Momentum broadening in the Glasma

David I. Müller

Institute for Theoretical Physics
TU Wien, Austria

dmueller@hep.itp.tuwien.ac.at

March 25, 2020

IGFAE Theory Seminar

based on [arXiv:2001.10001]
in collaboration with Andreas Ipp and Daniel Schuh

Overview

- Introduction
  - Heavy ion collisions
  - Jets
  - CGC and the Glasma
- Momentum broadening
  - General idea
  - Abelian background fields
  - Non-Abelian background fields
- Results
  - Dilute Glasma: semi-analytic results
  - Dense Glasma: lattice results
- Outlook

Introduction

Relativistic heavy ion collisions

Heavy ion collision experiments as a means to study the properties of nuclear matter at extremely high energies

Examples:

- Au+Au at RHIC, BNL with $\sqrt{s_{\mathrm{NN}}}$ up tp $200\,\mathrm{GeV}$.
- Pb+Pb at LHC, CERN with $\sqrt{s_{\mathrm{NN}}}$ up tp $5\,\mathrm{TeV}$.

Experimental data:
Number of particles, E and $\mathbf{p}$ distributions, ...
Flow coefficients v_n, correlations, ...

should be explainable by theory!

Relativistic heavy ion collisions

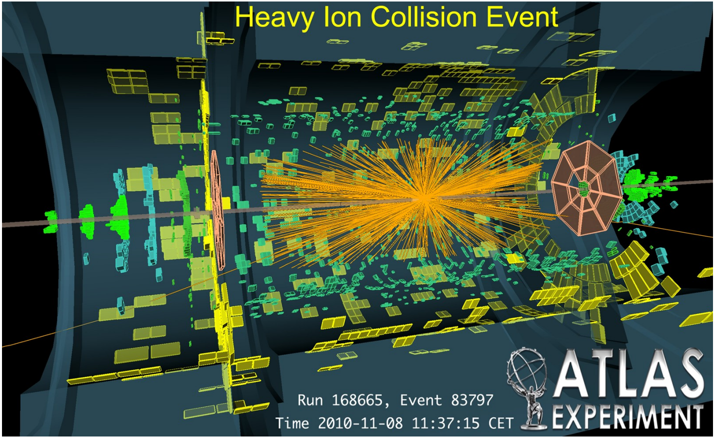


Image from ATLAS @ CERN (2010)

https://twiki.cern.ch/twiki/bin/view/AtlasPublic/EventDisplayHeavyIonCollisions

Jets

- Highly energetic, focused particle "sprays"
- Jets originate from hard scatterings of parton **during the collision**
- Seeds of jets are pairs of high energy partons
- Jets traversing a background/medium of soft partons
- Interactions with the medium:
  - momentum broadening
  - energy loss (quenching)

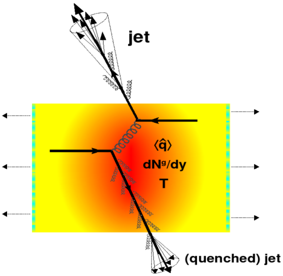


Fig. from [arXiv:0902.2011]

Jets

Momentum conservation: pairs of jets on opposite sides

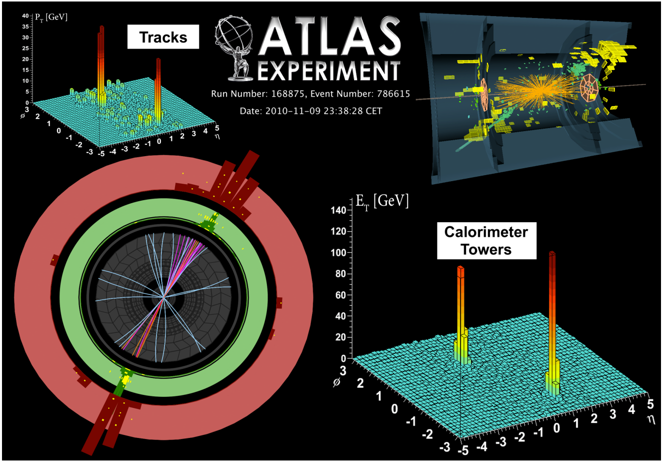


Image from ATLAS @ CERN (2010)

