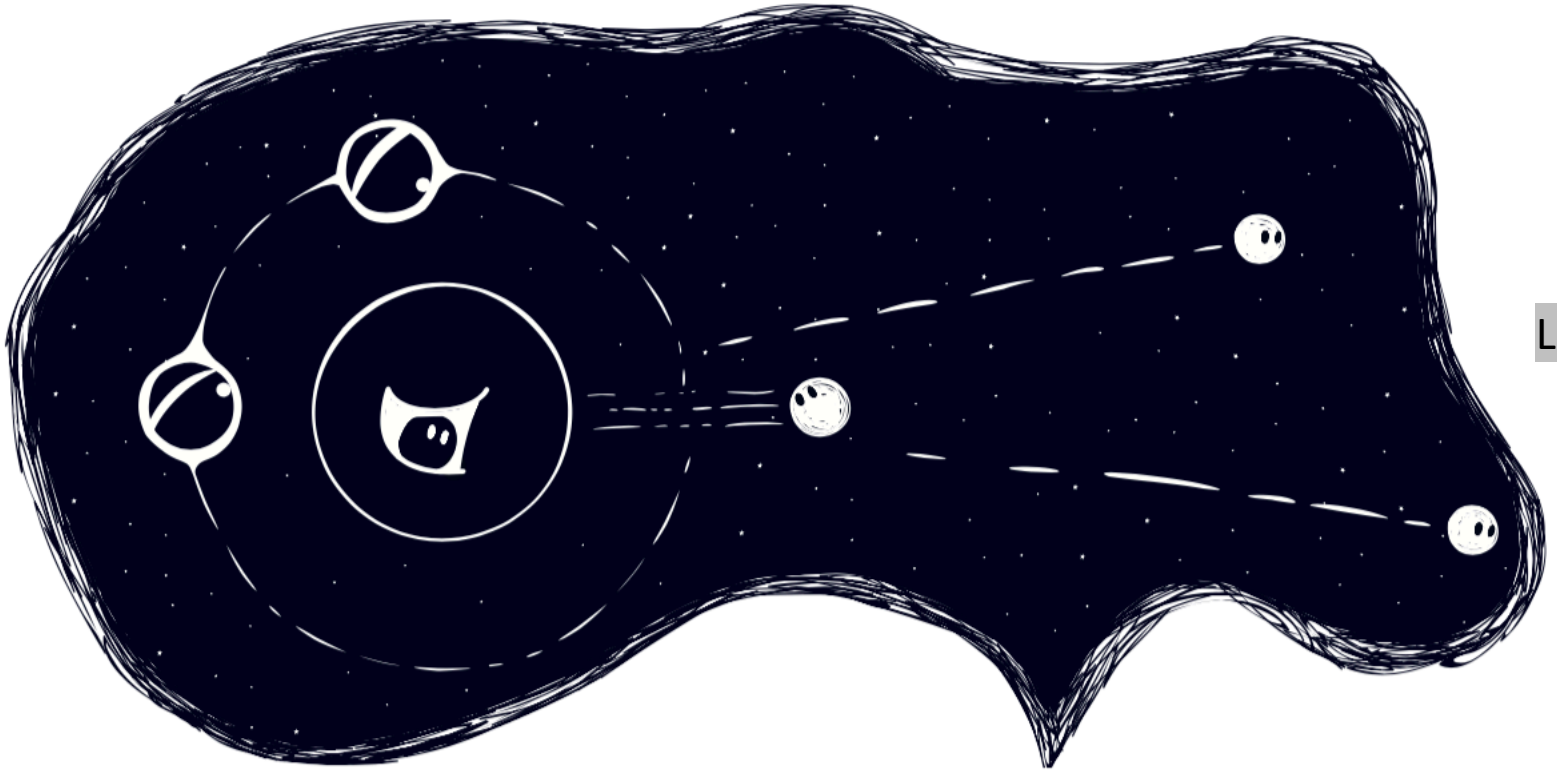
Complexity and Page curve of an evaporating black hole in flat space

Part I:
1911.06800
2001.06453
with
Lukas Schneiderbauer
and
Lárus Thorlacius



Part II:
2003.xxxxx
with
Friðrik Gautason,
Lukas Schneiderbauer
and
Lárus Thorlacius

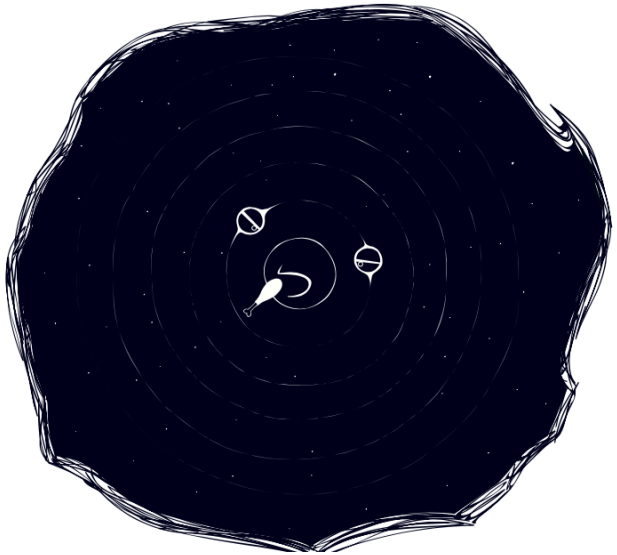
rannís

~~Santiago, 13th of March 2020~~
Santiago, 23rd or March 2020

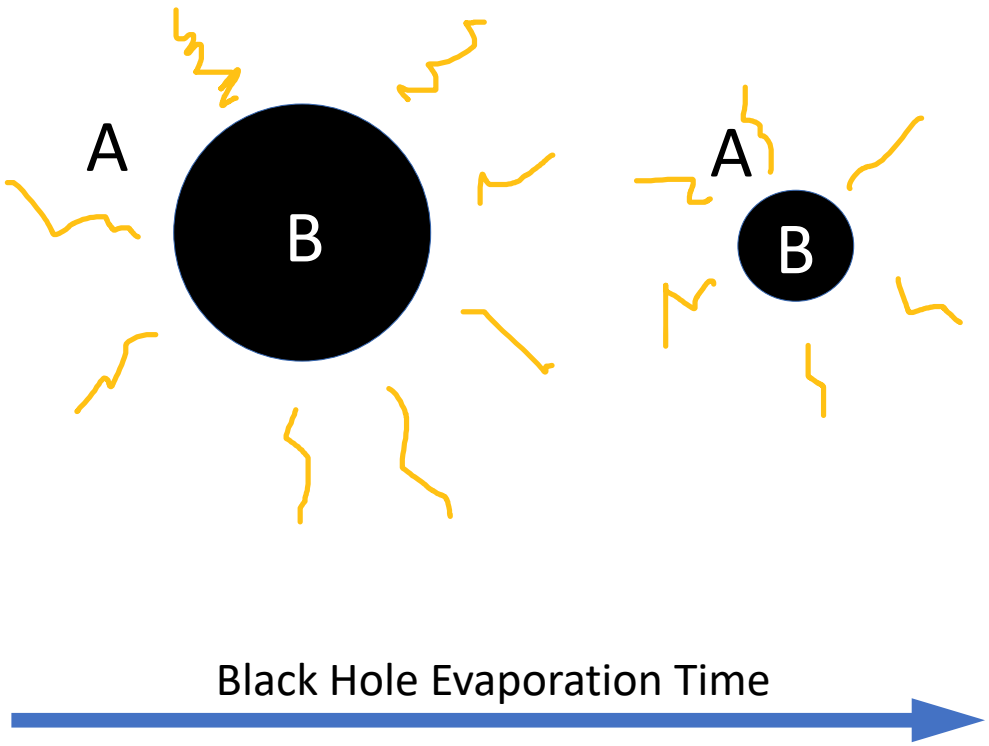
Watse Sybesma
University of Iceland

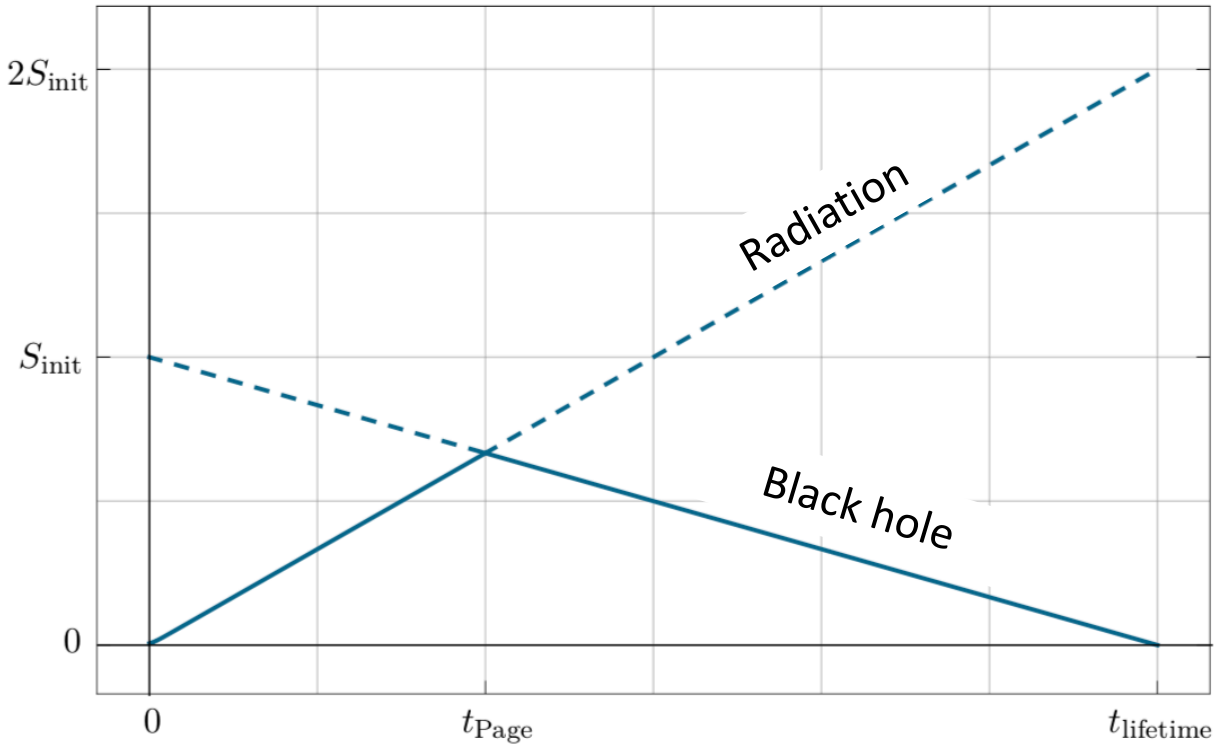
Outline

- Prelude to Part 2: Page curve
- Part 1: Complexity & semi-classical flat space black hole
- Part 2: flat space Page curve



