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Relativistic Fluid Dynamics:

From Particle Colliders to Neutron Star Mergers

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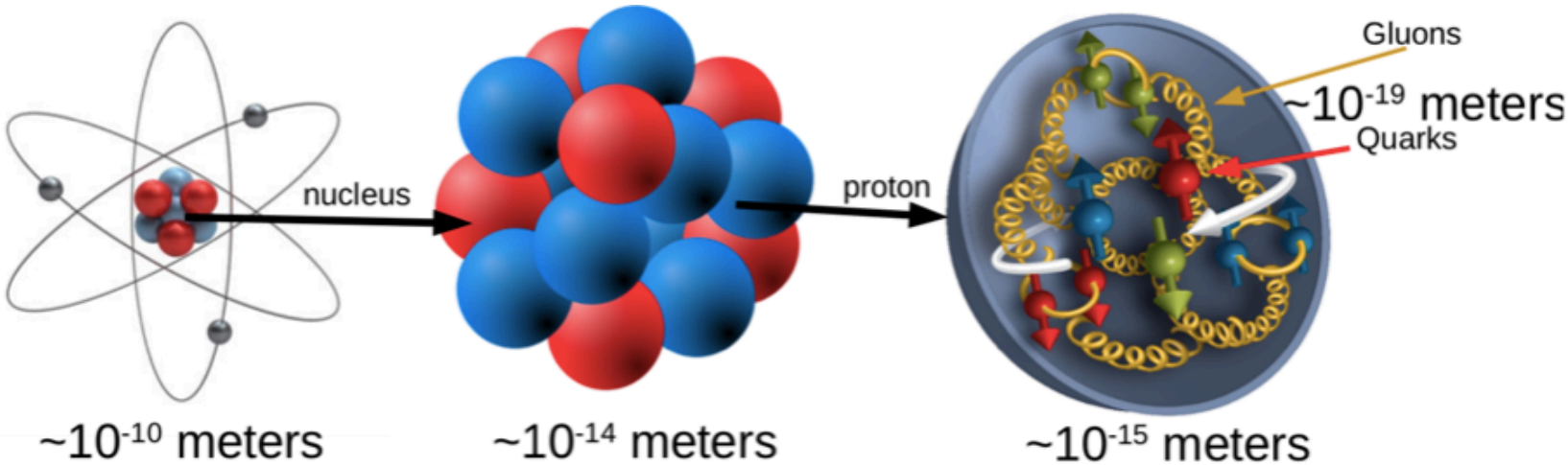
Virtual Seminar, Santiago de Compostela, Spain, March 2020

