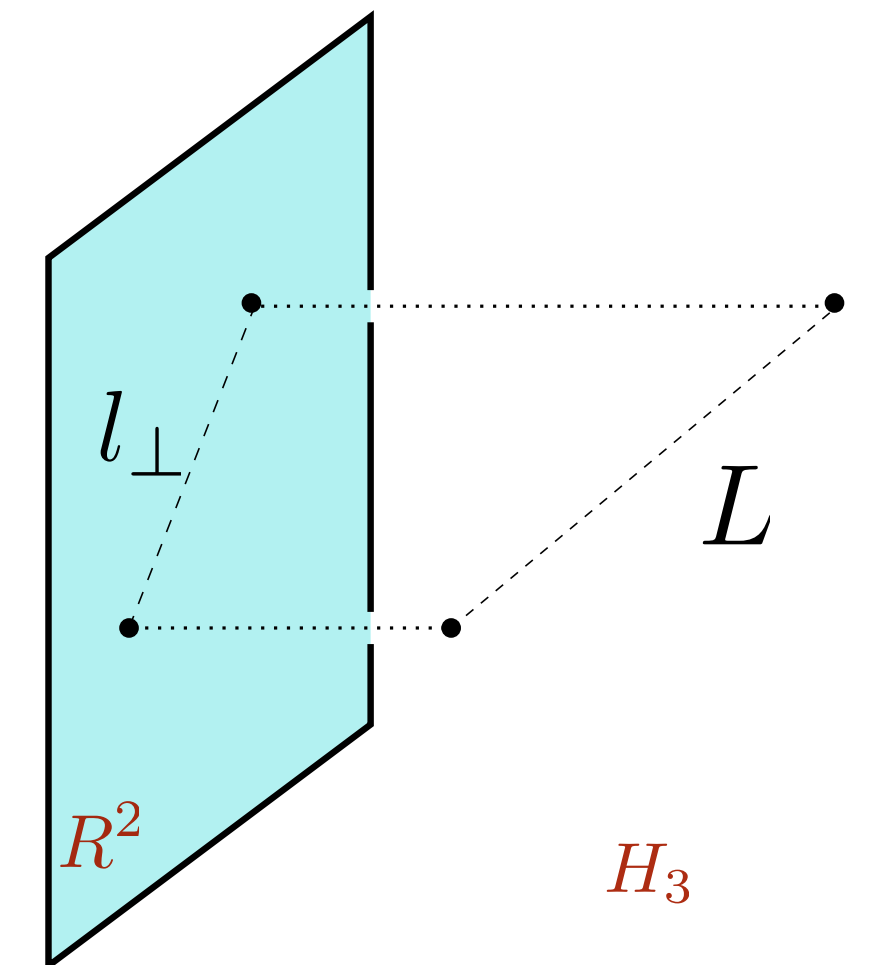
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- AdS impact parameter representation. In Regge limit

[Cornalba, MSC, Penedones, Schiappa 07]

$$A_J(s, t) \approx iV \kappa_J \kappa'_J s \int dl_{\perp} e^{iq_{\perp} \cdot l_{\perp}} \int \frac{dz}{z^3} \frac{dz'}{z'^3} \Phi_1(z) \Phi_3(z) \Phi_2(z') \Phi_4(z') S^{J-1} G_J(L)$$



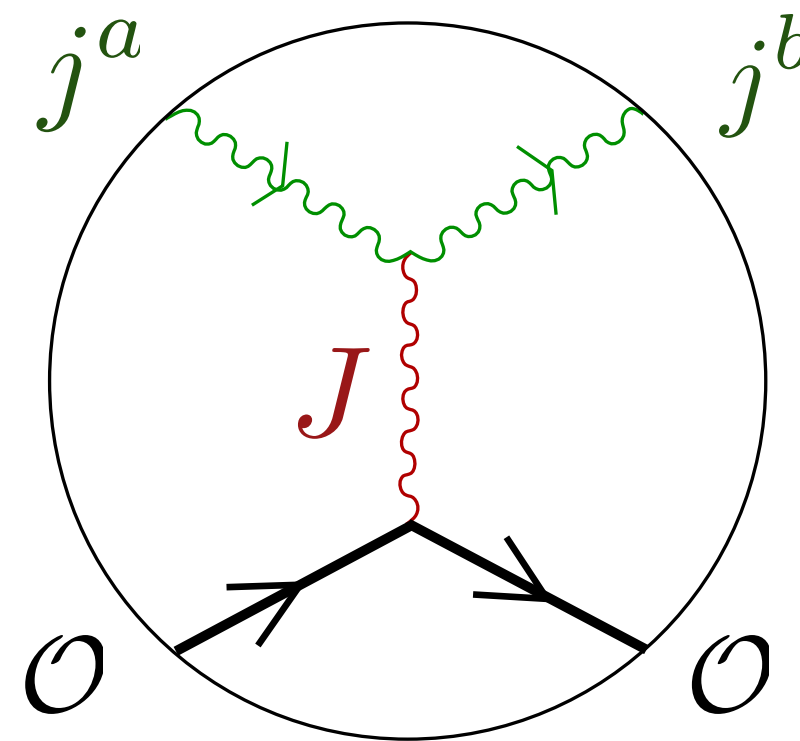
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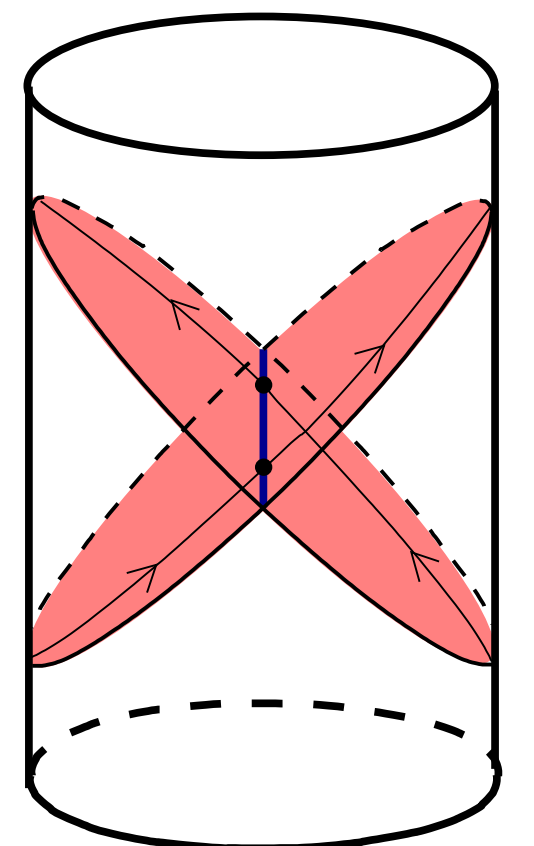
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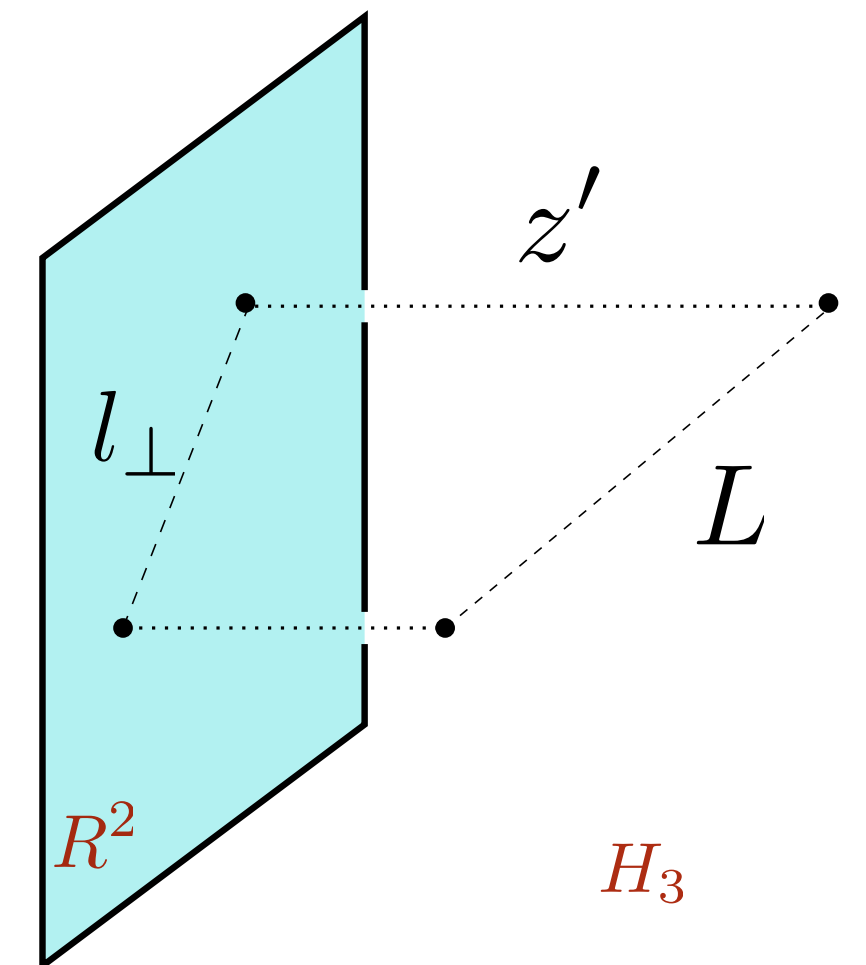
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$S = zz' s$ , AdS energy squared

$$\cosh L = \frac{z^2 + z'^2 + l_{\perp}^2}{2zz'}, \text{ impact parameter}$$

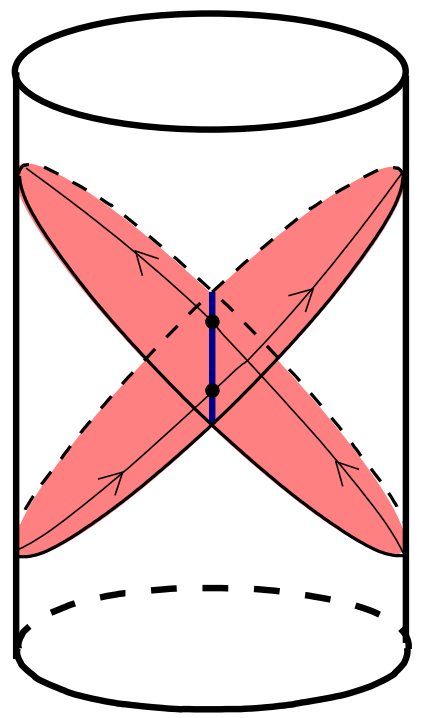


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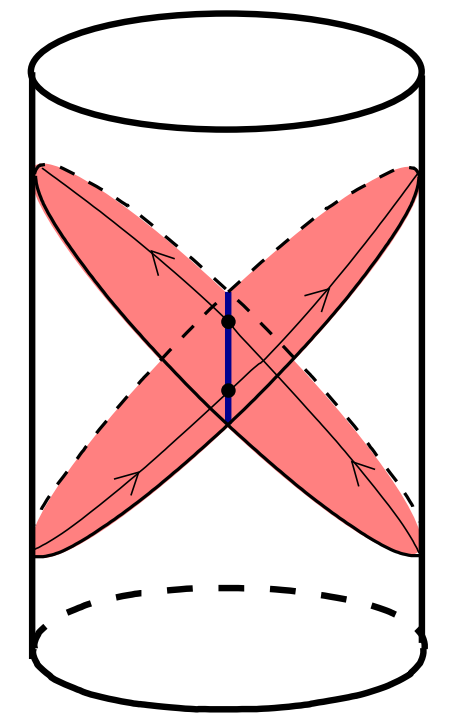
- $G_J(L)$  is the integrated propagator (  $w = x - x' = (w^+, w^-, l_{\perp})$  )

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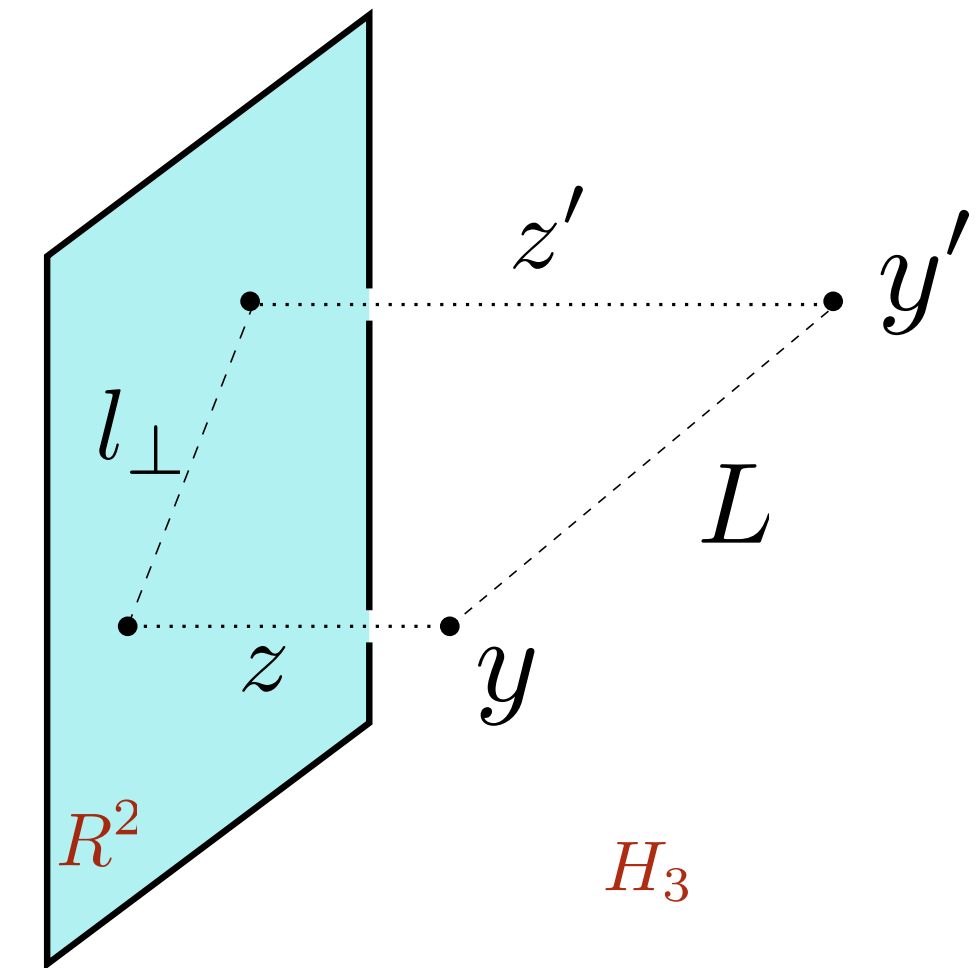


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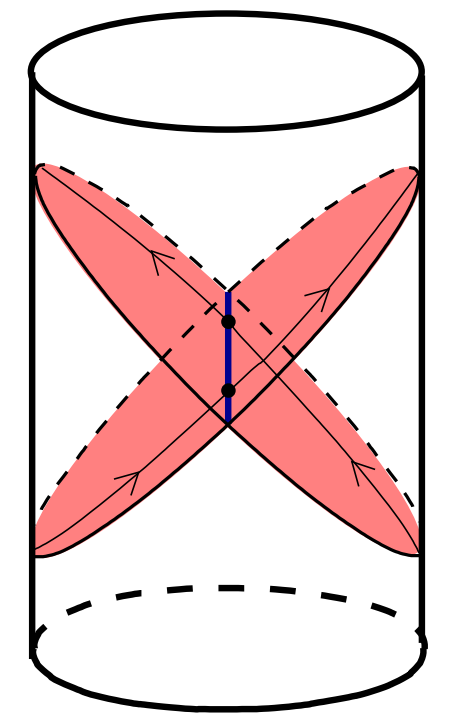
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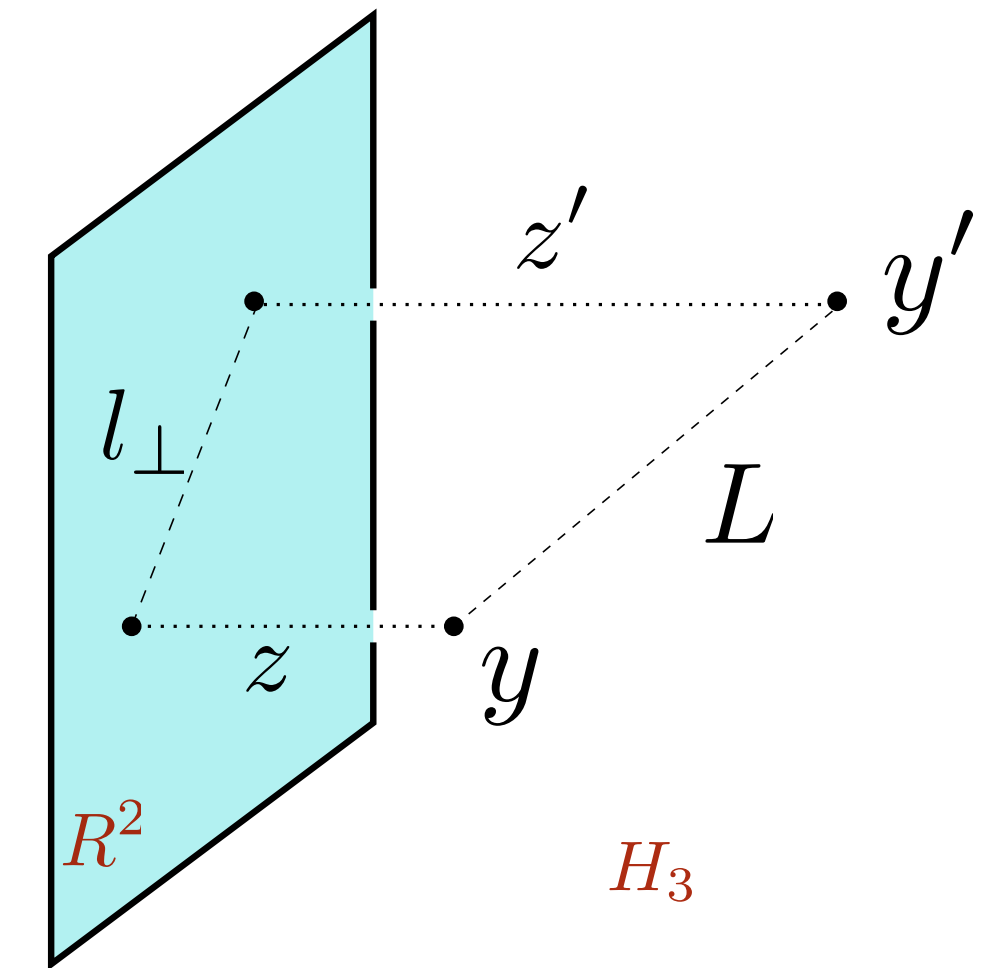


