Holographic hydrodynamics of gauge theory plasma: Beyond large-N and beyond Navier-Stokes

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$\Longrightarrow$ **Motivation:** we would like to understand dynamics of quantum gauge theories in non-equilibrium setting

- Heavy ion collision experiments (QGP dynamics)
- Cosmology (early Universe, signatures of physics beyond SM)

$\Longrightarrow$ **Standard tools:**

$$\begin{array}{ccc} kinetic\ theory & pertubative\ QFT & lattice\ formulation \\ \Uparrow & \Uparrow & \Uparrow \\ \text{quasiparticle} & \text{weak coupling} & \text{static aspects} \end{array}$$

$\Longrightarrow$ **Advocate:** gauge theory/string theory (holographic) correspondence: *a tool to study quantum gauge theories at strong coupling*

There is huge literature devoted to the subject, including:

- computation of the equation of state of QGP-like theories (conformal/non-conformal)
- hydrodynamics transport coefficients (viscosities, conductivities, $\cdots$)
- hydrodynamics as an effective theory (higher order derivative expansion, resummation, effective transport coefficients)
- dynamical simulations of out-of-equilibrium (holographic) QGP plasma (quantum quenches, approach to equilibrium, turbulence)
- gauge theory dynamics in de Sitter

$\Longrightarrow$ Most (but all not) analysis are done when the holographic duality between the gauge theory and string theory reduced to the correspondence with *classical supergravity*

For this to be true:

- quantum string loop corrections must be suppressed, *i.e.*,
  - $N \to \infty$ & $g^2_{YM} \to 0$ with $N g^2_{YM} =$ const (string loop corrections $\propto \frac{1}{N^2}$)
  - $c - a \propto \frac{1}{N^2} \to 0$ at the UV fixed point of the theory
- $N g^2_{YM} = \infty$ (higher derivative corrections to 10D type IIB SUGRA $\propto (N g^2_{YM})^{-3/2}$)

$\Longrightarrow$ Recently, there has been renewed interest in exploring *conformal* holographic QGP models with $c - a \neq 0$

$\Longrightarrow$ I report on results for *non-conformal* holographic QGP models

Outline:

- Non-conformal Gauss-Bonnet (GB) holographic model
  - how does $c - a \neq 0$ come about
  - holographic renormalization, EOS, speed of sound
- Hydrodynamic transport
  - shear viscosity
  - bulk viscosity
- Homogeneous and isotropic expansion of GB QGP
  - check on bulk viscosity
  - large-order behavior of the hydrodynamic expansion
- Causality of the GB hydrodynamics
- Conclusions and future directions
  - holographic viscoelastic materials (with Matteo Baggioli)

$\Longrightarrow$ Consider RG flows close to UV fixed point, with Lagrangian density $\mathcal{L}_{CFT}$ perturbed by a relevant operator of $\mathcal{O}_\Delta$ of dimension Δ:

$$\mathcal{L} = \mathcal{L}_{CFT} + \lambda_{4-\Delta}\mathcal{O}_\Delta$$

- UV CFT has finite (non-infinitesimal) $c - a \neq 0$
- by 'close' I mean

  $$\frac{|\lambda_{4-\Delta}|}{T^{4-\Delta}} \ll 1$$

  i.e., , the effects of the conformal symmetry breaking in thermal plasma state are small.

  this is a simplifying technical assumption.

$\Longrightarrow$ It is important to emphasize that we are discussing holographic models, rather than a top-down string theory construction — in real holography is in inconsistent to be within SUGRA approximation with finite $c - a \neq 0$

$\Longrightarrow$ The reason such model are nonetheless interesting, as they allow to explore effects of microscopic causality on the hydrodynamics

$\Longrightarrow$ Gravitational holographic model:

$$\mathcal{I} = \frac{1}{2\ell_P^3} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[\mathcal{L}_{CFT} + \delta\mathcal{L}\right]$$

with

$$\mathcal{L}_{CFT} = \frac{12}{L^2} + R + \frac{\lambda_{\rm GB}}{2} L^2 \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)$$

$$\delta\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

- $\mathcal{L}_{CFT}$ is the bulk Lagrangian of the UV conformal fixed point
- $\delta\mathcal{L}$ is the conformal symmetry breaking perturbation, $\phi \leftrightarrow \mathcal{O}_\Delta$ with

$$m^2 L^2 \beta_2 = \Delta(\Delta - 4)\,, \qquad \beta_2 \equiv \frac{1}{2} + \frac{1}{2}\sqrt{1 - 4\lambda_{\rm GB}}$$

- $\lambda_{\rm GB}$ — Gauss-Bonnet coupling constant
- L — asymptotic AdS curvature radius, related to the central charge (# of UV DOF, rank of the gauge group)