Fluid behavior under extreme conditions

The Frontiers of Fluid Dynamics

"Macro" scales → nuclear/particle physics



Quark-Gluon Plasma formed in ultrarelativistic hadron colliders

The Frontiers of Fluid Dynamics

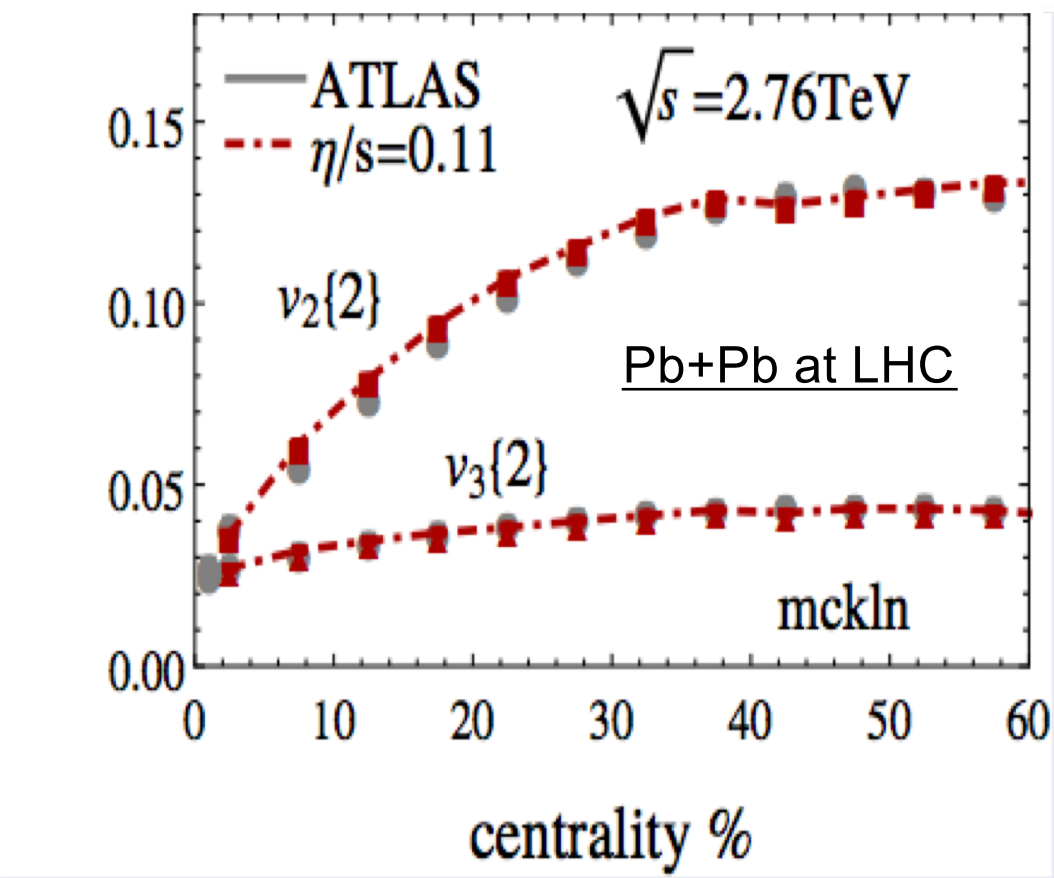
Viscous Fluid Dynamics in Strong Fields

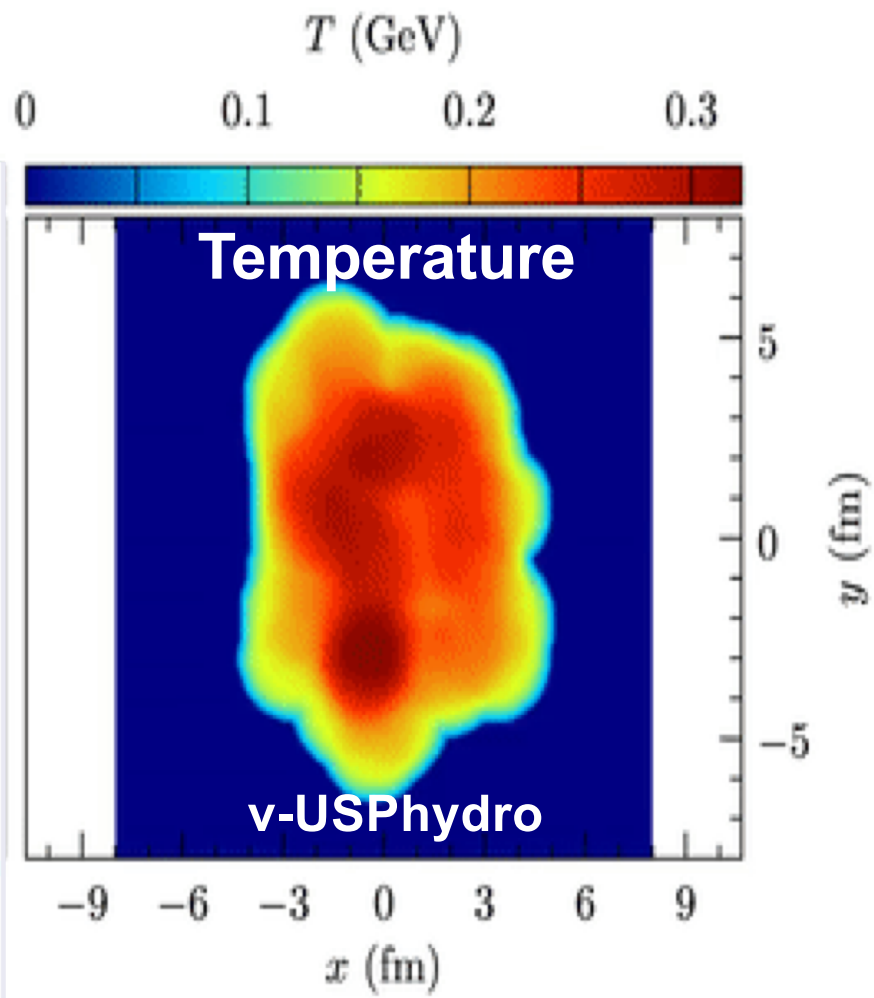
Neutron Star Mergers

(Nearly) Perfect Fluidity of the QGP in Colliders

QGP as a viscous relativistic fluid

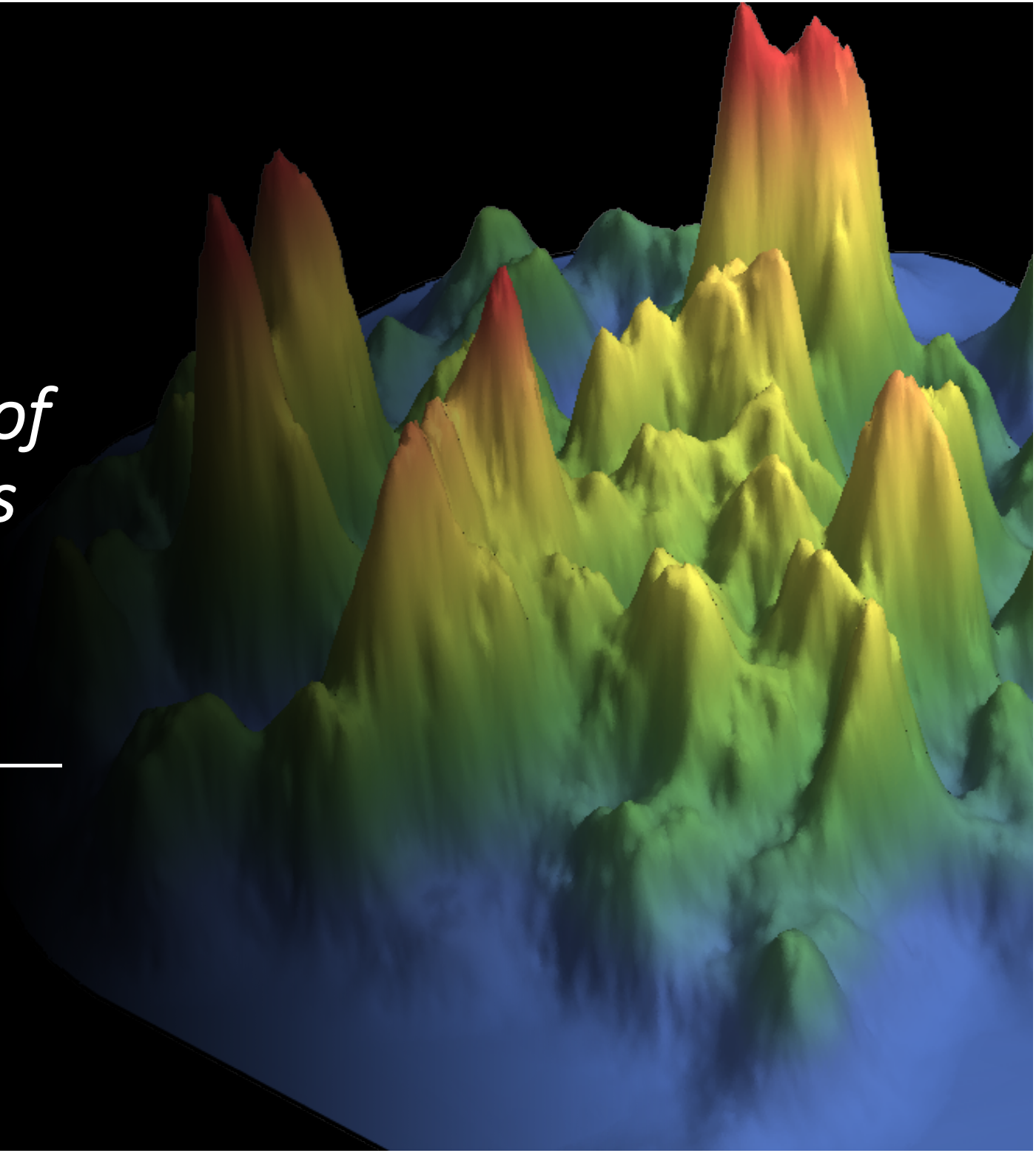
Anisotropic Flow





J. Noronha-Hostler, B. Betz, JN, M. Gyulassy, PRL 2016

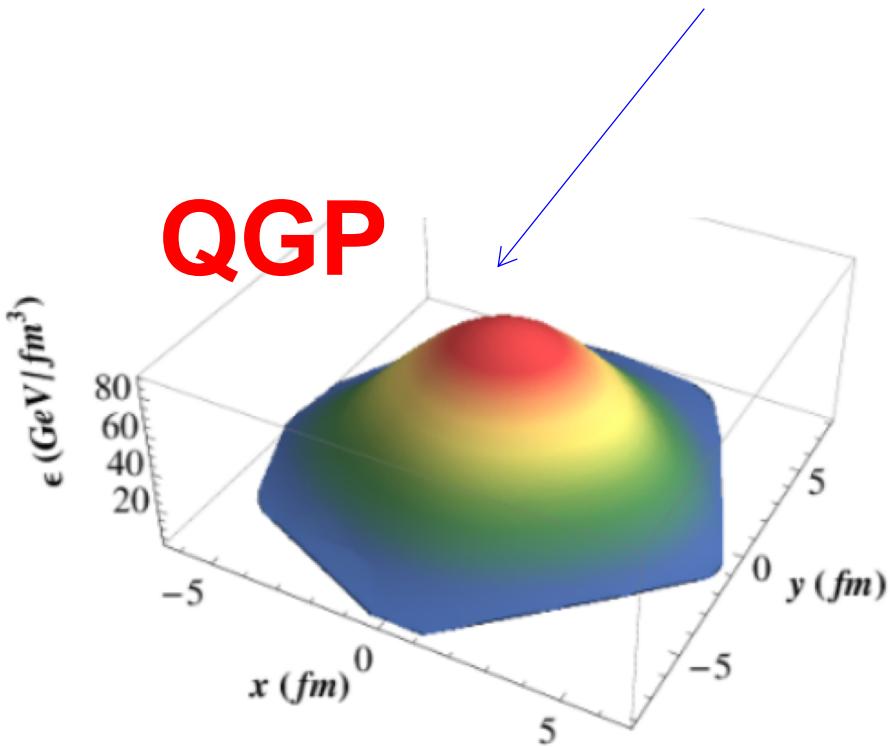
The unreasonable effectiveness of fluid dynamics in heavy-ion collisions



Quark-Gluon Plasma and Relativistic Fluid Dynamics

At first (< 2010), it seemed that hydrodynamics was justifiable

Very smooth fluid over nuclear length scales



near equilibrium dynamics

macro $\partial\varepsilon/\varepsilon_0 \sim 1/L$

micro $\ell \sim 1/T \sim 1/\Lambda_{QCD}$

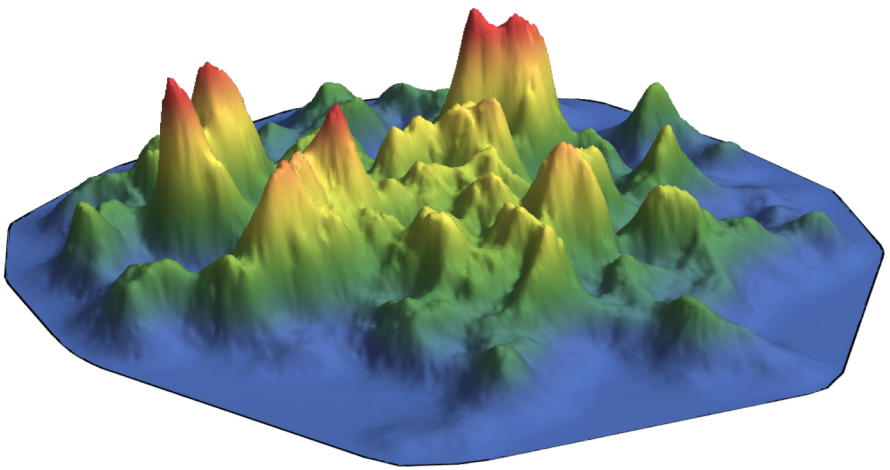
Knudsen number

$$K_N \sim \ell\,\partial\varepsilon < 0.1$$

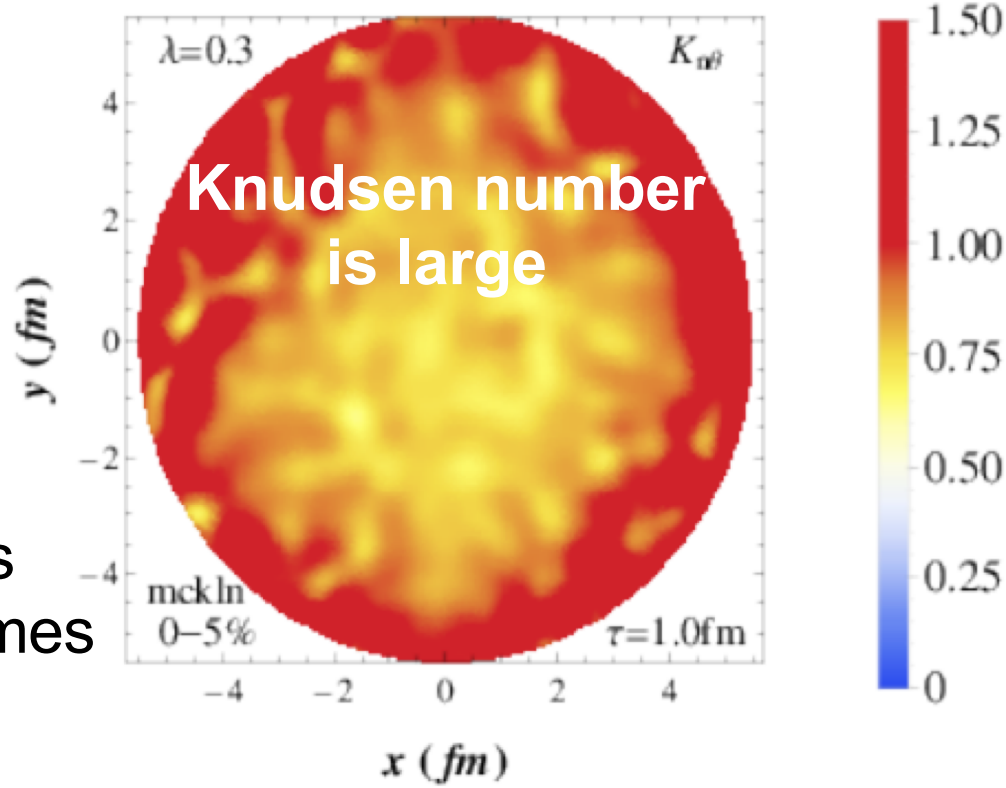
Fluid dynamics at scales of the size of a large nucleus

Reality is much more complicated …

QGP energy density



J. Noronha-Hostler, JN, M. Gyulassy, PRC 2016



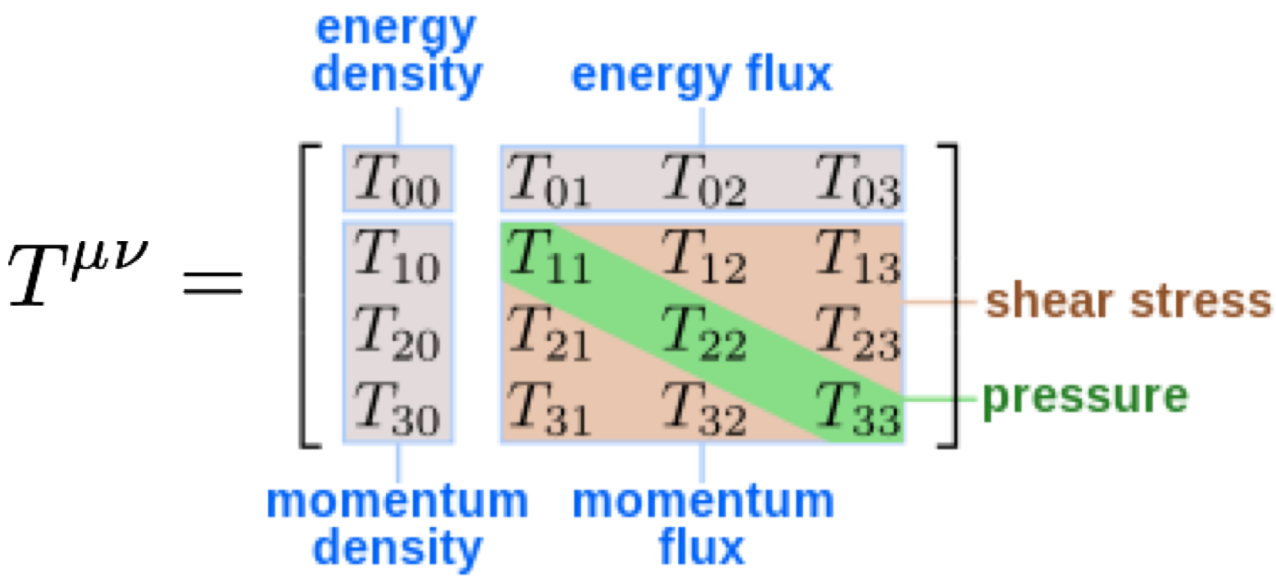
- Unavoidable quantum fluctuations
- Large spatial gradients at early times

PARADOX: Knudsen number is large but “hydro” still works

This issue must be understood …

How can fluid dynamics emerge even far from equilibrium?