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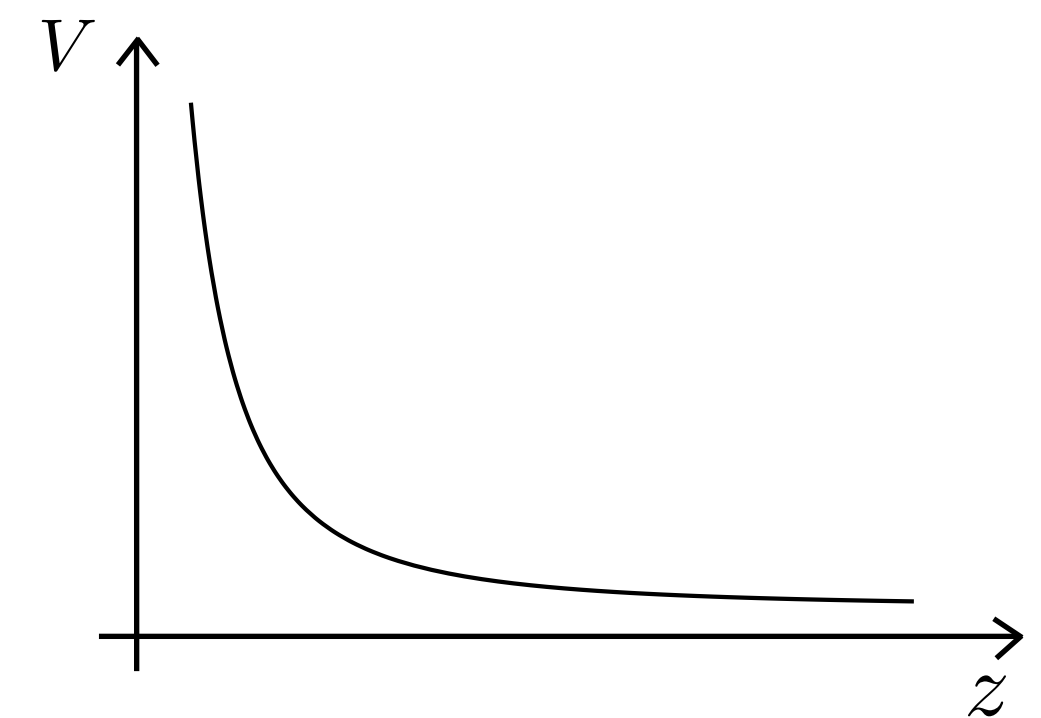
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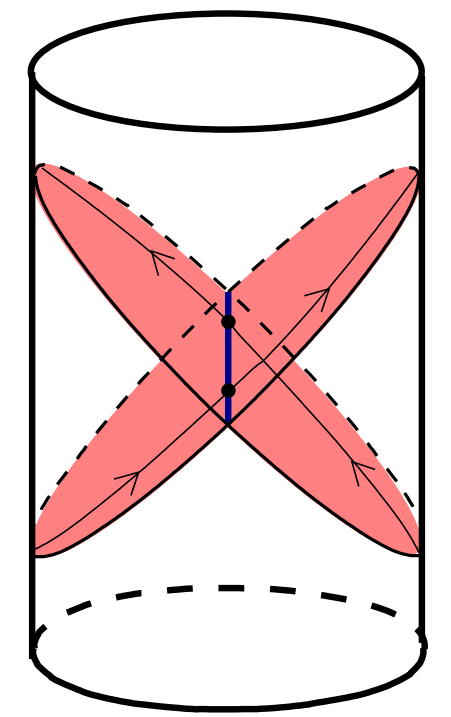


$G_J(L) = e^{iq_\perp \cdot l_\perp} \sqrt{z} \psi(z)$ , reduces to Schrodinger problem

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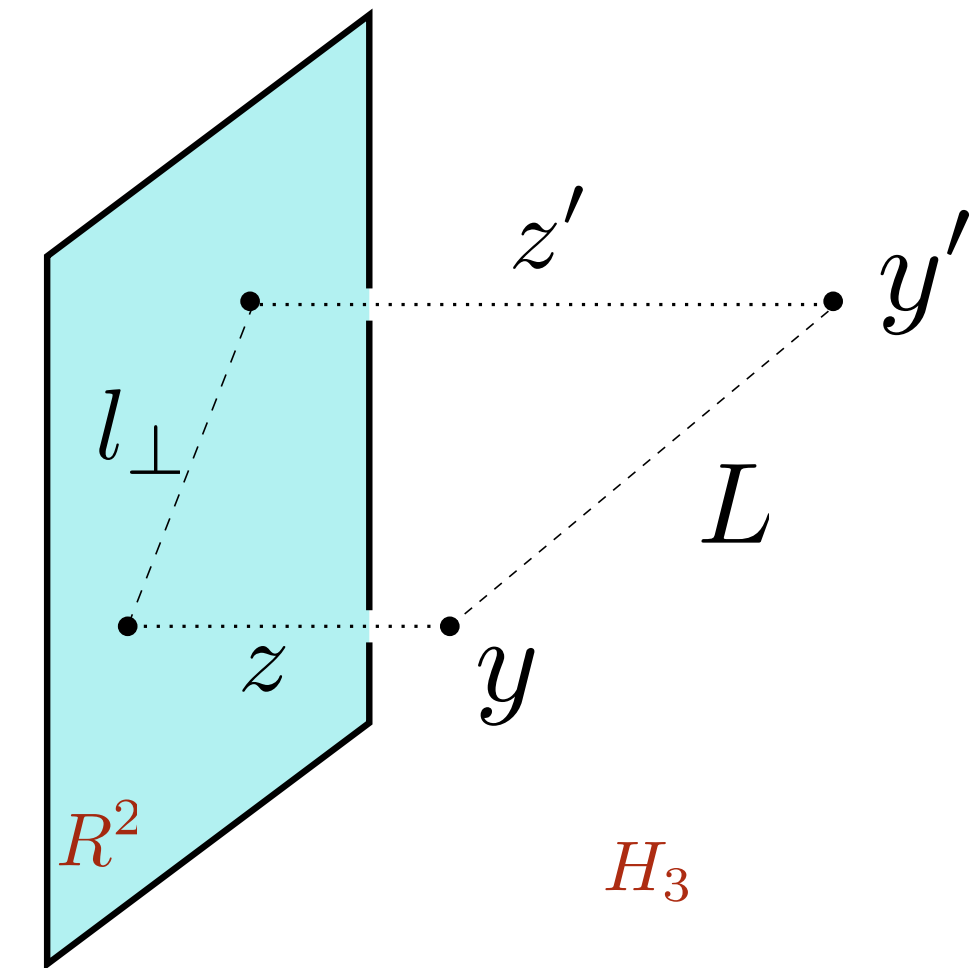


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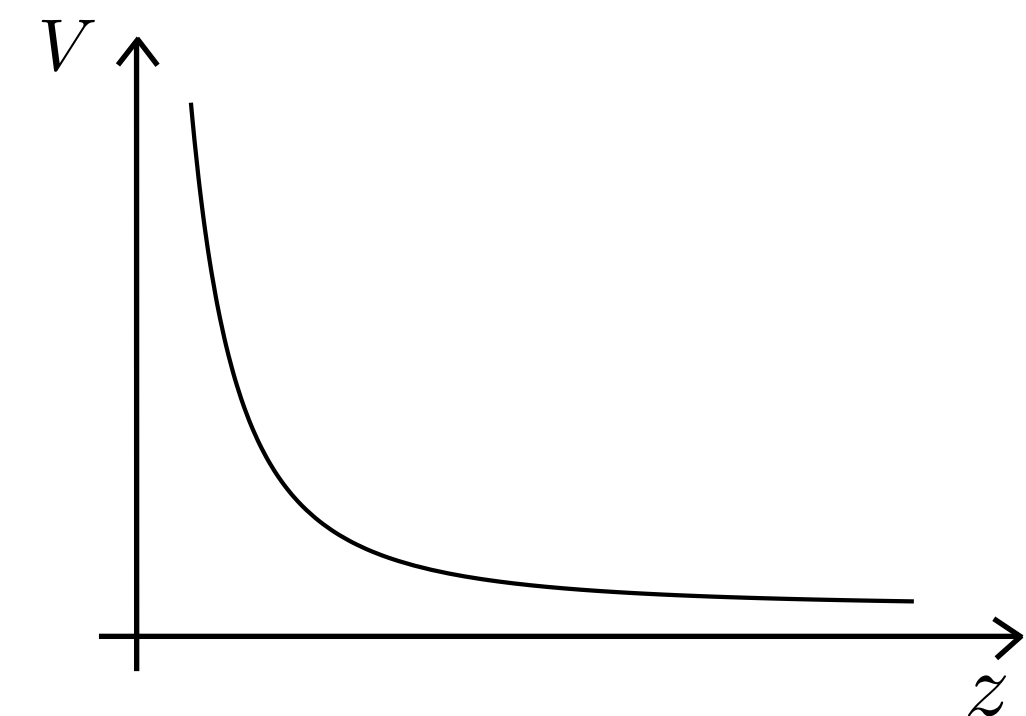
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# Application to low $x$ physics in QCD

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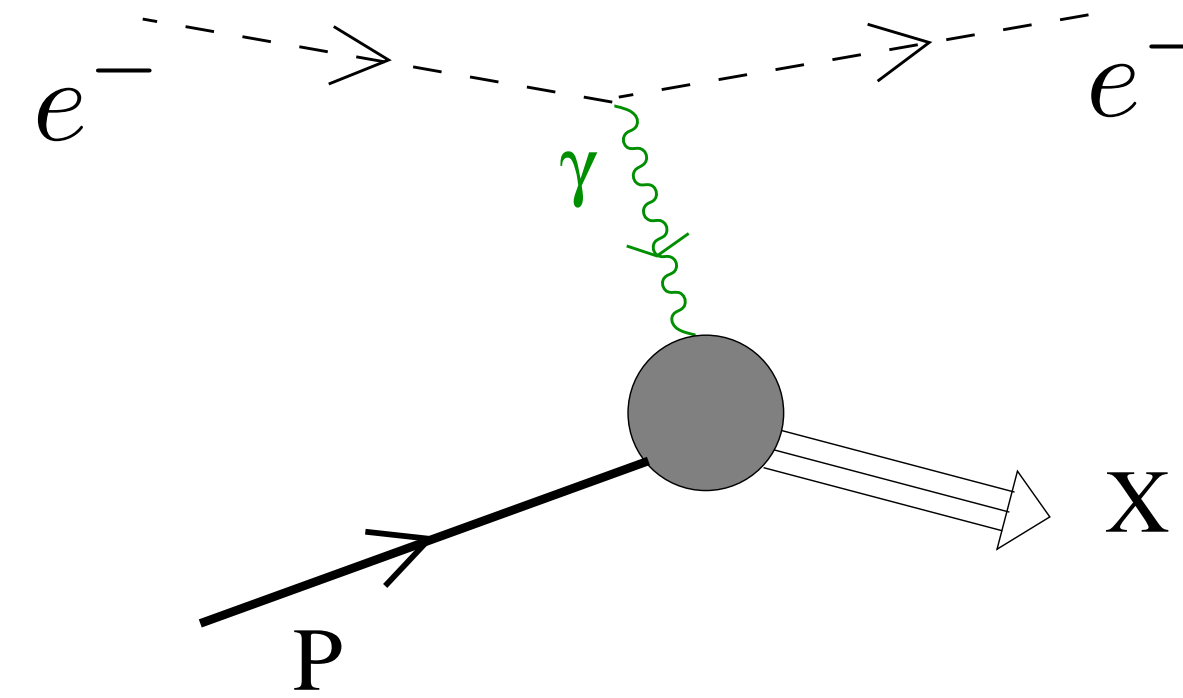
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- Deep inelastic scattering (DIS)

[Hatta, Iancu, Mueller 07;

Cornalba, MSC 08;

Brower, Djuric, Sarcevic, Tan 10]



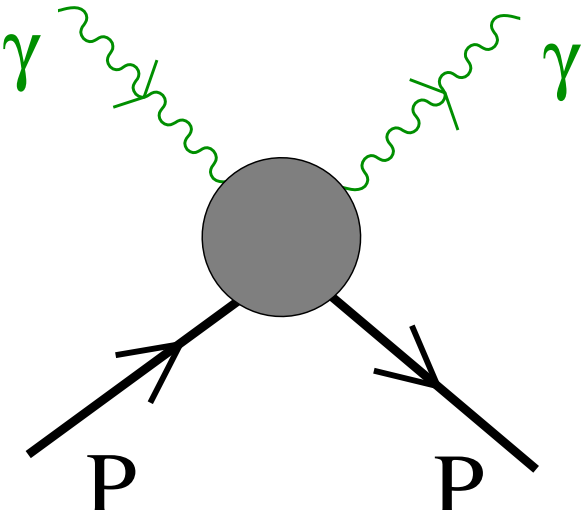
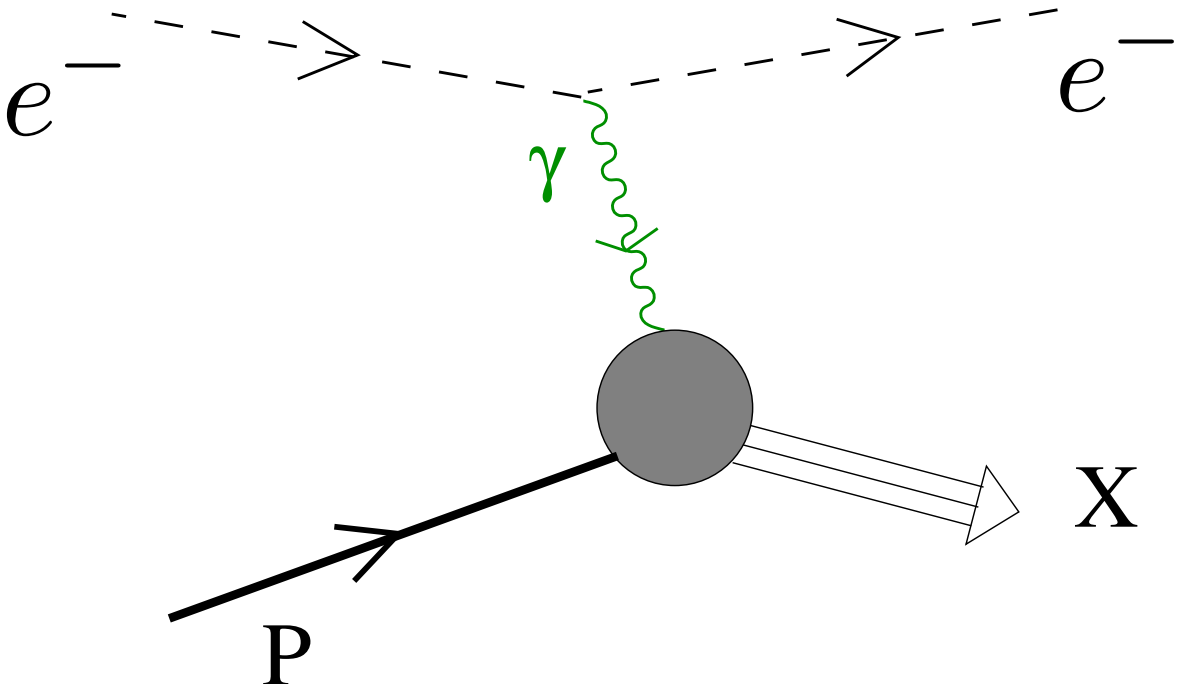
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Optical theorem

$$\sum_X \left| \begin{array}{c} \gamma \\ \nearrow \\ \bullet \\ \searrow \\ P \end{array} \begin{array}{c} \longrightarrow \\ \longrightarrow \\ X \end{array} \right|^2 = \text{Im}_{(t=0)} \begin{array}{c} \gamma \quad \gamma \\ \nearrow \quad \searrow \\ \bullet \\ \nearrow \quad \searrow \\ P \quad P \end{array}$$



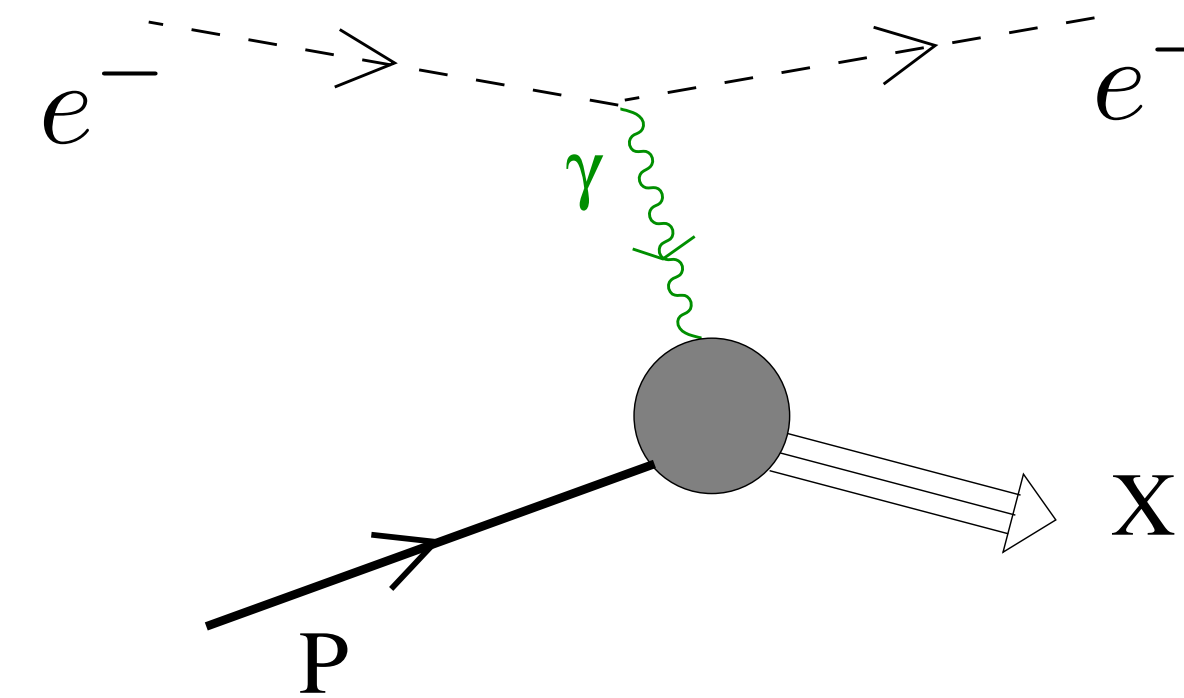
# Application to low $x$ physics in QCD

- Deep inelastic scattering (DIS)

[Hatta, Iancu, Mueller 07;

Cornalba, MSC 08;

Brower, Djuric, Sarcevic, Tan 10]