$\Longrightarrow$ Encoding gauge theory parameters in the model

- $\mathcal{L}_{CFT}$:

$$\langle T^{\mu}{}_{\mu}\rangle_{CFT} = \frac{c}{16\pi^2}\ I_4 - \frac{a}{16\pi^2}E_4$$

where $\{a, c\}$ are the two central charges, and the Euler density E_4 and the square of Weyl curvature I_4,

$$E_4 = R_{\nu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2\,, \qquad I_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

In our model

$$c = \frac{\pi^2}{2^{3/2}}\,\frac{L^3}{\ell_P^3}\,(1+\sqrt{1-4\lambda_{\rm GB}})^{3/2}\,\sqrt{1-4\lambda_{\rm GB}}$$

$$a = \frac{\pi^2}{2^{3/2}}\,\frac{L^3}{\ell_P^3}\,(1+\sqrt{1-4\lambda_{\rm GB}})^{3/2}\,\left(3\sqrt{1-4\lambda_{\rm GB}}-2\right)$$

- $\delta\mathcal{L}$:

To study equilibrium thermal states of the model we use the bulk metric ansatz

$$ds_5^2 = \frac{r_h^2}{x}\left(-f_1\beta_2\ dt^2 + \sum_{i=1}^{3} dx_i^2\right) + \frac{1}{f_2}\frac{dx^2}{4x^2}\,, \qquad x \in [0,1]$$

where $x = 1$ is the AdS Schwarzschild horizon and $x \to 0_+$ is the asymptotic AdS_5 (Poincare) boundary

- r_h determines the Hawking temperature of the horizon/plasma

$$T = \frac{\kappa}{2\pi} = \frac{r_h\beta_2^{1/2}}{\pi}\ \frac{\sqrt{f_1' f_2'}}{2}\Bigg|_{x=1}$$

- asymptotically near the boundary

$$\phi = \delta_\Delta \times \begin{cases} x^{1/2} + \mathcal{O}(x^{3/2})\,, & \Delta = 3\,, \\ x\ln x + \mathcal{O}(x)\,, & \Delta = 2 \end{cases}$$

$$\lambda_{4-\Delta} = \delta_\Delta r_h^{4-\Delta} \qquad \Longleftrightarrow \qquad \mathcal{L}_{CFT} + \lambda_{4-\Delta}\mathcal{O}_\Delta$$

$\Longrightarrow$ Holographic renormalization (cut-off at $x = \epsilon$):

$$\mathcal{I} \;\rightarrow\; \mathcal{I}_{renom,cut-off} \;\equiv\; \mathcal{I}_{cut-off} \;+\; S_{GB,cut-off} \;+\; S_{c.t,cut-off}$$

- generalized Gibbons-Hawking term ($K \equiv K_{\mu}{}^{\mu}$, $J \equiv J_{\mu}{}^{\mu}$):

$$S_{GH} = -\frac{1}{\ell_P^3}\int_{\partial\mathcal{M}_5} d^4x\sqrt{-\gamma}\left[K + (\beta_2 - \beta_2^2)\left(J - 2G_{\gamma}^{\mu\nu}K_{\mu\nu}\right)\right]$$

$$K_{\mu\nu} = -\frac{1}{2}\left(\nabla_{\mu}n_{\nu} + \nabla_{\nu}n_{\mu}\right)$$

$$J_{\mu\nu} = \frac{1}{3}\left(2KK_{\mu\rho}K^{\rho}_{\nu}\right) + K_{\rho\sigma}K^{\rho\sigma}K_{\mu\nu} - 2K_{\mu\rho}K^{\rho\sigma}K_{\sigma\nu} - K^2K_{\mu\nu}\,,$$

- counter-terms:

$$S_{c.t.} = \frac{1}{\ell_P^3}\int_{\partial\mathcal{M}_5} d^4x\sqrt{-\gamma}\left[\mathcal{L}_{c.t.,CFT} + \mathcal{L}_{c.t.,\Delta}\right]$$

with (known)

$$\mathcal{L}_{c.t.,CFT} = -\left(2\beta_2^{1/2} + \beta_2^{-1/2}\right) + \left(\frac{1}{2}b_2^{3/2} - \frac{3}{4}\beta_2^{1/2}\right) R_\gamma$$
$$+ \left(\frac{1}{8}\beta_2^{5/2} - \frac{1}{16}\beta_2^{3/2}\right) \mathcal{P}_{2,\gamma} \ \ln\epsilon$$
$$\mathcal{P}_{2,\gamma} = \mathcal{P}_\gamma^{\mu\nu}\mathcal{P}_{\mu\nu,\gamma} - \left(\gamma^{\mu\nu}\mathcal{P}_{\mu\nu}\right)^2 \,, \qquad \mathcal{P}_\gamma^{\mu\nu} = R_\gamma^{\mu\nu} - \frac{1}{6}R_\gamma\gamma^{\mu\nu}$$

