An order parameter for Inverse Magnetic Catalysis

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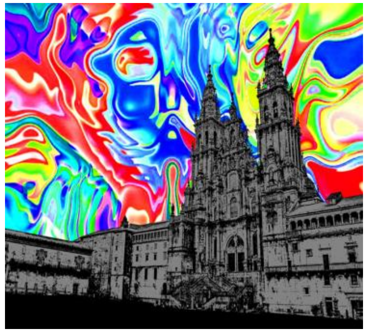


Outline

- Inverse Magnetic Catalysis (IMC) at zero density
- IMC at finite density
- Top-down holographic QCD : The Sakai-Sugimoto model
- IMC vs MC at finite density from holographic QCD
- An order parameter for IMC

Inverse Magnetic catalysis (IMC) at zero density $(\mu = 0)$

Magnetic catalysis (MC) at **T=0**:

A magnetic field enhances chiral symmetry breaking .

Gusynin, Miransky & Shovkovy 1994

$< \bar{\psi}\psi >$ increases with **B**

$< \bar{\psi}\psi >$ **:** chiral condensate , **B** : magnetic field.

If $< \bar{\psi}\psi > |_{B=0}$ = 0 a non-zero **B** generates a non-zero $< \bar{\psi}\psi >$.

Physical picture:

- In QM a magnetic field **B** along **z** quantizes the orbits on **(x,y)**.
- Landau levels for fermions :

$$\boldsymbol{E_n^2 = m^2 + p_z^2 + |qB|(2n + 1 - s)}$$

where $\boldsymbol{n} = (\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2}, \dots)$ and $\boldsymbol{s} = \pm\boldsymbol{1}$.

- Dynamics reduces from **3+1** dim to **1+1** dim and **B** behaves as a mass**.**

- Lowest Landau Level (**LLL**) : $\boldsymbol{n = 0}$, $\boldsymbol{s = 1}$.

- Degeneracy for each Landau level : $\boldsymbol{N = \frac{|qB|}{2\pi} \; L_x L_y}$

- In QFT the **LLL** acquire a dynamical mass from **B.**

- Banks-Casher relation :

$$< \bar{\psi}\psi > |_{m=0} \sim \rho(\lambda \to 0)$$

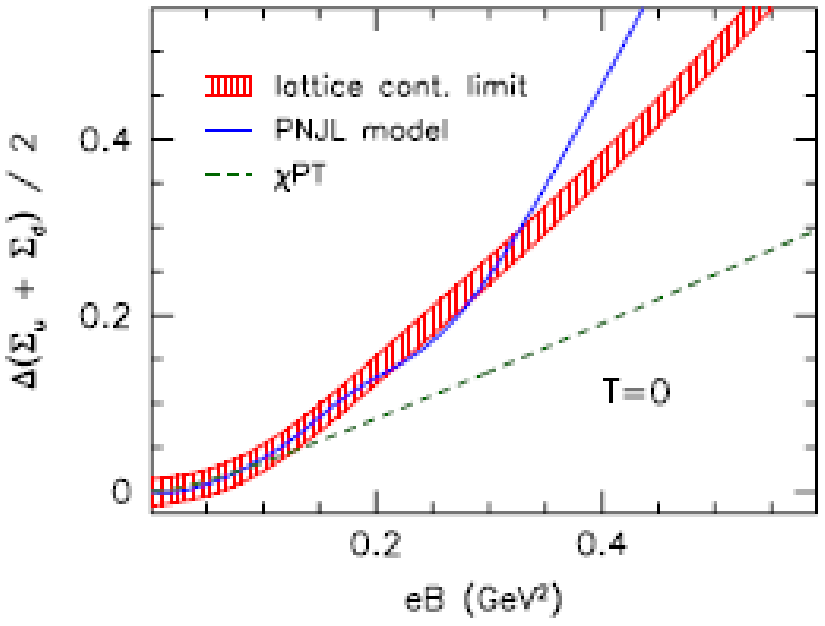
Banks & Casher 1980

where $\rho(\lambda)$ is the spectral density of the Dirac operator.

- $\rho(\lambda)$ is enhanced by **B** as consequence of the LLL degeneracy.

→ **B** catalyses chiral symmetry breaking at **T=0** (**Magnetic catalysis**).

- Consistent with effective models for QCD such as Nambu-Jona-Lasinio (**NJL**) and Chiral Perturbation Theory (χPT) .



Inverse Magnetic Catalysis (IMC) at finite **T**

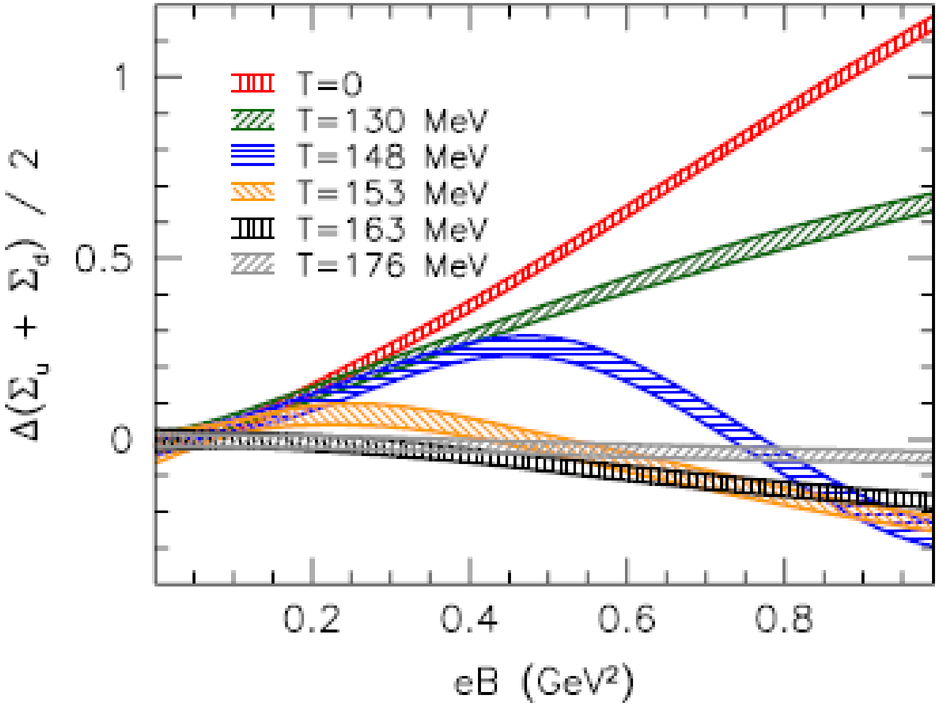
- As **T** increases a dramatic change of behaviour found in lattice QCD.

Bali et al 2012

- The magnetic field **B** now restores chiral symmetry ! (**IMC**)

- Conflict with results from effective models for QCD.

- What is the physical mechanism that turns MC into IMC ?