Prelude to Part 2: Page Curve

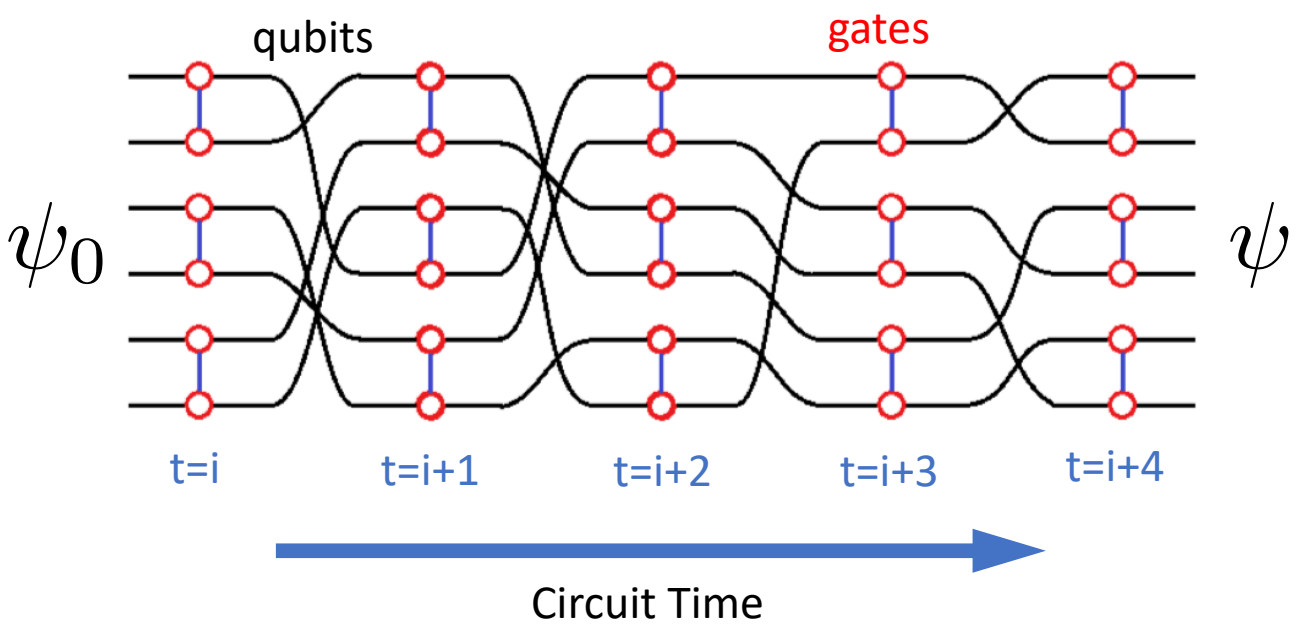




[Penington '19]
[Almheiri, Engelhardt, Marolf, Maxfield '19]

Part 1: Quantum Complexity

Complexity quantum circuit
governed by some Hamiltonian:



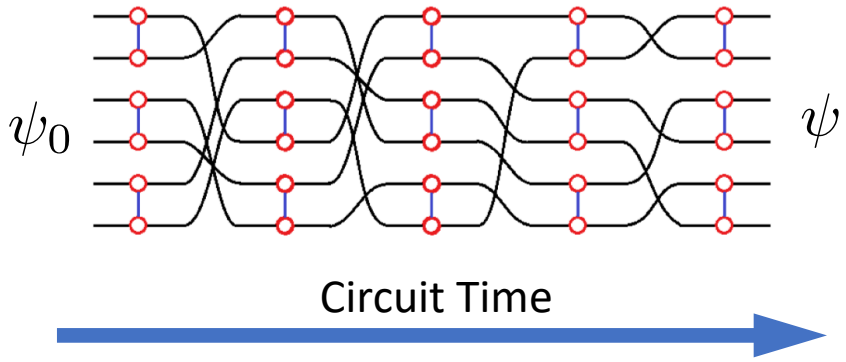
Complexity is 'distance' of state ψ to original ψ_0

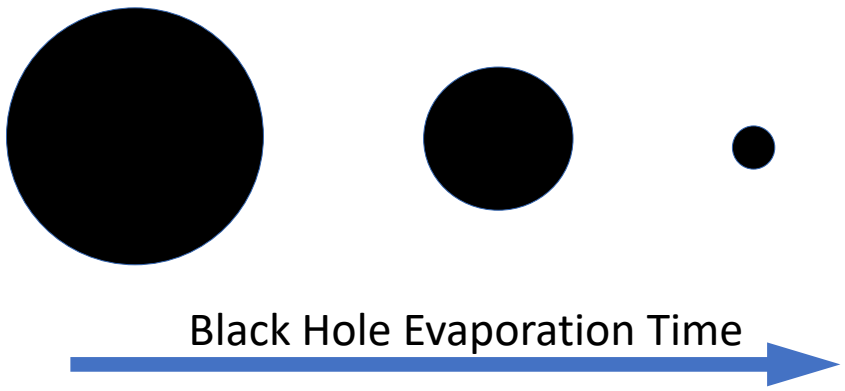
Number of qubits K $K \sim S$,

Maximal growth: $\frac{d\mathcal{C}}{dt} \sim S$

see e.g. [Susskind '18]

Quantum Complexity for a Black Hole?





Black Holes are Fastest Scramblers of Information

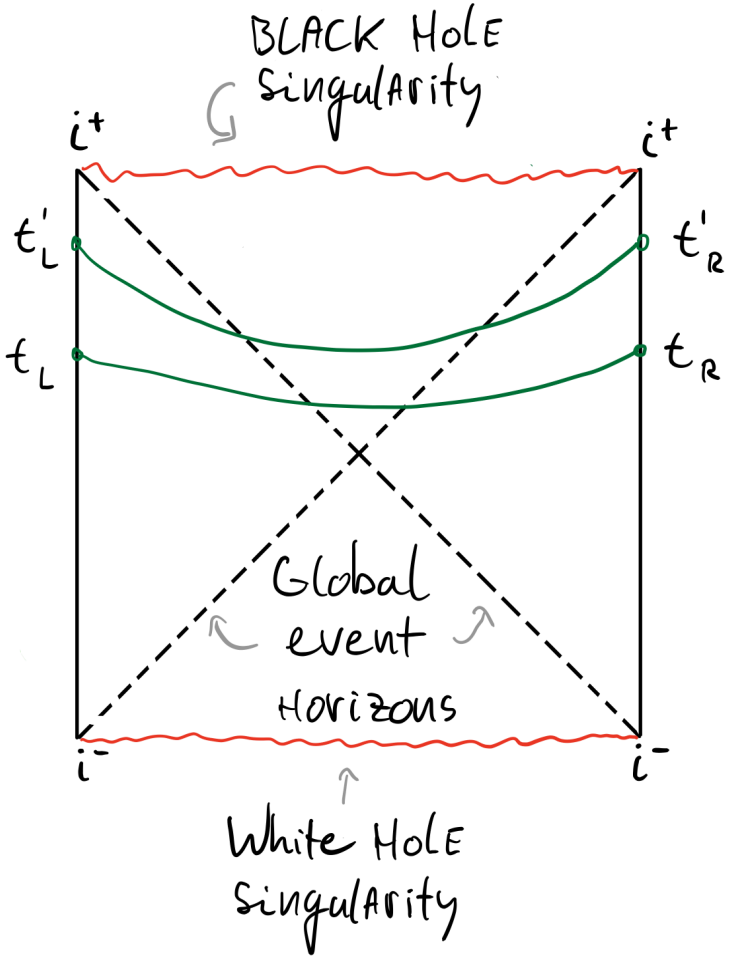
[Sekino, Susskind '08]

After onset growth: $\frac{d\mathcal{C}}{dt} \sim S$

[Susskind '16]

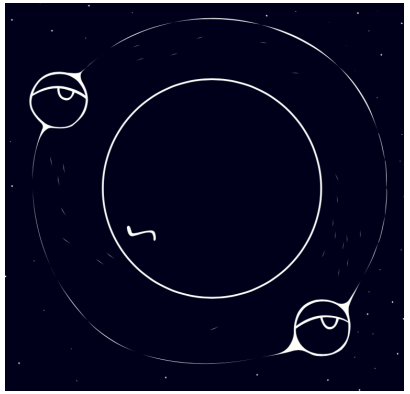
(How) can we compute complexity for a(n evaporating) black hole?
Does it satisfy the maximal growth?