and (previously unknown)

$$\mathcal{L}_{c.t.,\Delta} = \begin{cases} -\frac{1}{4}\beta_2^{-1/2}\phi^2 - \frac{\beta_2^{-1/2}}{48(2\beta_2-1)}\phi^4 \ \ln\epsilon - \frac{\beta_2^{1/2}}{48}R_\gamma\phi^2 \ \ln\epsilon & , \qquad \Delta = 3\,, \\ -\frac{1}{2}\beta_2^{-1/2}\phi^2 - \frac{1}{2}\beta_2^{-1/2}\phi^2 \ \frac{1}{\ln\epsilon} & , \qquad \Delta = 2 \end{cases}$$

$\Longrightarrow$ removing the cut-off, *i.e.*, , $\epsilon \to 0$, produces finite results of physics interest

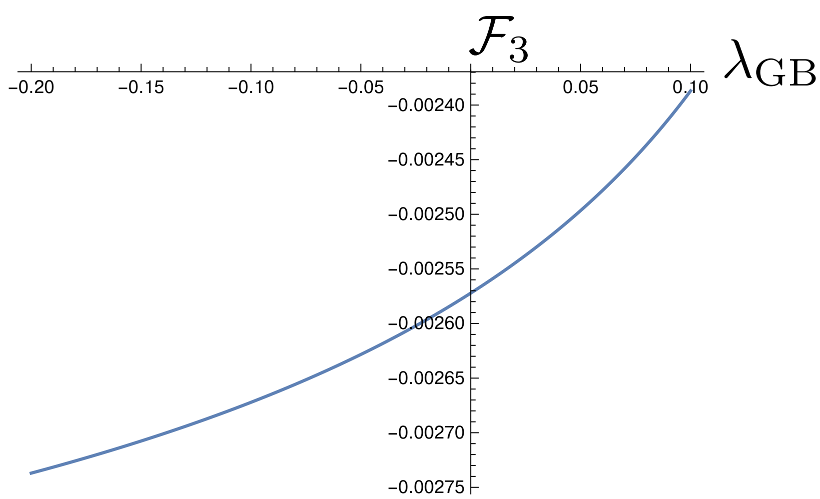
Results:

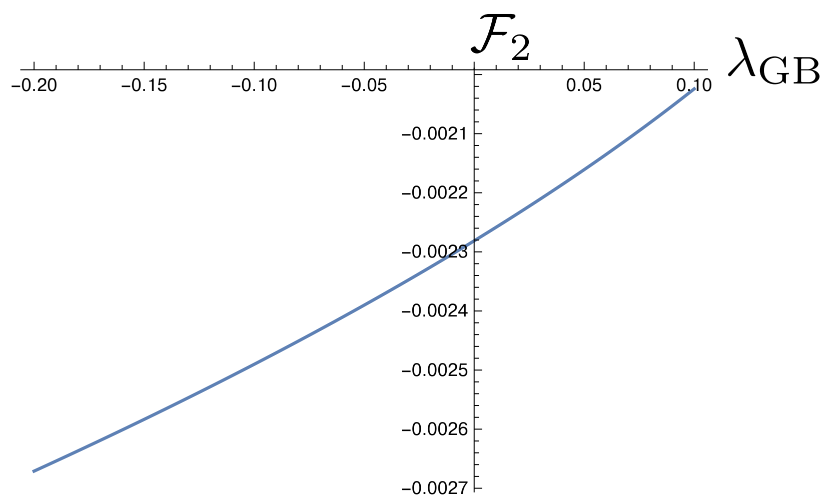
(we focus on $\Delta = \{2,3\}$ conformal symmetry breaking deformations)

- EOS

$$c_s^2 = \frac{\partial P}{\partial \mathcal{E}}$$

$$c_s^2 - \frac{1}{3} = \left(\frac{\lambda_{4-\Delta}}{T^{4-\Delta}}\right)^2 \mathcal{F}_\Delta(\lambda_{\rm GB})$$