- **Hint:** Two competing mechanism identified in lattice QCD.

Bruckmann, Endrodi & Kovacs 2013

$$< \bar{\psi}\psi > = \frac{1}{Z(B)} \int dU\, e^{-S_g} det[\gamma^\mu D_\mu(B) + m] Tr[\gamma^\mu D_\mu(B) + m]^{-1}.$$

Then

$$< \bar{\psi}\psi > \approx \; < \bar{\psi}\psi >^{\mathrm{val}} + \; < \bar{\psi}\psi >^{\mathrm{sea}} ,$$

where

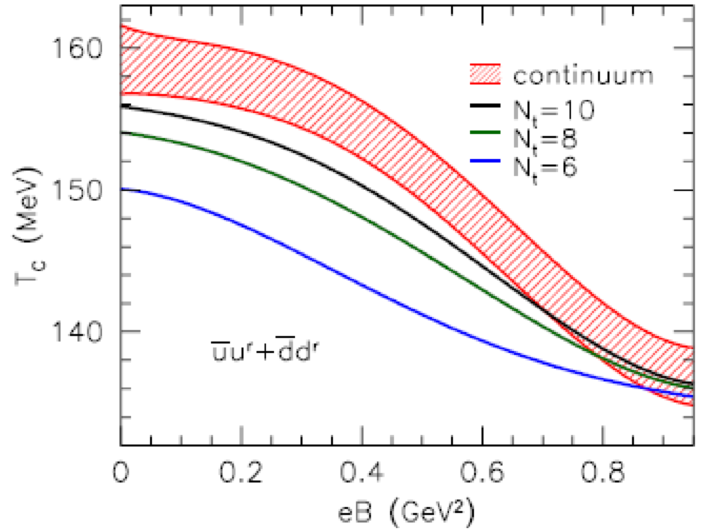
$$< \bar{\psi}\psi >^{\mathrm{val}} := \frac{1}{Z(0)} \int dU\, e^{-S_g} det[\gamma^\mu D_\mu(0) + m] Tr[\gamma^\mu D_\mu(B) + m]^{-1},$$

$$< \bar{\psi}\psi >^{\mathrm{sea}} := \frac{1}{Z(B)} \int dU\, e^{-S_g} det[\gamma^\mu D_\mu(B) + m] Tr[\gamma^\mu D_\mu(0) + m]^{-1}.$$

- The sea quark effect can be interpreted as a backreaction of the quarks on the gauge fields.

IMC and the deconfinement/chiral transition ($\mu = 0$)

- The transition critical temperature T_c is a decreasing function of **B**. ***Bali et al 2011***



Attempts to describe this behaviour in models for non-perturbative QCD :

- Magnetic MIT bag model ***Fraga & Palhares 2012***
- Large N Magnetic QCD ***Fraga, Noronha & Palhares 2012***
- Polyakov-Quark-Meson model ***Fraga, Mintz & Schaffner-Bielich 2013***
- NJL model with a B dependent coupling ***Farias et al 2014***
- Holographic QCD ***B-B 2013 Mamo 2015 Rougemont, Critelli & Noronha 2015 Fang 2016 Evans, Miller & Scott 2016 Gursoy, Jarvinen & Nijs 2016.***

IMC at finite density ($\mu \neq 0$)

- No available Lattice QCD due to the sign problem.
- Rely on models for non-perturbative QCD.
- First IMC result from the finite density **NJL** model.

Ebert et al 2000 Inagaki, Kimura & Murata 2004

- The NJL Lagrangian is

$$\boldsymbol{L = \bar{\psi}(i\,\gamma^{\mu} D_{\mu} - m + \mu\gamma_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\psi)^2]},$$

where $\boldsymbol{D_{\mu} = \partial_{\mu} + iq\,A_{\mu}}$, with $\boldsymbol{A_{\mu} = (0, yB, 0, 0)}$.

- Assumptions : $\boldsymbol{< \bar{\psi}\gamma_5\psi > = 0}$ and

$$\boldsymbol{(\bar{\psi}\psi)^2 \simeq < \bar{\psi}\,\psi >^2 + 2 < \bar{\psi}\,\psi > \bar{\psi}\,\psi}$$

(mean-field approx).

- Constituent quark mass: $$M := m - 2G < \bar{\psi}\,\psi >$$
- Thermodynamic potential : $$\Omega = \frac{(M-m)^2}{4G} - \frac{T}{V} TrLog[\frac{-i\omega_n+\mu-\epsilon}{T}],$$
- ϵ is a spectral decomposition for the Dirac Hamiltonian

$$H_D = \gamma^0\, \vec{\gamma} \cdot \left(-i\, \nabla - q\vec{A}\right) + \gamma^0 M$$

- Minimising the potential with respect to M one finds the gap equation

$$< \bar{\psi}\,\psi > = -\frac{T}{V} Tr[\frac{\gamma^0}{i\omega_n + \mu - \epsilon}]$$

- At large coupling **G** and small **B** the solution to the gap eq. takes the form

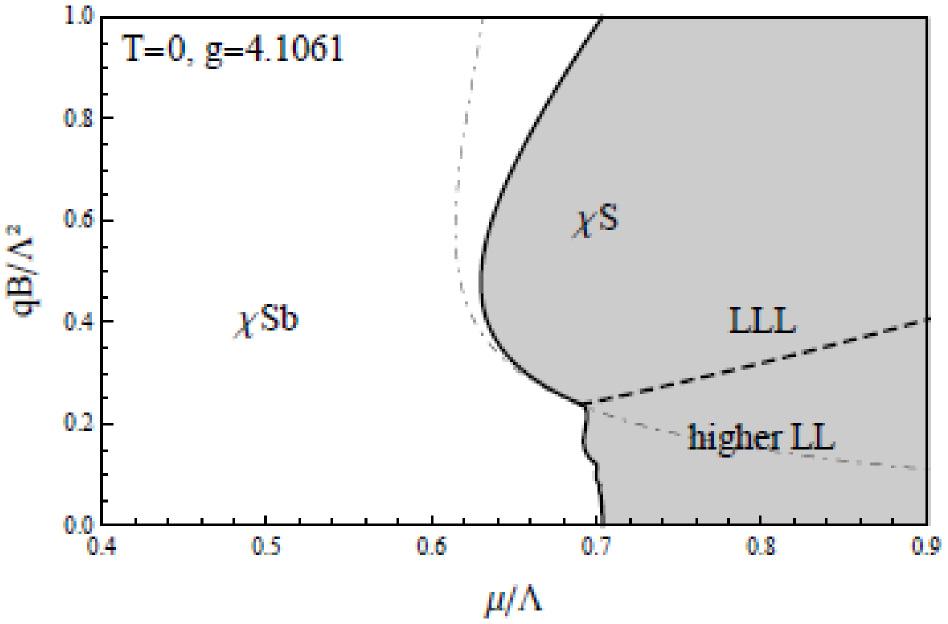
$$M \simeq M_0[1 + \frac{(qB)^2}{6M_0^4\Gamma(0,\frac{M_0^2}{\Lambda^2})}],$$ where M_0 is the mass gap for $\mu = B = 0$.