Relativistic Fluid Dynamics



Conservation Laws
(energy and momentum)

$$\nabla_\mu T^{\mu\nu} = 0$$

Energy-momentum tensor

$$T^{\mu\nu} = \underbrace{(\varepsilon + P(\varepsilon))u^\mu u^\nu + P(\varepsilon)g^{\mu\nu}}_{\text{ideal part}} + \pi^{\mu\nu}$$

dissipative part

What is the dissipative part?

Standard view (past 100 years): **Knudsen/gradient expansion**

Hydrodynamics → Effective theory for $\{T, u_\mu\}$ near local equilibrium

Dissipation included via a Knudsen/gradient expansion

$$T^{\mu\nu} = \underbrace{T^{\mu\nu}_{ideal}}_{\text{Zero viscosity}} + \underbrace{\mathcal{O}(\partial^\mu u^\nu)}_{\text{1st order in Kn Relativistic Navier-Stokes}} + \underbrace{\mathcal{O}(\partial^2 u, \partial^2 T)}_{\text{2nd order in Kn^2}} + \ldots$$

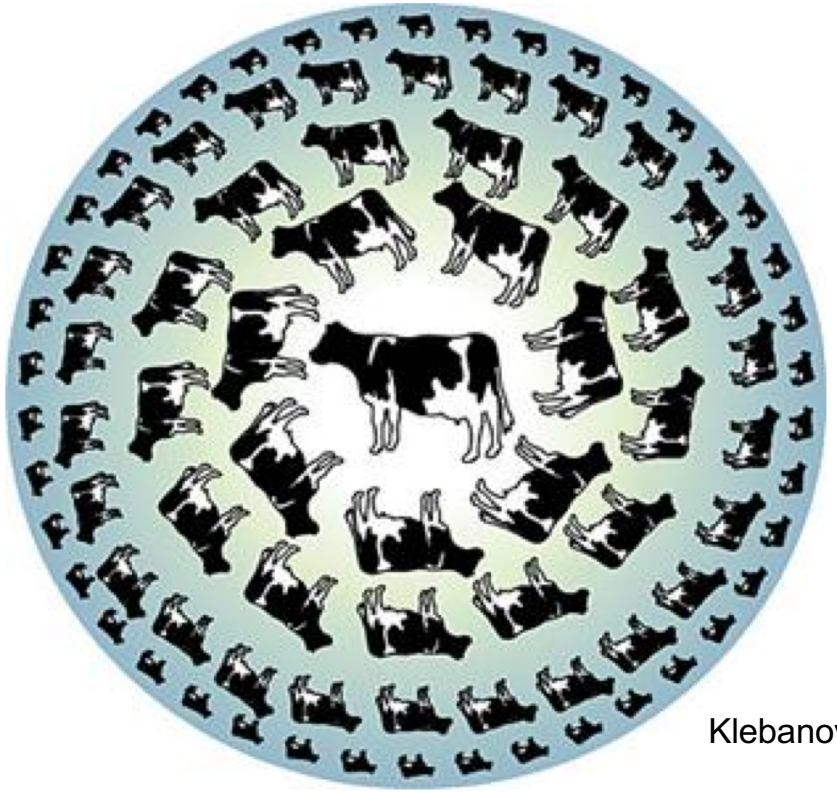
- Does this power series in Kn converge ? What if it doesn't?
- How does one approach the Kn ~ 1 limit ?

Does the Knudsen (gradient) series diverge?

Seminal work by Heller, Janik, Witaszczyk, PRL (2013)

Does the Knudsen (gradient) series diverge?

Holographic Duality



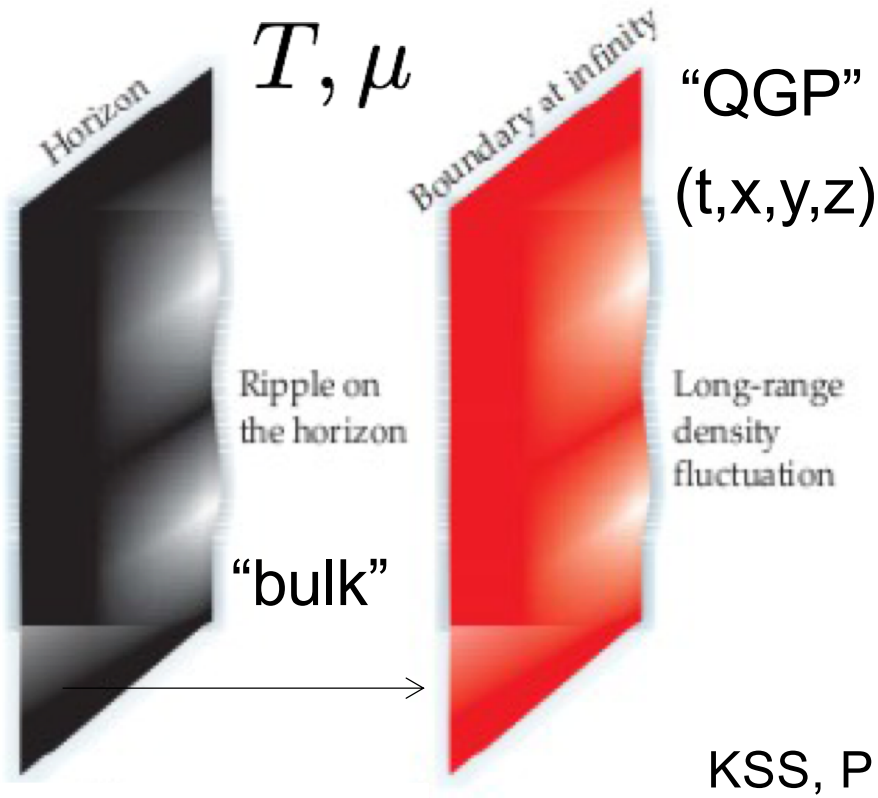
Klebanov, Maldacena, Physics Today (2009)

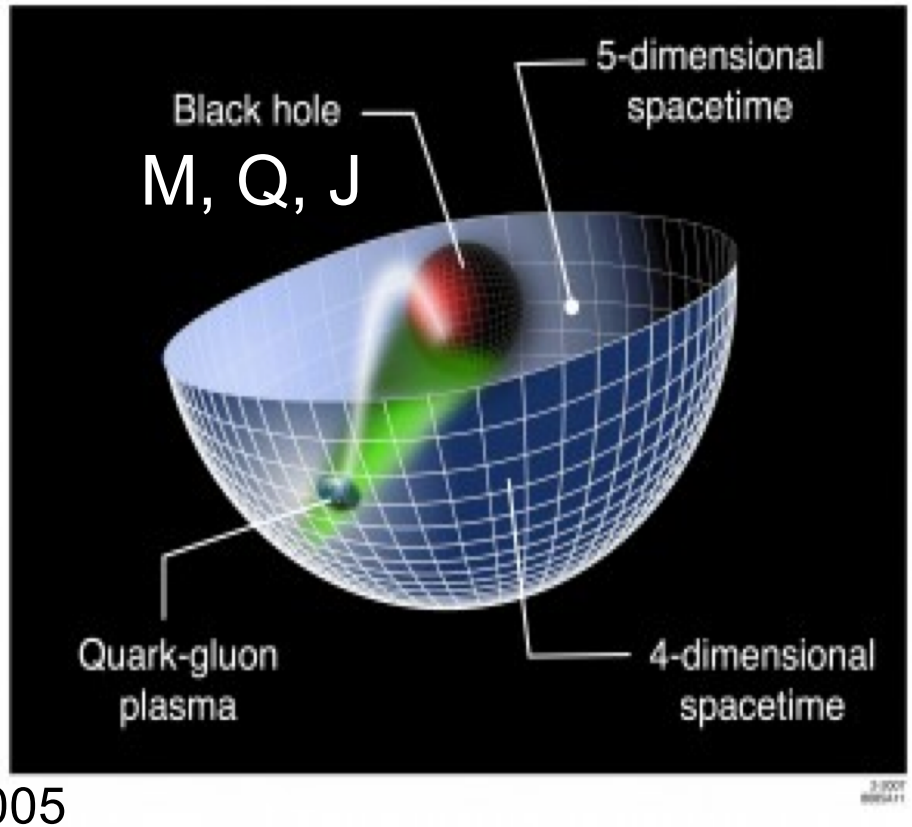
Model for the strongly coupled QGP

Holographic correspondence (gauge/gravity duality)

Maldacena 1997; Witten 1998; Gubser, Polyakov, Klebanov 1998

Strongly coupled gauge theory ⟷ String Theory/Classical Gravity





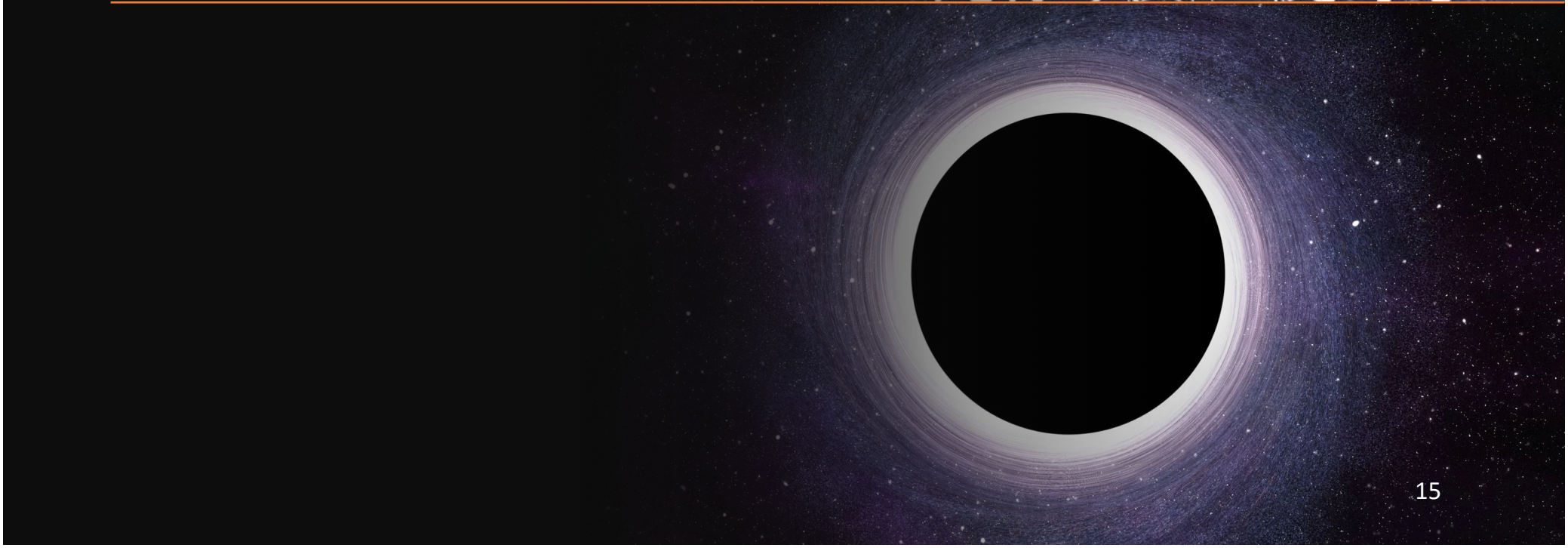
KSS, PRL 2005

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Fluid dynamics from black hole physics!

ballpark of QGP!