https://twiki.cern.ch/twiki/bin/view/AtlasPublic/EventDisplayHeavyIonCollisions

Jet quenching

Interactions with the medium: “quenching” of one of the jets

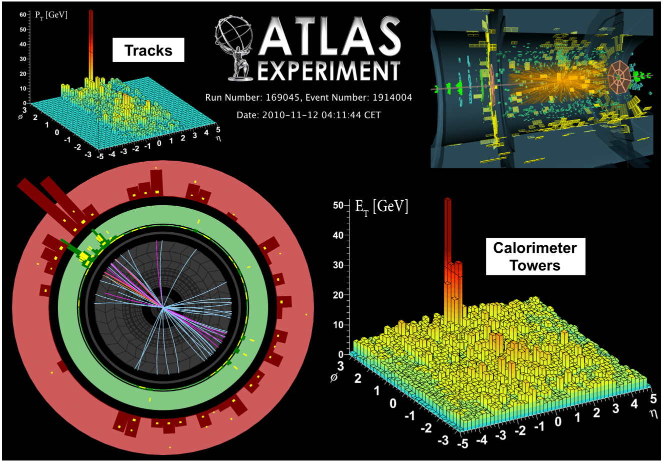


Image from ATLAS @ CERN (2010)

https://twiki.cern.ch/twiki/bin/view/AtlasPublic/EventDisplayHeavyIonCollisions

The medium: stages of heavy ion collisions

Theoretical description of the medium:
a series of effective theories and models ("stages")

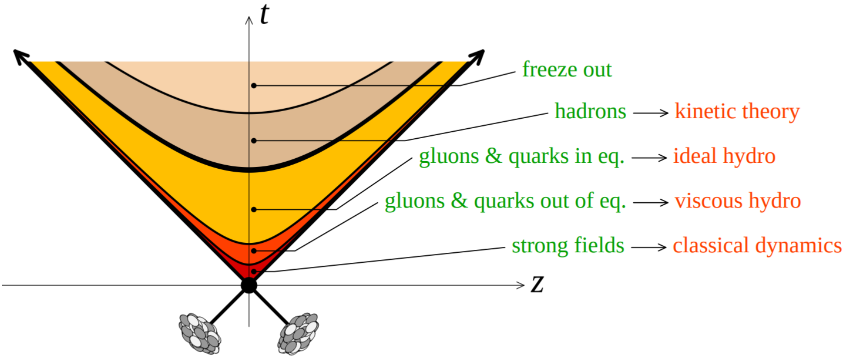


Figure from [arXiv:1110.1544]

Color Glass Condensate

CGC is an effective theory for high energy QCD

Nuclei are split into two types of degrees of freedom:

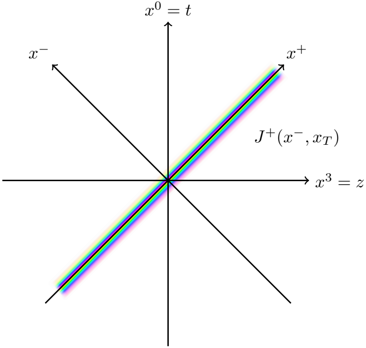
- quarks, high momentum gluons: classical color charges ρ
- low momentum gluons: classical color fields A_μ

Allows an effectively classical treatment of high energy nuclei and the earliest stages of the collision

- F. Gelis, "Color Glass Condensate and Glasma", Int. J. Mod. Phys. A 28, 1330001 (2013) [arXiv:1211.3327]

Color Glass Condensate

Nucleus "A" described by color current $J^+(x^-, x_T) = \rho_A(x^-, x_T)$ in terms of light cone coordinates $x^\pm = (x^0 \pm x^3)/\sqrt{2}$ and transverse coordinates $x_T = (x, y)$



- Yang-Mills equations

$$D_\mu F^{\mu\nu} = J^\nu$$

- Use covariant gauge $\partial_\mu A^\mu = 0$
- YM eqs. reduce to 2D Poisson eq.

$$-\Delta_T A^+(x^-, x_T) = \rho_A(x^-, x_T)$$

- Solve in Fourier space with infrared regulator m

$$A^+(x) = \int \frac{d^2 p_T}{(2\pi)^2} \frac{\widetilde{\rho_A}(x^-, p_T)}{p_T^2 + m^2} e^{-ip_T \cdot x_T}$$

Color Glass Condensate

Covariant gauge solution:

$$A^+(x) = \int \frac{d^2p_T}{(2\pi)^2} \frac{\widetilde{\rho_A}(x^-, p_T)}{p_T^2 + m^2} e^{-ip_T \cdot x_T}$$

All other components vanish ($A^- = A_i = 0$).