- At finite $\boldsymbol{\mu}$ there is another solution where $\boldsymbol{M = 0}$, corresponding to chiral symmetry restoration.

- The free energy difference between the chirally broken (XB) and chirally symmetric (XS) phases takes the form

$$\boldsymbol{\Delta\Omega \simeq -\frac{M_0^2\Lambda^2}{16\,\pi^2}\left(1-\frac{2\pi}{G\Lambda^2}\right)+\frac{|q|B}{4\,\pi^2}\mu^2+\cdots}$$

- First term: Energy gained from the chiral condensate
- Second term: Energy cost to form quark-antiquark pairs at finite $\boldsymbol{\mu}$.



-There is a regime where B favours chiral symmetry restoration ! (**IMC**).

Top down holographic QCD: The Witten-Sakai-Sugimoto model

Confinement and the Witten's model : In String Theory a 5-d $SU(N_c)$ SYM theory is described in terms of a stack of N_c D4-branes.

- Compactifying one coordinate, $\mathbf{0 < \tau < 2\pi R}$, one arrives at pure 4-d YM Theory.
- At strong coupling and large $\boldsymbol{N_c}$, the gravity dual is a 10-d background :

Witten 1998 Kruczenski, Mateos, Myers & Winters 2003

$$\mathbf{ds^2} = \frac{u^{3/2}}{R_{D4}^{3/2}}\left[-dt^2 + dx_i^2 + f(u)d\tau^2\right] + \frac{R_{D4}^{3/2}}{u^{3/2}}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right], \quad f(u) = 1 - \frac{u_{KK}^3}{u^3},$$

$$e^{\phi} = g_s \frac{u^{3/4}}{R_{D4}^{3/4}} \quad , \quad F_4 = \frac{2\pi N_c}{V_{S^4}} \epsilon_4 \quad , R_{D4}^3 = \pi g_s N_c \ell_s^3$$

$\boldsymbol{\phi}$: dilaton , $\boldsymbol{F_4}$: RR 4-form $\boldsymbol{g_s}$: string coupling , $\boldsymbol{\ell_s}$: string length.

- The 5-d and 4-d YM couplings are given by $g_5^2 = 4\pi^2 g_s \ell_s \quad , \quad g_4^2 = \frac{g_5^2}{2\pi R}$.

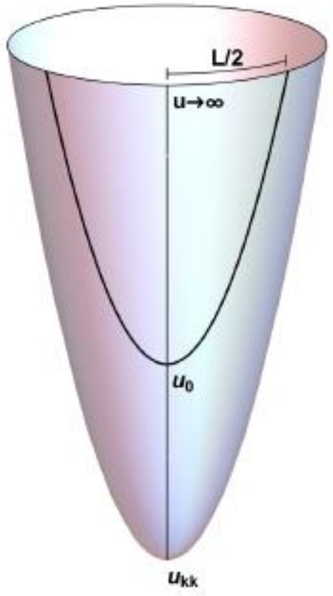
- The radial coordinate $\boldsymbol{u}$ has a minimum at $\boldsymbol{u_{KK}}$ where the circle in $\boldsymbol{\tau}$ shrinks to zero size (cigar geometry).

- Regularity implies that $\boldsymbol{R = \frac{2}{3}\frac{R_{D4}^{3/2}}{u_{KK}^{1/2}}}$, Useful units: $\boldsymbol{R_{D4} = u_{KK} = \frac{3}{2}R}$.

- Witten’s model leads to linear confinement, i.e. $\boldsymbol{V_{\bar{Q}Q} = \sigma\, L}$,

with $\boldsymbol{\sigma = \frac{1}{2\pi\ell_s^2} = \frac{2}{27\pi}\frac{\lambda}{R^2}}$.

Kinar, Schreiber & Sonnenschein 1998



Sakai-Sugimoto model: Addition of $N_f\ \overline{\boldsymbol{D8}}$ and $N_f\ \boldsymbol{D8}$ probe branes.

- At weak coupling this is a string realization

of chiral symmetry $SU(N_f)_L \times SU(N_f)_R$.

- At strong coupling the branes merge in the $(\boldsymbol{\tau}, \boldsymbol{u})$ plane.
- This the holographic dual of chiral symmetry breaking.

-

Sakai & Sugimoto 2004

The black brane, deconfinement and the chiral transition

Aharony, Sonnenschein & Yankielowicz 2006

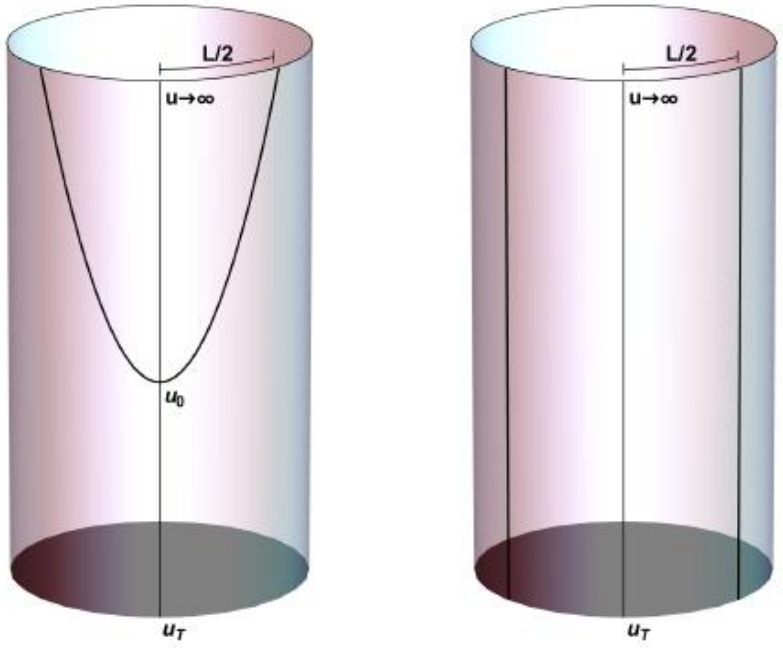
- At finite **T** a black brane solution appears, with metric

$$\mathrm{ds}^2 = \frac{u^{3/2}}{R_{D4}^{3/2}}\left[-h(u)dt^2 + dx_i^2 + d\tau^2\right] + \frac{R_{D4}^{3/2}}{u^{3/2}}\left[\frac{du^2}{h(u)} + u^2 d\Omega_4^2\right], \qquad h(u) = 1 - \frac{u_T^3}{u^3},$$

- This time regularity relates the temperature **T** to the horizon position u_T :

$$\frac{1}{T} = \frac{4\pi}{3}\frac{R_{D4}^{3/2}}{u_T^{1/2}}$$

- The N_f $\overline{D8}$ and N_f $D8$ probe branes now can merge (**XB phase**) or split (**XS phase**).
- At some critical T_c there is a chiral transition between the XB and XS regime.
- u_0 leads to a non-zero constituent quark mass.