Black Hole "Engineering"

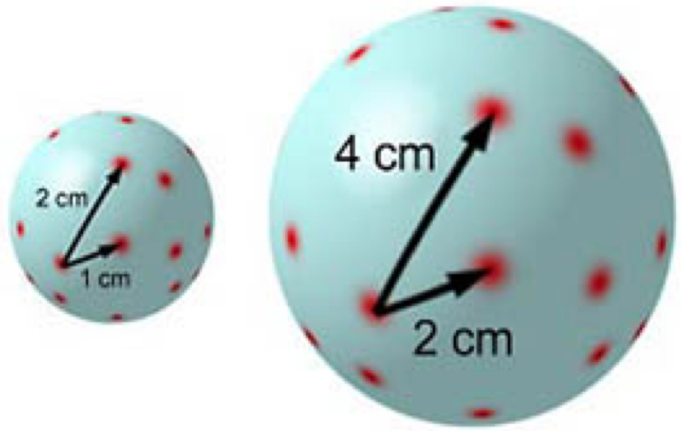


Example: QCD phase transition in the early universe

Buchel, Heller, JN, arXiv:1603.05344, PRD (2016)

Embed the strongly coupled plasma in an expanding universe
(N=2* gauge theory)

Friedmann-Robertson-Walker
(FRW) universe



- **Isotropy and homogeneity**
- **Flow velocity is known**

Knudsen number

$$K_N \sim H \sim \nabla_\mu u^\mu$$

Holography determines the entropy production

Example: QCD phase transition in the early universe

Buchel, Heller, JN, arXiv:1603.05344, PRD (2016)

Entropy production from holography – Expanding Universe

$$\frac{d\mathcal{S}}{dt} \sim \left[\sum_{n=0}^{\infty} c_n (K_N)^n\right]^2$$

$$c_n \sim n!$$

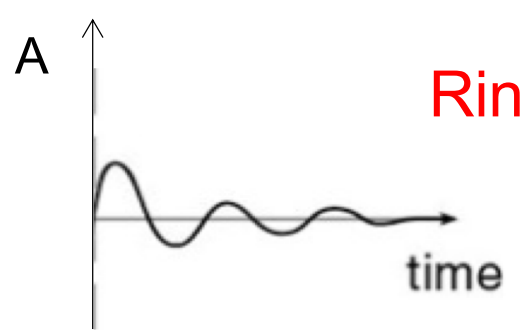
Series diverges!

After resummation of the divergent series → $\sim e^{-\omega_{QNM}/K_N}$

Non-perturbative!!

Dual to black hole

Quasinormal mode



Ringdown

"Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever"

Example: Harmonic series

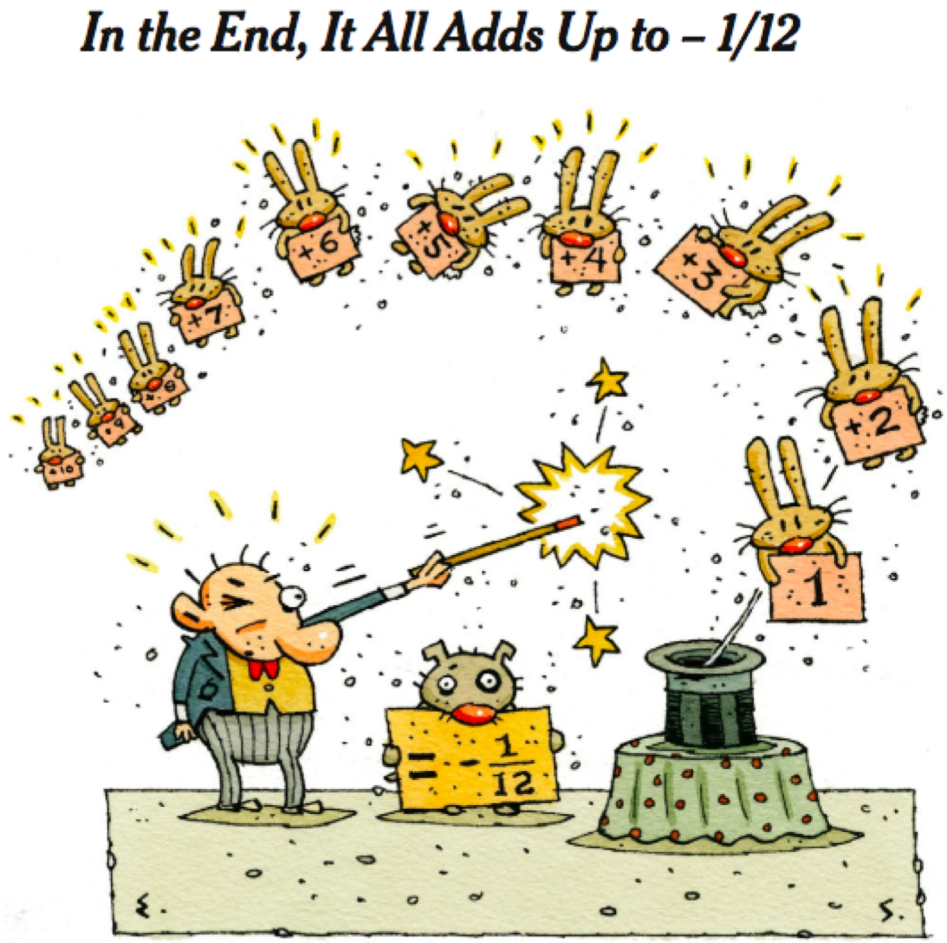
$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n}.$$

N. H. Abel (1802-1829)

How does one resum a divergent series?

How does one resum a divergent series?

MAGIC ?



New York Times
(2014)

$$1 + 2 + 3 + \ldots = -1/12$$

Trans-series and Resurgence Theory

Ecalle (1980), Constin, Dunne, Unsal, Schiappa, Heller and etc

Example: Consider the following asymptotic expansion

$$f(x)\Big|_{x\ll 1} \sim \underbrace{\sum_{n=0}^{\infty} f_n x^n}_{\text{perturbative}} + \underbrace{a e^{-1/x}(1 + c_1 x + c_2 x^2 + \ldots)}_{\text{non-perturbative}}$$

Hydrodynamics expansion should also have this generalized form

$$\pi^{\mu\nu} \sim \underbrace{\sum_{n=0}^{\infty} \pi_n^{\mu\nu}(K_N)^n}_{\text{perturbative}} + \underbrace{a^{\mu\nu} e^{-1/K_N}(1 + c_1 K_N + \ldots)}_{\text{non-perturbative}}$$

Israel-Stewart Theory

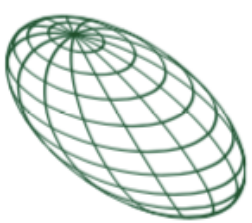
Israel, Stewart, Ann. Phys. 118, 341 (1979)

This already involves a resummation of gradients

$T_{\mu\nu}$ ➡ set of variables are $\varepsilon, u_\mu, \boxed{\pi_{\mu\nu}, \Pi}$

Dynamics is given by the <u>conservation laws</u> and

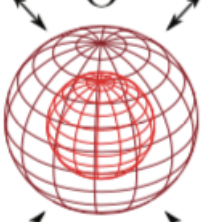
$\sigma_{\mu\nu}$



shear

$$u^\lambda \nabla_\lambda \pi^{\mu\nu} + F^{\mu\nu}(\varepsilon, \nabla_\alpha u_\beta, \pi^{\alpha\beta}, \Pi) = 0$$ (shear η)

Θ



expansion

$$u^\lambda \nabla_\lambda \Pi + F(\varepsilon, \nabla_\alpha u_\beta, \pi^{\alpha\beta}, \Pi) = 0$$ (bulk ζ)

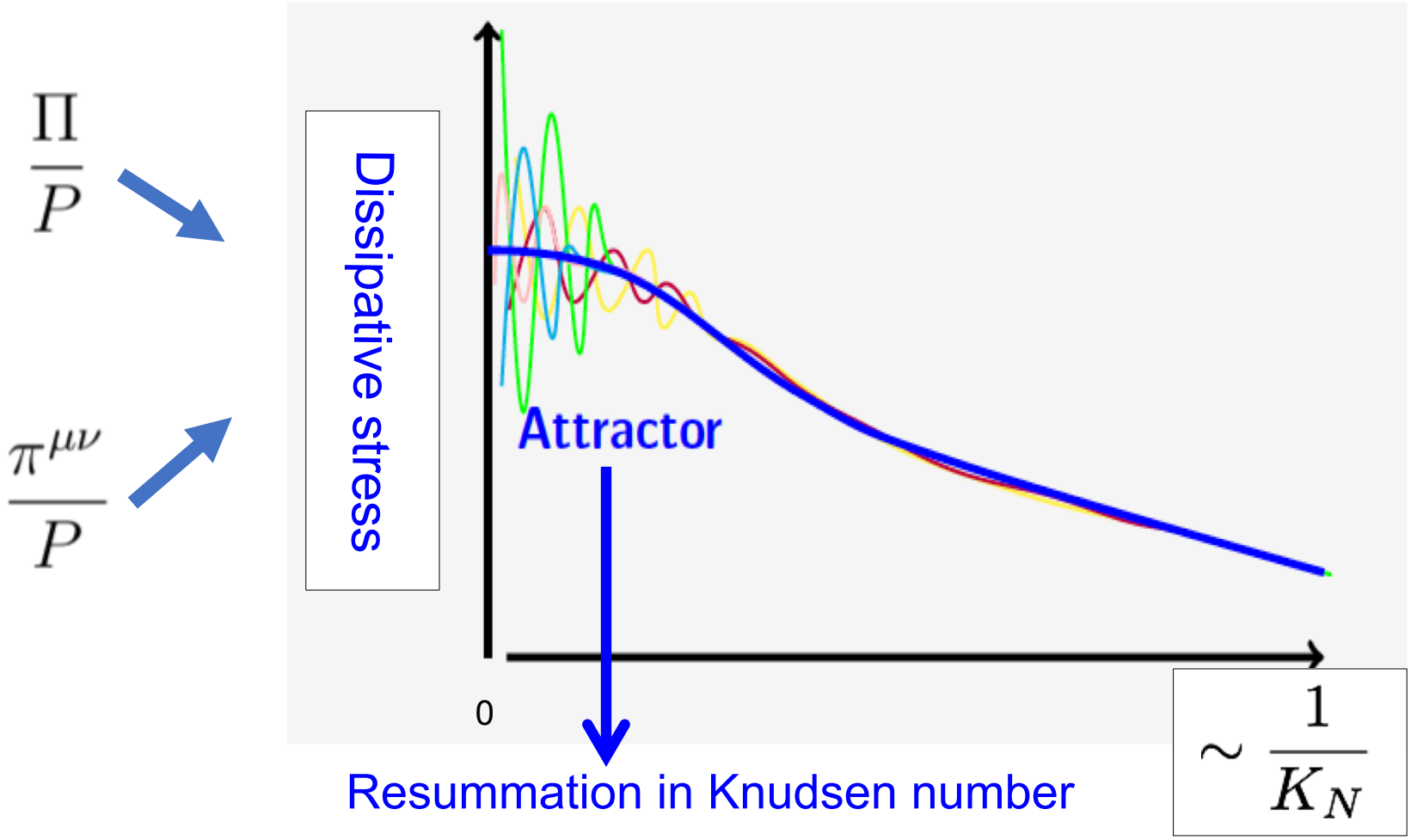
From Paradox to Paradigm

Is there universal behavior even far from equilibrium?