Perform gauge transformation to light cone gauge $A^+ = 0$

$$A_i(x) = \frac{1}{ig} V_A(x) \partial_i V_A^\dagger(x)$$

with light like Wilson line

$$V_A^\dagger(x) = \mathcal{P} \exp \left(-ig \int_{-\infty}^{x^-} dx'^- A^+(x'^-, x_T) \right)$$

Color Glass Condensate

McLerran-Venugopalan (MV) model:
Gaussian probability functional $W[\rho]$

$$W[\rho] = Z^{-1} \exp\left(- \int d^2 x_T \, dx^- \, \frac{\rho^a(x^-, x_T)\rho^a(x^-, x_T)}{2g^2\mu^2\lambda(x^-)} \right)$$

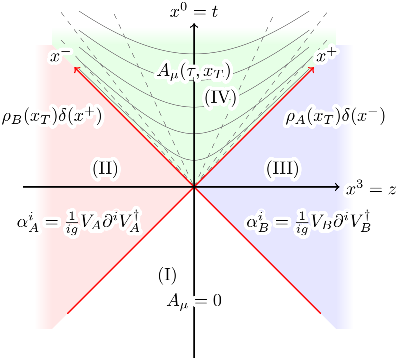
$$\langle \mathcal{O}(A_\mu) \rangle = \int \mathcal{D}\rho \, \mathcal{O}(A_\mu[\rho]) \, W[\rho]$$

$$\left\langle \rho^a(x^-, x_T)\rho^b(y^-, y_T) \right\rangle = g^2\mu^2\lambda(x^-)\delta^{ab}\delta(x^- - y^-)\delta^{(2)}(x_T - y_T)$$

No notion of finite radius $\Rightarrow$ suitable for central collisions of very large nuclei

Glasma

Collision of two CGCs: **Glasma**



- Yang-Mills (YM) equations

$$D_\mu F^{\mu\nu} = J^\nu_A + J^\nu_B$$

- Analytic solutions in (I) - (III)
- Future light cone spanned by

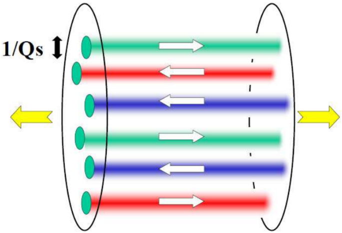
$$\tau = \sqrt{2x^+ x^-}, \quad \eta = \frac{1}{2} \ln\left(\frac{x^+}{x^-}\right)$$

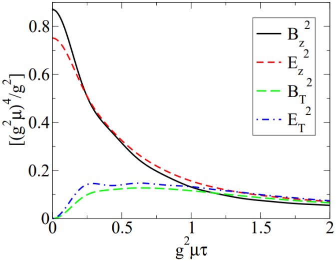
- Analytic solution at boundary of future light cone at $\tau = 0^+$ ("Glasma initial conditions")

$$A^i = \alpha^i_A + \alpha^i_B, \quad A^\eta = \frac{ig}{2}\left[\alpha^i_A, \alpha^i_B\right]$$

Glasma

Color-electric and -magnetic longitudinal flux tubes with typical transverse size of Q_s^{-1} (saturation momentum)





Figs. from [arXiv:0803.0410] and [arXiv:hep-ph/0602189]

Flux tubes expand and decay according to the YM eqs. until energy density components “equilibrate” (free streaming)

Jets as probes of earliest stages

- Jets affected by all stages of the medium
- Created **during the collision** $\tau \approx 0$
- Models/simulations of jet evolution usually start with the hydrodynamical QGP stage ($\tau \approx 0.6\,\mathrm{fm}/c$)
- Strong color fields of the Glasma might affect jets even before the hydrodynamical stage

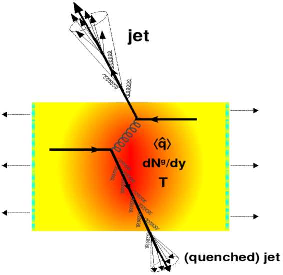
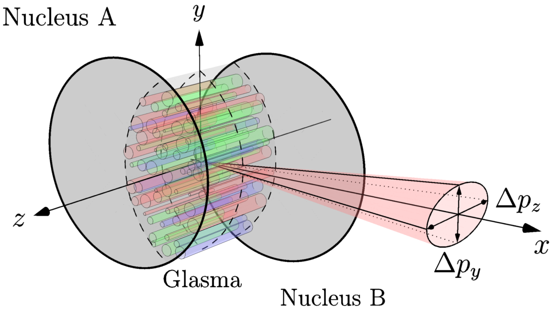