Brane dynamics: After integrating over $\boldsymbol{S^4}$, The $D8/\overline{D8}$ brane pair is described by a 5-d DBI-CS action

$$\boldsymbol{S = \int d^4x \int du \, \{-\gamma(u)\big[\sqrt{-E_L} + \sqrt{-E_R}\big] + \frac{\alpha}{4}\epsilon^{\ell mnpq}[A^L_\ell F^L_{mn} F^L_{pq} - A^R_\ell F^R_{mn} F^R_{pq}]} \, ,$$

$$\boldsymbol{E_{L(R)} = det[E^{L(R)}_{mn}] \, , \; E^{L(R)}_{mn} = g^{L(R)}_{mn} + \beta F^{L(R)}_{mn}}$$

where $\boldsymbol{\alpha = \frac{N_c}{24\pi^2} \, , \beta = 2\pi\ell_s^2 \, , \gamma(u) = \frac{\mu_8}{g_s} V_{S^4} R_{D4}^{15/4} u^{1/4}}$

and the 5-d induced metrics $\boldsymbol{g^{L(R)}_{mn}}$ are given by

$$\boldsymbol{g^{L(R)}_{mn} dx^m dx^n = \frac{u^{3/2}}{R_{D4}^{3/2}}\left[-h(u)dt^2 + dx_i^2\right] + \left[\frac{R_{D4}^{3/2}}{u^{3/2}h(u)} + \frac{u^{3/2}}{R_{D4}^{3/2}}\left(\partial_u \tau_{L(R)}\right)^2\right] du^2}$$

- Finite quark density and finite **B**

$$\boldsymbol{\tau_{L(R)} = \pm\tau(u) \quad , \qquad A^{L(R)}_u = 0 \quad , \qquad A^{L(R)}_0 = f_0(u) \quad , \qquad \vec{A} = \frac{1}{2}\vec{B} \times \vec{x} \pm \vec{f}(u)} \; ,$$

with $\boldsymbol{\vec{B} = B\,\hat{x}_3 \quad , \vec{f}(u) = f_3(u)\,\hat{x}_3}$.

(I)MC at finite density in the Witten-Sakai-Sugimoto model

Preis, Rebhan & Schmitt 2010

B-B, Ihl, Shock & Zoakos 2017

- At finite density and finite **B** the DBI-CS equations reduce to

$$\partial_v\left[\sqrt{\frac{Q_0}{Q_2}}\,v^{\frac{11}{2}}h(v)\partial_v\hat{\tau}\right]=0\quad,\quad \partial_v\left[\sqrt{\frac{Q_0}{Q_2}}\,v^{\frac{5}{2}}\partial_v\hat{f}_0\right]+3b\partial_v\hat{f}_3=0\,,$$

$$\partial_v\left[\sqrt{\frac{Q_0}{Q_2}}\,v^{\frac{5}{2}}h(v)\partial_v\hat{f}_3\right]+3b\partial_v\hat{f}_0=0$$

where

$$Q_0=1+\frac{b^2}{v^3}\quad,\quad Q_2=1+v^3h(v)(\partial_v\hat{\tau})^2-\left(\partial_v\hat{f}_0\right)^2+h(v)\left(\partial_v\hat{f}_3\right)^2,\quad h(v)=1-\frac{v_T^3}{v^3}$$

and we are defining the dimensionless fields

$$v=\frac{u}{u_{KK}}\quad,\quad \hat{f}_{0,3}=\frac{2\pi\ell_s^2}{R_{D4}}f_{0,3}\quad,\quad \hat{\tau}=\frac{\tau}{R_{D4}}\quad,\quad b=2\pi\ell_s^2B$$

- The horizon position v_T is related to the temperature by

$$v_T=\left(\frac{4\pi}{3}\hat{T}\right)^2,\qquad \hat{T}=TR_{D4}$$

Boundary conditions for the XB phase

- It is convenient to work with the Sakai-Sugimoto radial coordinate, defined by

$$\boldsymbol{v(z) = v_0 \left(1 + \frac{z^2}{v_0^2}\right)^{1/3}} .$$

- Then $\hat{\boldsymbol{\tau}}$ and $\hat{\boldsymbol{f}}_{\boldsymbol{3}}$ are odd in z whereas $\hat{\boldsymbol{f}}_{\boldsymbol{0}}$ is even in z. This leads to three non-trivial B.C. at $\boldsymbol{v_0}$.
- It turns out that $\hat{\boldsymbol{\tau}}'$ and $\hat{\boldsymbol{f}}_{\boldsymbol{3}}'$ diverge in the same way at $\boldsymbol{v_0}$ so it is convenient to define the parameter

$$\boldsymbol{\eta := v_0^{-\frac{3}{2}} \frac{\hat{f}_3'(v_0)}{\hat{\tau}'(v_0)}}$$

- At the boundary $\boldsymbol{v} = \infty$ we impose the B.C.

$$\boldsymbol{\hat{\tau}(\infty) = \frac{\ell}{2} \quad , \quad \hat{f}_0(\infty) = -\mu \ , \quad \hat{f}_3(\infty) = j}$$

where **j** will lead to a non-zero density.

Boundary conditions for the XS phase

- In the XS phase $\hat{\boldsymbol{\tau}}$ is a constant in $\boldsymbol{v}$. Then we only need to find $\hat{\boldsymbol{f}}_{\boldsymbol{0}}$ and $\hat{\boldsymbol{f}}_{\boldsymbol{3}}$.
- At the horizon $\boldsymbol{v_T}$ we impose the conditions

$$\hat{\boldsymbol{f}}_{\boldsymbol{0}}(\boldsymbol{v_T}) = \boldsymbol{0} \ , \quad \boldsymbol{Im}\left[\frac{\boldsymbol{Q_0}}{\boldsymbol{Q_2}}(\boldsymbol{v_T})\right] = \boldsymbol{0} \ .$$

- At the boundary $\boldsymbol{v} = \infty$ we impose the B.C.