Notice that

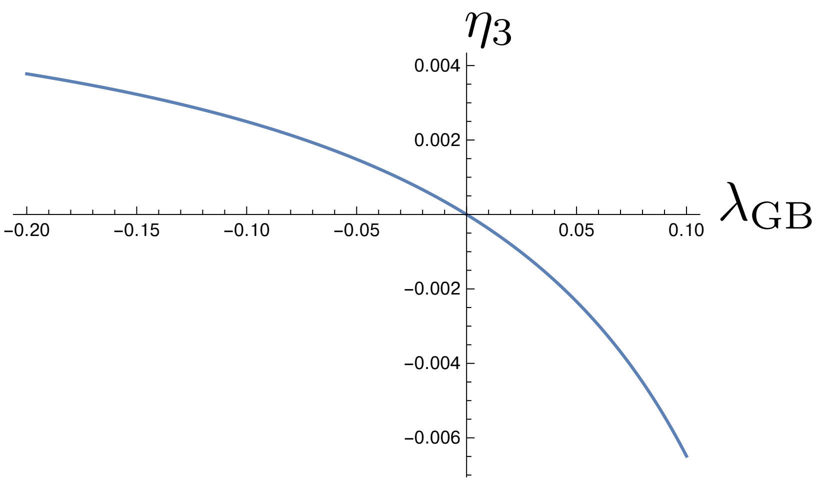
$$c_s^2 \; < \; \frac{1}{3} \; = \; c_{s,CFT}^2$$

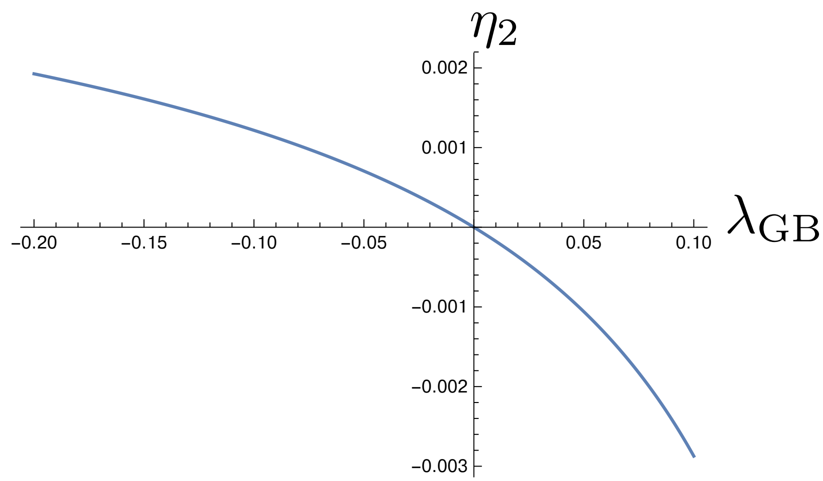
Results:

(we focus on $\Delta = \{2,3\}$ conformal symmetry breaking deformations)

- shear viscosity

$$\frac{\eta}{s} = \frac{(2\beta_2 - 1)^2}{4\pi} \left(1 + \eta_\Delta(\lambda_{\rm GB}) \left(\frac{\lambda_{4-\Delta}}{T^{4-\Delta}}\right)^2\right)$$





Notice:

$$\eta_\Delta(\lambda_{\rm GB} = 0) = 0 \qquad \Longleftrightarrow \qquad \text{universality at } a = c$$

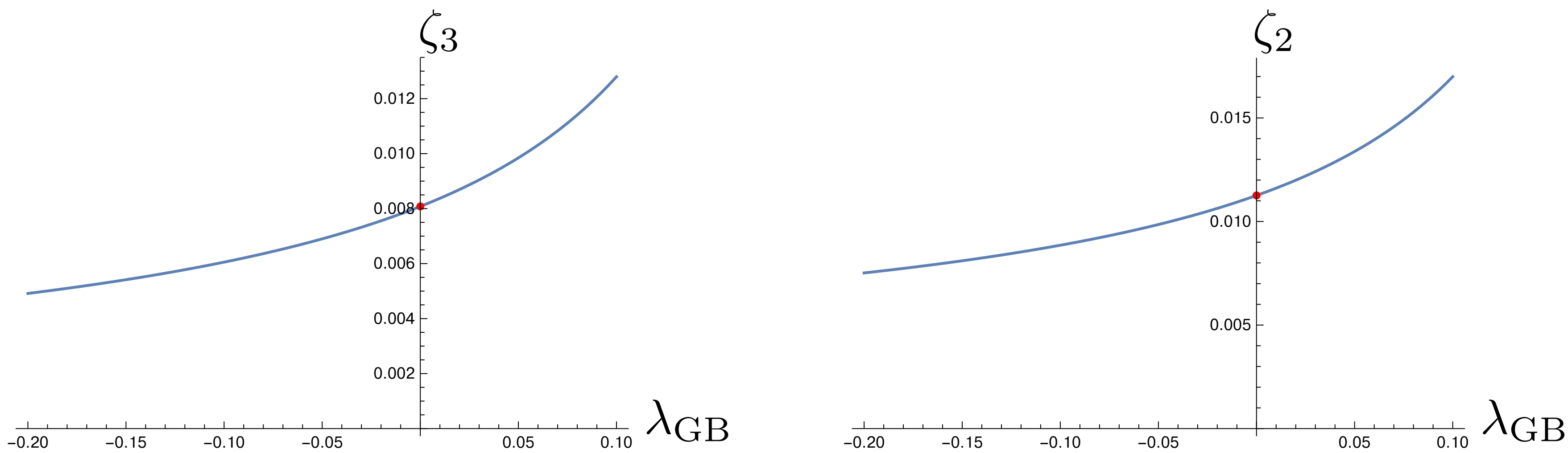
$$\frac{\eta}{s} \lesseqgtr \frac{1}{4\pi} \qquad \text{and} \qquad \frac{\eta}{s} \lesseqgtr \left.\frac{\eta}{s}\right|_{CFT}$$