Fig. from [arXiv:0902.2011]

Jets as probes of earliest stages



Momentum broadening

General idea

Classical model of a jet moving through the medium:

- Highly energetic particle moving along x-axis

$$p_0^\mu = (|p_0|, p_0, 0, 0)^\mu$$

- Interaction with medium modeled as stochastic force $\mathcal{F}_\mu(x)$

$$\langle \mathcal{F}_\mu(x) \rangle = 0, \qquad \langle \mathcal{F}_\mu(x) \mathcal{F}_\nu(y) \rangle \neq 0$$

- Trajectory unaffected by medium, but particle accumulates *transverse* momentum

$$\langle p_\perp^2 \rangle \ll |p_0|^2$$

- Diffusion in momentum space $\Rightarrow$ momentum broadening
- Neglect backreaction

General idea

Classical model of a jet moving through the medium:
Solve classical equations of motion to obtain $p_\perp$

$$x^\mu(t) = v^\mu t + x_0^\mu, \qquad v^\mu = (1,1,0,0)^\mu$$

$$\frac{dp_\mu}{dt} = \mathcal{F}_\mu(x(t)) \quad \Leftrightarrow \quad p_\mu(t) = p_\mu(0) + \int_0^t dt' \mathcal{F}_\mu(x(t'))$$

No initial transverse momentum $p_\perp(0) = 0$

$$\langle p_i^2(t) \rangle = \int_0^t dt' \int_0^t dt'' \langle \mathcal{F}_i(x(t')) \mathcal{F}_i(x(t'')) \rangle \qquad \text{(no sum over i)}$$

Accumulated transverse momentum: $\langle p_\perp^2(t) \rangle = \langle p_y^2(t) \rangle + \langle p_z^2(t) \rangle$

General idea

Classical model of a jet moving through the medium:
Transverse momentum broadening

$$\left\langle p_\perp^2(t)\right\rangle = \left\langle p_y^2(t)\right\rangle + \left\langle p_z^2(t)\right\rangle$$

Longitudinal momentum broadening

$$\left\langle p_\|^2(t)\right\rangle = \left\langle p_x^2(t)\right\rangle = p_0^2 + \int_0^t dt' \int_0^t dt'' \left\langle \mathcal{F}_x(x(t'))\mathcal{F}_x(x(t''))\right\rangle$$

Energy loss?

$$\left\langle p_x(t)\right\rangle = p_0 + \int_0^t dt' \left\langle \mathcal{F}_x(x(t'))\right\rangle = p_0$$

Formalism only captures momentum broadening, not energy loss

General idea

Classical model of a jet moving through the medium:
Transverse momentum broadening

$$\left\langle p_\perp^2(t)\right\rangle = \left\langle p_y^2(t)\right\rangle + \left\langle p_z^2(t)\right\rangle$$

Longitudinal momentum broadening

$$\left\langle p_\parallel^2(t)\right\rangle = \left\langle p_x^2(t)\right\rangle$$

Jet broadening parameter: squared momentum per unit time/length

$$\hat{q}_\perp = \frac{d\left\langle p_\perp^2(t)\right\rangle}{dt}, \qquad \hat{q}_\parallel = \frac{d\left\langle p_\parallel^2(t)\right\rangle}{dt}$$

Abelian background field

Example: random Lorentz force acting on a charged particle

$$\mathcal{F}_\mu(x(t)) = q\frac{dx^\nu}{dt}F_{\mu\nu}(x(t)) \approx qv^\nu F_{\mu\nu}(x(t)) = \sqrt{2}qF_{\mu+}(x(t))$$

Introduce light cone coordinates w.r.t. particle velocity

$$x^\pm = (x^0 \pm x^1)/\sqrt{2}$$

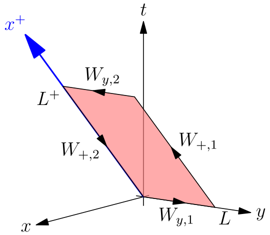
Accumulated transverse momentum

$$\left\langle p_i^2(t)\right\rangle = 2q^2\int_0^t dt'\int_0^t dt''\left\langle F_{i+}(x(t'))F_{i+}(x(t''))\right\rangle \qquad \text{(no sum over i)}$$