Proposals to Compute BH Complexity



$$C_V = \frac{\text{maximal wormhole volume}}{L},$$

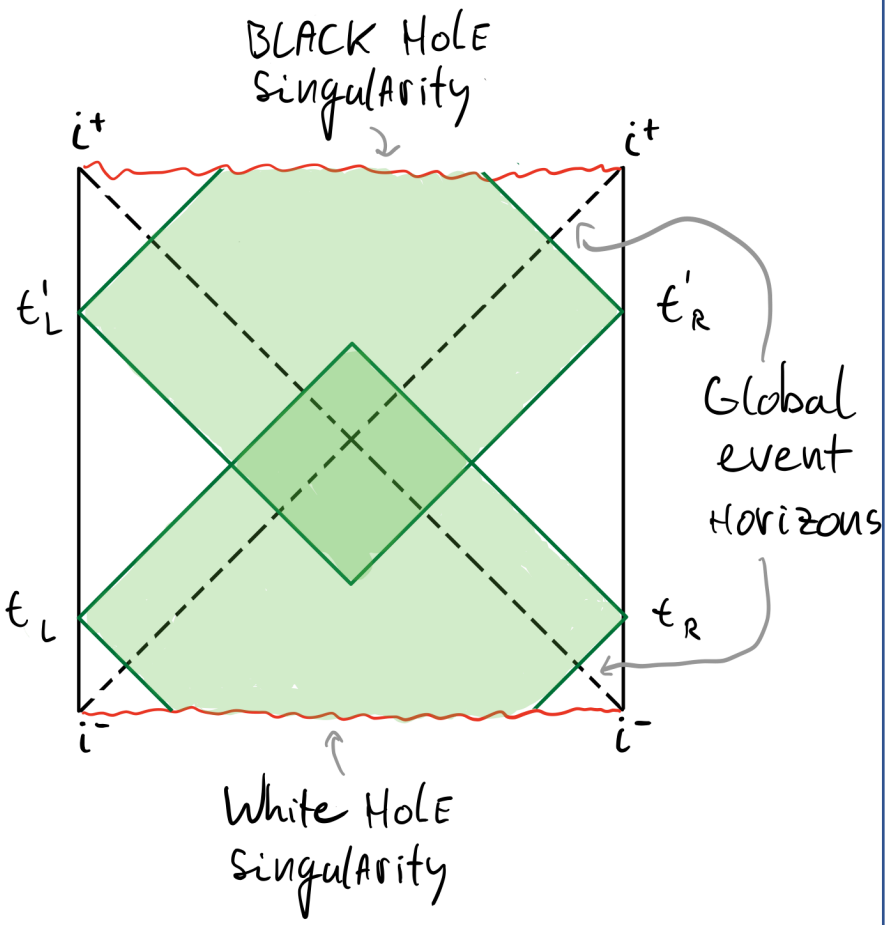
[Susskind '16] [Susskind, Zhao '14]



After onset:

$$\dot{C}_V \sim \dot{C}_A \sim S$$

Now all we need is an evaporating black hole!



$$C_A = \frac{\text{action evaluated on WdW}}{\pi},$$

[Brown, Roberts, Susskind, Swingle, Zhao '16 (x2)]

Classical CGHS Model

$$S_0 = \int_{\mathcal{M}} d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right]$$

+ boundary terms

[Callan, Giddings, Harvey, Strominger '92]

- Length scale 1/λ set by magnetic charge of 4-dimensional parent theory
- N scalars f, describing conformal matter fields
- $e^{-2\phi}$ can be identified with the area of the transverse 2-sphere of 4d theory.

Effective Newton's constant

$$\frac{1}{G_{\text{eff}}} = \frac{e^{-2\phi}}{G}$$

Entropy

$$S = 2e^{-2\phi}\big|_{\text{horizon}} = 2M$$

Temperature

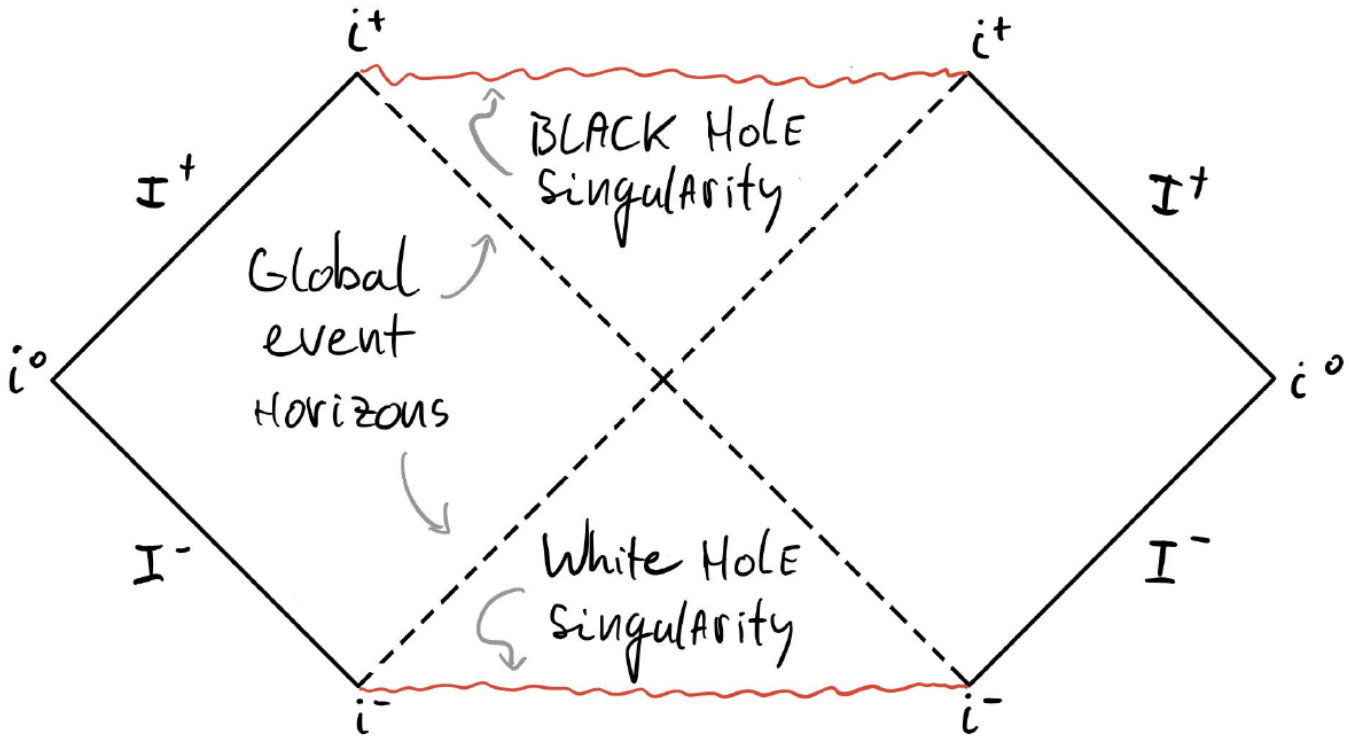
$$T = \frac{1}{2\pi}$$

Eternal Black Hole

Conformal and Kruskal gauge:

$$f_i = 0$$
$$ds^2 = -e^{2\phi} dx^+ dx^-$$
$$e^{-2\phi} = M - x^+ x^-$$

$$\Longrightarrow \quad R = \frac{4M}{M - x^+ x^-}$$



Classical Gravitational Collapse Black Hole

- For simplicity: thin shockwave of infalling matter

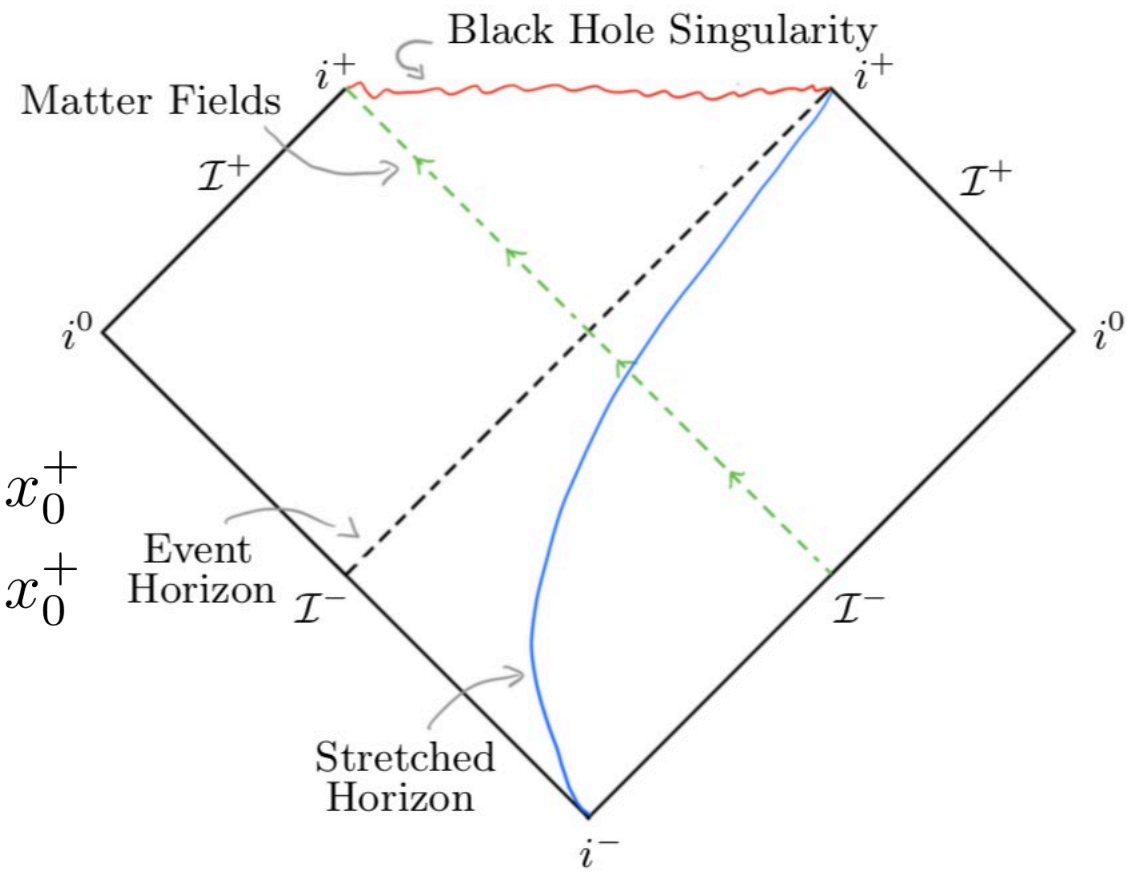
$$T^f_{++} = \frac{M}{x_0^+}\delta(x^+ - x_0^+)$$