Results:

(we focus on $\Delta = \{2, 3\}$ conformal symmetry breaking deformations)

- bulk viscosity

$$\frac{\zeta}{\eta} = \left(\frac{\lambda_{4-\Delta}}{T^{4-\Delta}}\right)^2 \zeta_\Delta(\lambda_{\rm GB})$$

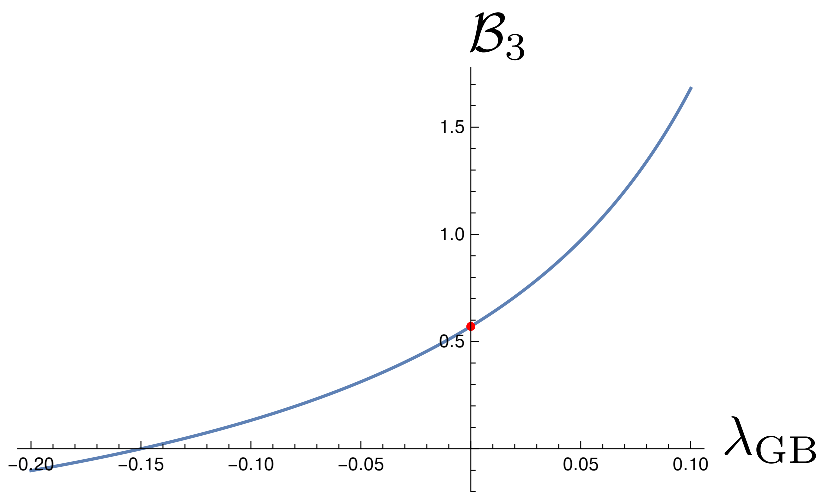


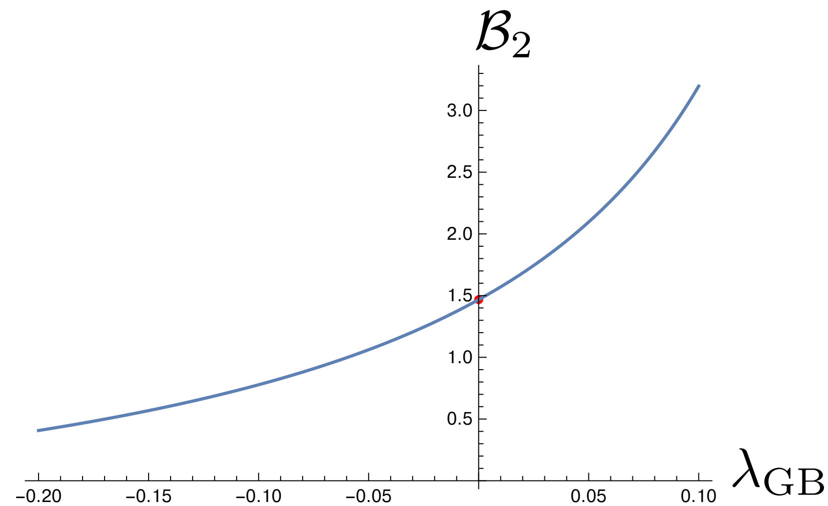
$\Longrightarrow$ Bulk viscosity bound:

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$$

$\Longrightarrow$ reparameterized bulk viscosity bound

$$\frac{\zeta}{\eta} = 2\left(\frac{1}{3} - c_s^2\right) \left(1 + \mathcal{B}_\Delta(\lambda_{\rm GB}) \right) , \qquad \mathcal{B}_\Delta \geq 0$$





- red dots demonstrate check on previously known result

$$\mathcal{B}_\Delta\Big|_{\lambda_{\rm GB}=0} = \begin{cases} \frac{\pi}{2} - 1\,, & \Delta = 3\,, \\ \frac{\pi^2}{4} - 1\,, & \Delta = 2 \end{cases}$$