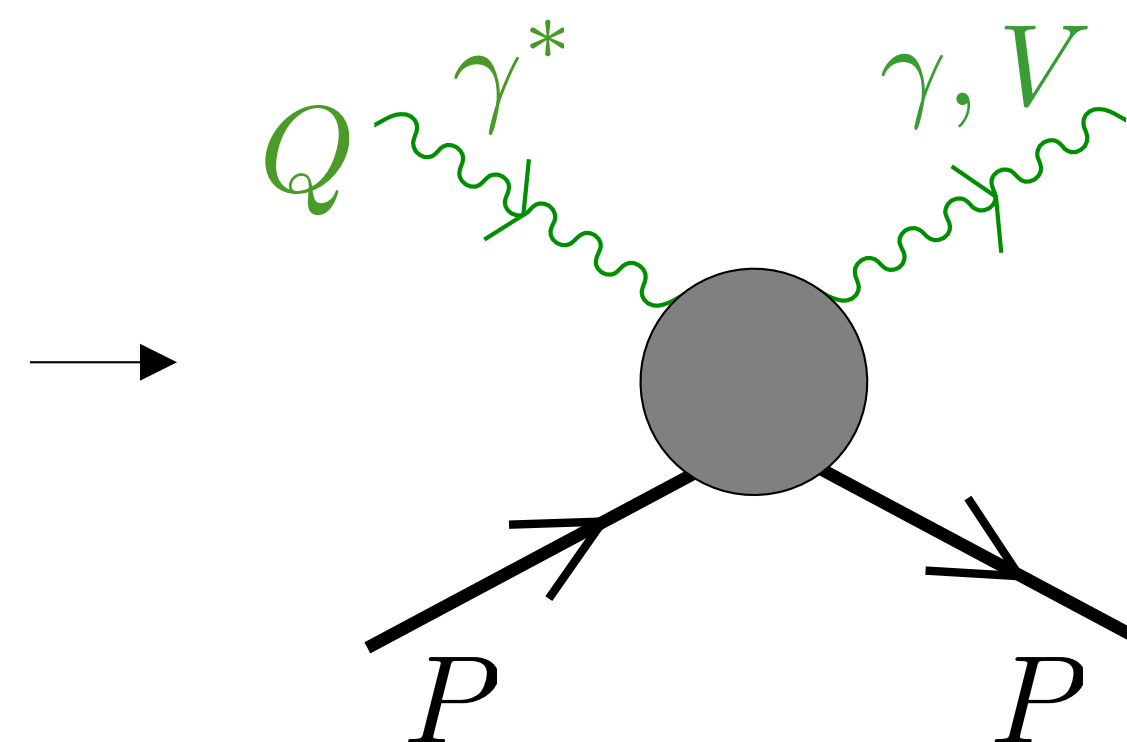
## Optical theorem

$$\sum_X \left| \begin{array}{c} \gamma \\ \nearrow \\ \text{---} \bullet \text{---} \\ \nearrow P \\ \text{---} X \end{array} \right|^2 = \text{Im} \begin{array}{c} \gamma \\ \nearrow \\ \text{---} \bullet \text{---} \\ \nearrow P \\ \searrow P \end{array} (t=0)$$

- DVCS & VMP

[MSC, Djuric 12;

MSC, Djuric, Evans 13]



$$\frac{d\sigma}{dt}(Q, x, t) \propto |W|^2$$

$$\sigma_{tot}(Q, x)$$

# Hard and soft pomeron are distinct Regge trajectories [Donnachie, Landshoff]

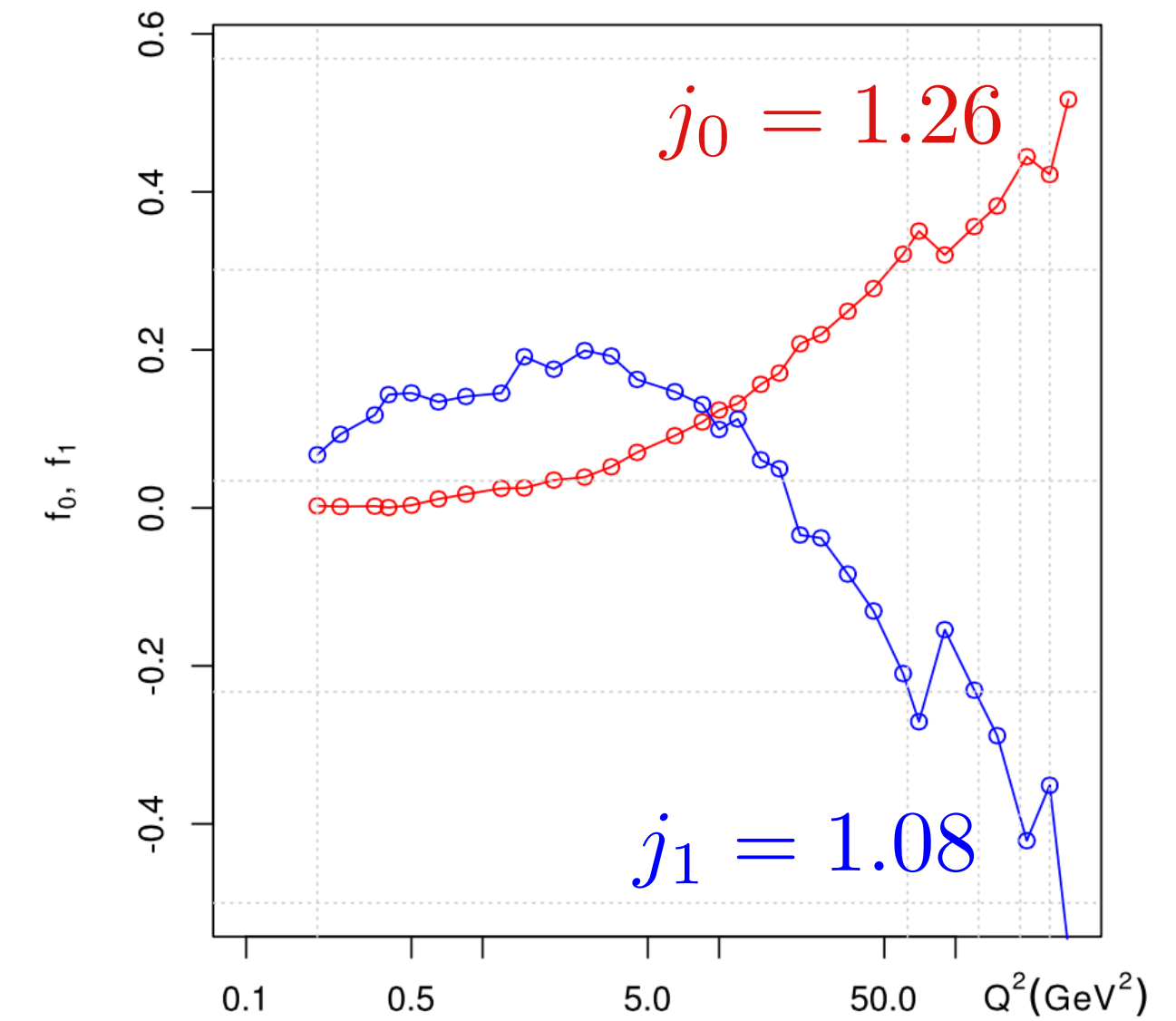
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# Hard and soft pomeron are distinct Regge trajectories [Donnachie, Landshoff]

- Explain DIS data with two Regge trajectories

$$\sigma(Q^2, x) \propto f_0(Q^2) x^{-j_0} + f_1(Q^2) x^{-j_1}$$



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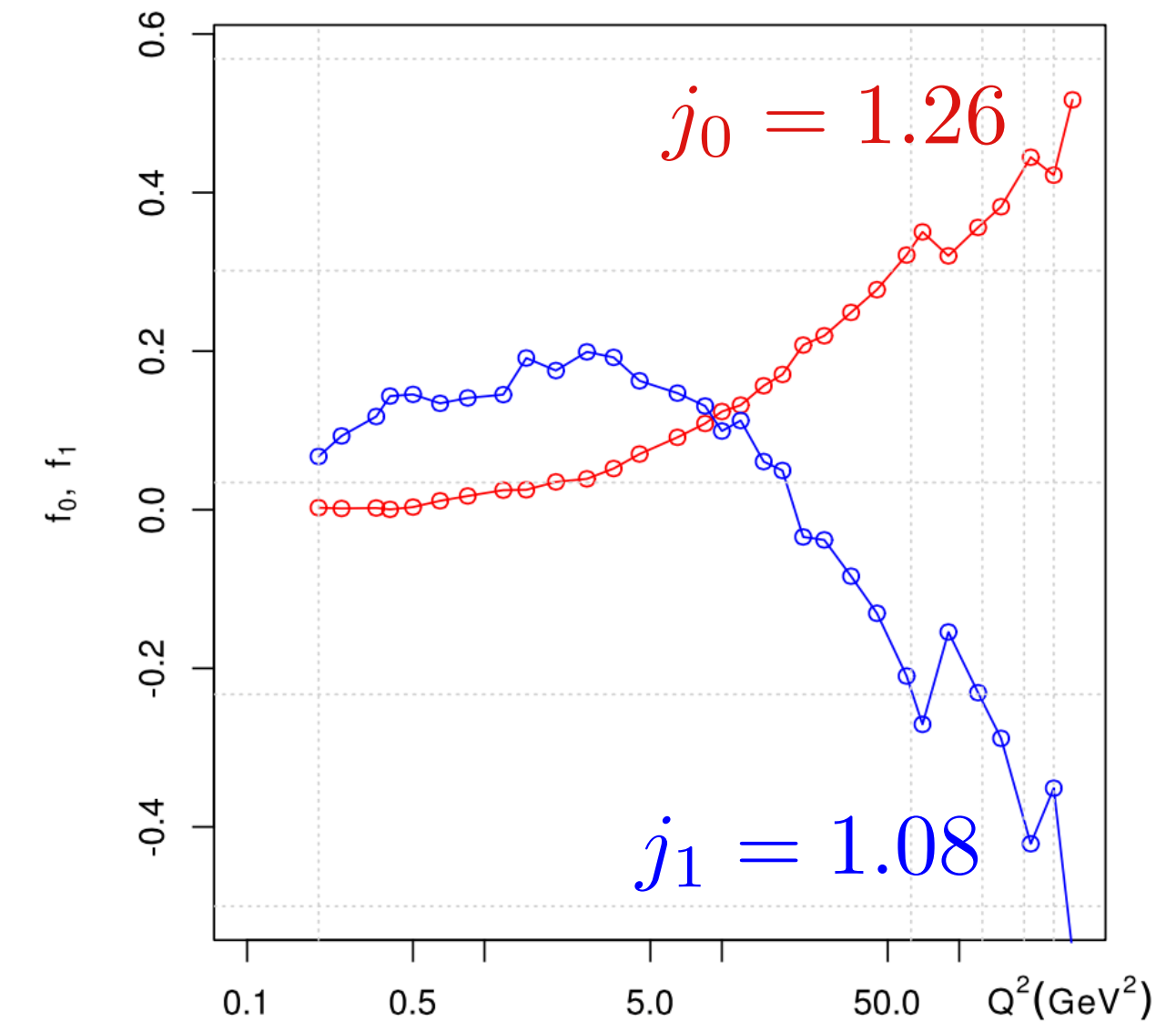
- Let us apply this idea to gauge/string duality [Bayona, MSC, Quevedo 17]

Holographic direction  $z \sim 1/Q$

$$f_k(Q^2) = P_k(Q^2) \varphi_k(Q^2)$$

Known function of  $Q^2$  and  $j_k$

Wave function of a 1D Schrodinger problem in holographic direction



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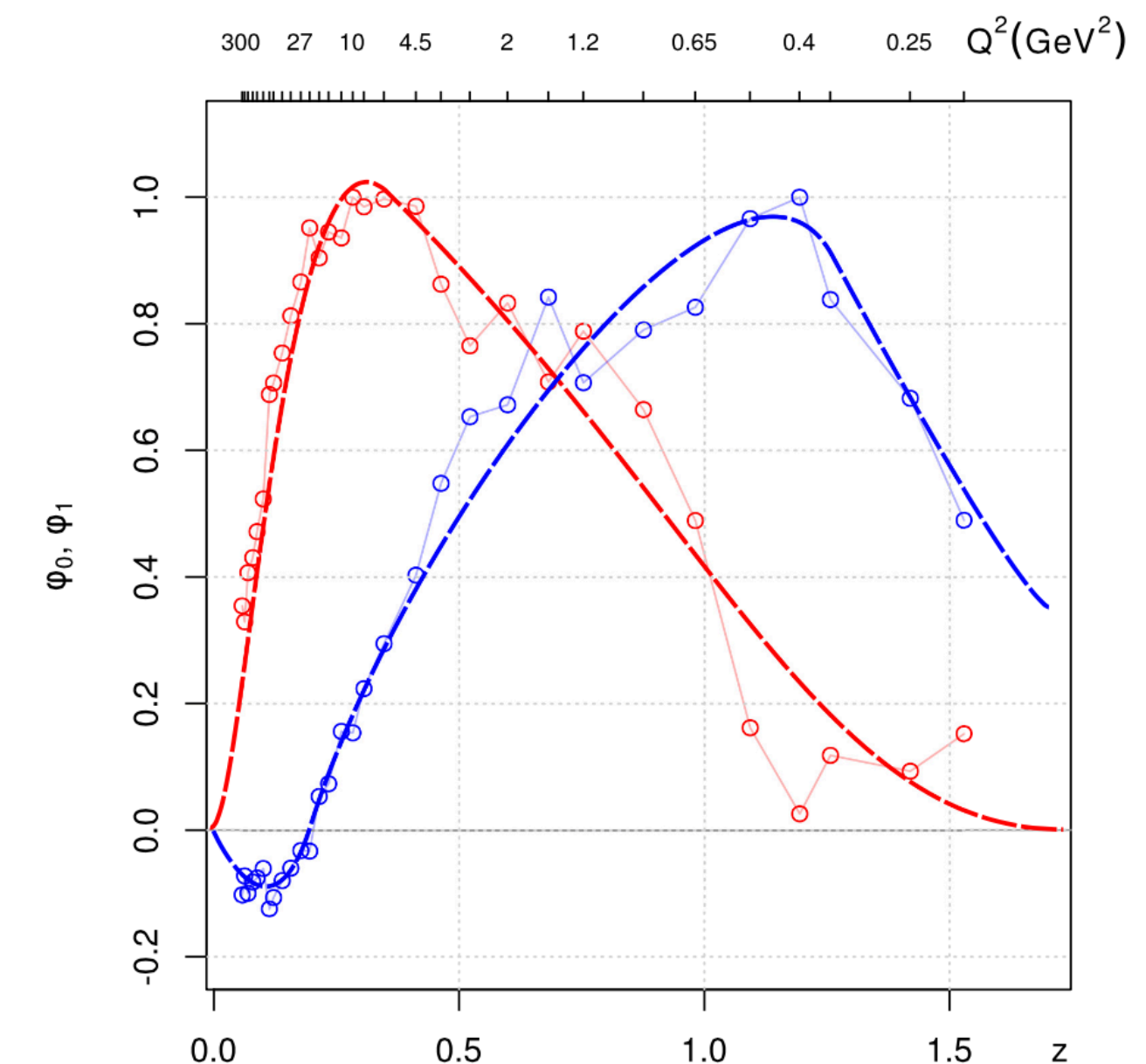
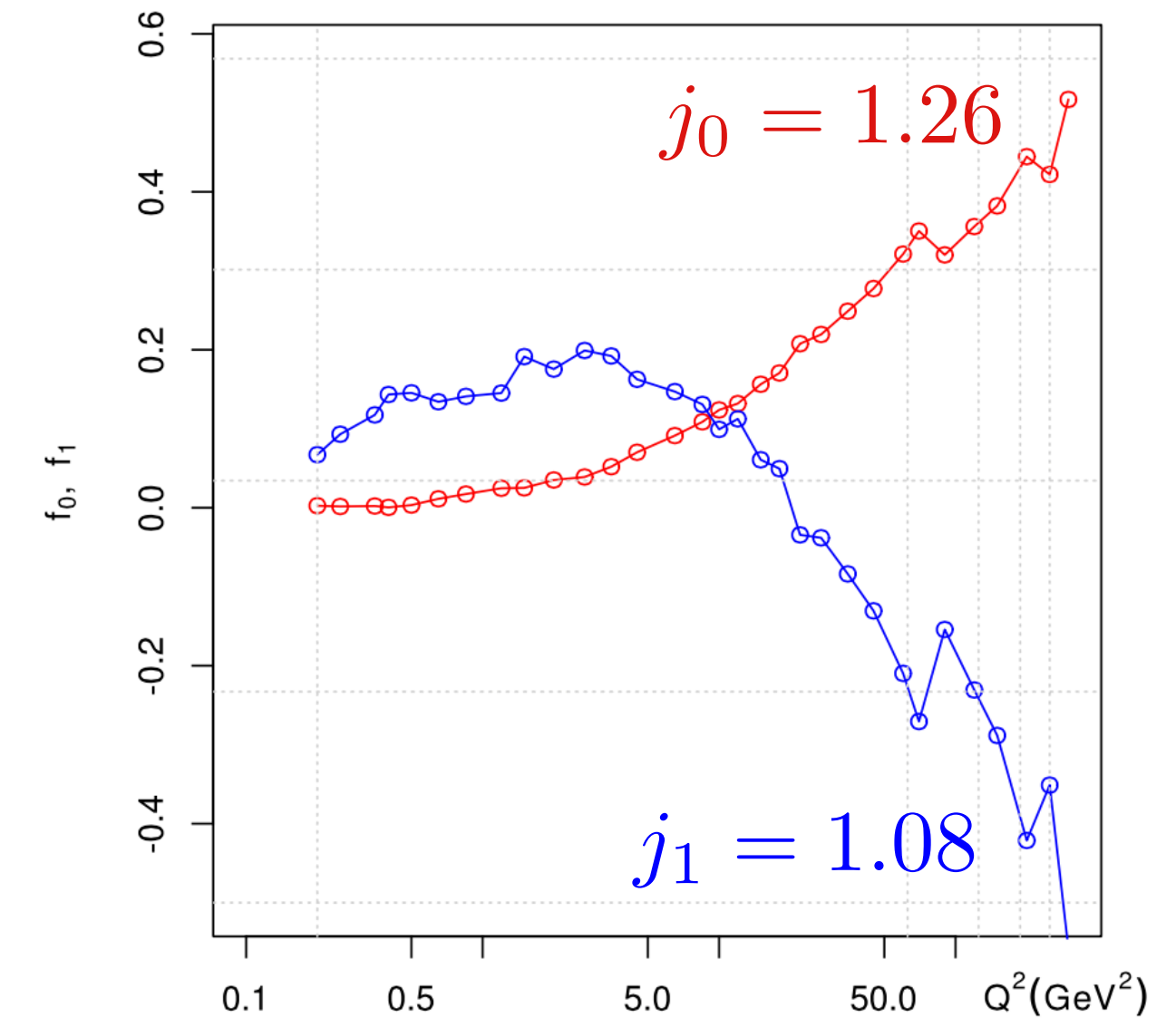
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**It seems data “knows” about holographic QCD!!**