Hydrodynamic Attractors

Heller, Spalinski, PRL (2015)

System behaves as a fluid, even though gradients are very large



The analytical hydrodynamic attractor

G. Denicol, JN, PRD (2018), See also arXiv:1908.09957 **[nucl-th]**

Israel-Stewart theory with constant relaxation time τ_R

$$D\varepsilon + (\varepsilon + P)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} = 0$$

$$(\varepsilon + P)Du^{\mu} - \Delta^{\mu}_{\lambda}\nabla^{\lambda}P + \Delta^{\mu}_{\lambda}\nabla_{\mu}\pi^{\mu\lambda} = 0$$

$$\tau_R\Delta^{\mu\nu}_{\alpha\beta}D\pi^{\alpha\beta} + \delta_{\pi\pi}\,\theta\pi^{\mu\nu} + \tau_{\pi\pi}\,\Delta^{\mu\nu}_{\alpha\beta}\pi^{\alpha\lambda}\sigma^{\beta}_{\lambda} - 2\,\tau_R\Delta^{\mu\nu}_{\alpha\beta}\pi^{\alpha}_{\lambda}\omega^{\beta\lambda} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}.$$

+ Bjorken flow

$$\varepsilon = 3P$$

can be FULLY solved analytically

Energy density

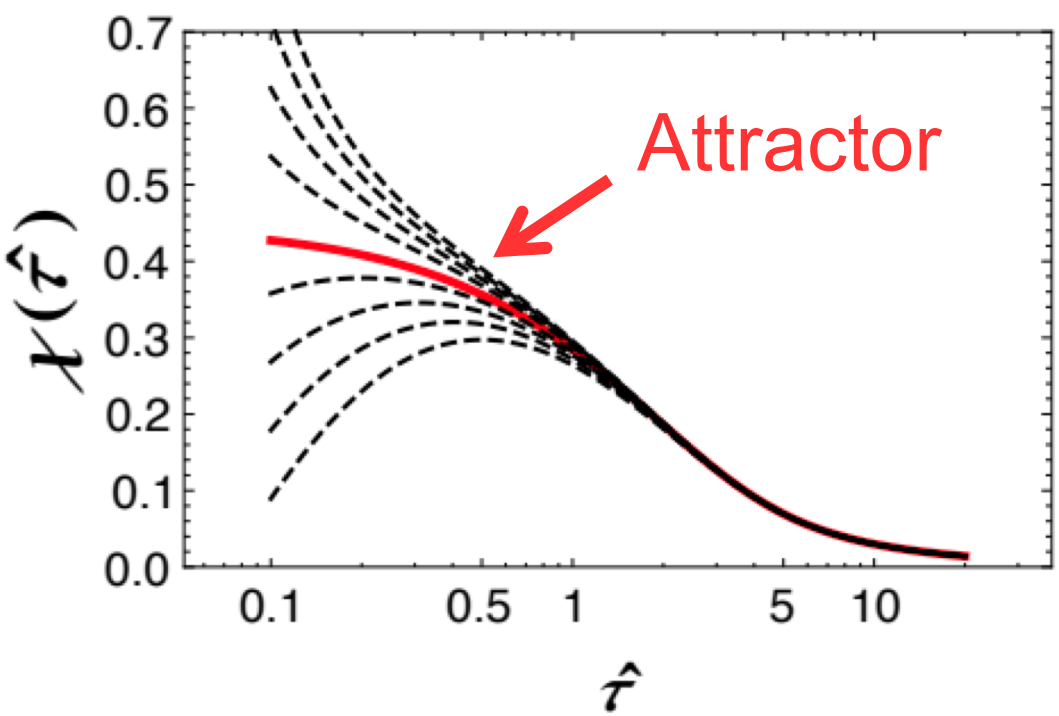
$$\varepsilon(\hat{\tau}) = \varepsilon_0\, e^{-\frac{1}{2}(\hat{\tau}-\hat{\tau}_0)}\left(\frac{\hat{\tau}_0}{\hat{\tau}}\right)^{\frac{5}{6}}\left[\frac{\alpha\left(K_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) - K_{\frac{1}{2}+\sqrt{a}}\left(\frac{\hat{\tau}}{2}\right)\right) + I_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) + I_{\frac{1}{2}+\sqrt{a}}\left(\frac{\hat{\tau}}{2}\right)}{\alpha\left(K_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}_0}{2}\right) - K_{\frac{1}{2}+\sqrt{a}}\left(\frac{\hat{\tau}_0}{2}\right)\right) + I_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}_0}{2}\right) + I_{\frac{1}{2}+\sqrt{a}}\left(\frac{\hat{\tau}_0}{2}\right)}\right]$$

Full solution for shear stress tensor

$$\chi(\hat{\tau}) = \frac{\pi}{\varepsilon + P} = \frac{3\sqrt{a}}{4}\left[\frac{\alpha\left(K_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) + K_{\sqrt{a}+\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right)\right) + I_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) - I_{\sqrt{a}+\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right)}{\alpha\left(K_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) - K_{\sqrt{a}+\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right)\right) + I_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) + I_{\sqrt{a}+\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right)}\right]$$

First analytical expression for a hydrodynamic attractor

$$\chi(\hat{\tau}) \to \chi_{att}(\hat{\tau}) = \frac{3\sqrt{a}}{4}\left[\frac{I_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) - I_{\sqrt{a}+\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right)}{I_{\sqrt{a}-\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right) + I_{\sqrt{a}+\frac{1}{2}}\left(\frac{\hat{\tau}}{2}\right)}\right]$$



Non-perturbative behavior

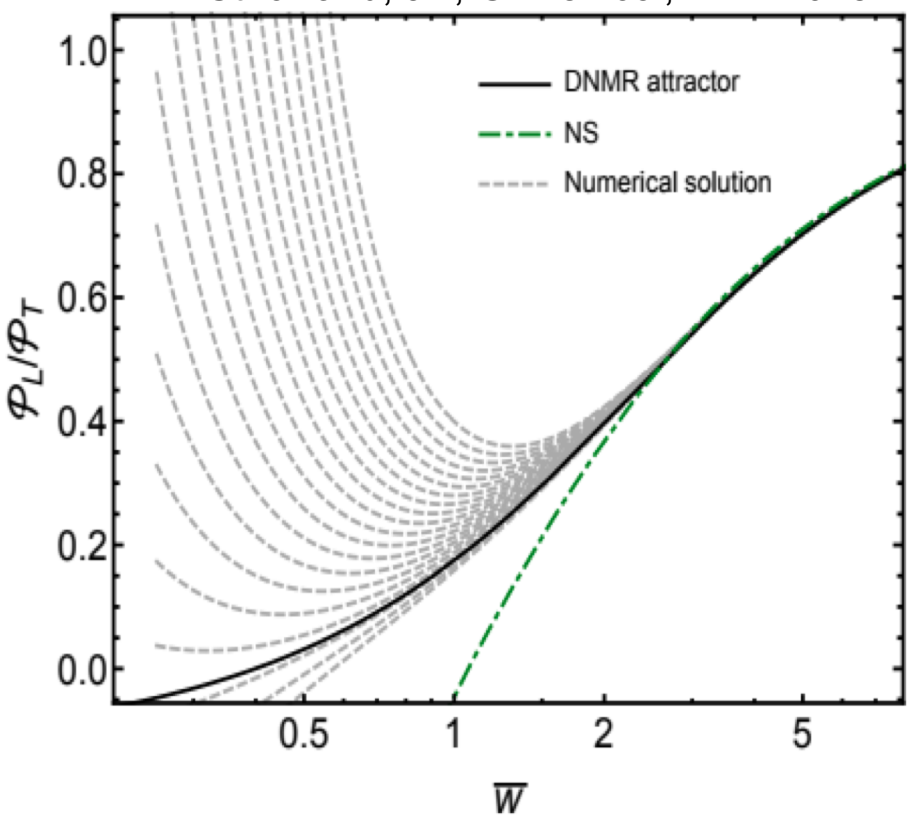
$$\exp\{-1/K_N\}$$

- Resummation of gradient series
- Trans-series

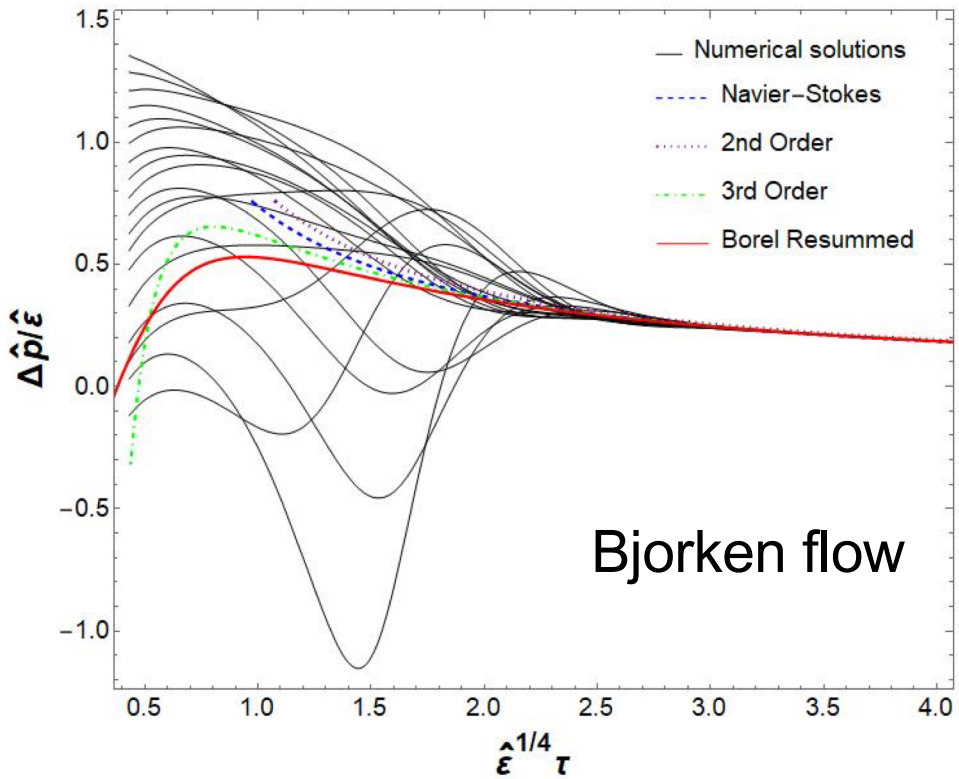
Far-From-Equilibrium Hydro – Attractor Behavior

Boltzmann (weak coupling)

M. Strickland, JN, G. Denicol, PRD 2018



N = 4 SYM at strong coupling



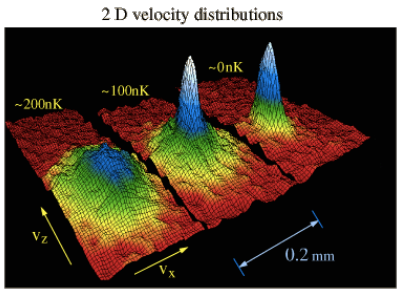
- Very different transient behavior at weak vs. strong coupling.
- Jets: ideal tool to characterize the transient regime.