Correlation function of the background field determines transverse momentum broadening $\Rightarrow$ coherence time/length

Dipole approximation:
Momentum broadening related to expectation value of Wilson loop

$$W_{y+} = W_{y,1} W_{+,1} W_{y,2} W_{+,2}$$
$$= \exp\left(-iq \oint_{\mathcal{C}} dx^{\mu} A_{\mu}(x)\right)$$



$$\left\langle \mathrm{Re}\left[W_{y+}\right] \right\rangle \simeq \exp\left(-\frac{L^2 \left\langle p_y^2(t) \right\rangle}{2}\right), \qquad L \ll L^+$$

(analogous for broadening along z)

Abelian background field: dipole approximation

Use Stokes' theorem:

$$W_{y+} = \exp\left(-iq \oint_{\mathcal{C}} dx^{\mu} A_{\mu}(x)\right) = \exp\left(iq \int_0^{L^+} dx^+ \int_0^{L} dy\, F_{y+}(x)\right)$$

Expand for $L \ll L^+$ and re-parameterize integral over x^+ with time t:

$$\langle \mathrm{Re}\,[W_{y+}]\,\rangle \simeq 1 - q^2 L^2 \int_0^t dt' \int_0^t dt'' \langle F_{i+}(x(t'))F_{i+}(x(t''))\rangle + \mathcal{O}(L^4)$$

Compare to:

$$\langle \mathrm{Re}\,[W_{y+}]\,\rangle = \exp\left(-\frac{L^2\langle p_y^2(t)\rangle}{2}\right) \simeq 1 - \frac{L^2\langle p_y^2(t)\rangle}{2} + \mathcal{O}(L^4)$$

coherence length in medium $\sim$ momentum broadening

Non-Abelian background field

Generalization to non-Abelian fields: **Wong equations**

$$\frac{dp^\mu(t)}{dt} = g\, Q^a(t) \frac{dx^\nu(t)}{dt} F^{a,\mu\nu}(x(t))$$
$$\frac{dQ^a(t)}{dt} = g \frac{dx^\mu(t)}{dt} f^{abc} A^b_\mu(x(t)) Q^c(t)$$

Time-dependent color charge of the particle $Q^a(t)$

$$Q^a(t) Q^a(t) = \text{const.}$$

General solution of the second Wong equation

$$Q(t) = U(t) Q(0) U^\dagger(t), \qquad U^\dagger(t) = \mathcal{P} \exp\Bigg(-ig \int_{x(0)}^{x(t)} dx^\mu A_\mu(x) \Bigg)$$

Non-Abelian background field

- Color charge of a parton (quark, gluon) is random color vector
- Modulus of Q depends on *representation* R ($F \triangleq$ quark, $A \triangleq$ gluon)

$$\left\langle p_i^2(t) \right\rangle_R = \frac{2g^2}{D_R} \int_0^t dt' \int_0^t dt'' \left\langle \mathrm{Tr} \left[\widetilde{F_{i+}}(x(t')) \widetilde{F_{i+}}(x(t'')) \right]_R \right\rangle$$

$$D_F = N_c, \qquad D_A = N_c^2 - 1$$

- Parallel transported field strength tensor

$$\widetilde{F_{i+}}(x(t)) = U(t)\, F_{i+}(x(t))\, U^\dagger(t)$$

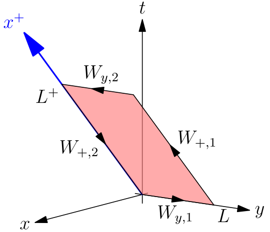
$$U^\dagger(t) = \mathcal{P} \exp\Big(- ig \int_{x^+(0)}^{x^+(t)} dx'^+ A_+(x'^+) \Big)$$

- Manifestly gauge invariant result

Non-Abelian background field: dipole approximation

Dipole approximation: non-Abelian Wilson loop

$$\frac{1}{D_R}\left\langle \mathrm{Re}\mathrm{Tr}\left[W_{i+}\right]_R \right\rangle \simeq \exp\left(-\frac{L^2}{2}\left\langle p_i^2(t)\right\rangle_R\right)$$