## DIS from gauge/string duality

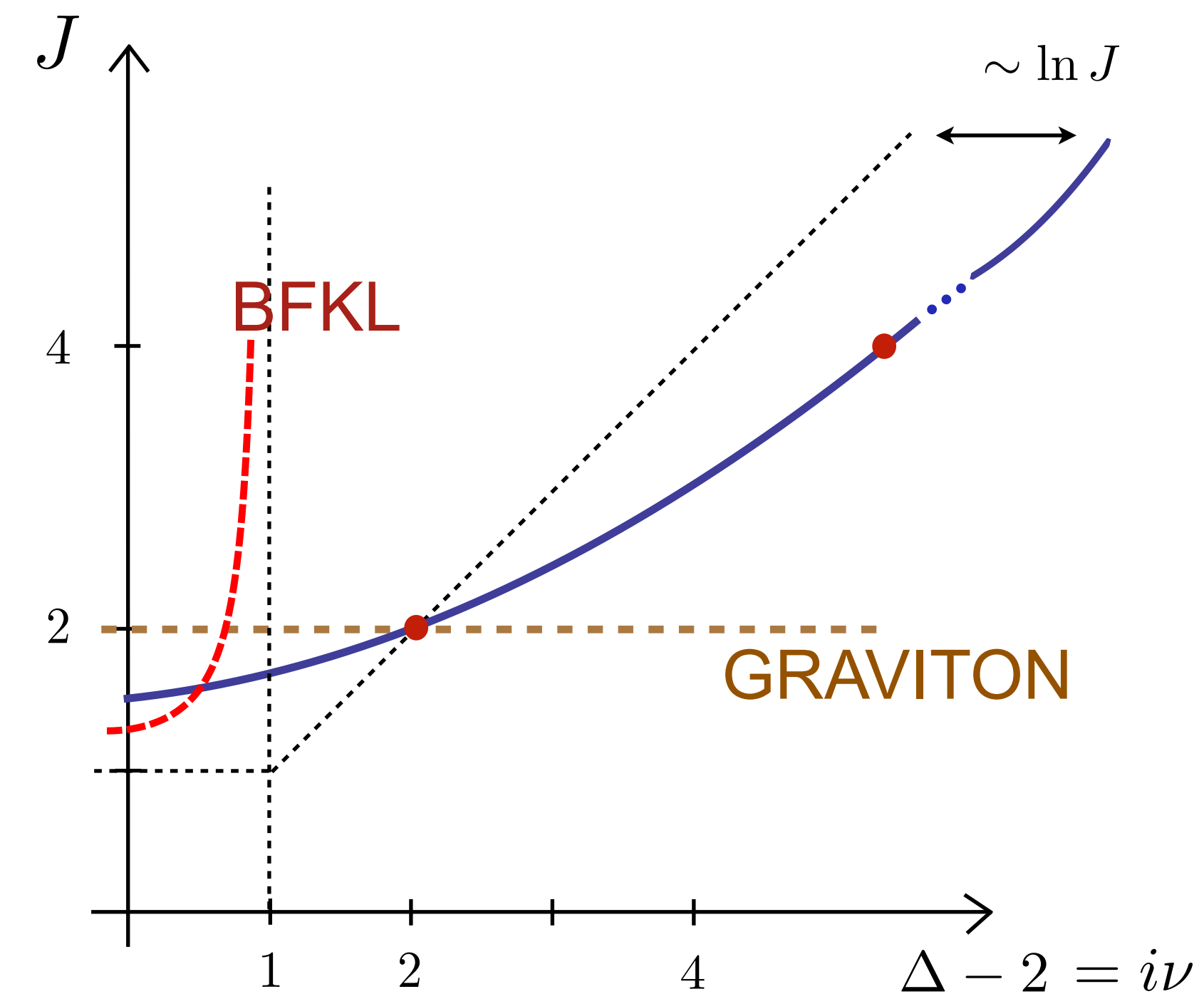
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- Hadronic tensor  $W^{ab}(x, Q, t) = i \int d^4y e^{iq \cdot y} \langle P | T \{ j^a(y) j^b(0) \} | P' \rangle$

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$$W = \int dz dz' \phi_1(z) \phi_3(z) \mathcal{K}_P(s, t, z, z') \phi_2(z') \phi_4(z')$$

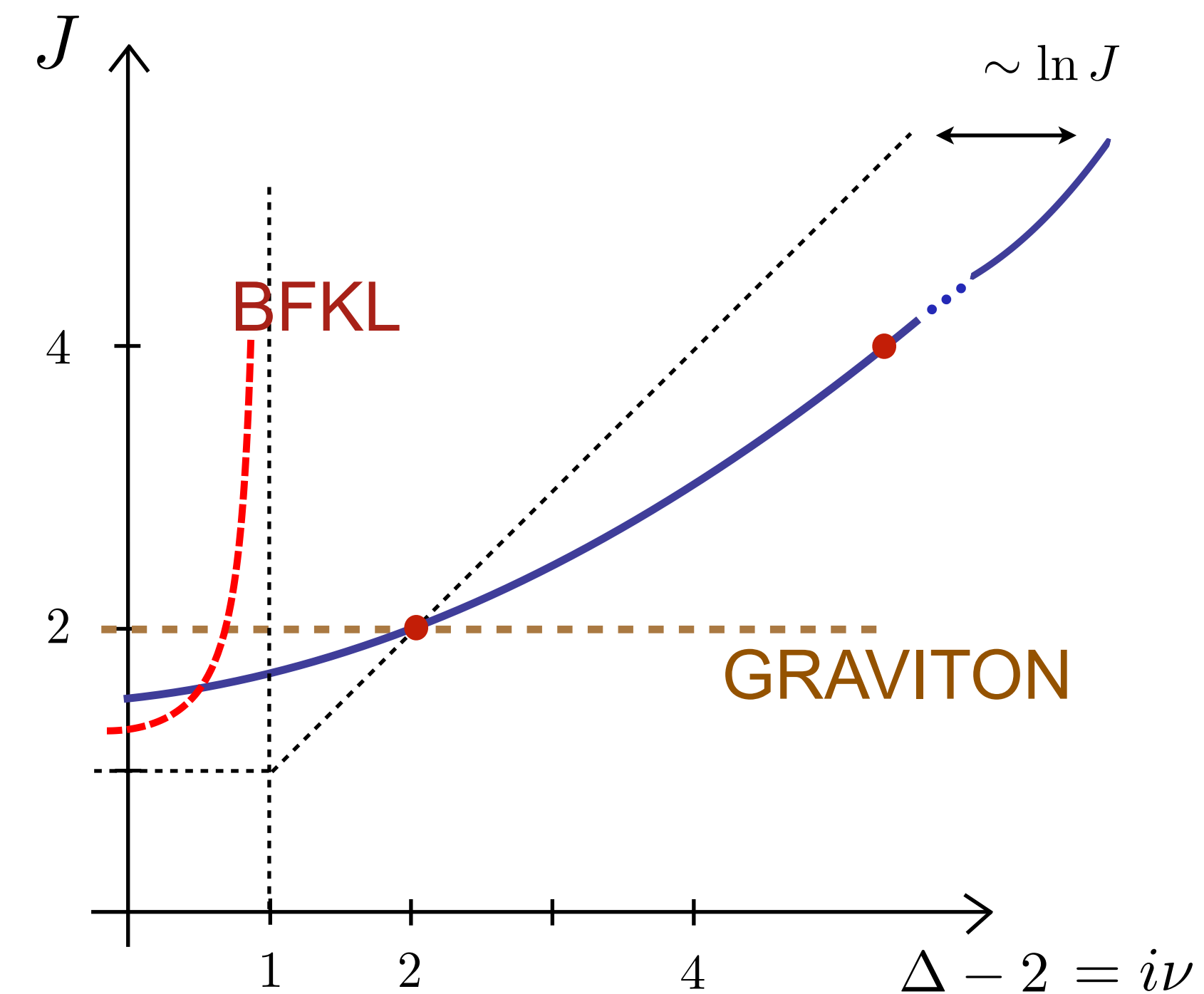


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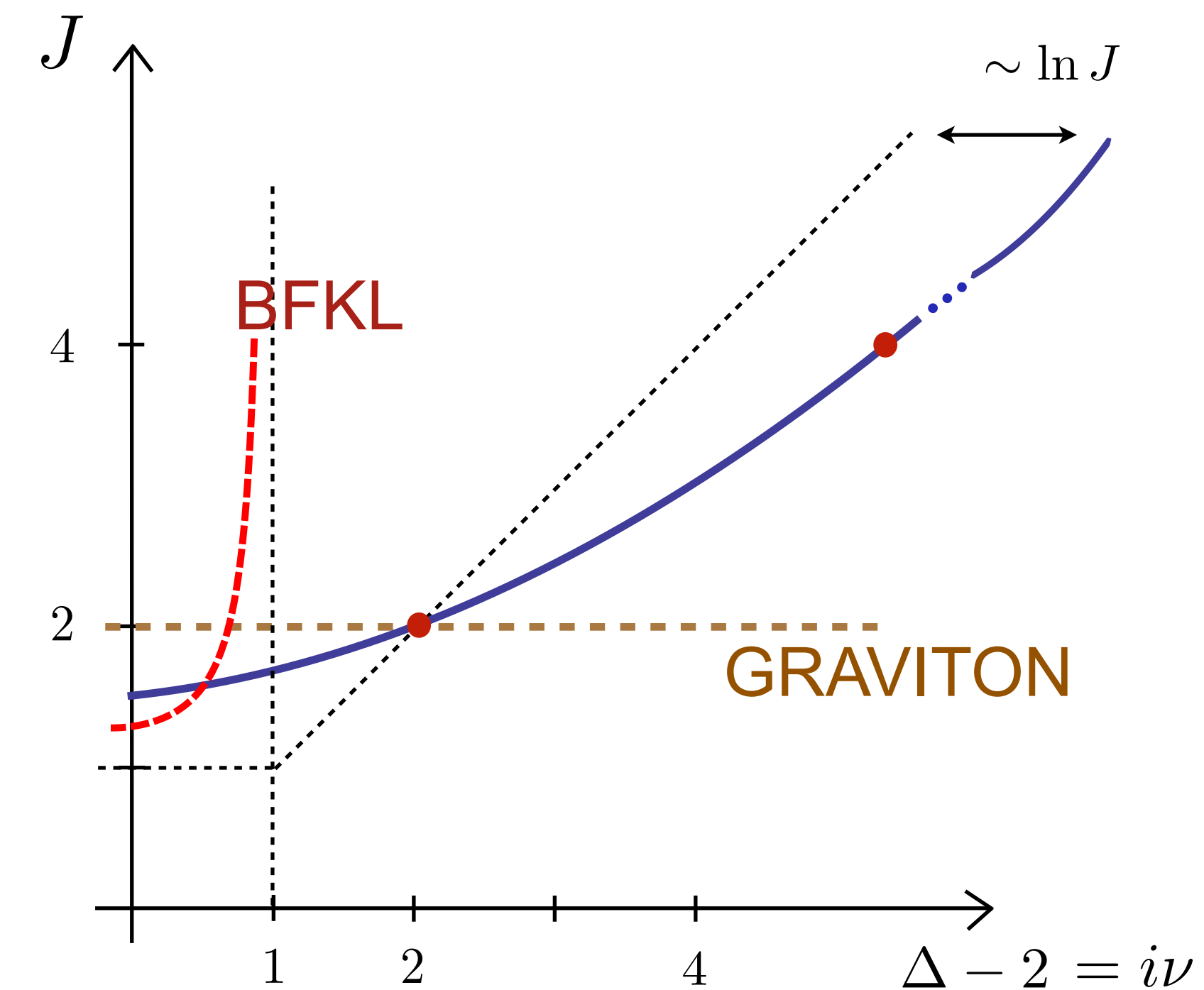


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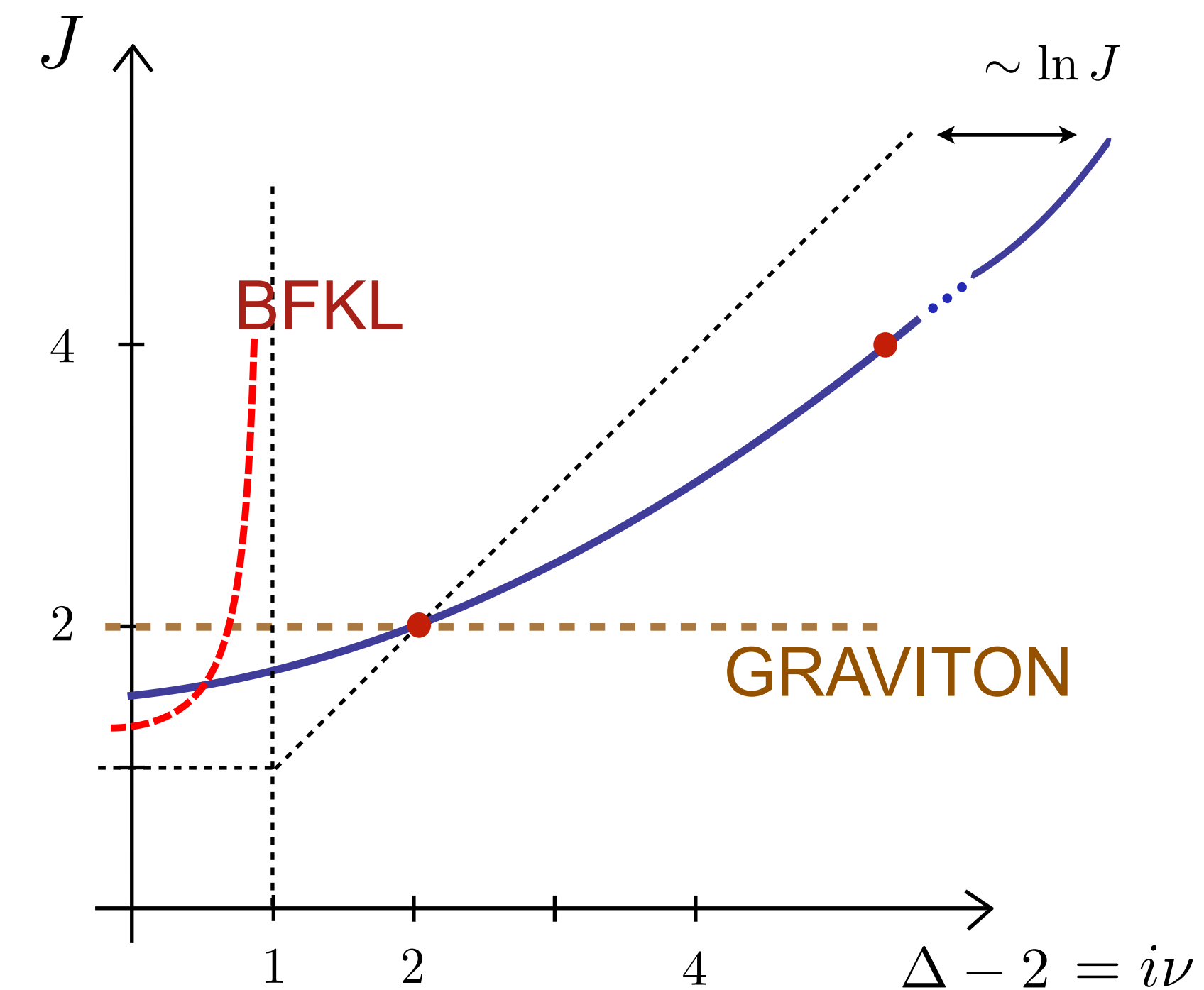


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# Holographic QCD

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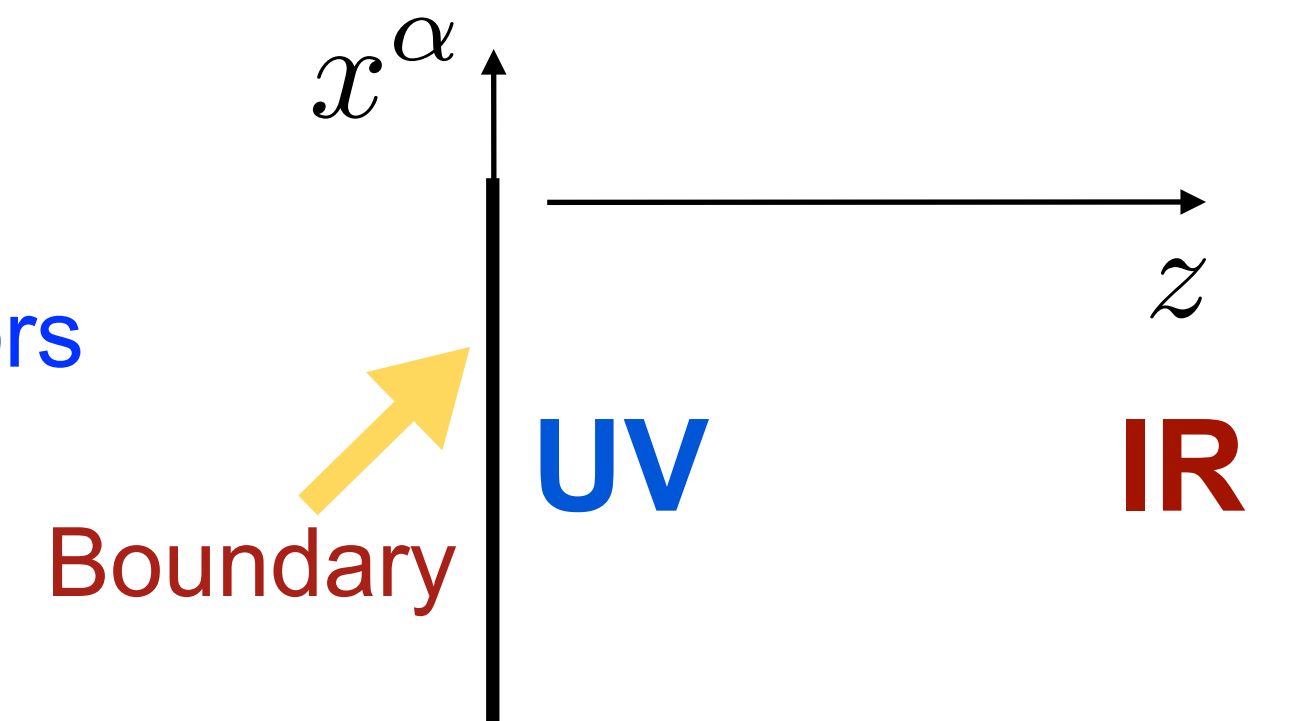
- QCD dual is a 5D theory with a graviton and a dilaton

$$ds^2 = e^{2A(z)} (dz^2 + \eta_{\alpha\beta} dx^\alpha dx^\beta)$$
$$\Phi = \Phi(z)$$