$$ds^2 = -e^{2\phi}dx^+dx^-$$

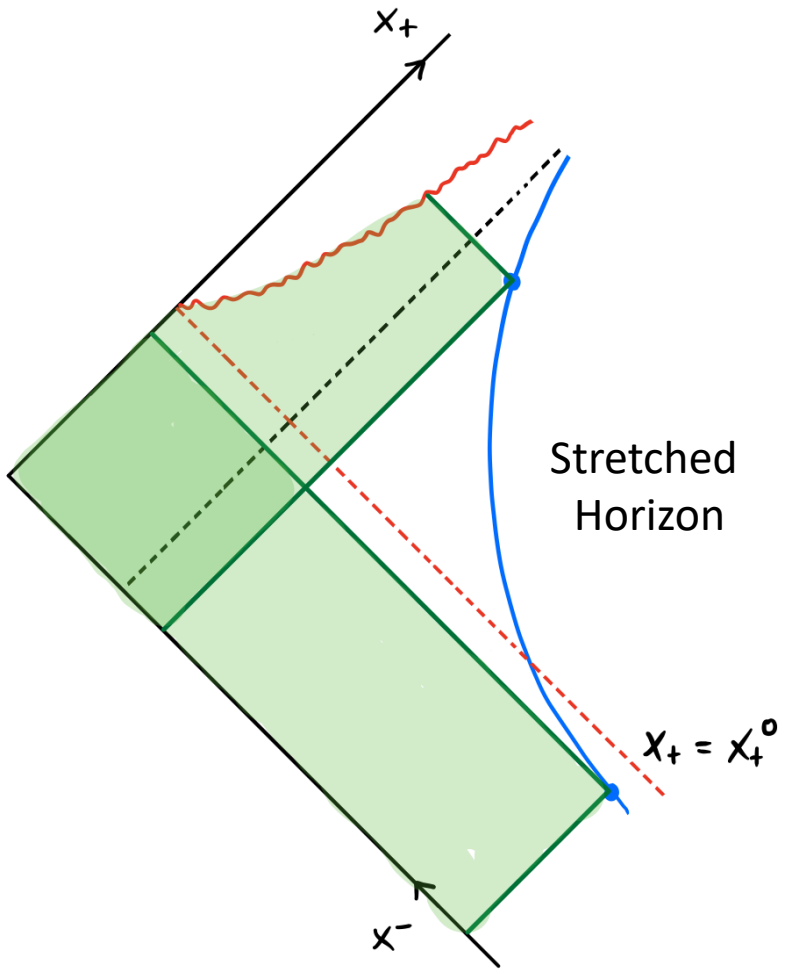
$$e^{-2\phi} = \begin{cases} -x^+x^- & \text{if } x^+ < x_0^+ \\ -x^+\left(x^- + \frac{M}{x_0^+}\right) + M & \text{if } x^+ \geq x_0^+ \end{cases}$$

Stretched horizon:

$$e^{-2\phi}\big|_{\text{stretched horizon}} := e^{-2\phi}\big|_{\text{horizon}} + 1$$



Classical Collapse BH



Kruskal diagram

$$\mathcal{L} = \sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\Lambda^2\right) - \frac{1}{2}\sum_{i=i}^{N}(\nabla f_i)^2\right],$$

$$I_{\mathrm{WdW}} = \int_{\mathrm{WdW}} \mathcal{L} + \sum_{\text{line elements}} 2\sigma \int_{\partial\mathrm{WdW}} \sqrt{|h|}e^{-2\phi}K + \sum_{\text{joints}} 2\sigma e^{-2\phi}\log(|n_1 \cdot n_2|),$$

[Lehner, Myers, Poisson, Sorkin '16]

Vacuum:

growth contribution:		
bulk	surface	joints
0	0	0

Black hole:

growth contribution:		
bulk	surface	joints
$4M$	$-2M$	0

Semi-Classical Recipe

$$S_0 = \int_{\mathcal{M}} d^2x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right) - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right]$$

(1) Backreaction due to conformal anomaly

$$\langle T^{\mu}{}_{\mu}\rangle = \frac{N}{12}R$$

[Christensen, Fulling '77]

(2) Large N to suppress other loops

$$S_{\mathrm{q}} = -\frac{N}{48}\int_{\mathcal{M}} d^2x\sqrt{-g}\left(R\frac{1}{\nabla^2}R\right)$$

[Callan, Giddings, Harvey, Strominger '92]

(3) M>>N>>1 to keep classical part leading

(4) Include counter term ct to make analytically solvable

$$S_{\mathrm{ct}} = -\frac{N}{12}\int_{\mathcal{M}} d^2x\sqrt{-g}\phi R$$

[Russo, Susskind, Thorlacius '92]

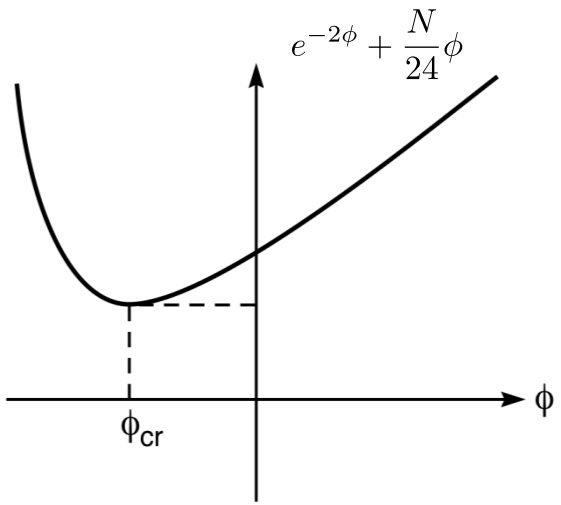
$$S_{\mathrm{RST}} = S_0 + \hbar S_{\mathrm{q}} + \hbar S_{\mathrm{ct}}$$ + <u>boundary terms</u>

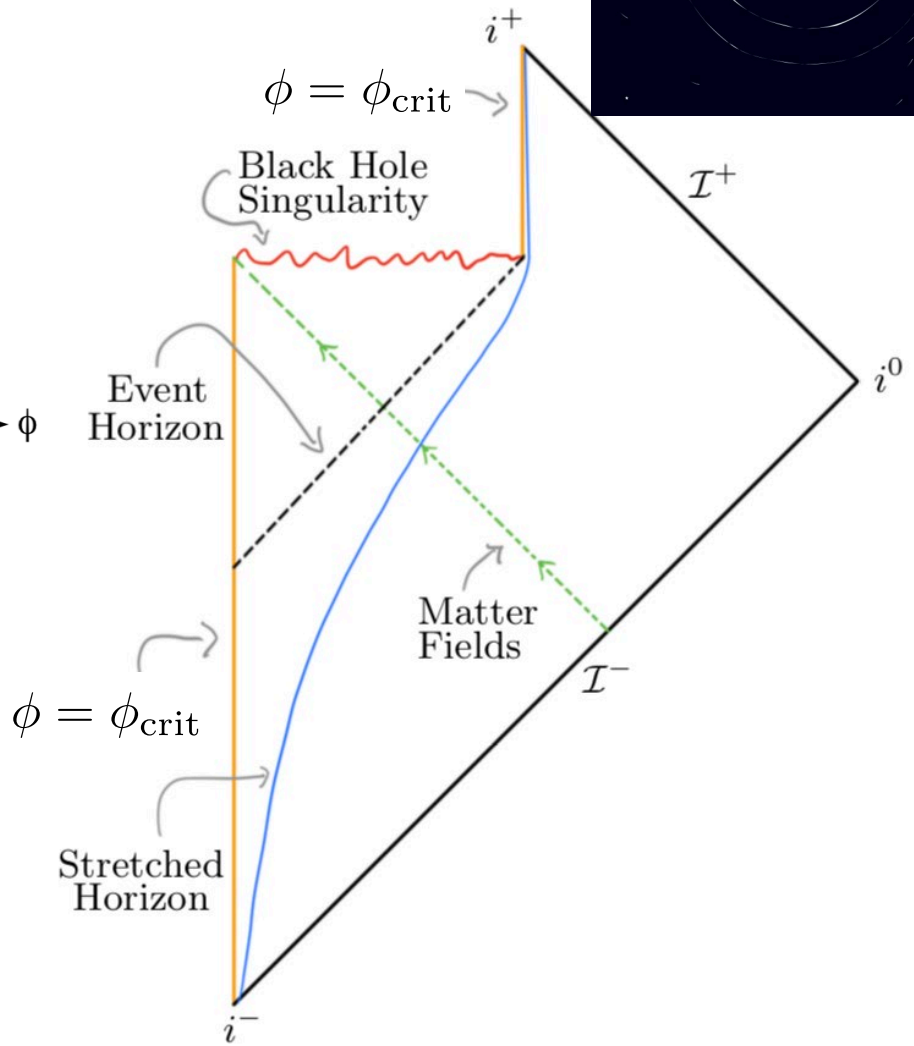
Semi-Classical Solution

Equations of motion:

$$\partial_+\partial_-\left(e^{-2\phi}+\frac{N}{24}\phi\right)=-1$$

$$-\partial_\pm^2\left(e^{-2\phi}+\frac{N}{24}\phi\right)=T_{\pm\pm}-t_\pm$$





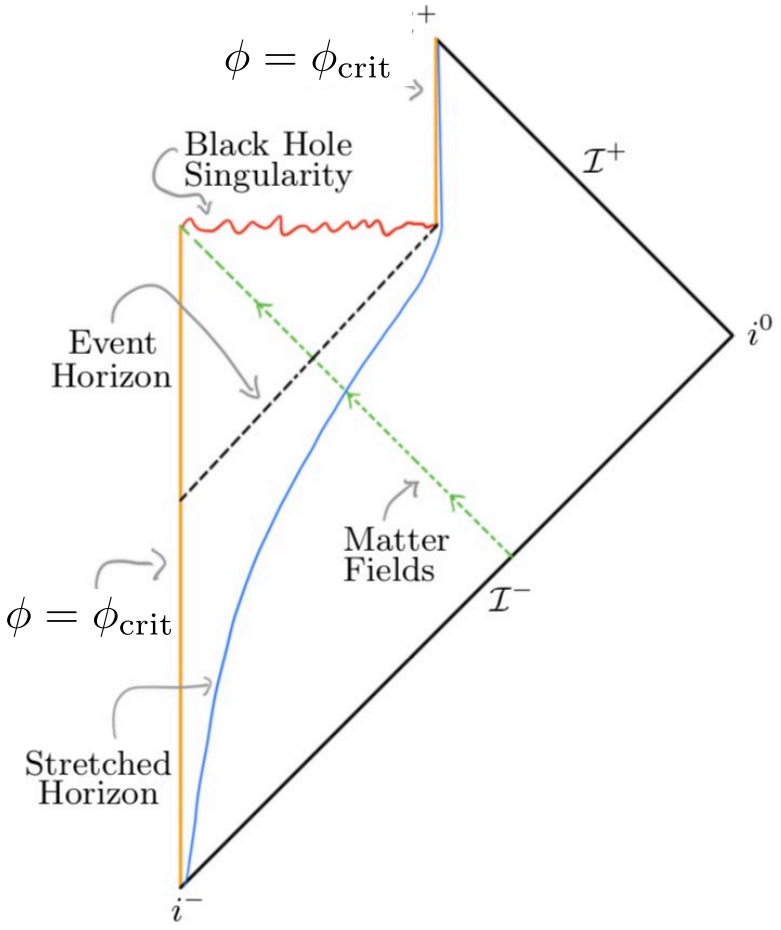
Solutions:

$$ds^2=-e^{2\phi}dx^+dx^-$$

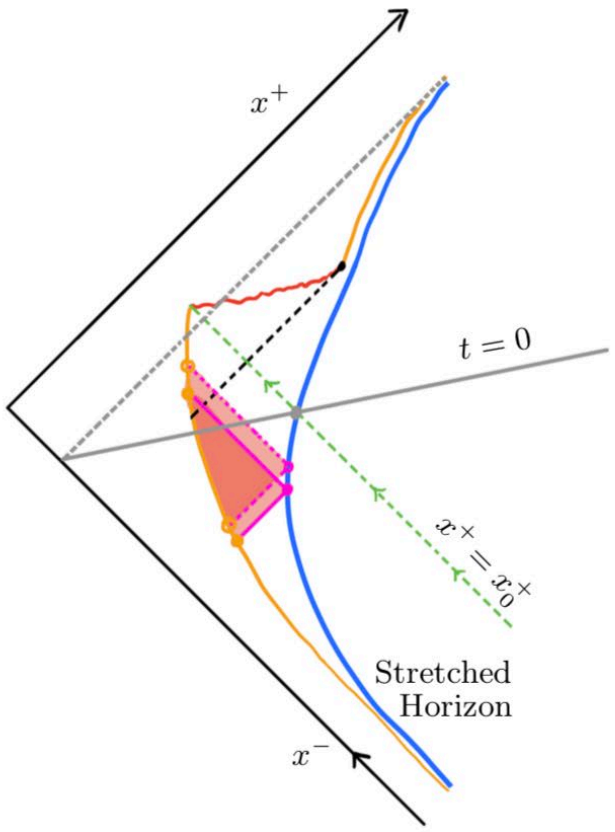
$$e^{-2\phi}+\frac{N}{24}\phi=\begin{cases}-x^+x^-\ -\ \frac{N}{48}\log\left(-x^+x^-\right) & \text{if } x^+<x_0^+\\ M-x^+\left(x^-+\frac{M}{x_0^+}\right)-\frac{N}{48}\log\left(-x^+x^-\right) & \text{if } x^+\geq x_0^+\end{cases}$$

We can infer: $S(t)=2M-\frac{N}{24}t$

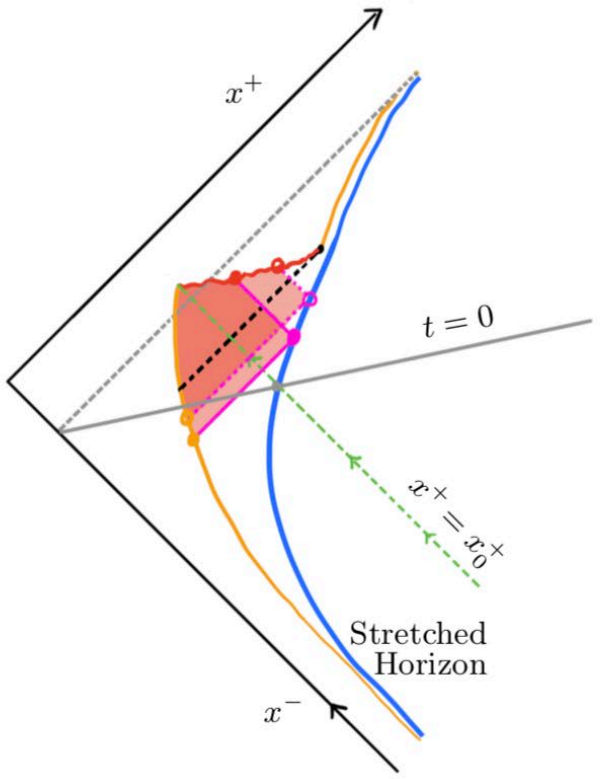
Wheeler-DeWitt Patches



Poincare Patch



Kruskal diagram



Kruskal diagram

RST boundary terms

Making backreaction term local:

$$S_{\rm q} = -\frac{N}{48}\int_{\mathcal{M}} d^2x\sqrt{-g}\left(R\frac{1}{\nabla^2}R\right) \longrightarrow S_q = \frac{N}{24}\int d^2x\sqrt{-g}\left(ZR - \frac{1}{2}(\nabla Z)^2\right)$$

Sensitive to boundary conditions:

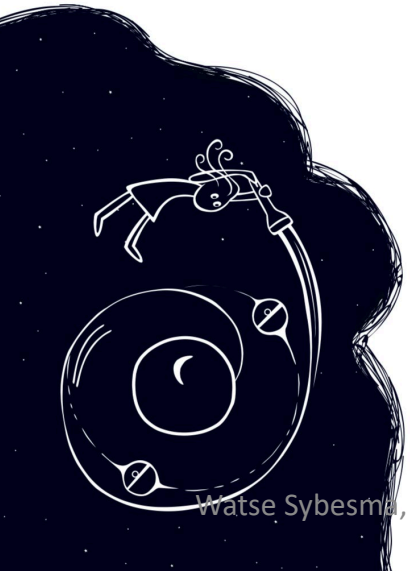
$$\Box Z = -R$$

Introduces 'arbitrary' harmonic field:

$$\eta$$

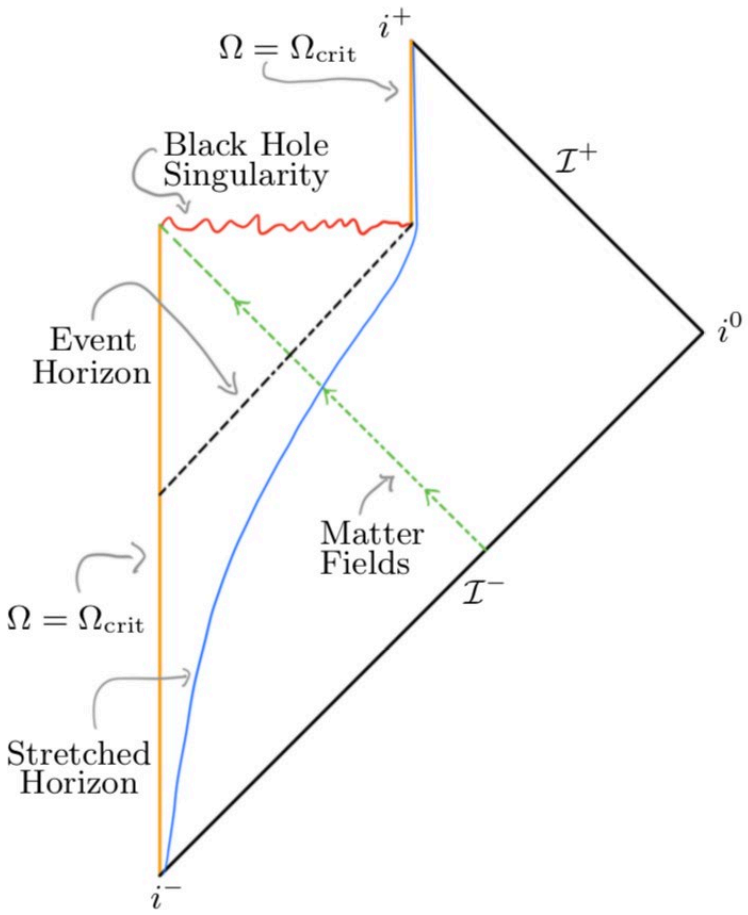
This is $t_\pm$ in disguise

$$S_{\rm boundary} = -\frac{N}{48}\int d^2x\sqrt{-g}\left[\nabla(\eta\nabla\eta)\right]$$



Complexity for an Evaporating black hole

[Schneiderbauer, WS, Thorlacius'20]



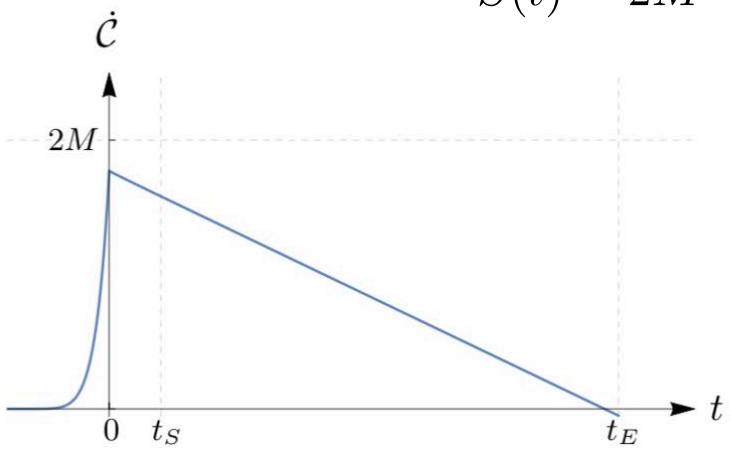
$$\dot{C} = 2M - \frac{N}{24}\left(t + \log\frac{4M}{\kappa} + \mathcal{O}\left(\frac{N}{M}e^{-t}\right)\right)$$