Expansion $L \ll L^+$ yields same result as integrating Wong's equations

Representation R determines type of particle (quark, gluon)

Results for the Glasma

Momentum broadening in the Glasma

Glasma: classical model fits naturally, $\langle \mathcal{F}_\mu \rangle = 0$

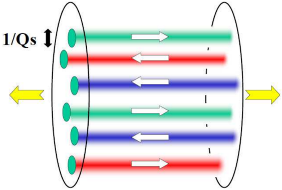
Problems:

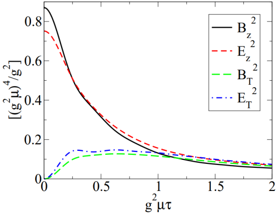
- no analytic solutions A_μ for YM eqs.
- $\langle p_\perp^2(t) \rangle$ is non-linear in A_μ
- medium average $\langle \dots \rangle$ often intractable to compute

Approximations:

- **Weak field approximation**:
  - "$g\rho \ll 1$", dilute Glasma, linearized YM eqs.
  - semi-analytical results (interpretability)
  - analytical medium average $\langle \dots \rangle$
  - not applicable to heavy ion collisions
- **Lattice gauge theory**:
  - "$g\rho \sim 1$", dense Glasma, lattice YM
  - purely numerical results, crosscheck with dilute case
  - medium average $\langle \dots \rangle$ using MC integration
  - applicable to heavy ion collisions

Momentum broadening in the Glasma



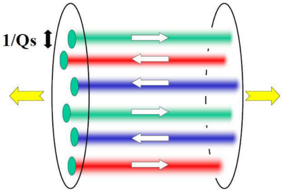


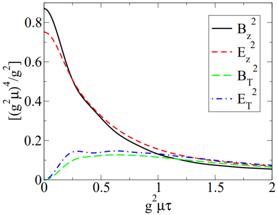
Figs. from [arXiv:0803.0410] and [arXiv:hep-ph/0602189]

At $\tau = 0$:

- electric and magnetic longitudinal flux tubes
- roughly similar energy density
- Lorentz force for a particle moving along x:
  - longitudinal electric E_L: broadening in p_z
  - longitudinal magnetic B_L: broadening in p_y

Momentum broadening in the Glasma





Figs. from [arXiv:0803.0410] and [arXiv:hep-ph/0602189]

For $\tau > 0$:

- flux tubes expand and decay, $\varepsilon \propto 1/\tau$
- E_L generates B_T: broadening in p_z
- B_L generates E_T: broadening in p_y
- expect roughly isotropic broadening
- strong time dependence of $\langle p_\perp^2(t) \rangle$

Dilute Glasma: weak field limit

Weak field approximation:
assume color charges $\rho_{A/B}$ of colliding nuclei are small

- allows expansion of Glasma initial conditions
- linearize Yang-Mills equations

$$D_\mu F^{\mu\nu} = 0, \quad \Rightarrow \quad \partial_\mu F^{\mu\nu} = 0$$

- effectively Abelian field equations
- medium average (MV model) can be performed analytically

Dilute Glasma: momentum broadening

Result: accumulated transverse momenta

$$\left\langle p_y^2(t)\right\rangle = \int_0^\infty dk\, g(\tau,k)c_B(k), \quad \left\langle p_z^2(t)\right\rangle = \int_0^\infty dk\, g(\tau,k)c_E(k),$$

where $g(\tau, k)$ is function that describes the (linearized) time evolution of the Glasma; no closed form but can be expressed as integral over Bessel functions

Initial correlators at $\tau = 0$

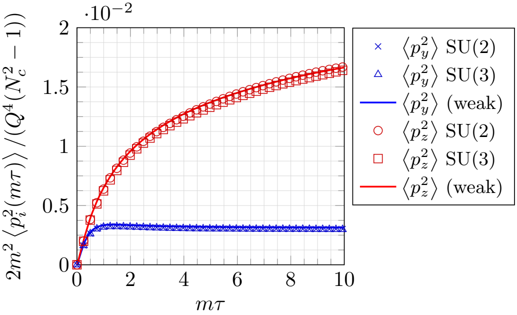
$$c_E(r) = \left\langle \mathrm{Tr}\left[E_z(x_T)E_z(y_T)\right]\right\rangle, \quad c_B(r) = \left\langle \mathrm{Tr}\left[B_z(x_T)B_z(y_T)\right]\right\rangle,$$

which can be computed analytically

Two separate contributions (polarization states)