AdS fields  $\leftrightarrow$  single trace operators

$$g_{ab} \leftrightarrow T_{\alpha\beta}$$

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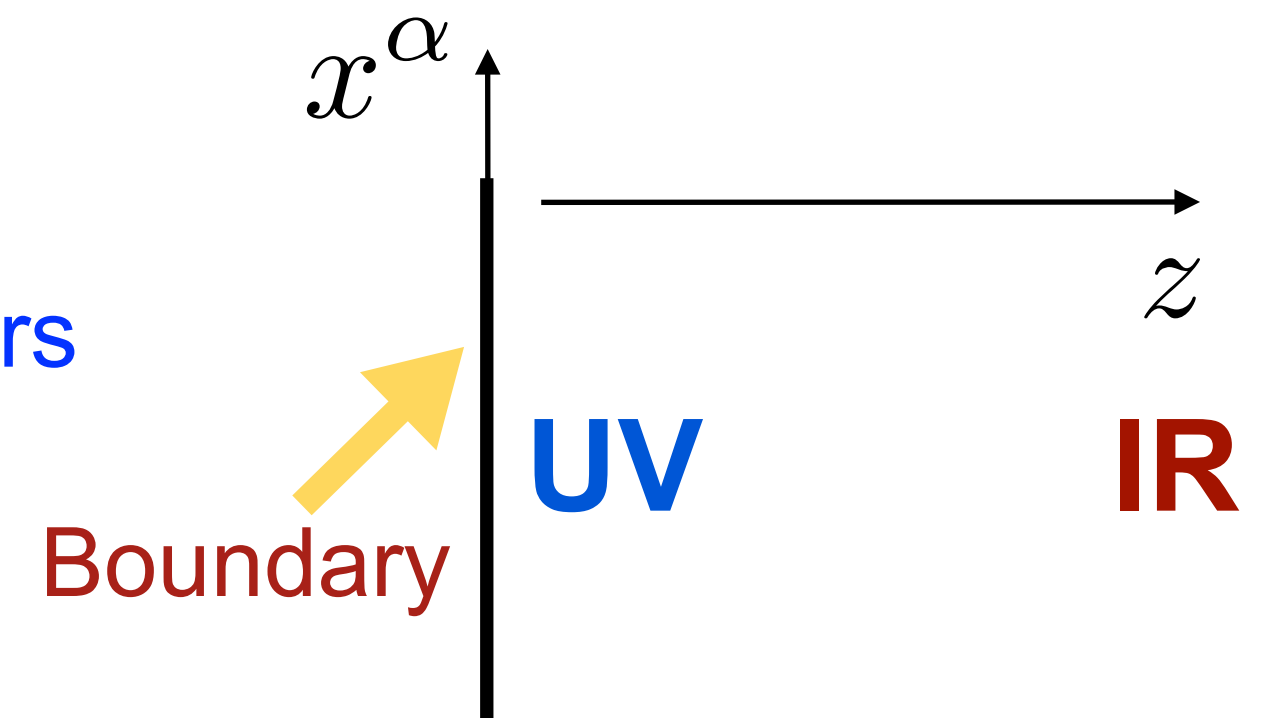
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- Test our ideas with a 5D dilaton-gravity model [Gursoy, Kiritsis, Nitti 07]

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} e^{-2\Phi} \left[ R + 4(\partial\phi)^2 + V(\phi) \right]$$

Judicious choice of potential with only 2 free parameters

Constructed to match QCD perturbative beta function

Reproduces: heavy quark-antiquark linear potential; glueball spectrum from lattice simulations; thermodynamic properties of QGP (bulk viscosity, drag force and jet quenching parameters)

## Spin J field in holographic QCD [Bayona, MSC, Djuric, Quevedo 15]

---

- Construct spin J field dual to gluon operator  $\mathcal{O}_J \sim \text{Tr} (F_{\alpha\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J}{}^\alpha)$

Decompose symmetric, traceless, transverse field  $h_{a_1 \dots a_J}$  with respect to global  $SO(1, 3)$  boundary symmetry. Propagating modes have boundary indices  $h_{\alpha_1 \dots \alpha_J}$

- Spin J equation must:
- In AdS limit reduce to  $(D^2 - m^2) h_{a_1 \dots a_J} \equiv 0 \quad m^2 = \Delta(\Delta - 4) - J, \quad \Delta = \Delta(J)$
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Equation for propagating mode in effective field theory

$$\left( \nabla^2 - 2\dot{\Phi}\nabla_z + J\dot{A}^2 e^{-2A} - \Delta(\Delta - 4) + (J - 2)e^{-2A} \left[ \textcircled{a}\ddot{\Phi} + \textcircled{b}\dot{\Phi}^2 + \textcircled{c}(\ddot{A} - \dot{A}^2) \right] \right) h_{\alpha_1 \dots \alpha_J} = 0$$

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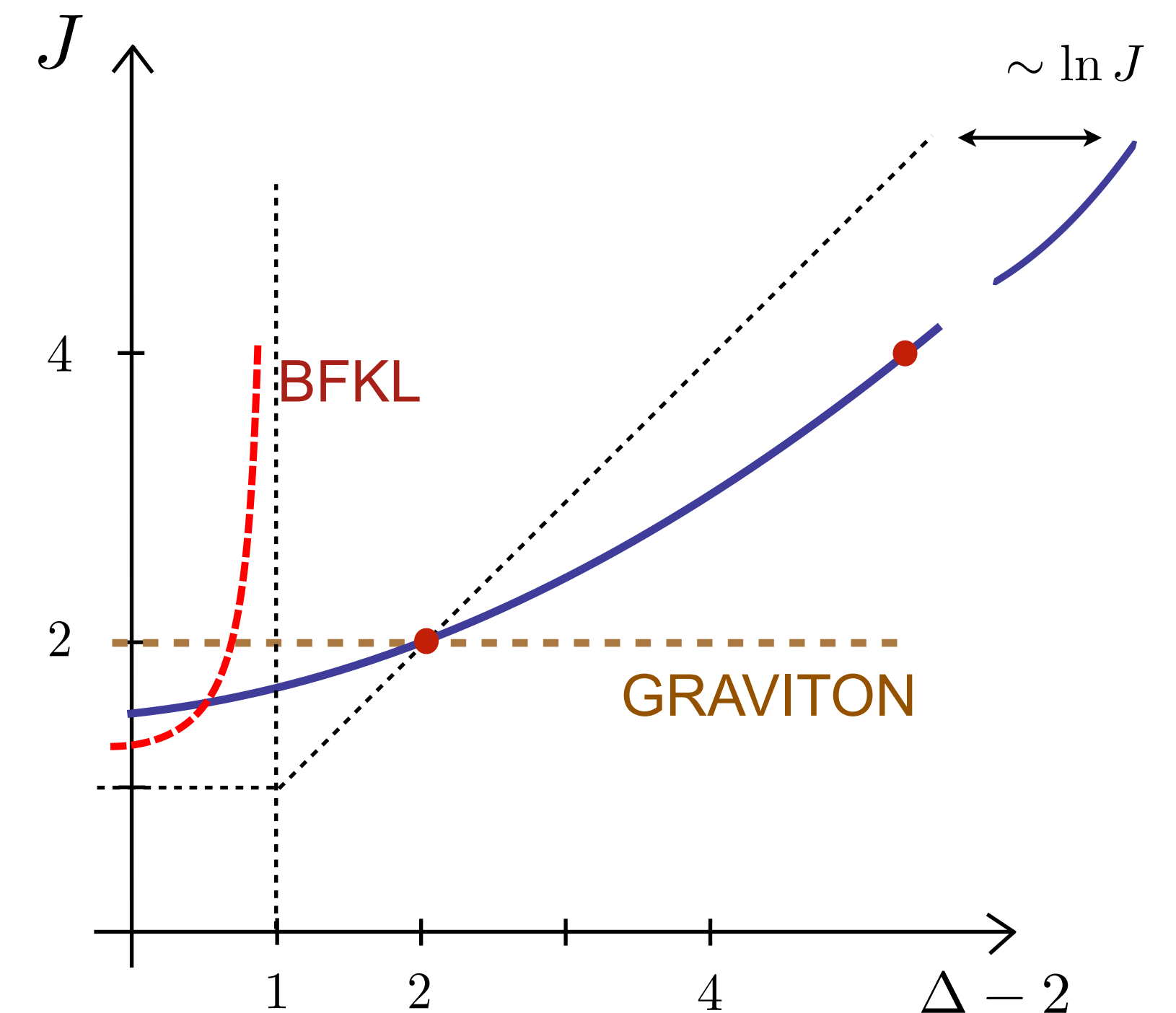
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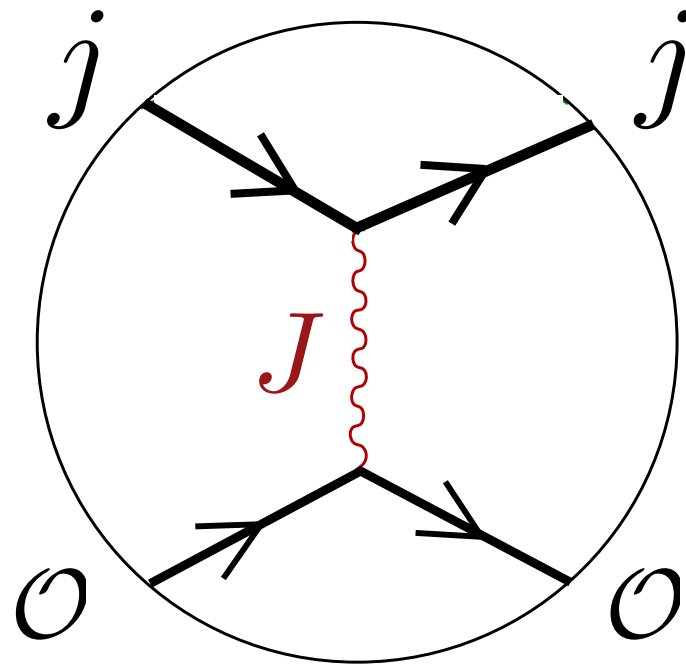
$$\Delta(\Delta - 4) \approx \underbrace{\left( \frac{2}{l_s^2} \right) (J - 2) \left( 1 + d e^{-\Phi/2} \right)}_{\text{IR described by graviton trajectory}} + \underbrace{e^{-4\Phi/3} (J^2 - 4)}_{\text{UV free theory unitarity bound}}$$



## Many Regge trajectories

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- Consider 5D exchange of spin  $J$  field in the Regge limit

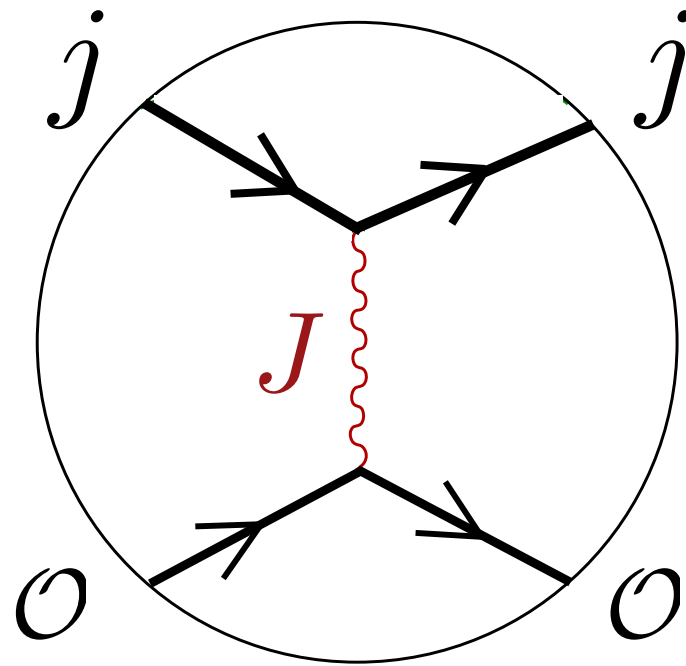


$$A_J(s, t) = iV \frac{\kappa_J \kappa'_J}{(-2)^J} s \int dz dz' e^{3A+3A'-\Phi-\Phi'} |v_1|^2 |v'_2|^2 \left( s e^{-A-A'} \right)^{J-1} G_J(z, z', t)$$

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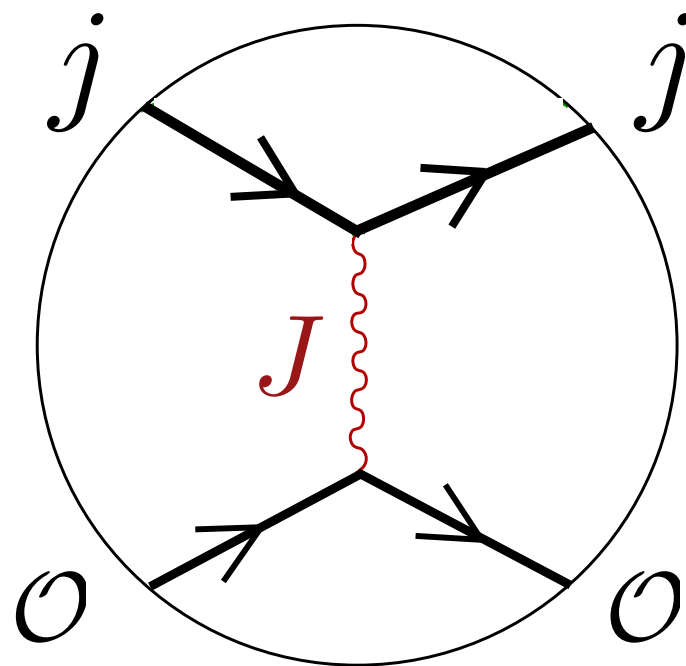
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$G_J(z, z', t)$  is the FT of integrated propagator

$$G_J(z, z', l_\perp) \sim i e^{(1-J)(A+A')} \int dw^+ dw^- \Pi_{+\dots+.-\dots-}(z, z', w)$$

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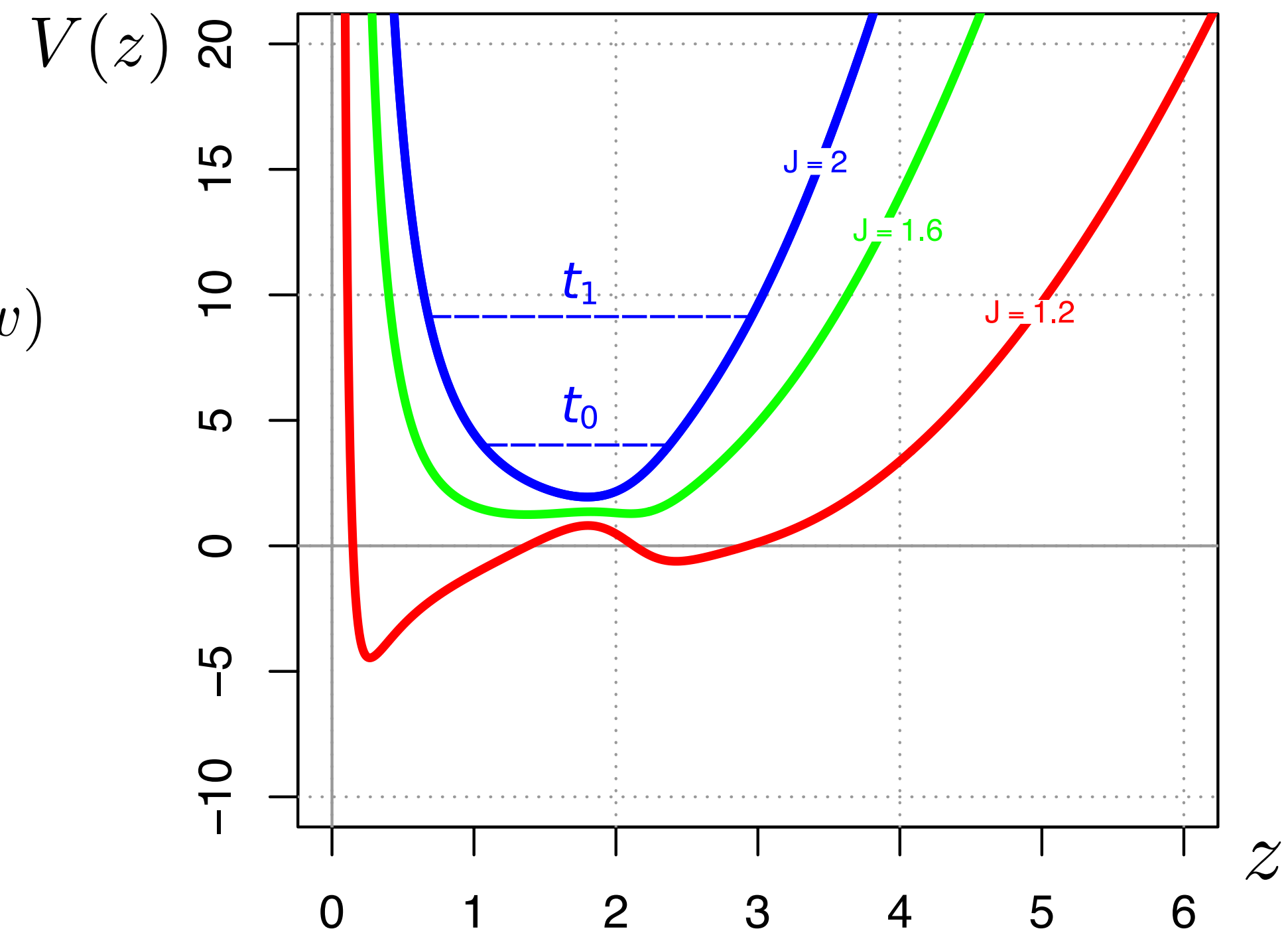
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Reduces to a Schrodinger problem (spectral representation)

$$G_J(z, z', t) = e^{\Phi - \frac{A}{2} + \Phi' - \frac{A'}{2}} \sum_n \frac{\psi_n(z) \psi_n^*(z')}{t_n(J) - t}$$