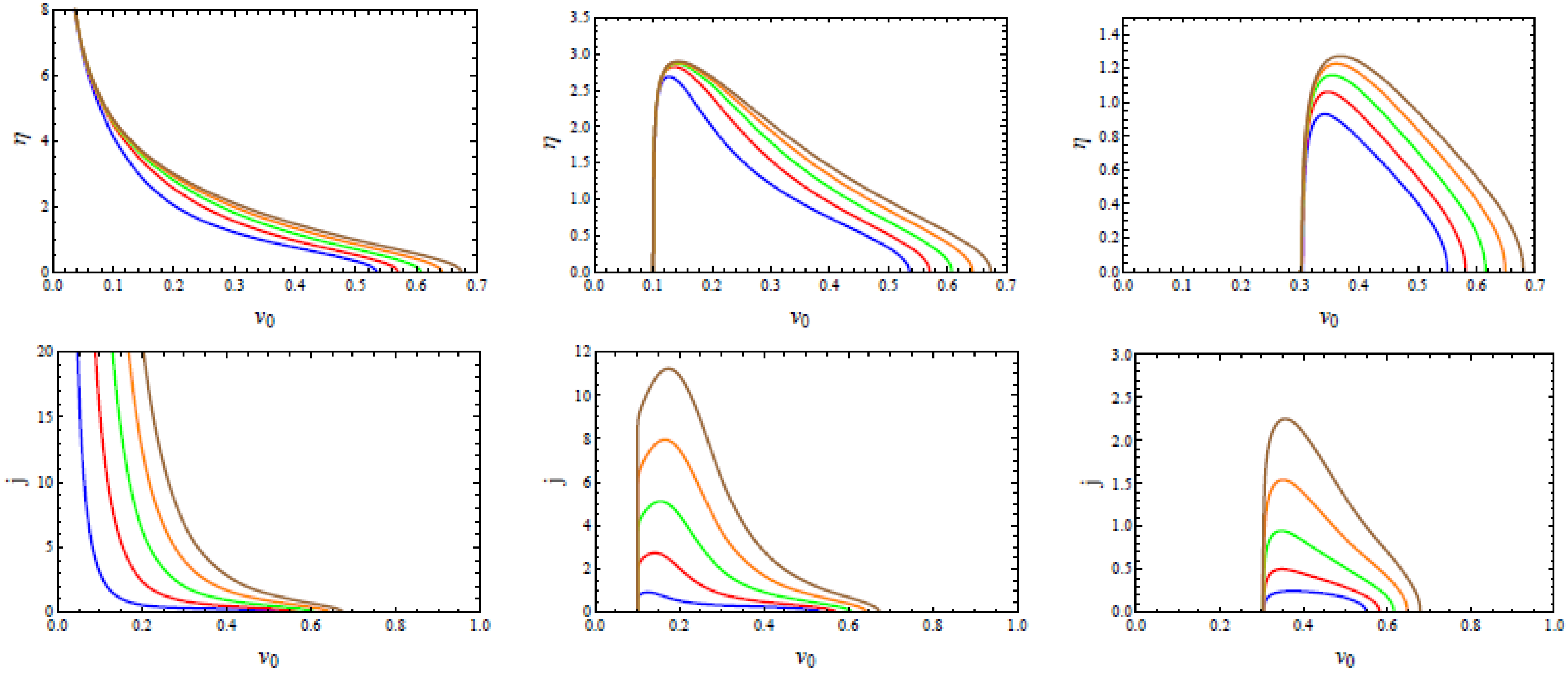
$$\hat{\boldsymbol{f}}_{\boldsymbol{0}}(\infty) = -\boldsymbol{\mu} \ , \quad \hat{\boldsymbol{f}}_{\boldsymbol{3}}(\infty) = \boldsymbol{0} \, .$$

- The XS profiles are determined by the quark density

$$\boldsymbol{\rho} \sim \boldsymbol{B}\,\boldsymbol{\mu}\,\boldsymbol{Coth}[\boldsymbol{z}_\infty] \, ,$$

where $\boldsymbol{z}_\infty$ is a convenient parameter.

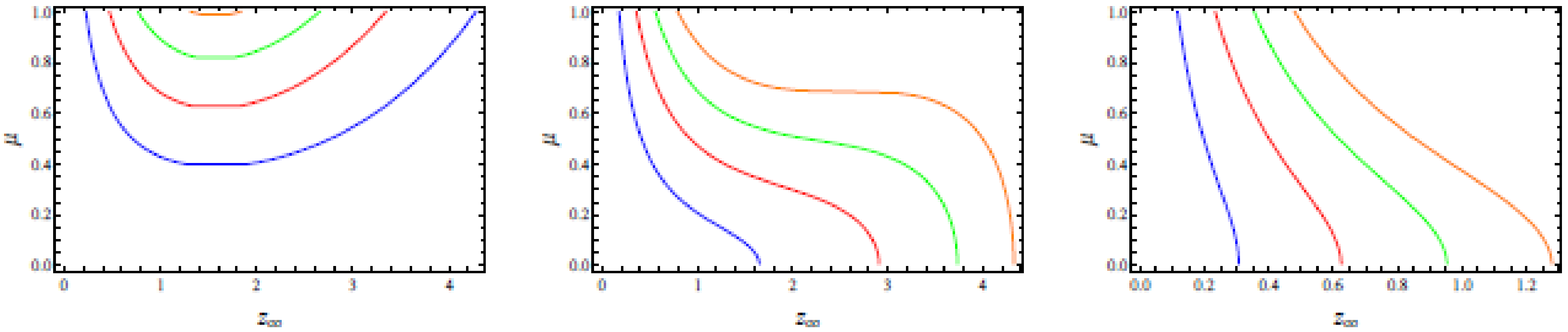
Numerical solutions : XB phase



T increases from left to right. **B** increases from blue to brown.

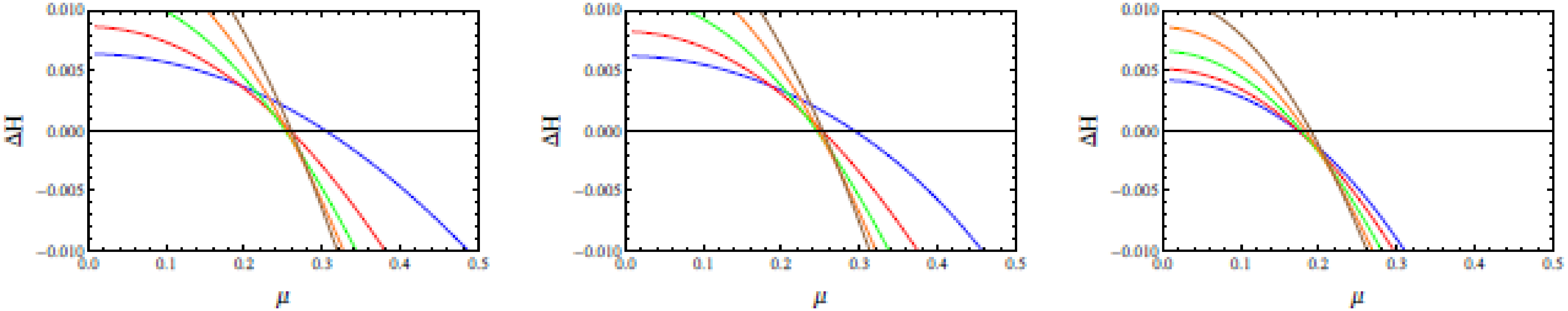
- Ground state is found by minimizing the energy with respect to $\boldsymbol{v_0}$.