- p_y determined by (B_z, E_T) polarization
- p_z determined by (E_z, B_T) polarization

Dilute Glasma: momentum broadening

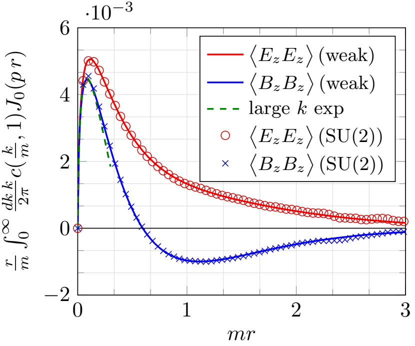


- Very anisotropic momentum broadening
- More efficient broadening along z compared to y. Why?
- From analytic result: differences in the initial correlators!

$$c_E(r) = \left\langle \mathrm{Tr}\left[E_z(x_T)E_z(y_T)\right] \right\rangle \neq c_B(r) = \left\langle \mathrm{Tr}\left[B_z(x_T)B_z(y_T)\right] \right\rangle,$$

Dilute Glasma: initial flux tubes and broadening

Plots of initial correlators

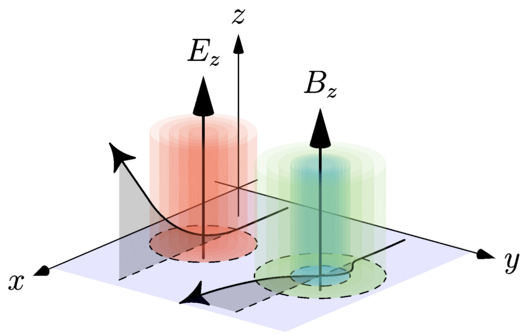


- Color-electric correlator: positive correlation everywhere
- Color-magnetic correlator: negative correlations around $r \approx m^{-1}$

How does this affect the jet?

Dilute Glasma: initial flux tubes and broadening

How structure of flux tubes affects jets:



Electric flux tubes are more efficient at accelerating particles compared to magnetic flux tubes $\Rightarrow$ momentum broadening anisotropy

Momentum broadening probes small-scale structure of color flux tubes!

Dilute Glasma: summary

Quick summary:

- Momentum broadening is directly related to flux tube structure
- Two separate, independent polarization states (linearized YM)
- Differences in correlators lead to anisotropy in broadening
- Weak field limit / dilute Glasma is not realistic
- Strong infrared dependence (artificial scale)

Dense Glasma: lattice gauge theory

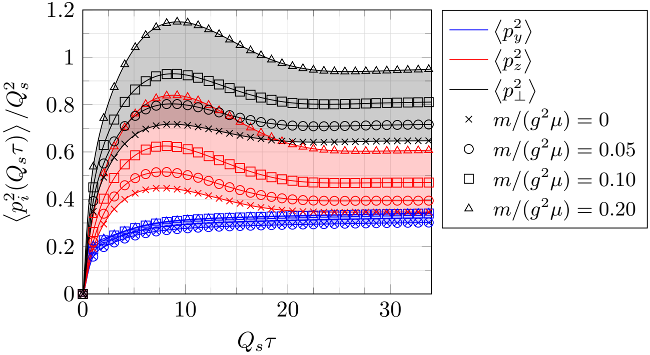
In order to investigate a realistic Glasma, we need numerical simulations.

- Use standard real-time lattice gauge theory methods for boost-invariant Glasma
- Discretize formulae derived earlier (for details see our paper)
- Allows us to investigate large color charge densities ρ, i.e. the relevant physics for heavy ion collisions

  Publicly available code (Python/CUDA):
  curraun (https://gitlab.com/openpixi/curraun)

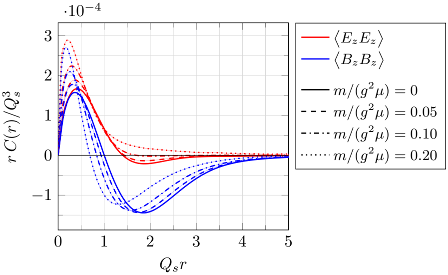
  Repository includes code, working examples, plots, ...