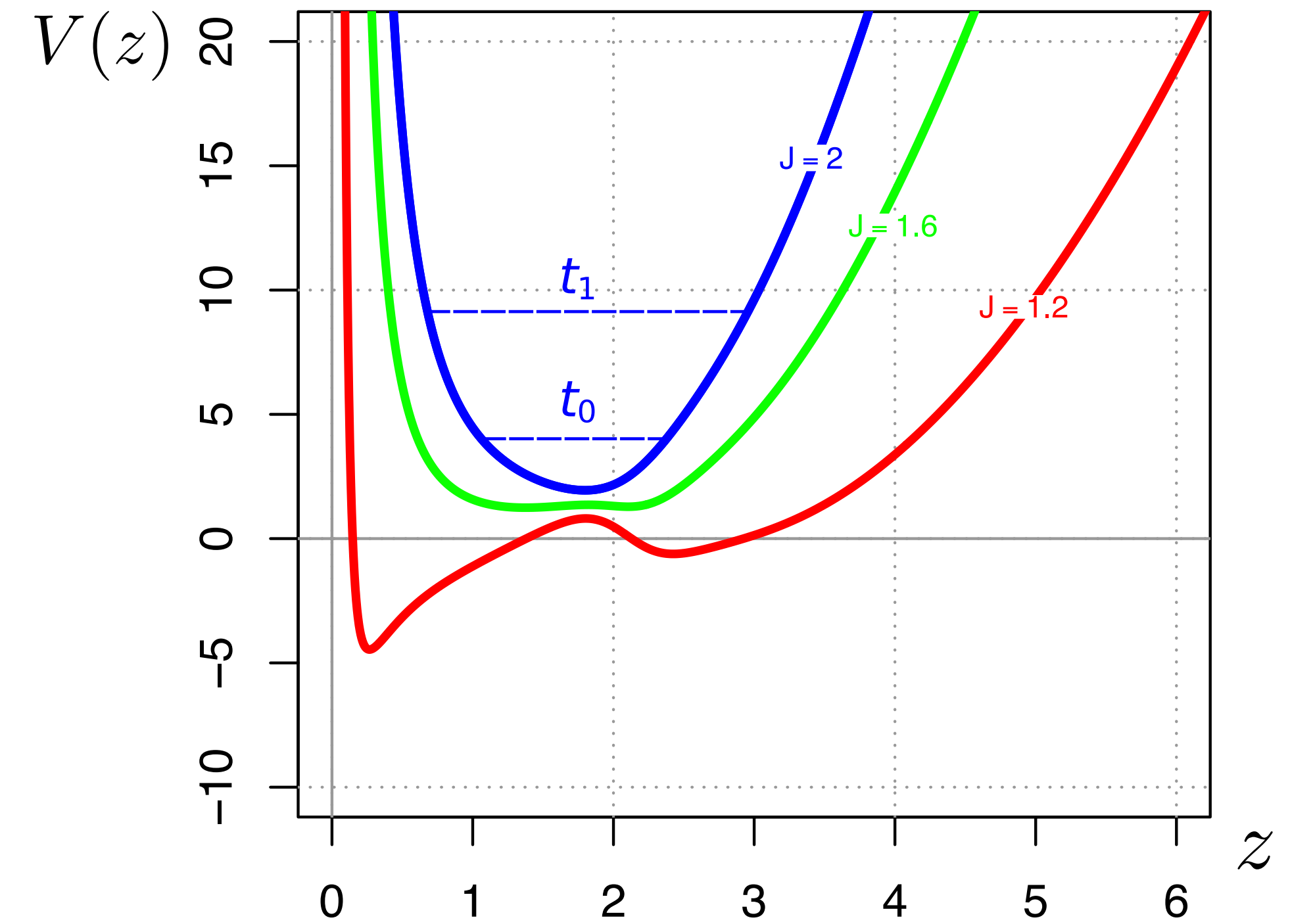
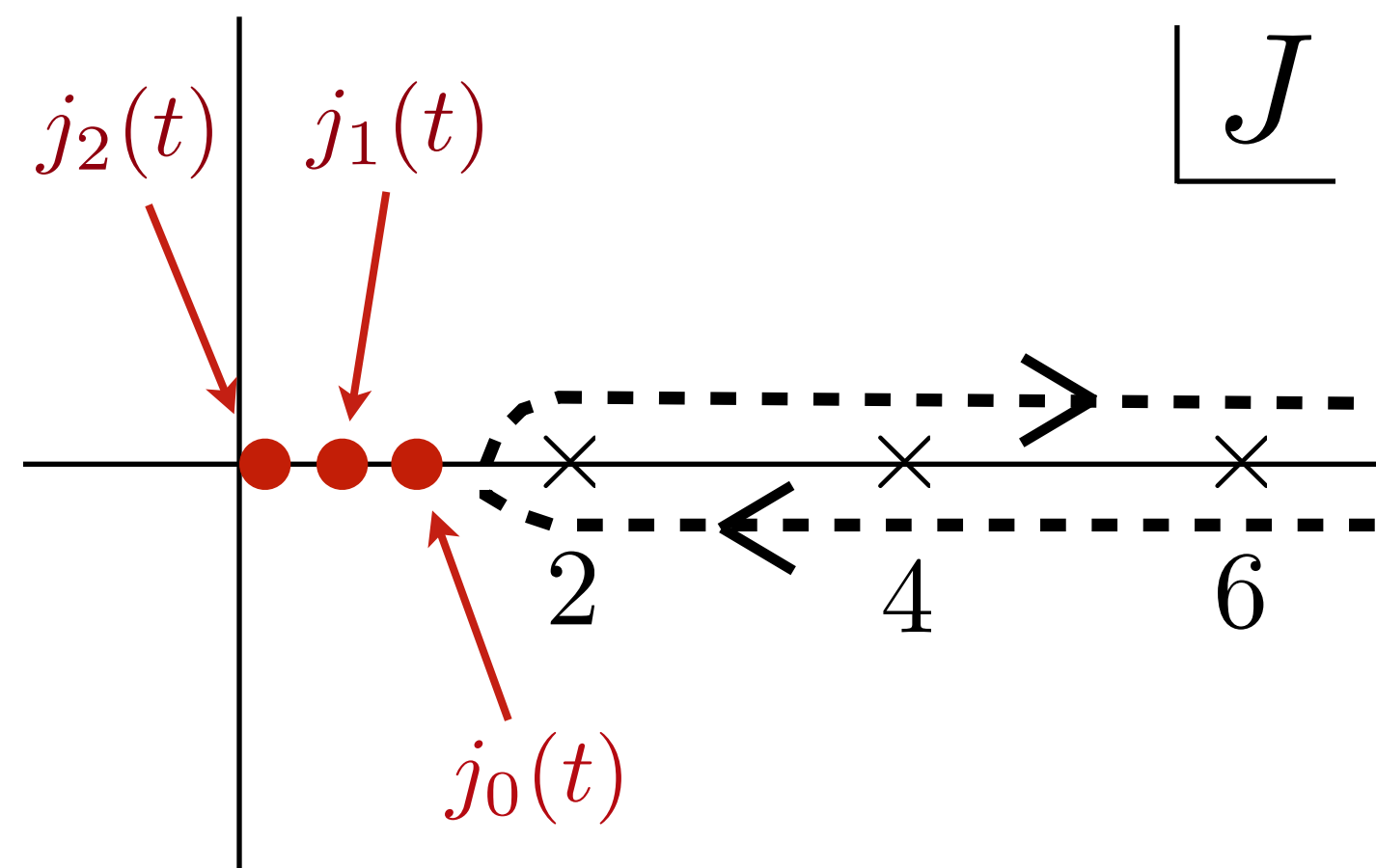


- Sum over spin J exchanges in 5D dual theory

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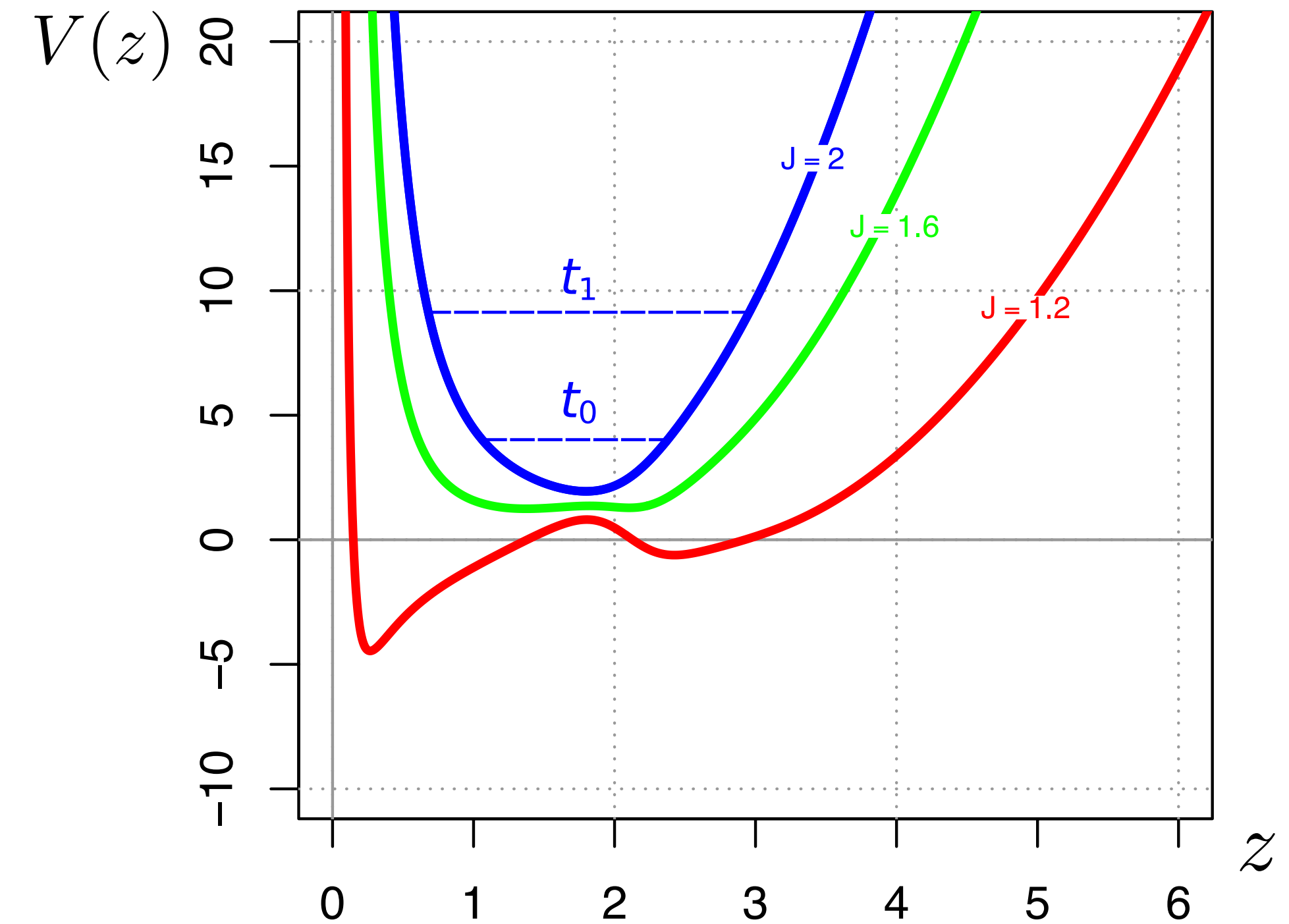
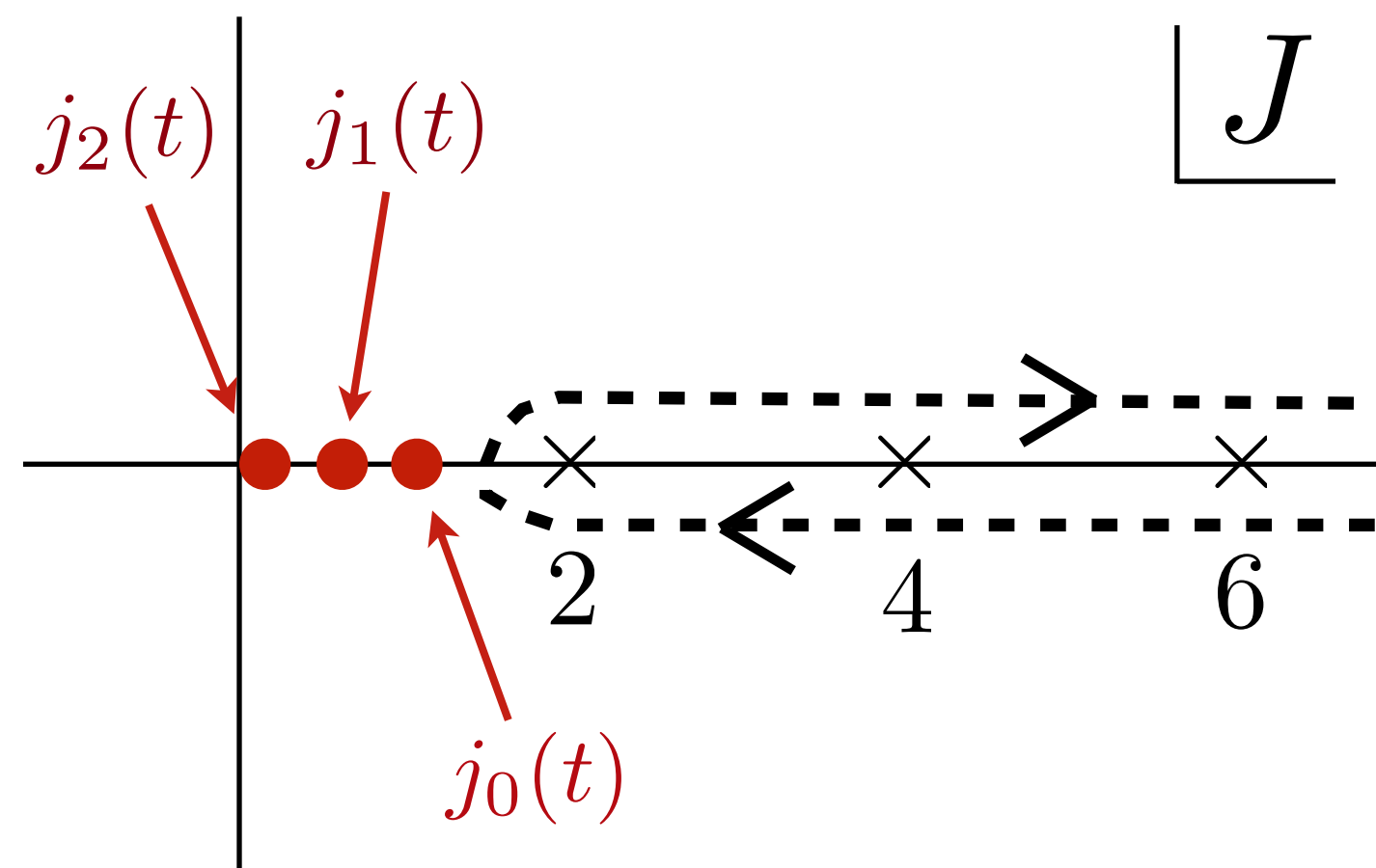


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- At the end of the day, structure function is of the form

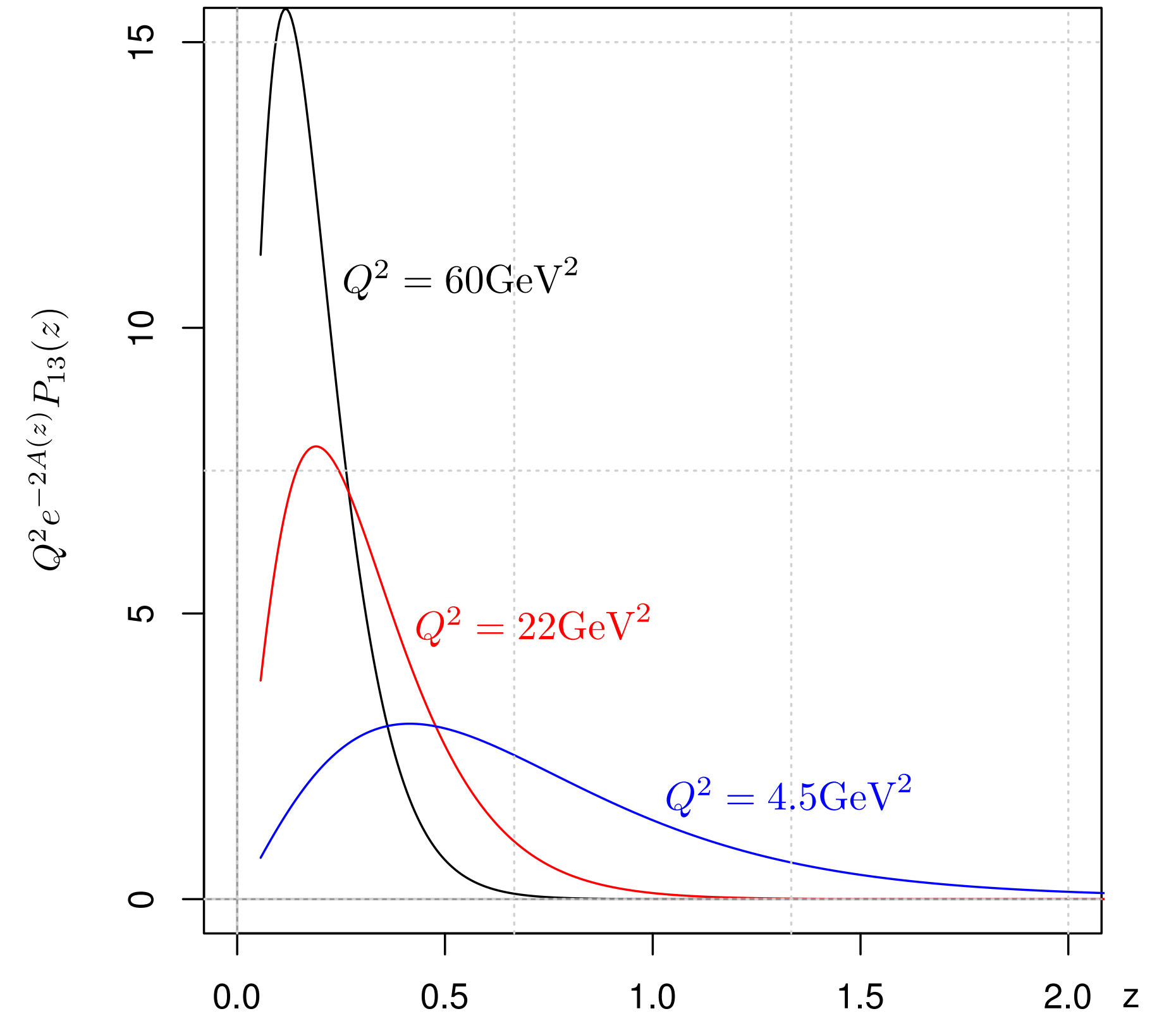
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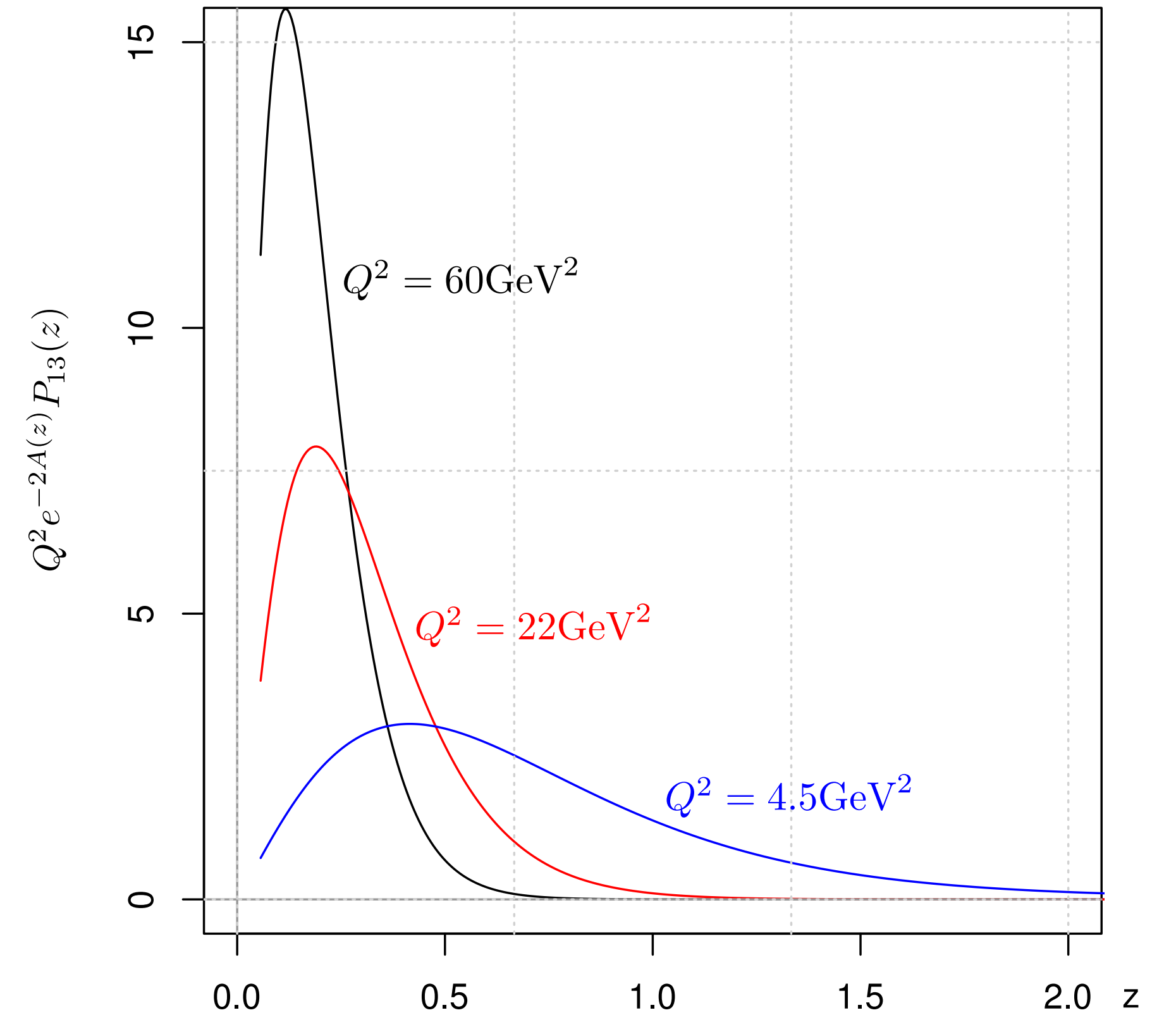
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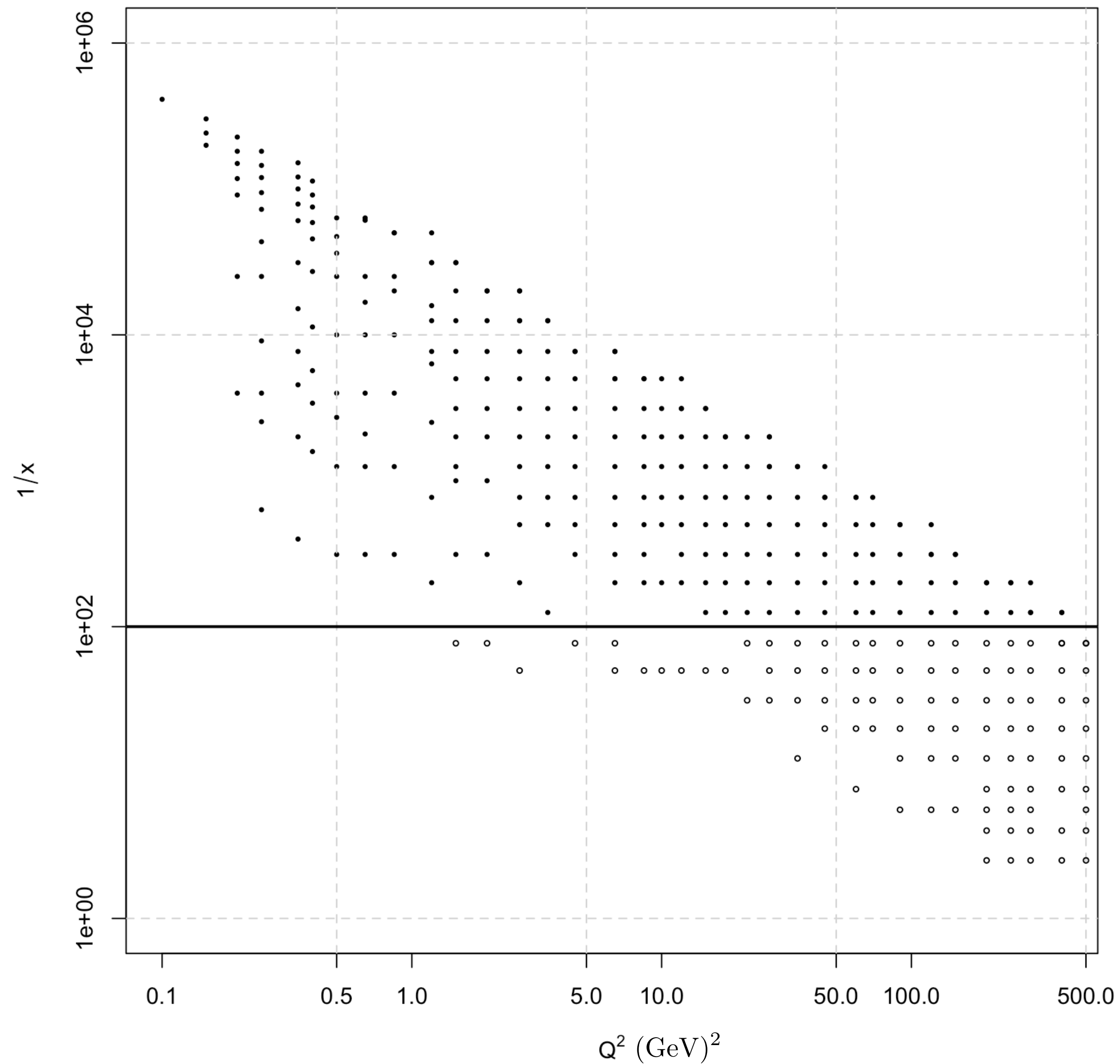
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- Dependence on fixed target absorbed in coupling