Numerical solutions : XS phase



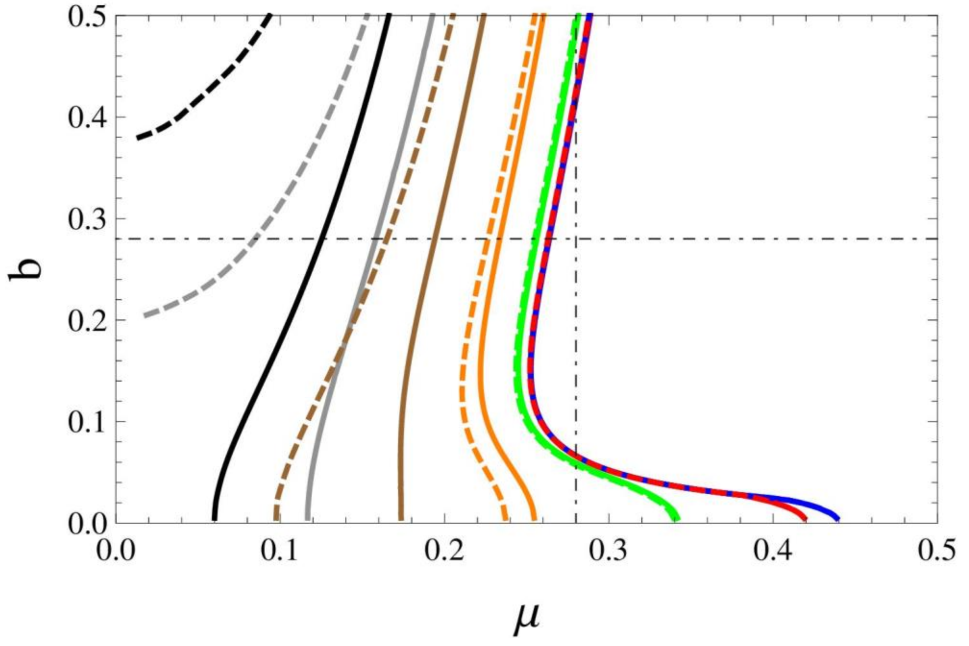
T increases from left to right. **B** increases from blue to orange. Ground state found by minimizing the energy with respect to $\boldsymbol{z_\infty}$ at fixed $\boldsymbol{\mu}$.

The chiral transition: Found from $\Delta H = H_{XS} - H_{XB}$



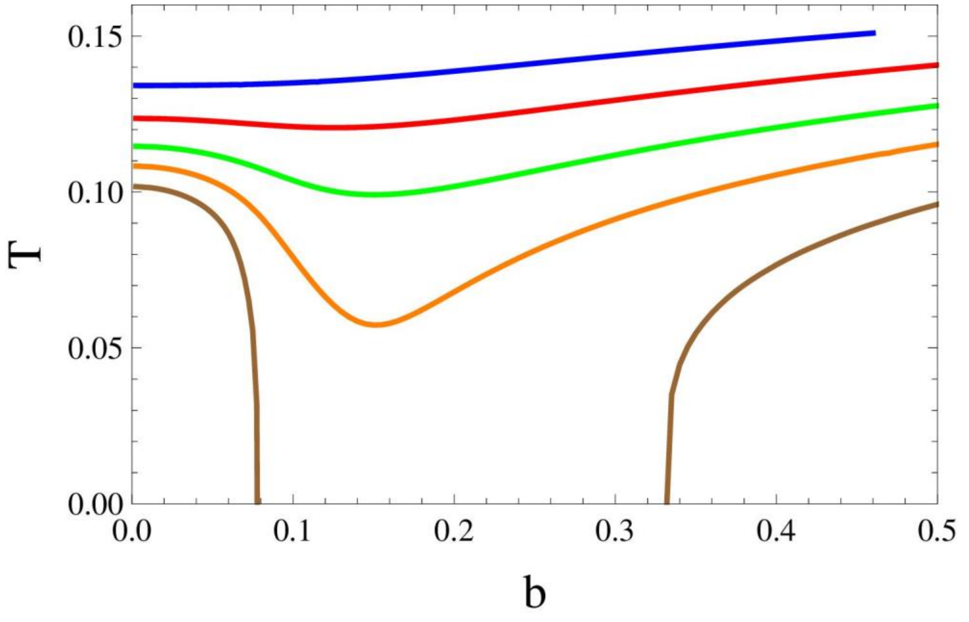
<u>**b** vs $\boldsymbol{\mu}$ phase diagram :</u>

- At low **T** there is a regime at small **b** where **IMC** dominates.
- At high **T** the **IMC** regime disappears.

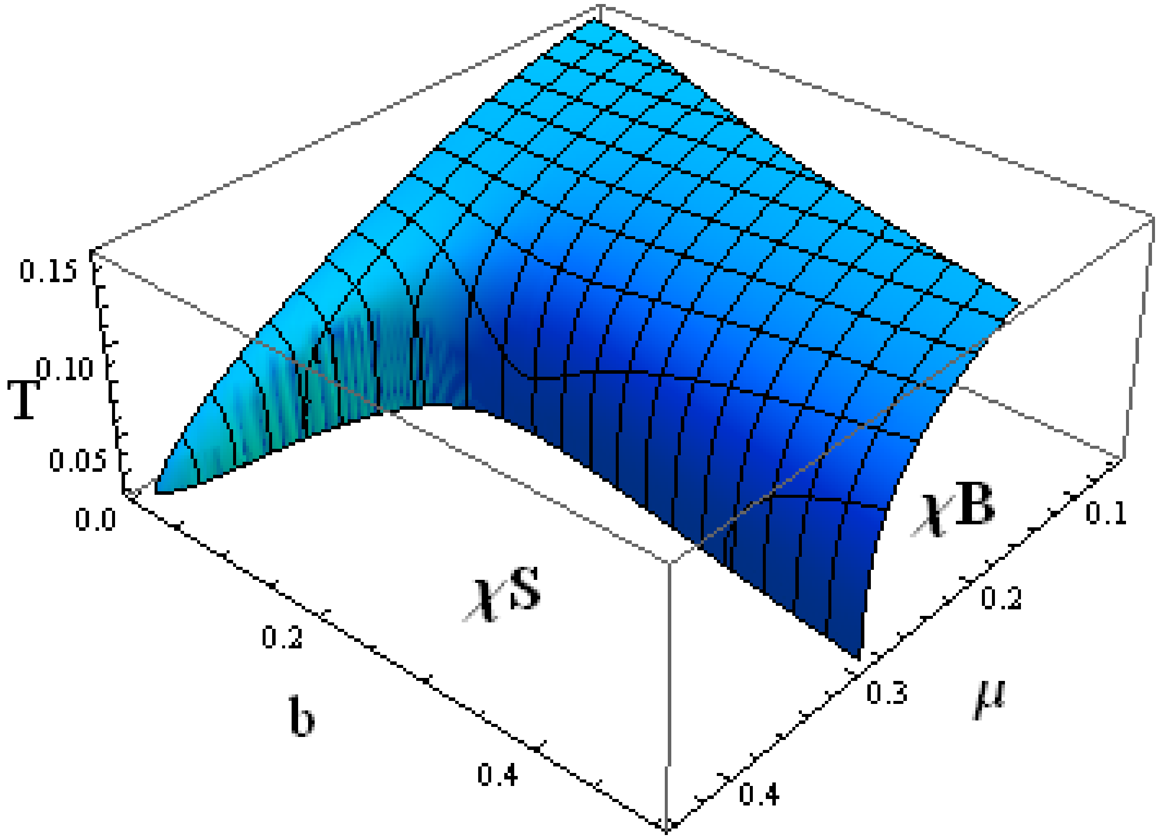


<u>**T** vs $\boldsymbol{b}$ phase diagram :</u>

- At low $\boldsymbol{\mu}$ there is only **MC**.
- As $\boldsymbol{\mu}$ increases the **IMC** regime appears small **b.**