- violation of bulk viscosity bound occurs for $a - c > 0 \sim \mathcal{O}(c)$; while shear viscosity bound is violated for $c - a > 0 \sim o(c)$

A question:

Why in all plots $\lambda_{\mathrm{GB}} \in (-0.2, 0.1)$*?*

The answer:

Causality of GB holographic plasma

$\Longrightarrow$ Consider a plasma at thermodynamic equilibrium. A spectrum of fluctuations in the plasma:

$$\mathfrak{w} = \mathfrak{w}(\mathfrak{q})$$

The speed with which a wave-front propagates out from a discontinuity in any initial data is governed by

$$\lim_{|\mathfrak{q}|\to\infty} \frac{\mathrm{Re}\,(\mathfrak{w})}{\mathfrak{q}} = v^{front}$$

$\Longrightarrow$ Statement of causality:

$$v^{front} \le 1$$

for <u>all</u> branches of the excitations in plasma

$\Longrightarrow$ Early studies (Hofman-Maldacena & Buchel-Myers) found that for $\mathcal{L}_{CFT}$, dual to GB gravity, causality in the bulk graviton QNM towers lead to

$$-\frac{7}{36} \le \lambda_{\rm GB} \le \frac{9}{100} \qquad \Longleftrightarrow \qquad -\frac{1}{2} \le \frac{c-a}{c} \le \frac{1}{2}$$

$\Longrightarrow$ Can this result be changed when

$$\mathcal{L}_{CFT} \ \to \ \mathcal{L} = \mathcal{L}_{CFT} + \delta\mathcal{L} \ ?$$

$\Longrightarrow$ The question of micro-causality is the question of the deep UV properties of the theory; thus one expects:

▪ breaking the scale invariance with $\Delta \le 4$ operator, should not affect the UV CFT result

▪ causality should not depend on the state of the theory, for example, the temperature compare to the coupling strength $\lambda_{4-\Delta}$.

$\Longrightarrow$ However, in principle,

- If several relevant couplings are present, causality can be affected by the dimensionless ratio of these couplings
- different channels of the fluctuations in plasma affect causality differently: the scalar channel of the bulk graviton fluctuations constraints

$$\lambda_{\rm GB} \le \lambda_{\rm GB}^{scalar} = \frac{9}{100}$$

while the shear and the sound channels constraint correspondingly:

$$\lambda_{\rm GB} \ge \lambda_{\rm GB}^{shear} = -\frac{3}{4}\,, \qquad \lambda_{\rm GB} \ge \lambda_{\rm GB}^{sound} = -\frac{7}{36}$$

- it is only the union of all the constraints that determines full causality range
- if the theory is non-conformal, additional branches of the QNMs appear which can further constraint the microscopic causality of the model.

$\Longrightarrow$ Analysis of the new towers of QNMs due to $\delta\mathcal{L}$ shows that

there are no further constrains

on λ_{GB} on top of the one provided by graviton QNM towers of $\mathcal{L}_{CFT}$

* Interplay of different relevant $\mathcal{O}_\Delta$ operators on causality is an open question

Homogeneous and isotropic expansion of GB plasma

Motivation:

- we would like to have an independent computation of the bulk viscosity;
- we would like to understand the interplay between the large-order behavior of the hydrodynamic expansion and causality

Methodology:

- put GB plasma in expanding FLRW Universe, *i.e.*, , the background metric is ($a(t)$ is the scale factor)

$$ds_4^2 = \hat{g}_{\alpha\beta} dx^\alpha dx^\beta = -dt^2 + a(t)^2 \sum_{i=1}^{3} dx_i^2$$

- In the FLRW geometry the matter expansion is locally static

$$u^\alpha = (1,0,0,0) \qquad \underline{\text{but}} \qquad \Theta \equiv \nabla_\alpha u^\alpha = 3\dot{a}/a$$

- effective hydrodynamic expansion is the series in Θ^n and $d^n/dt^n(\Theta)$; when $a(t) = \exp(Ht)$ (de Sitter), the hydrodynamic expansion is a series in H^n

- The corresponding gravitational geometry is:

$$ds_5^2 = 2dt\ \ (dr - Adt) + \Sigma^2\ \sum_{i=1}^{3} dx_i^2$$

where A, Σ, ϕ are functions of $\{t, r\}$

- AdS-boundary asymptotics encode the data:

$$\Sigma = a\ r + \mathcal{O}(r^{-1})\,, \qquad A = \frac{r^2}{2\beta_2} - \frac{\dot{a}r}{a} + \mathcal{O}(r^0)$$

$$\phi = \lambda_{4-\Delta} \begin{cases} \frac{1}{r} + \mathcal{O}\left(r^{-2}\right)\ , & \Delta = 3\,, \\ -\frac{\ln r^2}{r^2} + \mathcal{O}\left(r^{-2}\right)\ , & \Delta = 2 \end{cases}$$