$$g_n = -2\pi^2 \frac{\kappa_{j_n(0)} \bar{\kappa}_{j_n(0)}}{2j_n(0)} j_n'(0) \int dz P_{24}(P^2, z) e^{(1-j_n(0))A(z)} e^{B(z)} \psi_n^*(j_n(0), z)$$

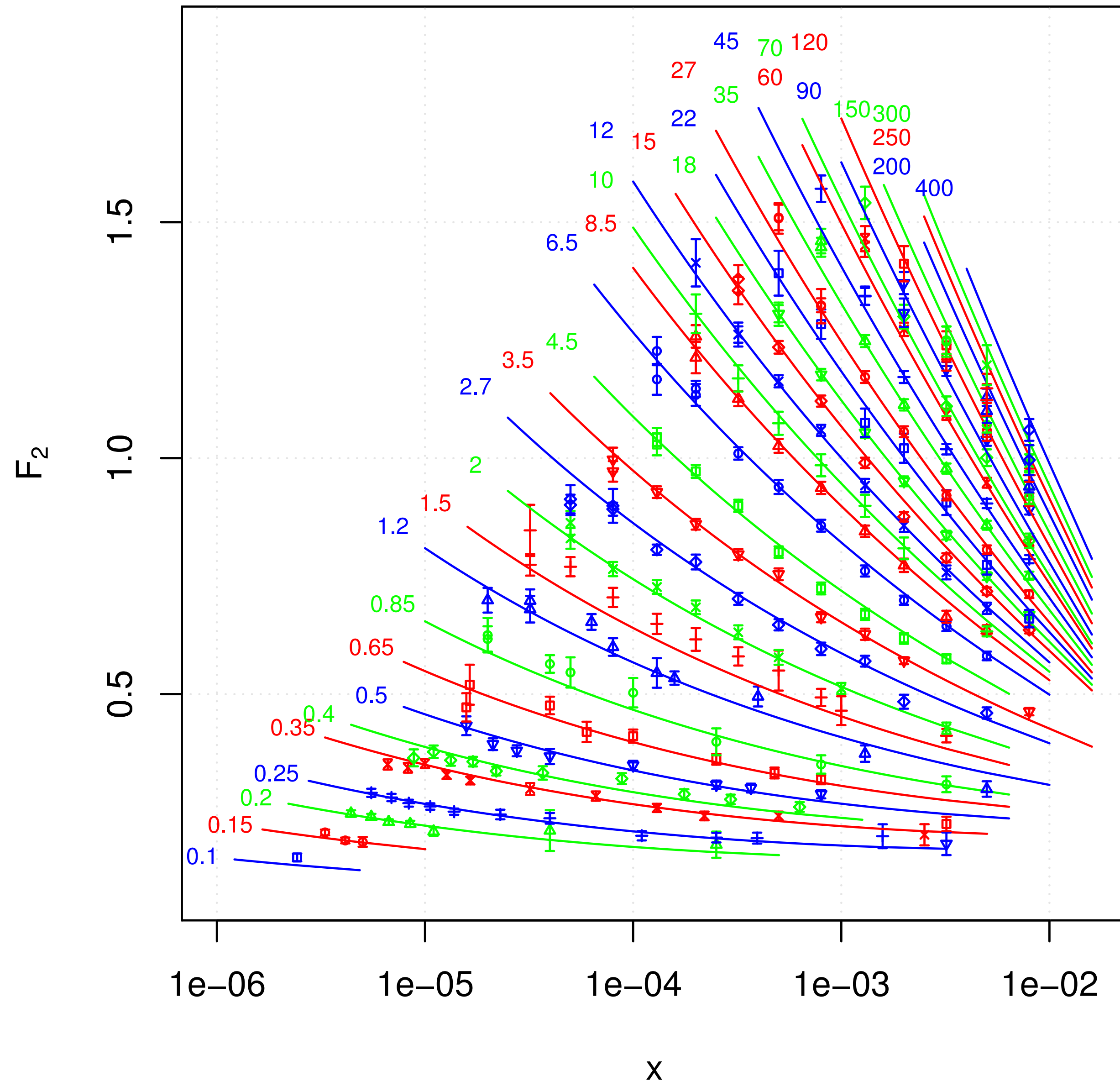


# Test model against low x DIS data from HERA



Truncated data to  $x < 0.01$  region.  
Has 249 data points and large range in Q  
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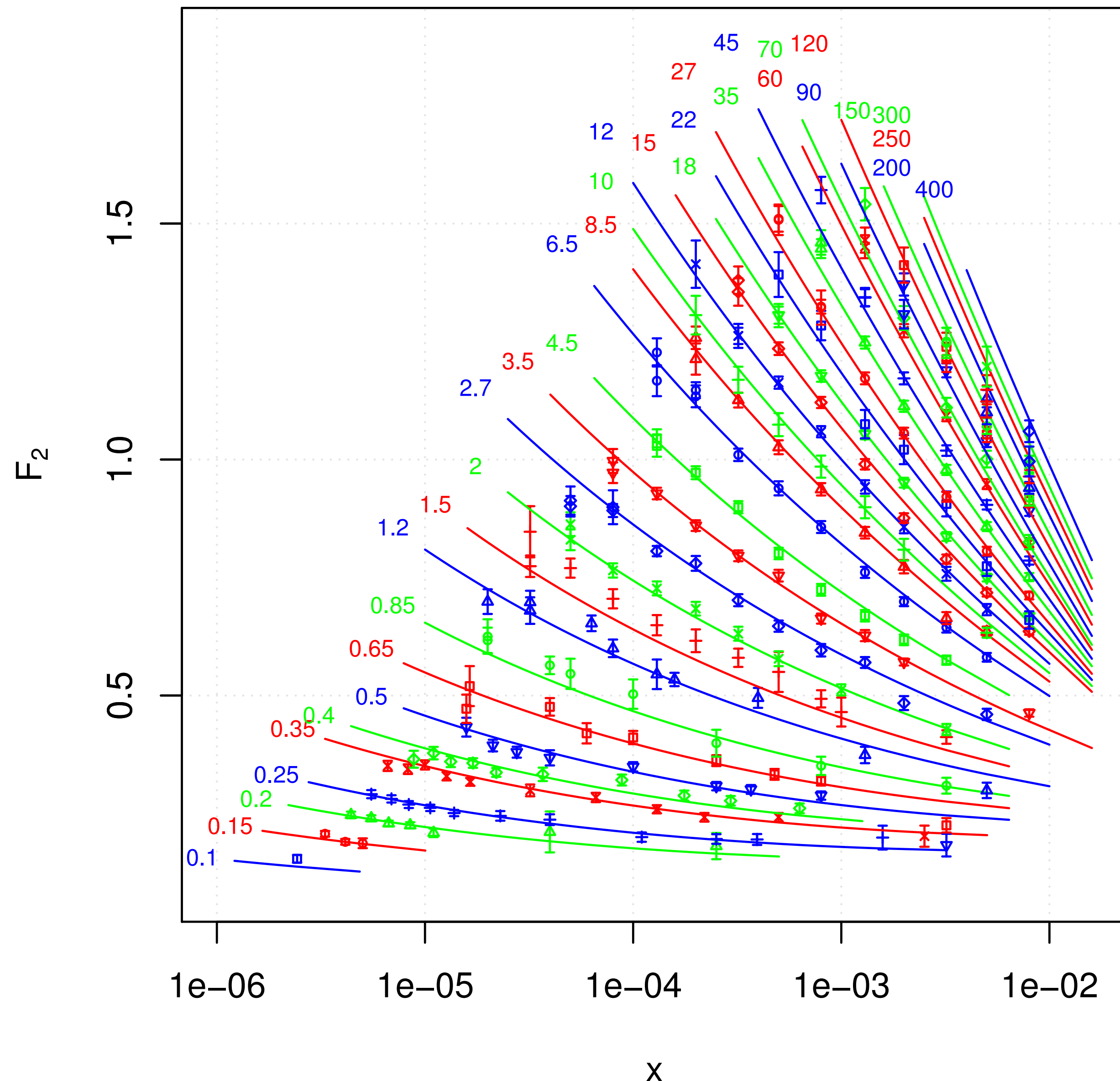


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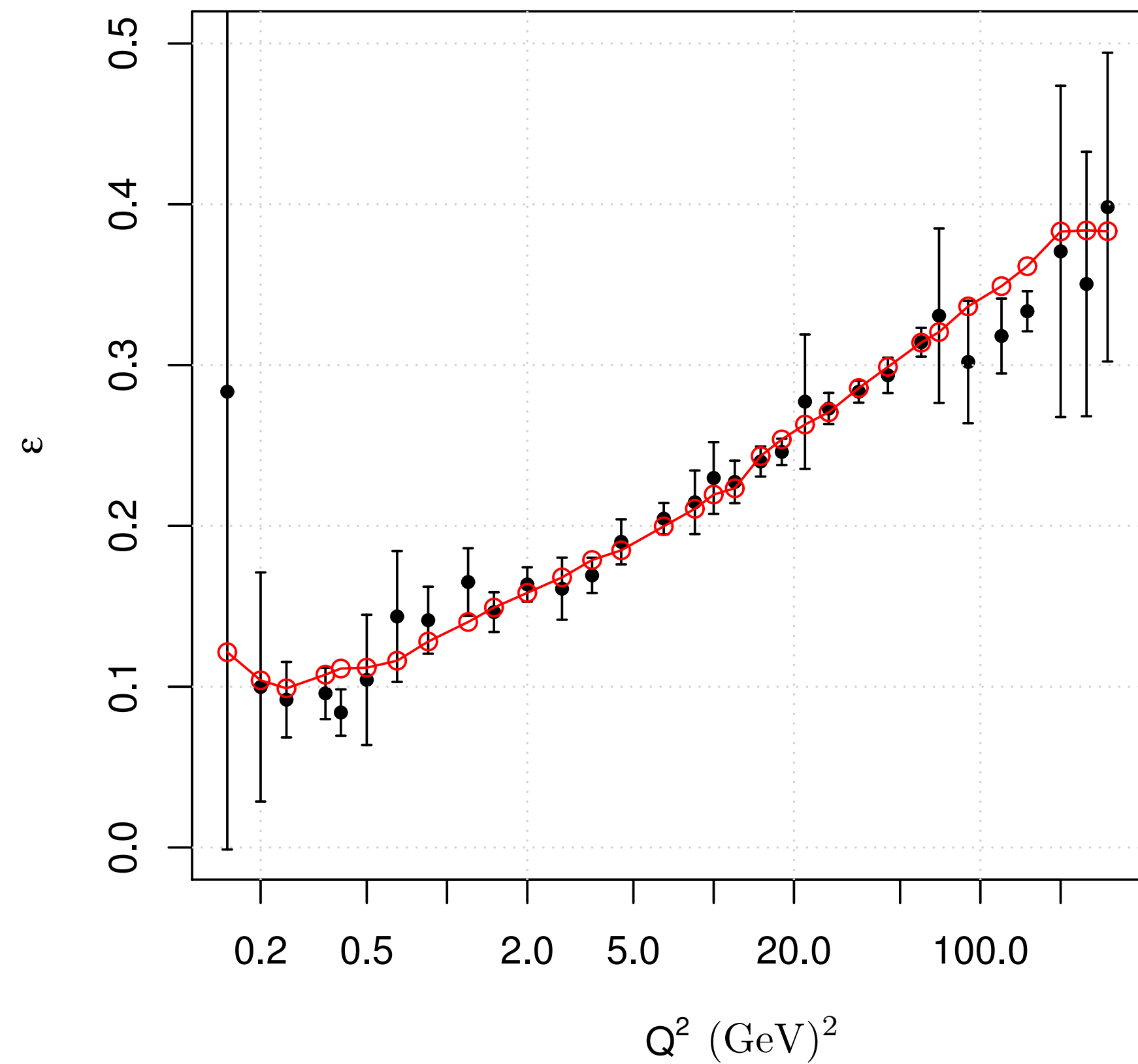
Parameters fixed with  $\chi^2 = 1.7$

Pomeron equation coefficients	coupling	Intercept
$a = -4.35$	$g_0 = 0.175$	$j_0 = 1.17$
$b = 1.41$	$g_1 = 0.121$	$j_1 = 1.09$
$c = 0.626$	$g_2 = 0.297$	$j_2 = 0.969$
$d = -0.117$	$g_3 = -1.63$	$j_3 = 0.900$
$l_s = 0.153 \text{ GeV}^{-1}$	–	–