- The surprising effectiveness of fluid dynamics in heavy ion collisions → Hydrodynamic Attractors

- This is the leading hydro-like justification for the collective phenomena observed in ultrarelativistic heavy-ion collisions.

- What are the consequences of these ideas to other areas of physics?

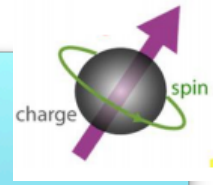
Emergence of fluid dynamics: A bird's-eye view



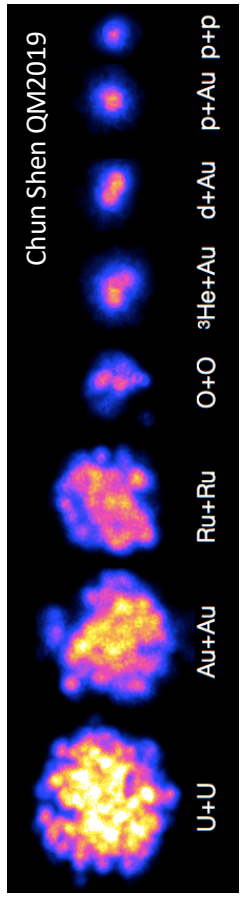
Bose-Einstein condensate

$\hbar$

Quantumness



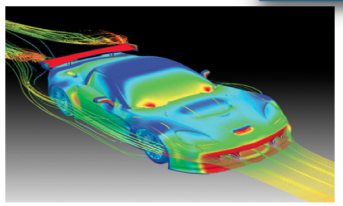
"Fluid"



small systems

HIC

Large systems

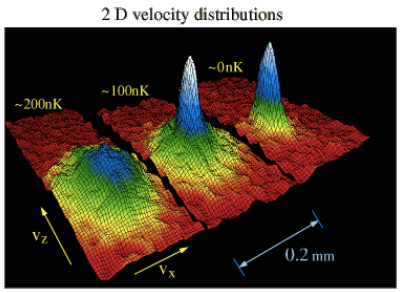


Daily life

Velocity

c

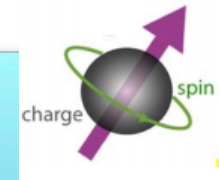
Emergence of fluid dynamics: A bird's-eye view

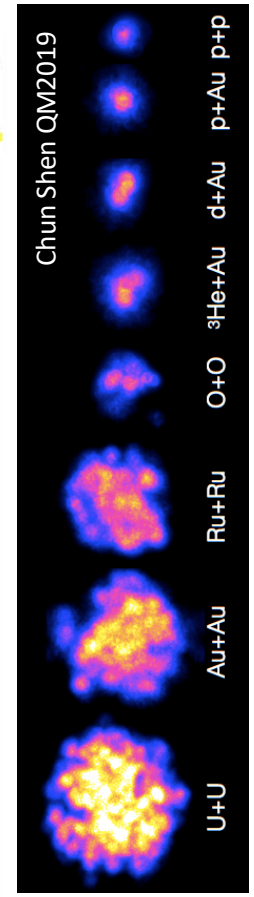


Bose-Einstein condensate

$\hbar$

Quantumness





small systems

HIC

Large systems

Neutron star mergers

0

Velocity

c

What new phenomena appear when viscous fluids are coupled to strong gravitational fields?

Neutron Star Mergers

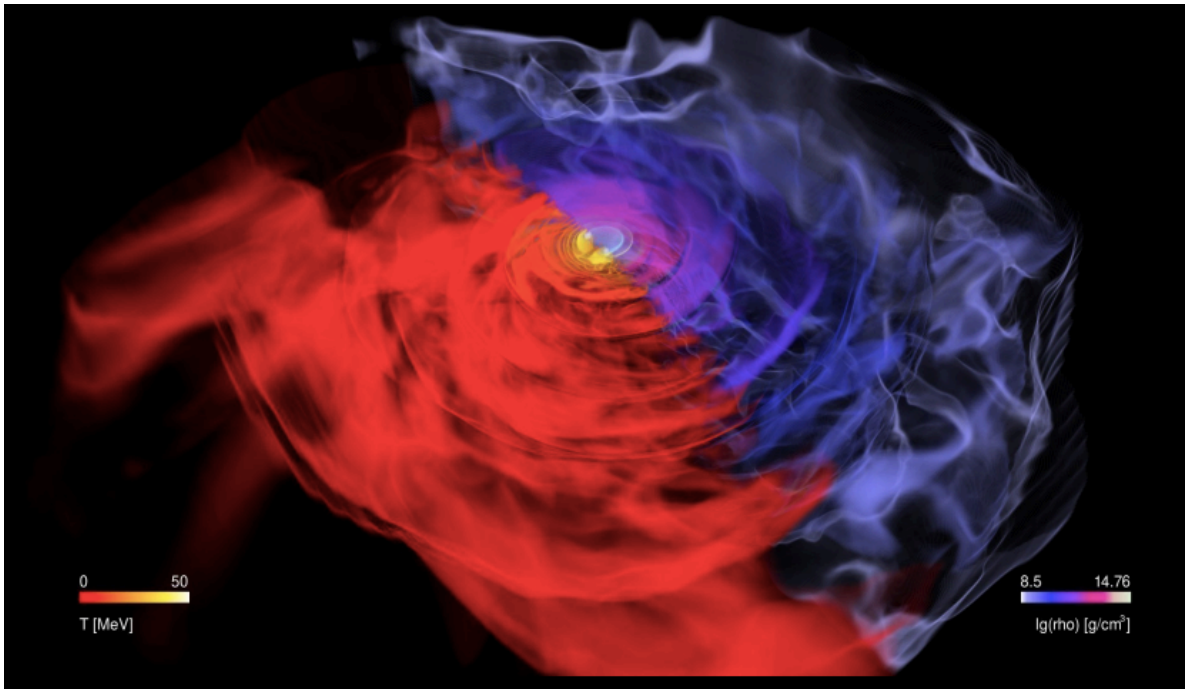


Fig. from L. Rezzolla's talk at QM2019

Reference: Most, Papenfort, Dexheimer, Hanauske, Schramm, Stöcker, Rezzolla, PRL (2019)

How does a lump of baryon rich QCD matter flow under strong gravitational fields?

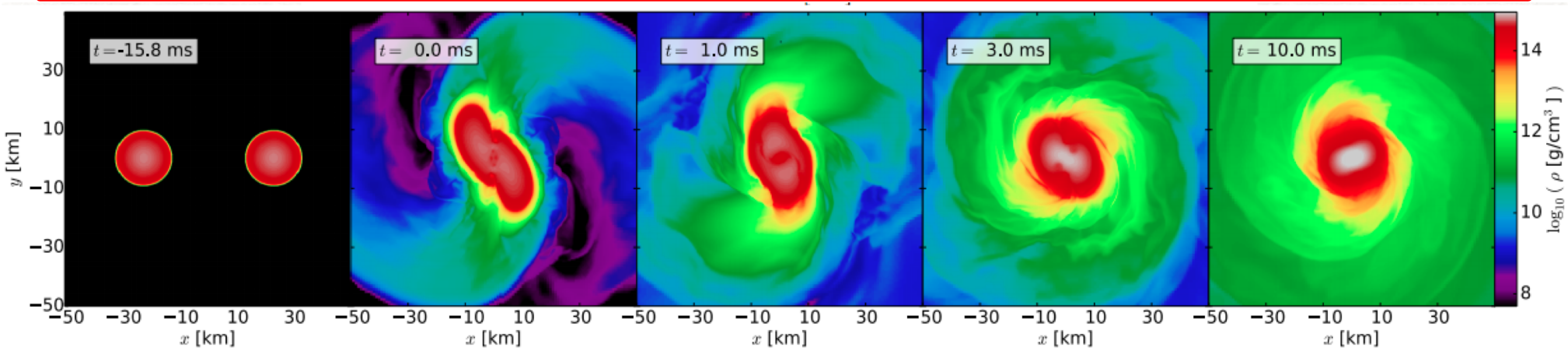


Fig. by L. Rezzolla

New signatures for deconfinement/phase transitions?

e.g. Most et al., PRL (2019)

Viscous fluid dynamics + strong gravitational fields?

Viscous effects in neutron star mergers?

Duez et al PRD (2004), Shibata et al. PRD (2017), Alford et al. PRL (2018)

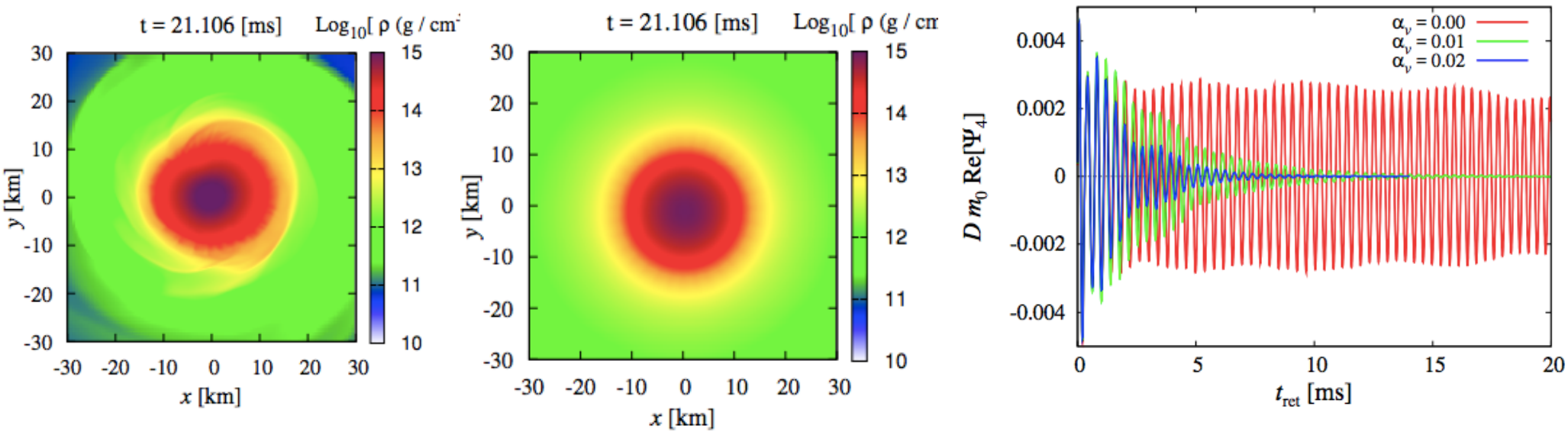
What about the evolution of hypermassive remnants?