- An interesting observable to focus is the *dynamical/non-equilibrium* co-moving entropy density

$$a(t)^3 s(t)$$

identified with the Bekenstein-Hawking entropy density of the apparent horizon in the bulk geometry

$$a^3 s = \frac{2\pi}{\ell_P^3}\ \Sigma^3\Big|_{r=r_h}$$

- From the holographic bulk Einstein equations, the co-moving entropy production rate is

$$\frac{d(a^3 s)}{dt} = \frac{4\pi}{\ell_P^3}\ (\Sigma^3)'\ \frac{(d_+\phi)^2}{24 - m^2\phi^2}\Big|_{r=r_h}$$

where $' \equiv \partial_r$ and $d_+ \equiv \partial_t + A\partial_r$

$\Longrightarrow$ To be specific, from now on we focus on de Sitter expansion (generalization to other $a(t)$ is straightforward)

$$a(t) = e^{Ht}\,, \qquad H \ = \ \text{constant}$$

- Contribution to the production rate in plasma of local temperature $T = \frac{T_0}{a(t)}$ from operator of dimension Δ in de-Sitter cosmology reads:

$$\frac{d(a^3 s)}{dt} = N^2 (aT)^2 \; a^{7-2\Delta} \times \Omega_\Delta^2$$

where

$$\Omega_\Delta \equiv \sum_{n=0}^{\infty} c_n(\Delta) \left(\frac{H}{T}\right)^n$$

- c_0 coefficient describes entropy production due to bulk viscosity; explicitly

$$\frac{d}{dt} \ln\left(a^3 s\right)\Big|_{hydro} \approx \frac{1}{T} (\nabla \cdot u)^2 \; \frac{\zeta}{s} = \frac{1}{T} \; (3H)^2 \; \frac{\zeta}{s}$$

- holography allows to express Ω_Δ (semi-analytically) through the behavior of ϕ at the apparent horizon

$\Longrightarrow$ Computation of Ω_Δ

- to order $\mathcal{O}(\lambda_{4-\Delta})$, the bulk geometry is known analytically:

$$A = -\frac{r\dot{a}}{a} + \frac{r^2}{4\beta_2(1-\beta_2)}\left(1 - \sqrt{(2\beta_2-1)^2 - \frac{4\beta_2(\beta_2-1)(\pi T_0)^4}{r^4 a^4}}\right)$$

$$\Sigma = ra$$

Note, apparent horizon is located at

$$r_h = \frac{\pi T_0}{a(t)}$$

so

$$r \ \in \ (r_h, +\infty) \qquad \Longleftrightarrow \qquad z \equiv \frac{\pi T_0}{ra(t)} \ \in \ (0,1)$$

- to order $\mathcal{O}(\lambda_{4-\Delta})$, the scalar field equation

$$\phi = \phi\left(t, z \equiv \frac{\pi T_0 x}{a}\right)$$

on the above geometry is

$$0 = \frac{\partial^2 \phi}{\partial z^2} + \frac{4a\beta_2(\beta_2 - 1)}{\mu(1-\sqrt{G})} \frac{\partial^2 \phi}{\partial t \partial z} + \frac{(\sqrt{G}(3-\sqrt{G}) - 2(2\beta_2 - 1)^2)}{z(\sqrt{G}-1)\sqrt{G}} \frac{\partial \phi}{\partial z}$$

$$+ \frac{6\beta_2 a(\beta_2 - 1)}{z\mu(\sqrt{G}-1)} \frac{\partial \phi}{\partial t} - \frac{2\Delta(\Delta - 4)(\beta_2 - 1)}{(\sqrt{G}-1)z^2} \phi$$

where

$$G \equiv (2\beta_2 - 1)^2 - 4z^4\beta_2(\beta_2 - 1)$$

$\Longrightarrow$ turns out scalar PDE can be organized into a series of successive (coupled) ODEs