Entropy as function of time

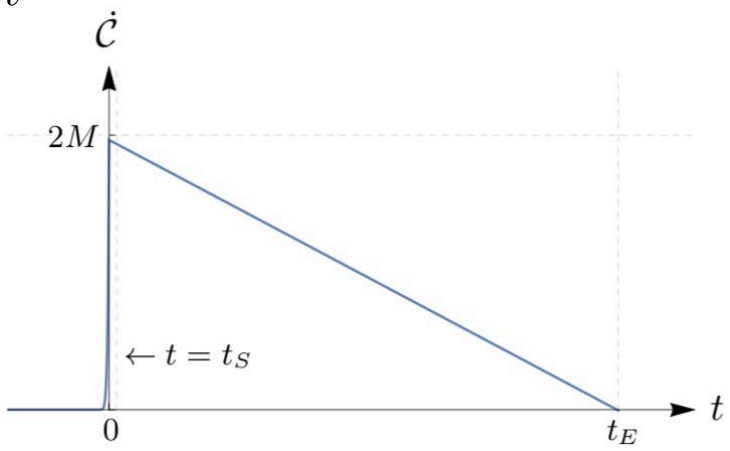
$$S(t) = 2M - \frac{N}{24}t$$

Matching Log

[Myers'94][Solodukhin'96]



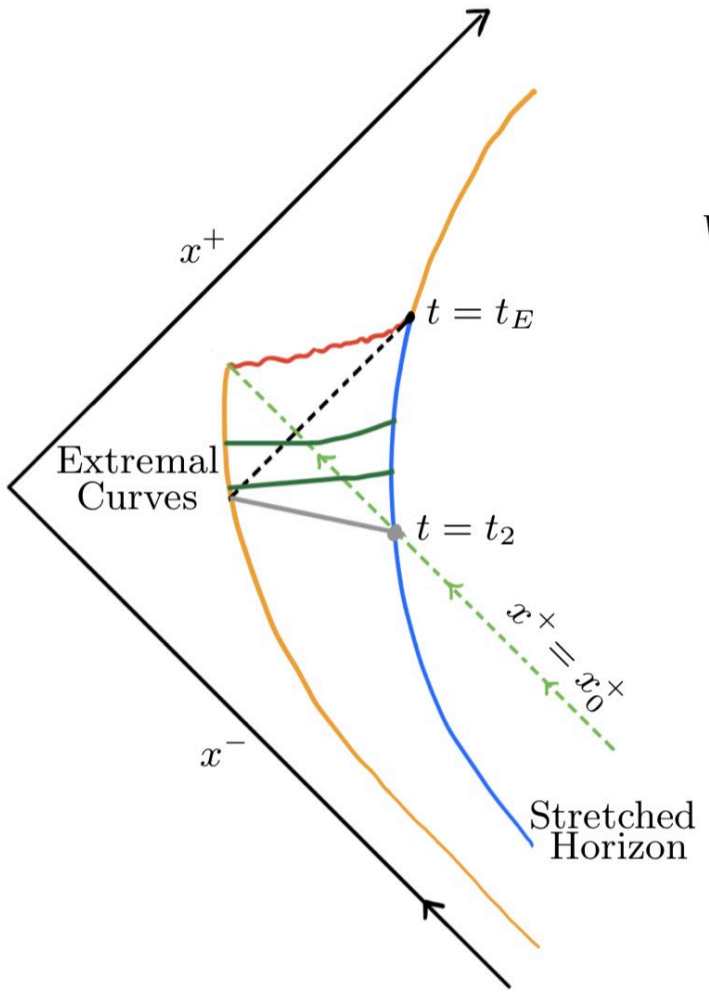
M/N~100

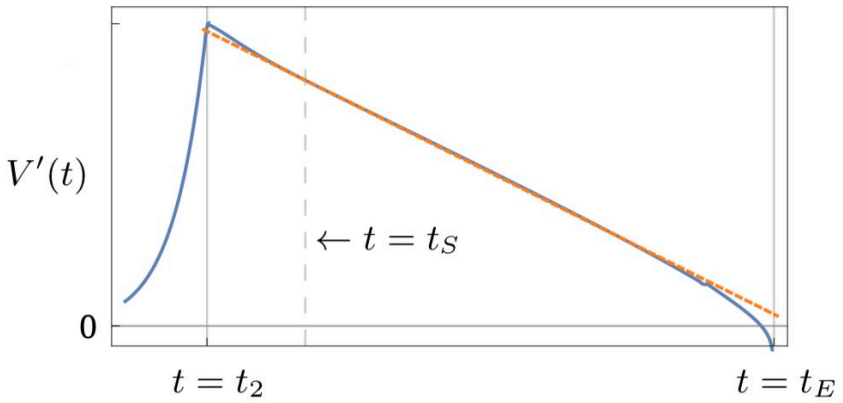


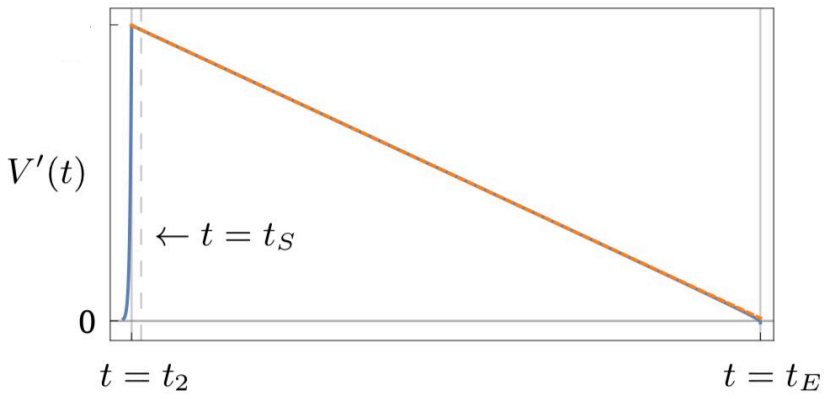
M/N~1000

Wormhole Result

[Schneiderbauer, WS, Thorlacius'19]