Duez, Liu, Shapiro, Stephens, PRD (2004)

- Viscosity and magnetic fields affect differentially rotating remnants
- Effective shear viscosity driven by local MHD turbulence

Shibata, Kiuchi, PRD (2017):

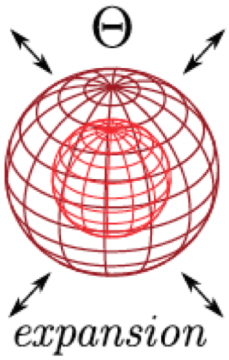
Gravitational wave form



Viscous effects in binary neutron-star mergers?

Alford, Bovard, Hanauske, Rezzolla, Schwenzer, PRL (2018)

Bulk viscous damping in neutron star mergers



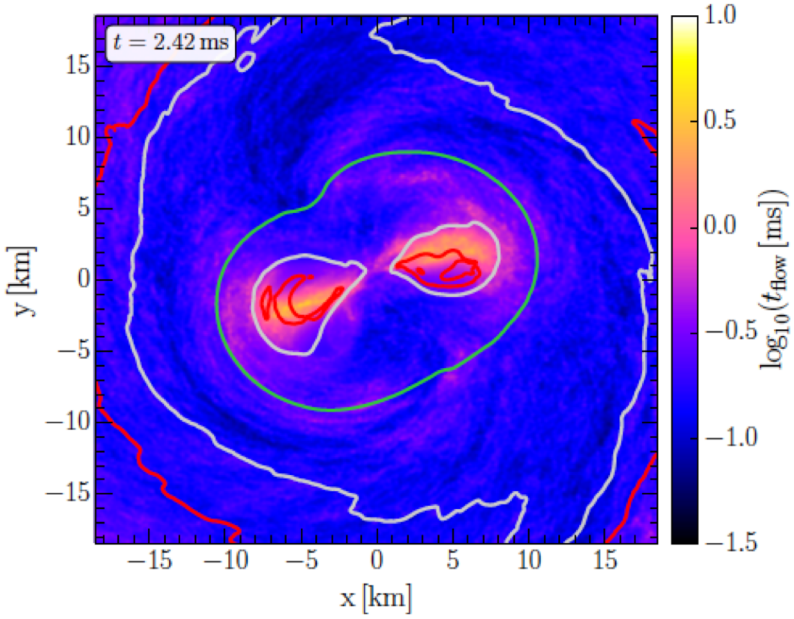


Figure 3: The flow timescale $t_{\rm flow}$ obtained from a numerical-relativity simulation of two 1.35 $M_\odot$ neutron stars [40]. The red (4 MeV) and gray (7 MeV) contours show the boundaries of the temperature range in which the bulk viscosity roughly takes its maximum value, while the green contour shows the inner region where the rest-mass density exceeds nuclear saturation density.

Alford and Harris, PRC (2019)

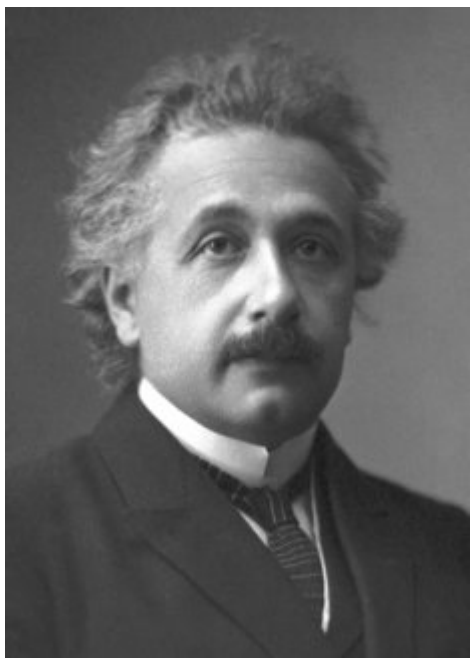
- Low densities (EOS)
- Neutrino transparency (low T)
- High frequencies (f > 1 kHz)

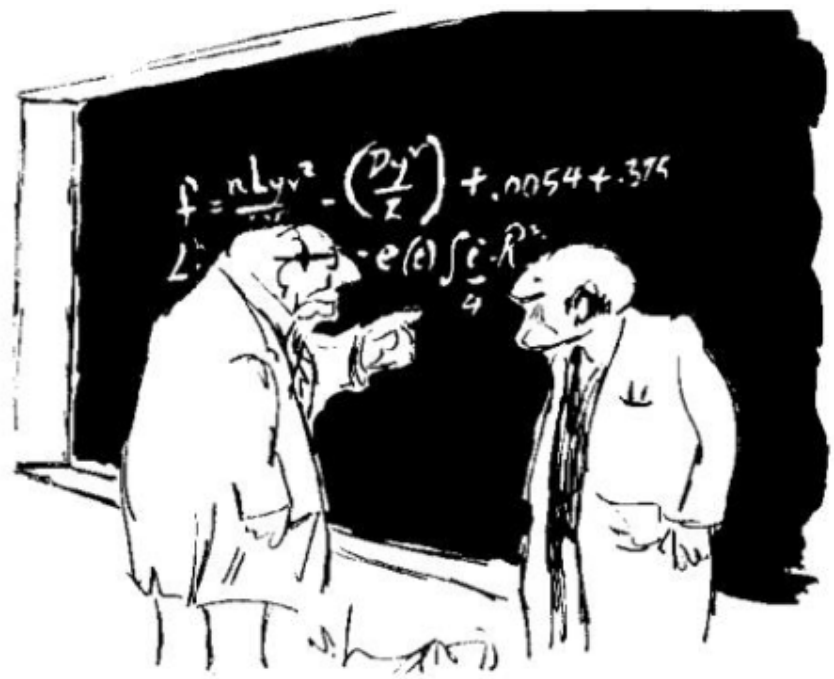
Viscous Fluid Effects in General Relativity

Challenge: Prove that the solutions $\{g_{\mu\nu}, T_{\mu\nu}, J^{\mu}_{B}\}$ are **well posed** (existence, uniqueness) and **causal in the nonlinear regime**

Conservation Laws + Viscosity

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \nabla_{\mu} T^{\mu\nu} = 0 \qquad \nabla_{\mu} J^{\mu}_{B} = 0$$

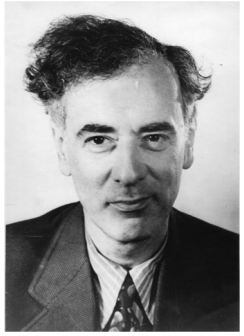




"Does this apply always, sometimes, or never?"

Eckart
1940

Relativistic Navier-Stokes Equations



Landau

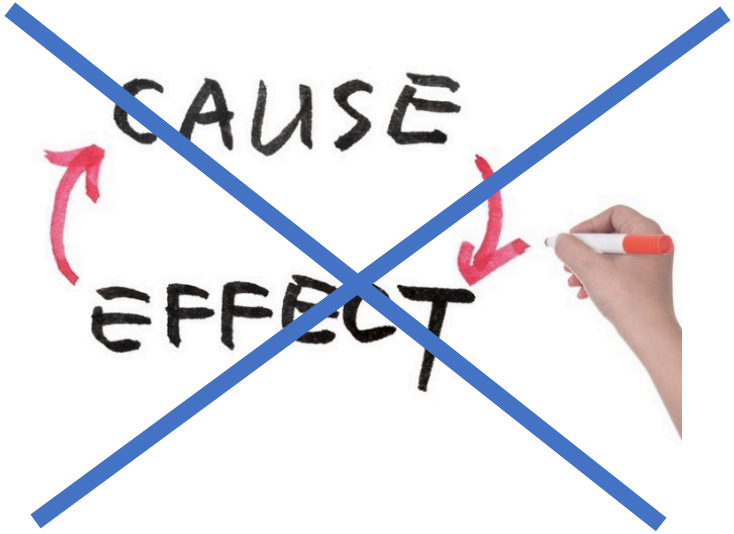
Energy-momentum tensor with bulk viscosity

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P - \zeta \nabla_\alpha u^\alpha)\Delta^{\mu\nu}$$

Conservation laws:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu (n_B u^\mu) = 0$$

This theory is **acausal**
(see Pichon, 1965)



Navier-Stokes ~ Nonlinear diffusion

$$\partial_t u = \frac{\zeta}{sT}\nabla^2 u + \ldots$$

Also UNSTABLE!

What about Israel-Stewart theory?

- 16 coupled nonlinear PDE's (Einstein + fluid equations)

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu}$$

$\Pi \longrightarrow$ Bulk scalar

Israel-Stewart Equation

$$\tau_\Pi u^\alpha \nabla_\alpha \Pi + \Pi + \lambda \Pi^2 + \zeta \nabla_\alpha u^\alpha = 0$$

- Causality? Existence? Uniqueness? Nonlinearity?

- Open problem in physics and mathematics since the 1970's.

1st Proof of CAUSALITY, EXISTENCE, AND UNIQUENESS in the full nonlinear regime

Theorem 1. *Let* $(\varepsilon, u^\alpha, \Pi, n, g_{\alpha\beta})$ *be a solution to the EIS equations defined on a globally hyperbolic spacetime* M*, and let* Σ *be a Cauchy surface. Suppose* $\frac{\zeta}{\tau_\Pi(\varepsilon+P+\Pi)} + \alpha_1 + \frac{\alpha_2 n}{\varepsilon+P+\Pi} \geq 0$ *and that* (1) *holds. Then, for any* $p \in M$ *in the future of* Σ*,* $(\varepsilon(p), u^\alpha(p), \Pi(p), n(p), g_{\alpha\beta}(p))$ *depends only on* $(\varepsilon, u^\alpha, \Pi, n, g_{\alpha\beta}, \kappa_{\alpha\beta})|_{\Sigma\cap J^-(p)}$*, where* $J^-(p)$ *is the causal past of* p *and* κ *is the extrinsic curvature of* Σ *in* M*.*