- Reproduced long sought running of effective exponent

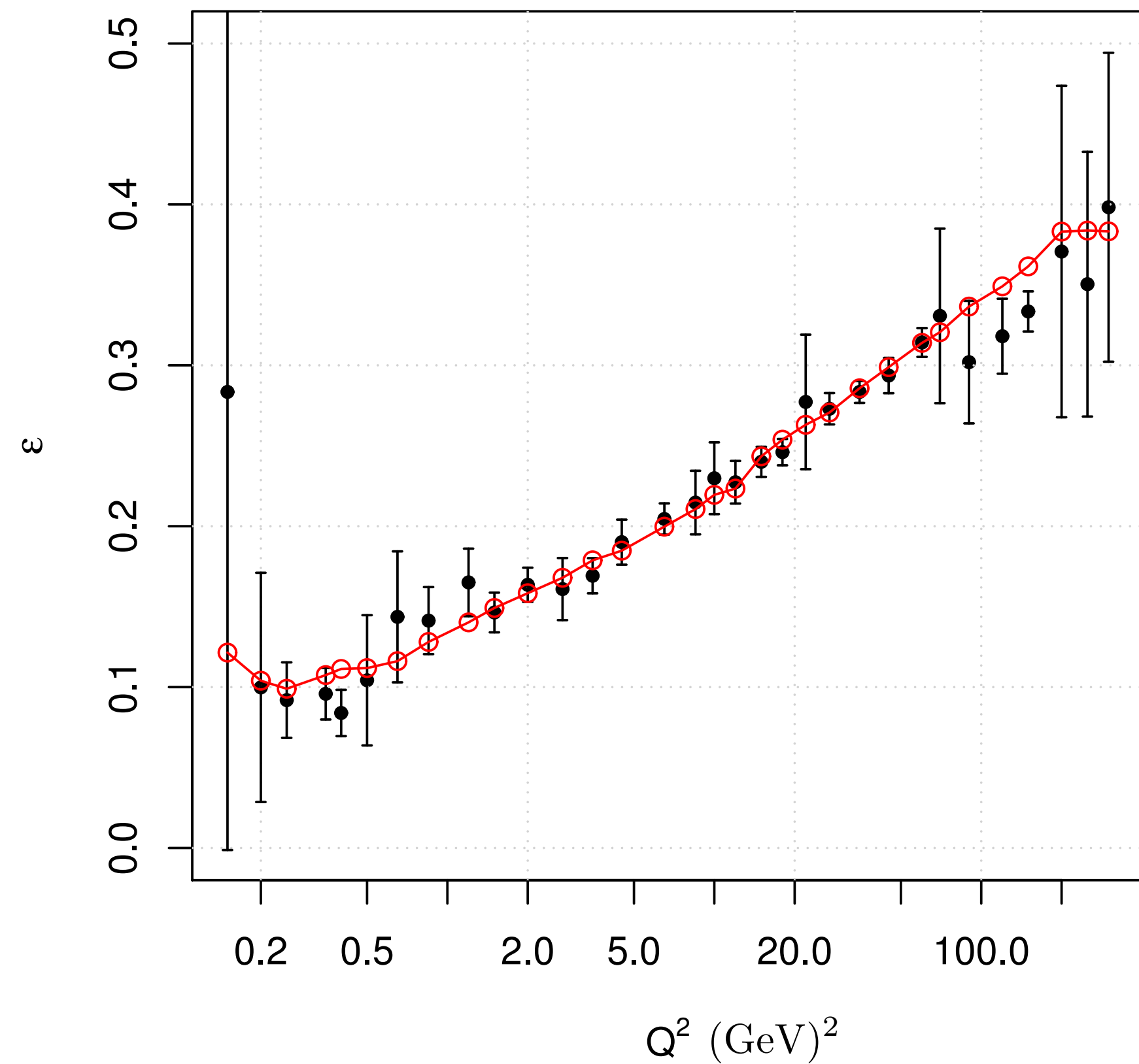
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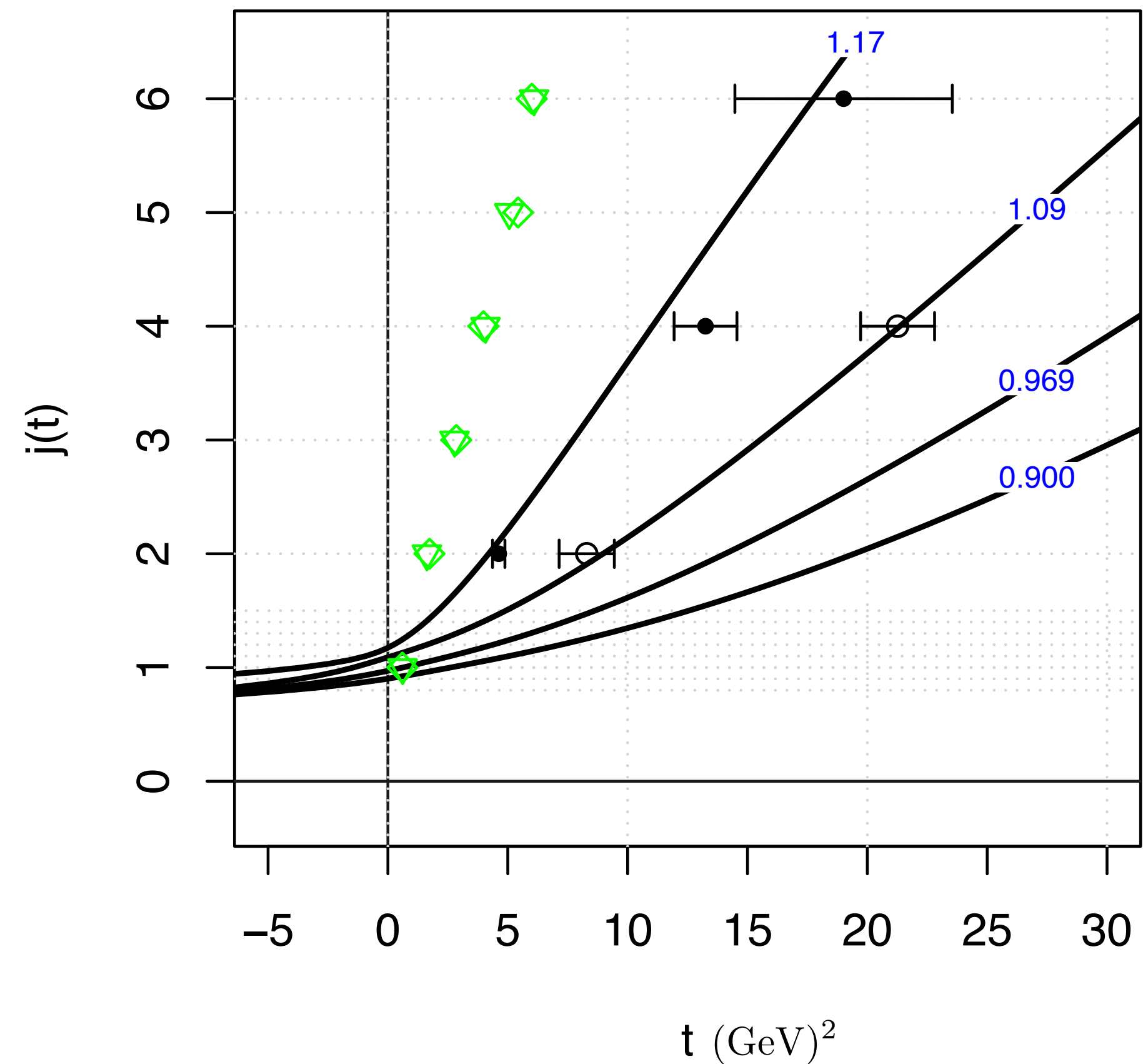
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- Regge trajectories consistent with lattice [Meyer 05] QCD glueball spectrum!



In green meson trajectories  
Shape matches [Caron-Huot, Komargodski, Sever, Zhiboedov et al 16]

## EMG current and Reggeon non-minimal coupling [Amorim, MSC, Quevedo 18]

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- So far considered minimal coupling between U(1) gauge field and graviton trajectory. But for graviton perturbations in AdS there are two possible couplings

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$$A_\mu^\lambda(X; k) = n_\mu^\lambda f_k(z) e^{ik \cdot x}$$

$$f_n^{\text{NMC}}(Q^2) = \tilde{g}_n Q^{2j_n} \int dz e^{-(j_n - \frac{3}{2})A} \left( f_Q^2 \tilde{\mathcal{D}}_\perp + \frac{\dot{f}_Q^2}{Q^2} \tilde{\mathcal{D}}_\parallel \right) \psi_n$$

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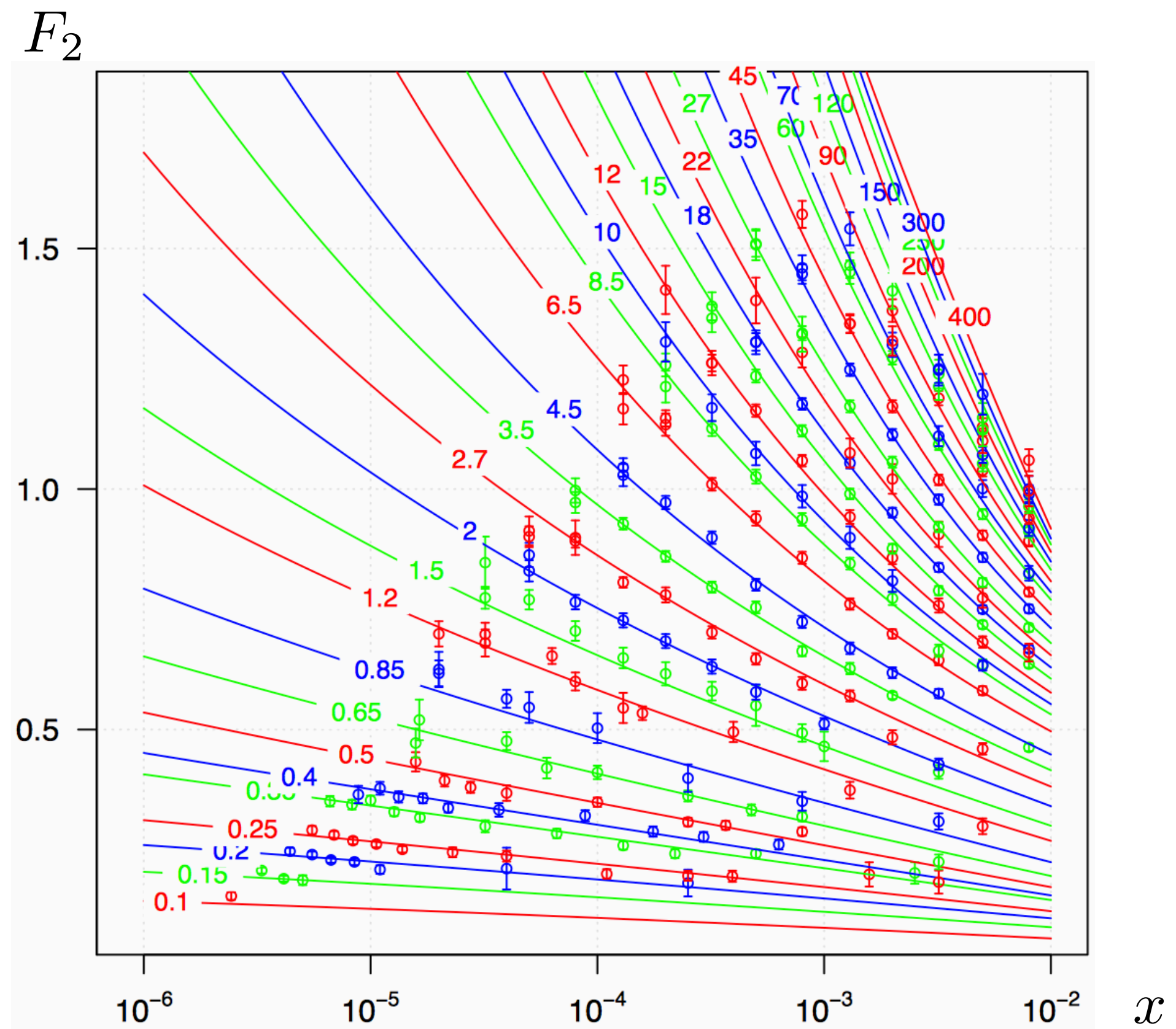
$$f_n^{\text{NMC}}(Q^2) = \tilde{g}_n Q^{2j_n} \int dz e^{-(j_n - \frac{3}{2})A} \left( f_Q^2 \tilde{\mathcal{D}}_\perp + \frac{\dot{f}_Q^2}{Q^2} \tilde{\mathcal{D}}_\parallel \right) \psi_n$$

$$\tilde{\mathcal{D}}_\perp = e^{-2A} \left( \dot{A} \partial_z + \dot{A}^2 + \dot{A} \dot{B} \right)$$

$$\tilde{\mathcal{D}}_\parallel = e^{-2A} \left( \partial_z^2 - (\dot{A} - 2\dot{B}) \partial_z + \ddot{B} + \ddot{A} + \dot{B}^2 - \dot{A} \dot{B} \right)$$

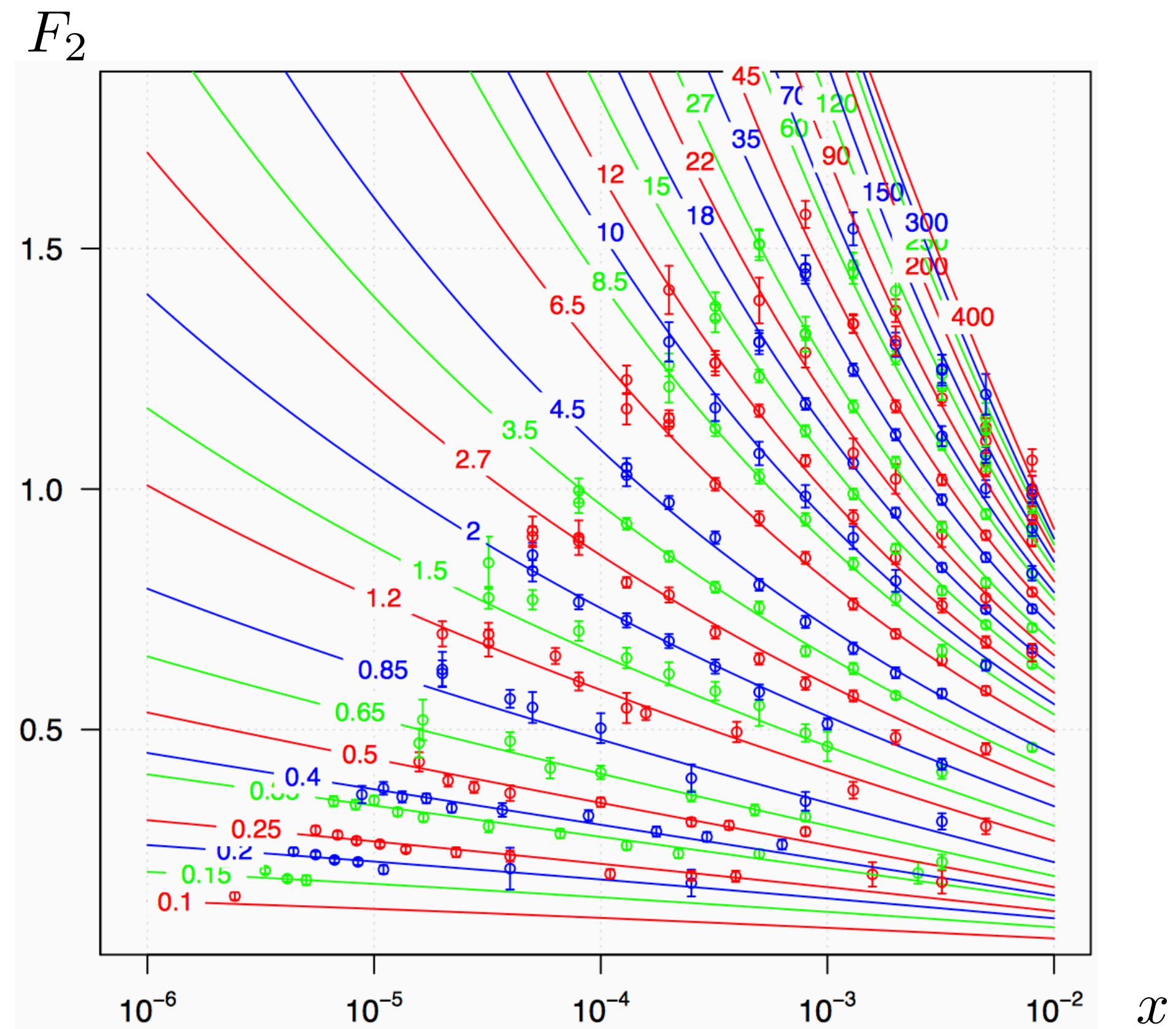
- Quality of fit improved significantly!

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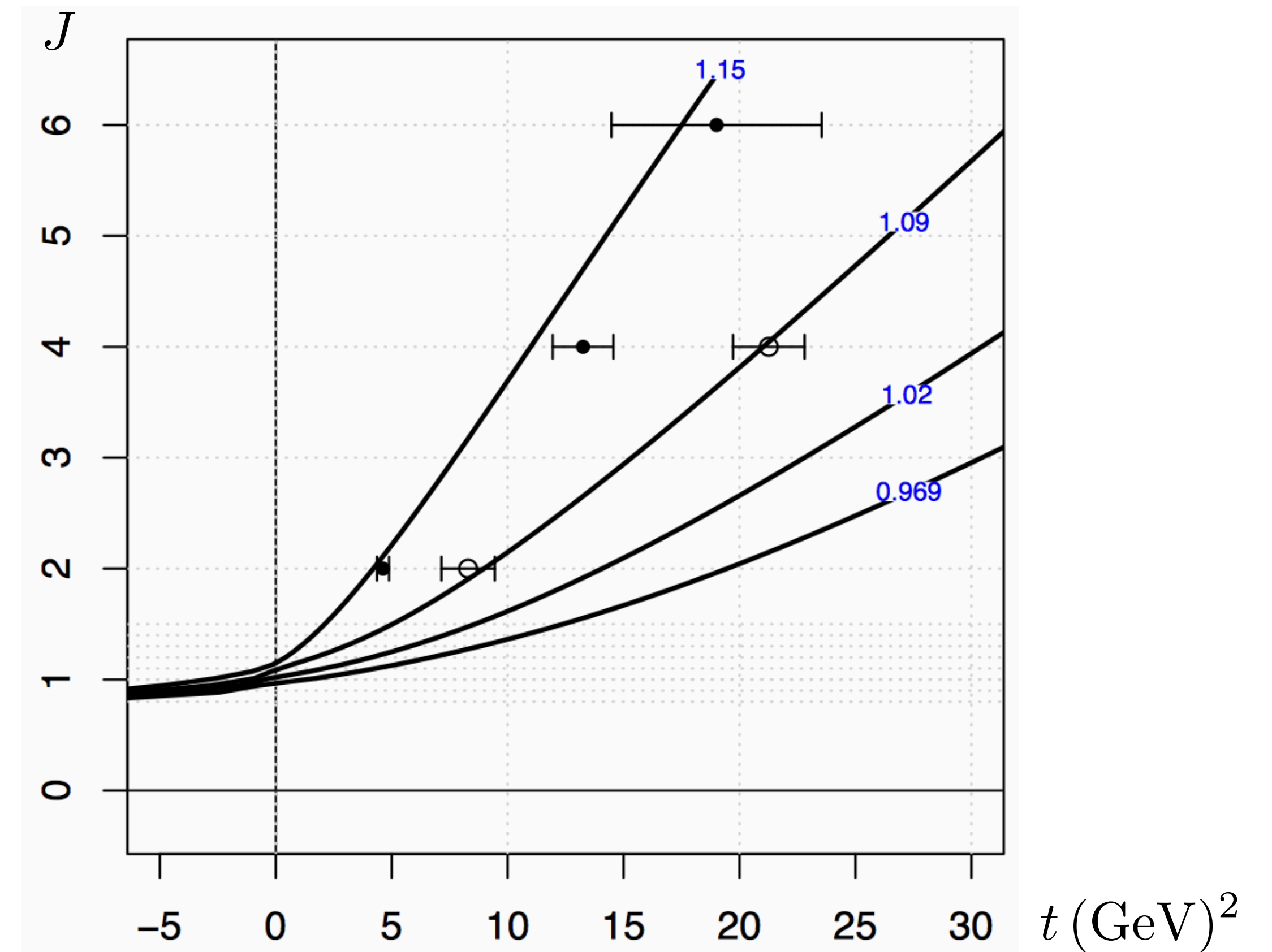


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- Non-minimal coupling has dimensions and defines scale of 1-10 GeV. Matches order of magnitude of gap between spin 4 and 2 glueballs [CEMZ 14]



# Concluding Remarks

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- Include meson trajectories.
- Coupling of Pomeron to gluon jets.
- How generic are our results? Should try other holographic QCD models...

**THANK YOU**