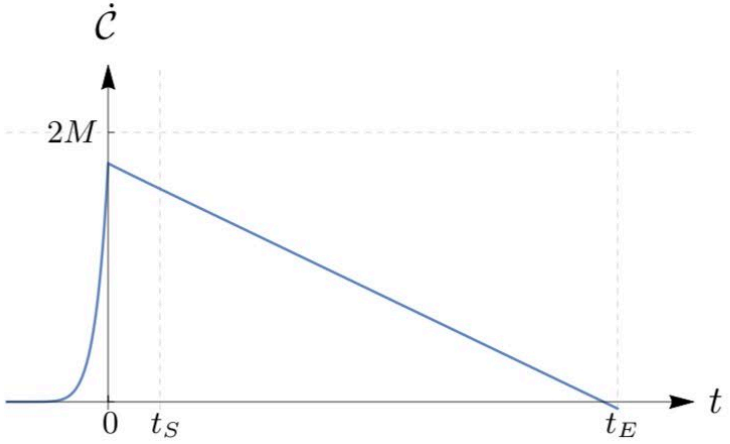


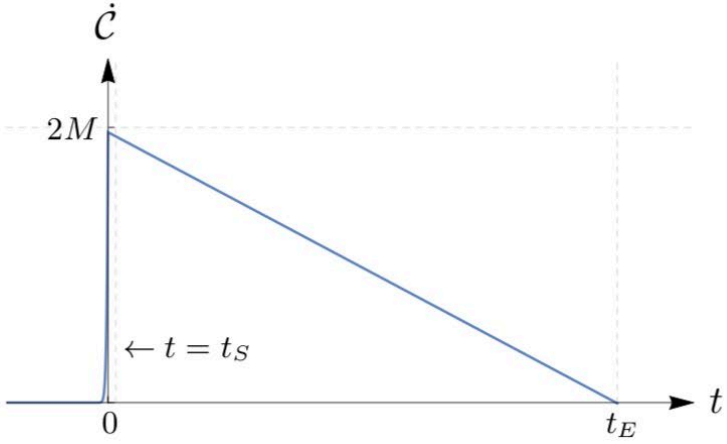




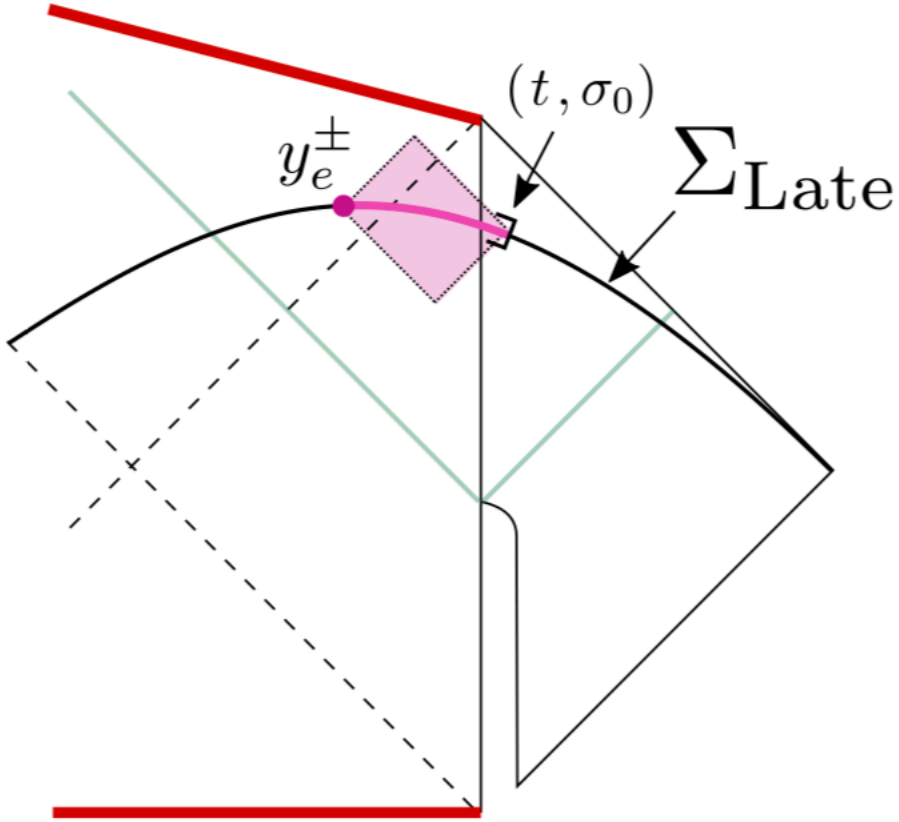
M/N~100

M/N~1000

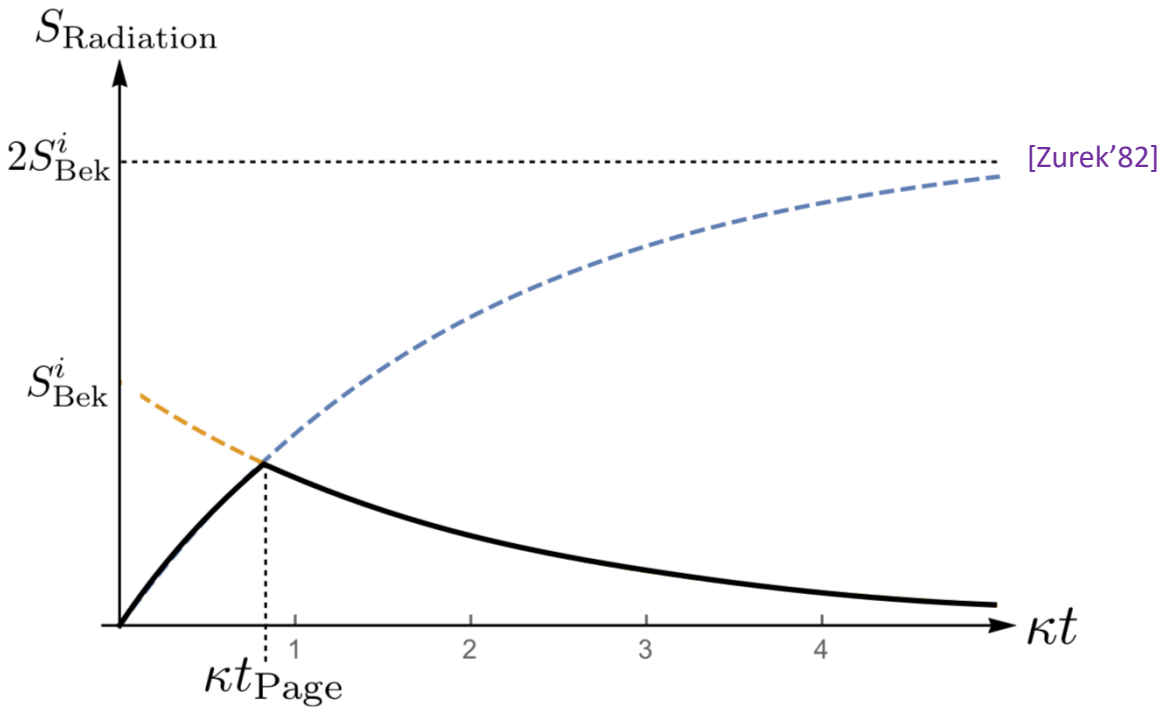




Part 2: Information Paradox and Page Curve



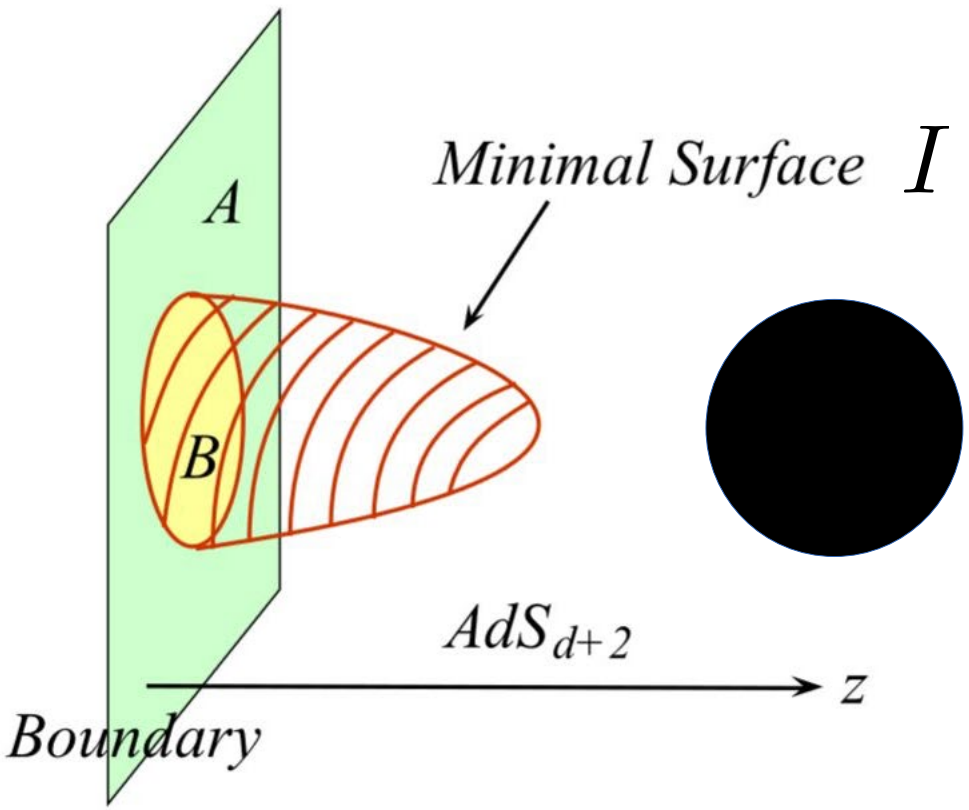
[Rocha'08]



[Zurek'82]

[Penington'19][Almheiri, Engelhardt, Marolf, Maxfield'19]
[Almheiri, Mahajan, Maldacena, Zhao'19]

Quantum Ryu-Takayanagi



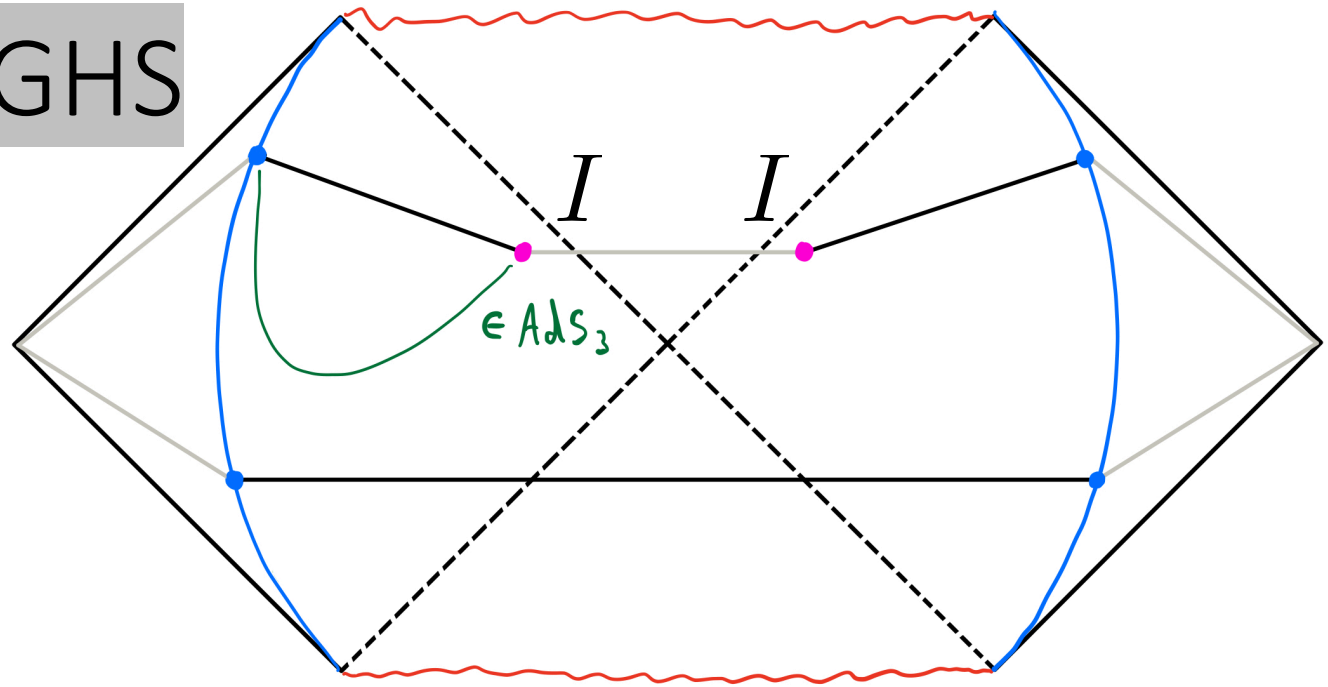
$$S_{\text{gen}} = \frac{\text{Area}(I)}{4G} + S_{\text{Bulk}}[\text{Volume}_{B \bigcup I}]$$

[Hubeny, Rangamani, Takayanagi '07]
[Faulkner, Lewkowycz, Maldacena '13]
[Engelhardt, Wall '14]

The holographic calculation of entanglement entropy via AdS/CFT.

[Ryu, Takayanagi '06]

Islands Outside Eternal CGHS



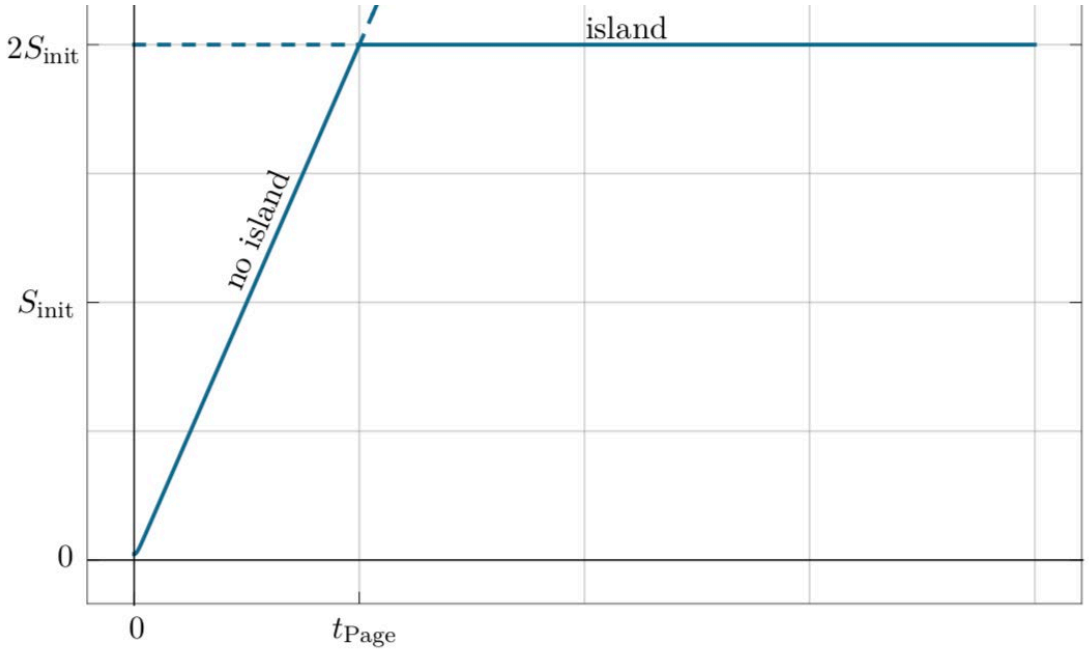
$$S_{\text{gen}} = \frac{\text{Area}(I)}{4G} + S_{\text{Bulk}}[\text{Volume}_{B \bigcup I}]$$