Theorem 2. *Let* $\mathcal{I} = (\Sigma, \mathring{\varepsilon}, \mathring{u}^i, \mathring{\Pi}, \mathring{n}, \mathring{g}_{ij}, \mathring{\kappa}_{ij})$ *be an initial data set for the EIS system, with an equation of state* $P = P(\varepsilon, n)$*, a bulk viscosity* $\zeta = \zeta(\varepsilon, n)$*, and a relaxation time* $\tau_\Pi = \tau_\Pi(\varepsilon, n)$*. Assume that* $\mathring{\varepsilon} + P(\mathring{\varepsilon}, \mathring{n}) + \mathring{\Pi}$*,* $\tau_\Pi(\mathring{\varepsilon}, \mathring{n})$*,* $\zeta(\mathring{\varepsilon}, \mathring{n}) > 0$*,* $\frac{\partial P}{\partial \varepsilon}(\mathring{\varepsilon}, \mathring{n}) + \frac{\partial P}{\partial n}(\mathring{\varepsilon}, \mathring{n})\mathring{n}/(\mathring{\varepsilon} + P(\mathring{\varepsilon}, \mathring{n}) + \mathring{\Pi}) \geq 0$*, and that* $\mathring{n}$*,* $\frac{\partial P}{\partial \varepsilon}(\mathring{\varepsilon}, \mathring{n})$*, and* $\frac{\partial P}{\partial n}(\mathring{\varepsilon}, \mathring{n})$ *are nonzero. Suppose that* $\mathring{g}_{ij} \in H^{s+1}_{ul}(\Sigma)$*,* $\mathring{\varepsilon}, \mathring{u}^i, \mathring{\Pi}, \mathring{n}, \mathring{\kappa}_{ij} \in H^s_{ul}(\Sigma)$*, and that* $P, \zeta, \lambda, \tau_\Pi \in C^s(\mathbb{R}^2)$*, where* $s \geq 3$*. Suppose that* (1) *holds for* $\mathcal{I}$*. Then, there exists a globally hyperbolic development of* $\mathcal{I}$*.*

Bemfica, Disconzi, Noronha

PRL (2019)

Proved that Einstein-Israel-Stewart equations can be written as

$$A_0(\Phi)\nabla_t\Phi + A^i(\Phi)\nabla_i\Phi + B(\Phi)\Phi = 0$$

where $\Phi = (\varepsilon, n, u_\mu, \Pi, g_{\mu\nu}, \partial g_{\mu\nu})$

Main Condition

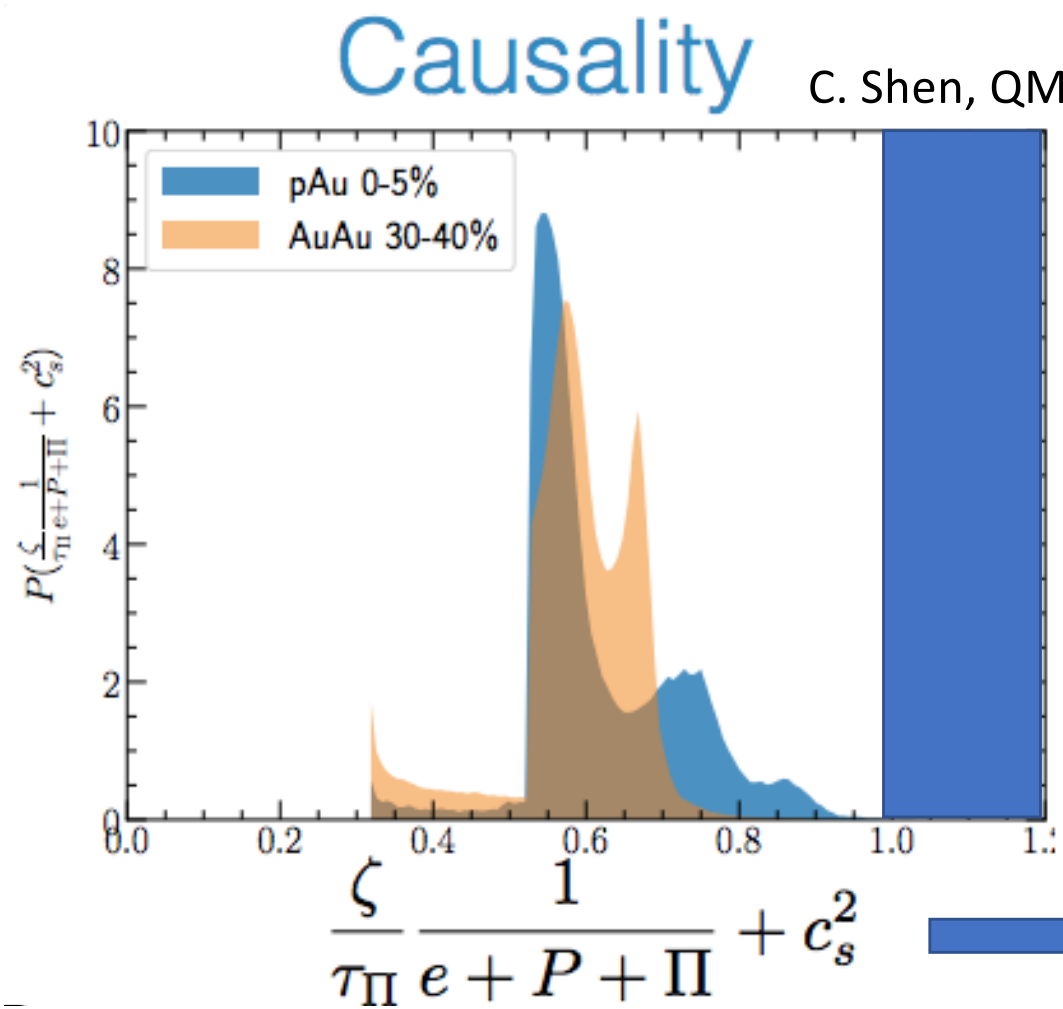
$$\left[\frac{\zeta}{\tau_\Pi} + n\left(\frac{\partial P}{\partial n}\right)_\varepsilon\right]\frac{1}{\varepsilon + P + \Pi} < 1 - \left(\frac{\partial P}{\partial \varepsilon}\right)_n$$

- Full generalization to include shear viscosity: Done!
- Investigate consequences to neutron star mergers and heavy-ion collisions (HIC).

Causality constraint and small systems in heavy ions

$$\frac{\zeta}{\tau_\Pi(\varepsilon + P + \Pi)} < 1 - c_s^2$$

Zero baryon density



Further constraints will appear when shear effects are included!

Dependence on transport coefficients !!!

Is the description of relativistic viscous fluids unique?

Is the description of relativistic fluid dynamics unique?

- Israel-Stewart "theory" not unique (eg. rBRSSS vs. DNMR).

- **Transient** behavior of strongly coupled (holographic) liquids has different properties than Israel-Stewart theory.

Denicol, Niemi, JN, Rischke, PRD (2011)
Heller, Janik, Spalinski, Witaszczyk, PRL (2014)
See also Grozdanov, Lucas, Poovuttikul, PRD (2019)

Is there another way to describe the motion of viscous fluids that is compatible with (general) relativity?

Revisiting the gradient expansion

Hydrodynamics: Simplest effective theory for $\{T, u^\mu\}$

$\mu_B = 0$

$$T^{\mu\nu} = T^{\mu\nu}_{eq}(T, u^\lambda) + T^{\mu\nu}_{viscous}$$

Viscous correction? Gradient expansion

Chapman-Enskog, 1930's
BRSSS, JHEP (2008)
BMHR, JHEP (2008)

$$T^{\mu\nu}_{viscous} = \underbrace{T^{\mu\nu}_{viscous}(\nabla T, \nabla u)}_{\text{All possible 1st order terms with no time derivatives in the local rest frame}} + \underbrace{\mathcal{O}(\nabla^2)}_{\text{higher orders}}$$

Revisiting the gradient expansion

IMPORTANT: **<u>If</u>** Landau definition is assumed to be valid throughout → $u_\mu T^{\mu\nu} = -\varepsilon u^\nu$

- First order truncation leads to relativistic Navier-Stokes theory, which is **acausal and unstable**.

- This is not fixed by going to 2nd order in spatial gradients (note this is **NOT** Israel-Stewart theory).

- Can one make the gradient expansion causal and stable?

A new approach to relativistic viscous fluid dynamics

Based on Bemfica, Disconzi, JN, PRD (2017) and PRD (2019)
See also P. Kovtun, JHEP (2019).

Effective theory: Space-time derivative expansion

$$T^{\mu\nu}_{viscous}(\nabla T, \nabla u) \Rightarrow$$

Most general **derivative** expansion compatible with symmetries

- Definition of T and u^μ not unique out of equilibrium.

- No reason to expect a priori that Landau's definition is a reasonable choice of variables.

Tsumura, Kunihiro, PLB (2008)
Van, Biro, EPJ ST (2008)

A new approach to relativistic viscous fluid dynamics

Based on Bemfica, Disconzi, JN, PRD (2017) and PRD (2019)
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Most general derivative expansion compatible with symmetries

$$T^{\mu\nu} = (\varepsilon + \mathcal{A}_1)u^\mu u^\nu + (P(\varepsilon) + \mathcal{A}_2)\,\Delta^{\mu\nu} - 2\eta\sigma^{\mu\nu} + u^\mu \mathcal{Q}^\nu + u^\nu \mathcal{Q}^\mu$$

where to 1st order in derivatives

$$\mathcal{A}_1 = \chi_1 \frac{u^\alpha \nabla_\alpha \varepsilon}{\varepsilon + P} + \chi_2 \nabla_\alpha u^\alpha, \qquad \mathcal{A}_2 = \chi_3 \frac{u^\alpha \nabla_\alpha \varepsilon}{\varepsilon + P} + \chi_4 \nabla_\alpha u^\alpha, \qquad \mathcal{Q}_\mu = \lambda \left(\frac{c_s^2 \Delta_\mu^\nu \nabla_\nu \varepsilon}{\varepsilon + P} + u^\alpha \nabla_\alpha u_\mu \right)$$

Energy density correction **Pressure correction** **Heat flow**

Equations of motion: $\nabla_\mu T^{\mu\nu} = 0$

Causality and well-posedness are valid in the full nonlinear regime, also including Einstein's equations, when

Rigorous theorems in Bemfica, Disconzi, JN, PRD (2019)

$$\lambda, \chi_1 > 0, \ \eta \geq 0$$

$$\lambda \geq \eta,$$

$$3\chi_4 \geq 4\eta,$$

$$\lambda\chi_1 + c_s^2\lambda\left(\chi_4 - \frac{4\eta}{3}\right) \geq c_s^2\lambda\chi_2 + \lambda\chi_3 + \chi_2\chi_3 - \chi_1\left(\chi_4 - \frac{4}{3}\eta\right) > 0.$$

Arbitrary EOS

Linear stability also holds:

P. Kovtun, JHEP (2019).
Bemfica, Disconzi, JN, PRD (2019).

- Heat flow coefficient lower bound from causality.
- Only 6 coefficients (Israel-Stewart > 10).
- No additional fields besides $\{T, u^\mu\}$.
- Very different δf at freeze-out (scalar, vector, tensor).

Conclusions

- QGP formed in heavy ions forces us to explore fluid dynamics in the far from equilibrium regime.

- New understanding about the emergence of fluid dynamics under extreme conditions: attractors!

- Many connections to string theory, cosmology, astrophysics, and mathematics.

- New formulation of relativistic viscous fluids.

- New results pave the way for the systematic study of of viscous effects in HIC and neutron star mergers.