3D phase diagram for the chiral transition

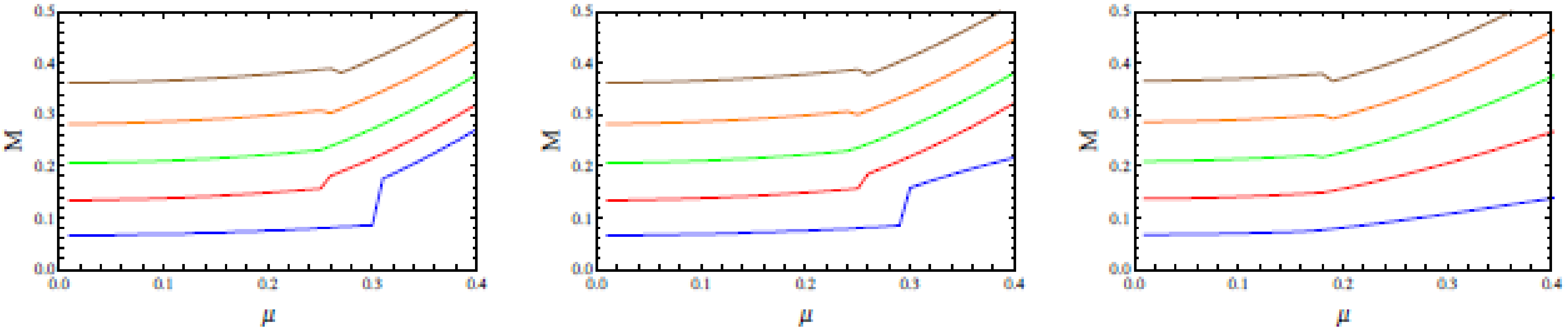


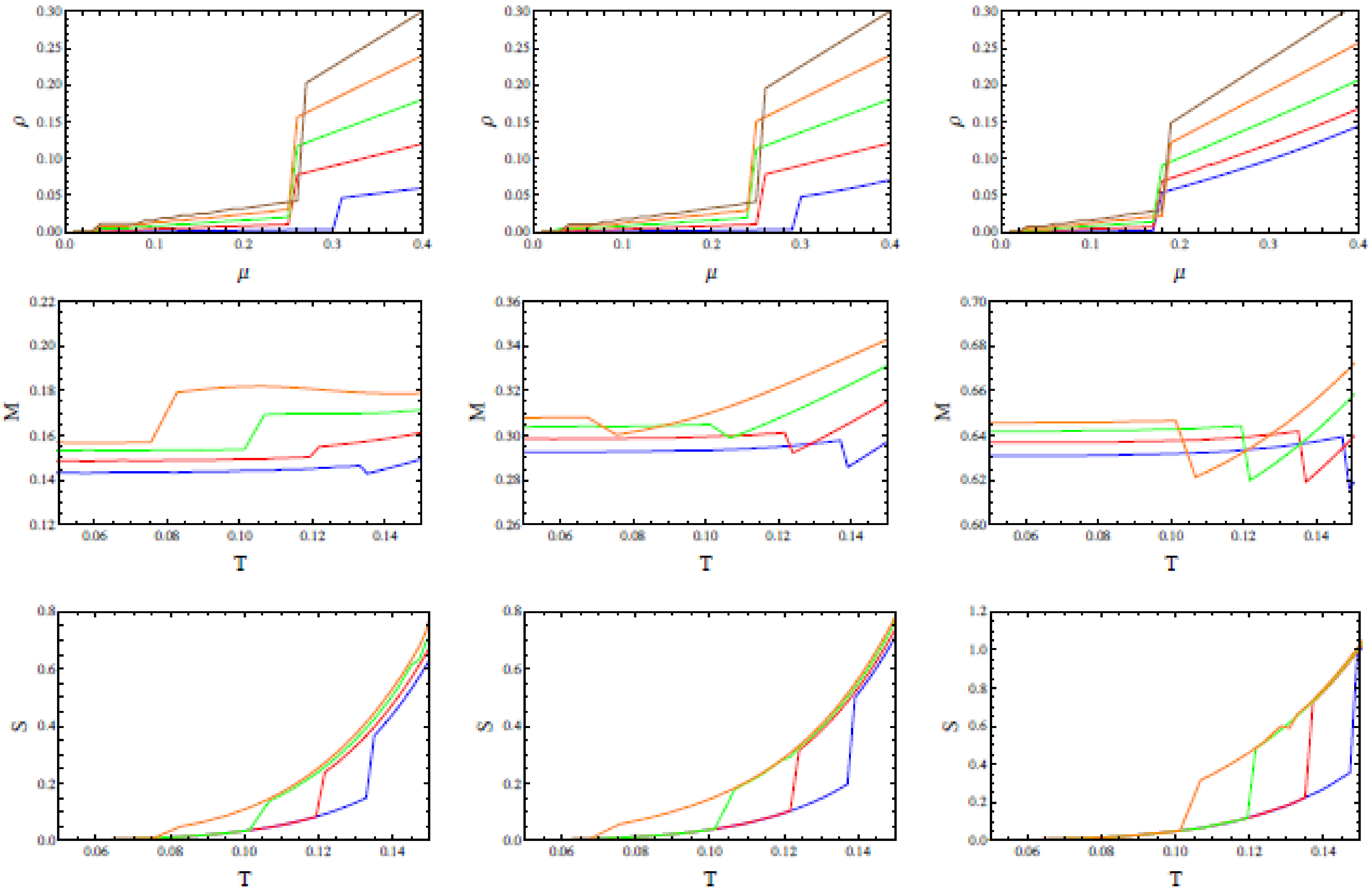
An order parameter for IMC

- The magnetisation, charge density and entropy density are defined by

$$M = -\frac{\partial H}{\partial b}\Big|_{T,\mu} \quad , \quad \rho = -\frac{\partial H}{\partial \mu}\Big|_{T,b} \quad , \quad S = -\frac{\partial H}{\partial T}\Big|_{\mu,b}$$

- Since the chiral transition is 1st order these quantities behave as order parameters.
- However, whereas $\boldsymbol{\rho}$ and $\boldsymbol{S}$ always increase at the transition **M** behaves differently in the **IMC** and **MC** regimes.