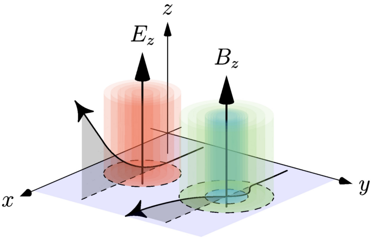
Dense Glasma: momentum broadening



- Momentum broadening anisotropy also in the dense Glasma
- Infrared dependence of $\langle p_z^2 \rangle$
- Strong initial accumulation up until $\tau \approx Q_s^{-1}$
- Details: 1024^2 lattice, $N_s = 50$ sheets, 50 random configurations

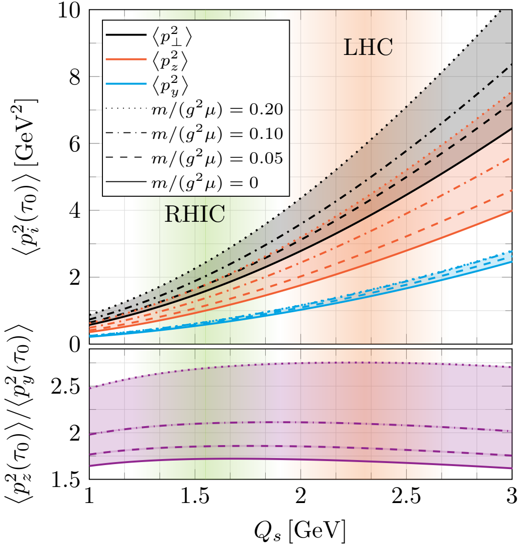
Dense Glasma: initial correlations



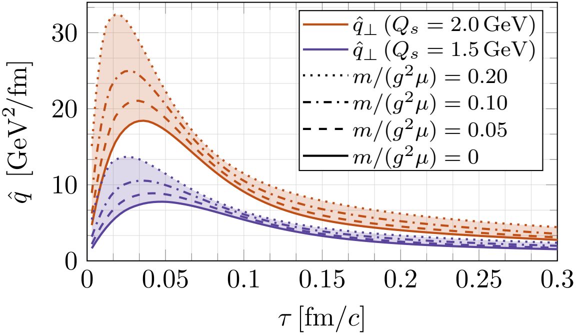


- Connection between $\langle p_\perp^2 \rangle$ not as clear as in dilute Glasma
- Magnetic fields: anti-correlated regions around $r \approx 2Q_s^{-1}$
- Electric fields: small anti-correlations too for very strong fields
- Similar mechanism as in dilute Glasma

Dense Glasma: quantitative results at $\tau = 0.6\,\mathrm{fm}/c$



Dense Glasma: jet broadening parameter



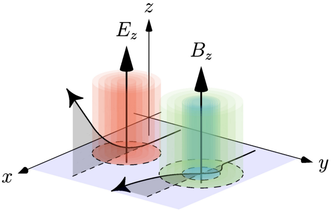
- Jet broadening parameter

$$\hat{q}_\perp = \frac{d\langle p_\perp^2(t)\rangle}{dt}$$

- Strong time dependence: most $p_\perp$ accumulated within $0.1\,\mathrm{fm}/c$

Dense Glasma: summary

- Gauge-invariant method for jet broadening in dense Glasma
- Applied to boost-invariant MV model (central collisions, large nuclei)
- Transverse momentum: $\langle p_\perp^2 \rangle \approx Q_s^2$ within $\tau = 0.6\,\mathrm{fm}/c$
- Strong time dependence, largest broadening within $\tau < Q_s^{-1}$
- Anisotropic broadening: color-electric and color-magnetic flux tubes have different structure (anti-correlated regions)



- Anisotropy: $\langle p_z^2 \rangle / \langle p_y^2 \rangle \approx 1.5 \sim 2.5$

Outlook

Outlook

This work:

- Fixed, light like trajectory (infinite momentum limit)
- No energy loss
- 2+1D MV model: no centrality dependence, rapidity dependence

Possible extensions, open questions:

- More complicated initial conditions (IP-Glasma, JIMWLK, ...)
- Energy loss
  - Change in momentum: gluon radiation (bremsstrahlung)
  - Polarization of Glasma: backreaction
  - Deviations from fixed trajectory: non-eikonal corrections
- How to integrate these methods with existing jet simulations?
- Is the momentum broadening anisotropy observable?
  (also seen in anisotropic QGP)

Thank you!