Matter passing through B U I. Go to flat coordinates:

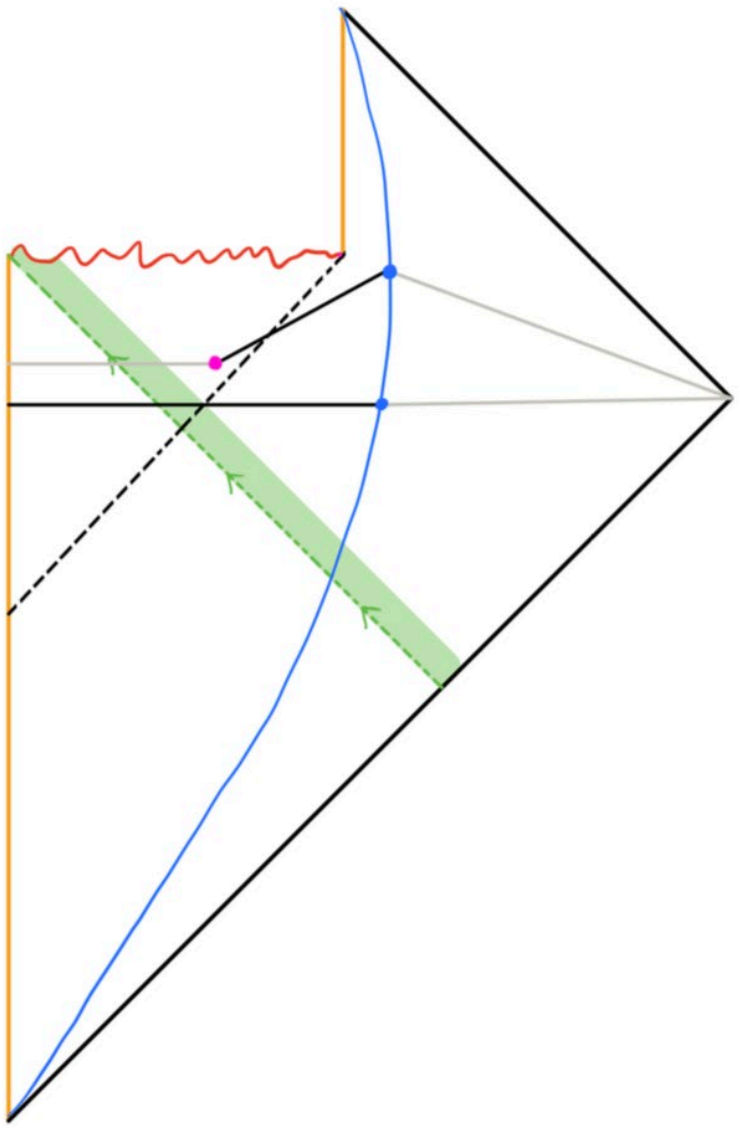
$$S_{\text{Bulk}}[\text{Volume}_{B \bigcup I}] \approx \frac{\text{Length}}{4G}$$

[Almheiri, Mahajan, Maldacena, Zhao'19]

$$S_{\text{gen}} = \min(S_{\text{island}}, S_{\text{no island}})$$



Island inside evaporating RST



$$S_{\text{gen}} = \frac{\text{Area}(I)}{4G} + S_{\text{Bulk}}[\text{Volume}_{B \bigcup I}]$$

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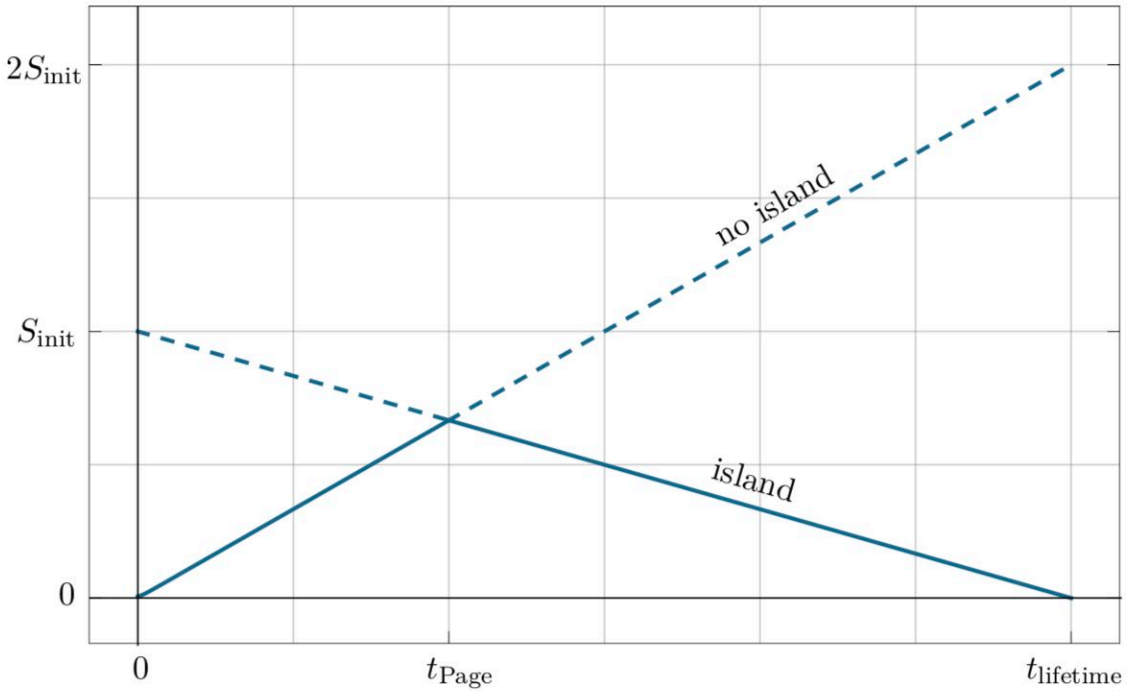


Summary

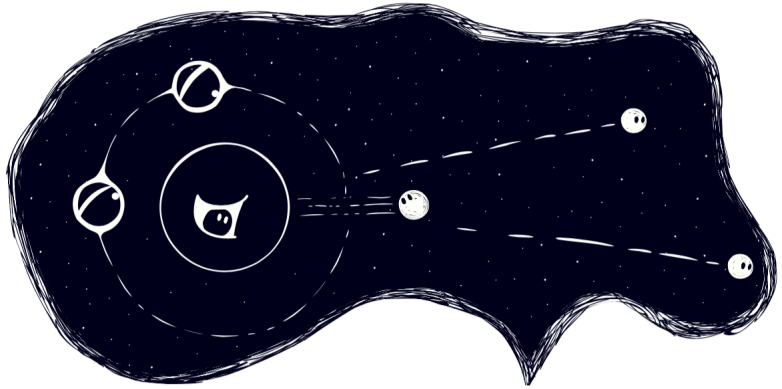
Complexity:

$$\frac{d\mathcal{C}}{dt} \sim S$$

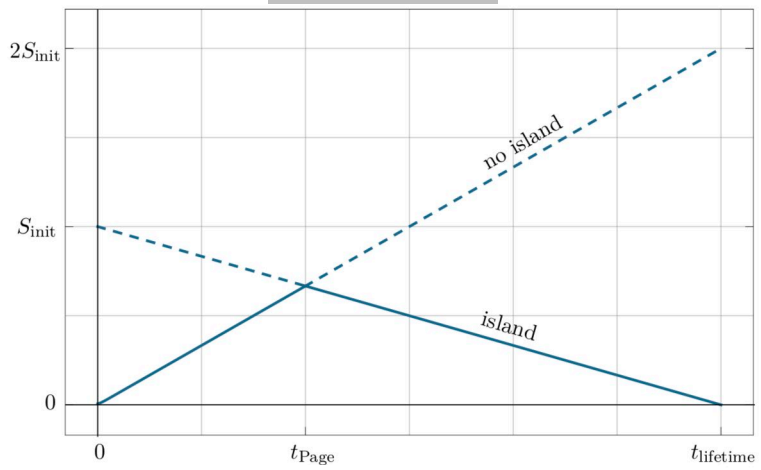
$$\dot{C} = 2M - \frac{N}{24}\left(t + \log\frac{4M}{\kappa} + \mathcal{O}\left(\frac{N}{M}e^{-t}\right)\right)$$

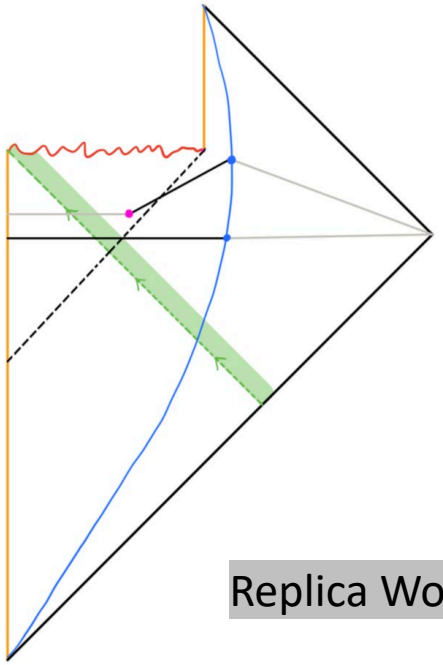
Out of equilibrium entropy

2d AdS setup, Schwarzschild in collapse



Page Curve:





Replica Wormholes

Thanks for your Attention!!