ρ
μ
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.1
0.2
0.3
0.4
M
T
0.06
0.08
0.10
0.12
0.14
0.12
0.14
0.16
0.18
0.20
0.22
0.26
0.28
0.30
0.32
0.34
0.36
0.60
0.62
0.64
0.66
0.68
0.70
S
0.0
0.2
0.4
0.6
0.8
1.0
1.2

- Consider a perturbative expansion around the critical transition, at fixed **T**,

$$\boldsymbol{H(\mu_c+\Delta\mu_c, b+\Delta \mathrm{b}) = \mathrm{H}(\mu_c, b) - \rho(\mu_c, b)\Delta\mu_c - M(\mu_c, b)\Delta \mathrm{b}}$$

- Since $(\boldsymbol{b}, \boldsymbol{\mu_c})$ and $(\boldsymbol{b}+\boldsymbol{\Delta b}, \boldsymbol{\mu_c}+\boldsymbol{\Delta\mu_c})$ are points along the critical line, the energy satisfies the relations

$$\boldsymbol{H_{\chi S}(\mu_c, b) = H_{\chi B}(\mu_c, b), \qquad H_{\chi S}(\mu_c+\Delta\mu_c, b+\Delta \mathrm{b}) = H_{\chi B}(\mu_c+\Delta\mu_c, b+\Delta \mathrm{b})}$$

- Then we find the following formula

$$\boldsymbol{\frac{\Delta\mu_c}{\Delta \mathrm{b}} = -\frac{M_{\chi S}-M_{\chi B}}{\rho_{\chi S}-\rho_{\chi B}} = -\frac{\Delta \mathrm{M}}{\Delta\rho}}$$

- Similarly, at fixed $\boldsymbol{\mu}$ we find the form

$$\boldsymbol{\frac{\Delta T_c}{\Delta \mathrm{b}} = -\frac{M_{\chi S}-M_{\chi B}}{S_{\chi S}-S_{\chi B}} = -\frac{\Delta \mathrm{M}}{\Delta S}},$$

These formulas can be also obtained from the evolution of the thermodynamic potential

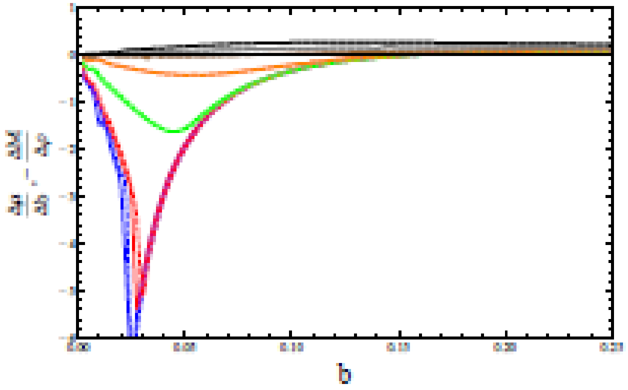
$$\boldsymbol{d\Omega = -SdT - MdB - \rho d\mu}$$

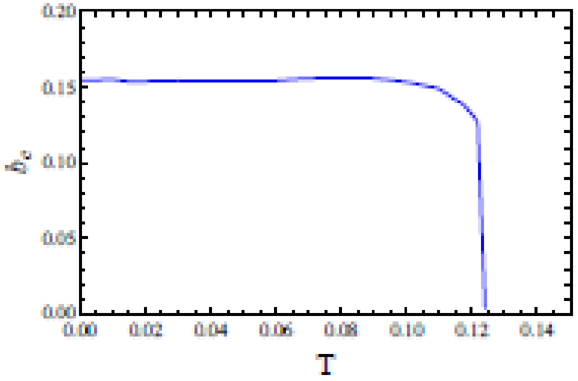
<u>Numerical check and the critical **b** where **IMC** becomes **MC**</u>

- For $\mathbf{b} < \boldsymbol{b_c}$ **IMC** occurs

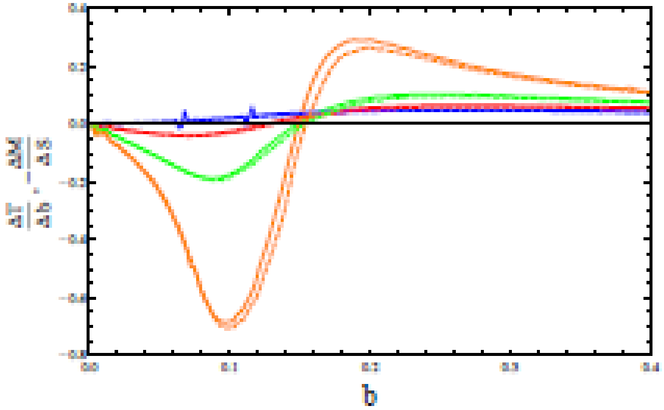
And for $\mathbf{b} > \boldsymbol{b_c}$ **MC** dominates.

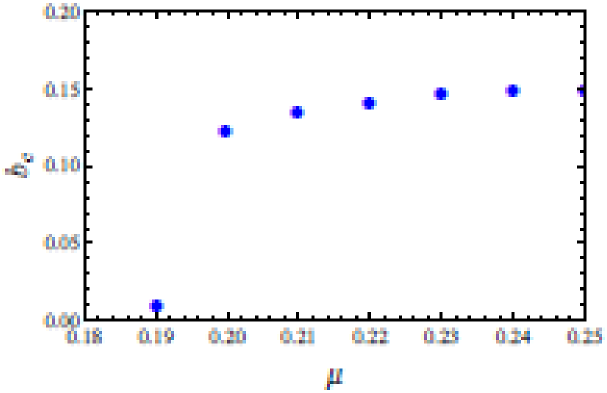
- There is critical **T** where **IMC** disappears.





- Similarly, there is a critical $\boldsymbol{\mu}$ where **IMC** is triggered.





Conclusions

- The influence of a magnetic field on QCD is a fascinating problem that has not been fully solved yet.
- Recent surprising results from lattice QCD, at $\boldsymbol{\mu = 0}$, revealed non-trivial aspects of the chiral condensate and the transition from **MC** to **IMC**.
- There is an interesting exchange between lattice QCD and non-perturbative models for QCD.
- Despite these efforts, a full description of **IMC** at $\boldsymbol{\mu = 0}$ has not been achieved yet.
- At finite $\boldsymbol{\mu}$, the results from the NJL model and holographic QCD agree qualitatively and a physical picture is emerging.
- It would be interesting to investigate how **IMC** may affect the critical point in the $(\boldsymbol{T}, \boldsymbol{\mu})$ phase diagram.