- A general solution for ϕ can be represented as a series expansion in the successive derivatives of the FLRW boundary metric scalar factor $a(t)$:

$$\phi = \hat{\delta}_\Delta \ a^{4-\Delta} \sum_{n=0}^{\infty} \frac{\mathcal{T}_{\Delta,n}[a]}{(\pi T_0)^n} \ F_{\Delta,n}(z)\,, \qquad \hat{\delta} \equiv \frac{\lambda_{4-\Delta}}{(\pi T_0)^{4-\Delta}}\,,$$

with $\mathcal{T}_{\Delta,0} = 1$ and

$$\mathcal{T}_{\Delta,n} = \frac{1}{4}\left(a\dot{\mathcal{T}}_{\Delta,n-1} + (4-\Delta)\dot{a}\mathcal{T}_{\Delta,n-1}\right), \qquad n \geq 1$$

and

$$0 = F''_{\Delta,0} + \frac{\sqrt{G}(3-\sqrt{G}) - 2(2\beta_2-1)^2}{z(\sqrt{G}-1)\sqrt{G}} \ F'_{\Delta,0} - \frac{2\Delta(\Delta-4)(\beta_2-1)}{(\sqrt{G}-1)z^2} \ F_{\Delta,0}$$

$$0 = F''_{\Delta,n} + \frac{\sqrt{G}(3-\sqrt{G}) - 2(2\beta_2-1)^2}{z(\sqrt{G}-1)\sqrt{G}} \ F'_{\Delta,n} - \frac{2\Delta(\Delta-4)(\beta_2-1)}{(\sqrt{G}-1)z^2} \ F_{\Delta,n}$$
$$-\frac{16\beta_2(\beta_2-1)}{\sqrt{G}-1}\left(F'_{\Delta,n-1} - \frac{3}{2z}F_{\Delta,n-1}\right), \qquad n \geq 1$$

with boundary conditions

$$F_{\Delta,0} = \begin{cases} z + \mathcal{O}(z^2)\,, & \Delta = 3\,, \\ z^2 \ln z^2 + \mathcal{O}(z^2)\,, & \Delta = 2\,, \end{cases} \qquad F_{\Delta,n\geq 1} = \mathcal{O}\left(zF_{\Delta,0}\right)$$

- in de Sitter, we find analytically

$$\mathcal{T}_{\Delta,n} = \frac{\Gamma(n+4-\Delta)H^n a^n}{4^n\Gamma(4-\Delta)}\,, \qquad n \geq 0$$

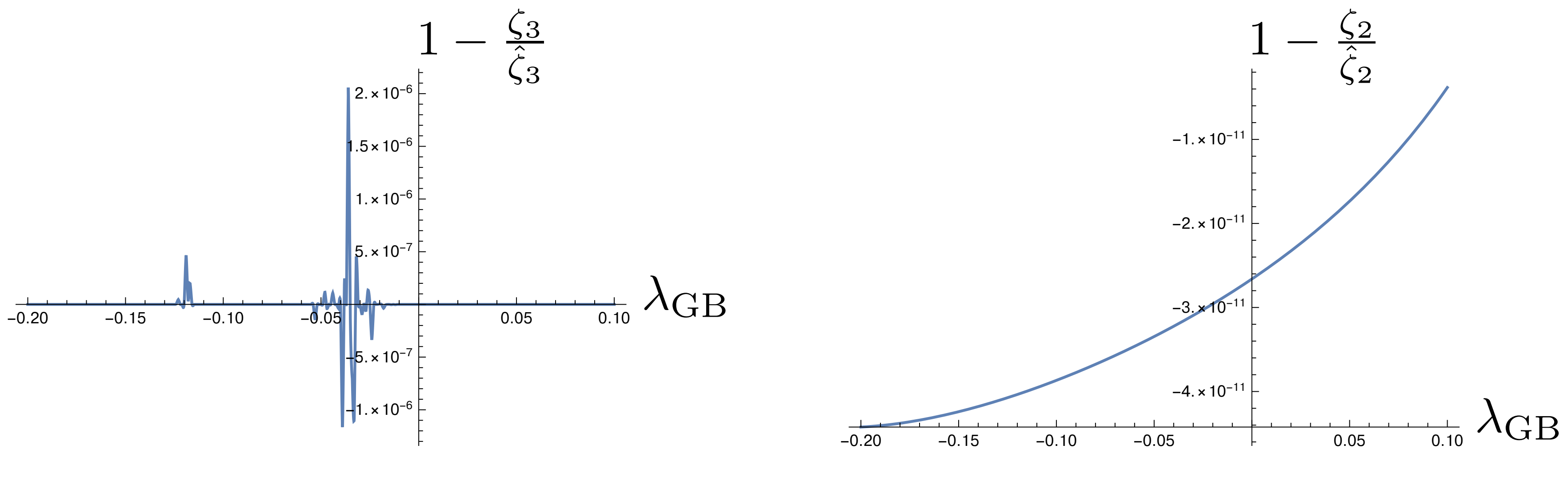
- the equations for $F_{\Delta,n}$ has to be solved numerically
- at the end of the day:

$$\Omega_\Delta = \sum_{n=0}^{\infty} c_n(\Delta)\left(\frac{H}{T}\right)^n\,, \qquad c_n = \frac{\Gamma(n+4-\Delta)H^n a^n}{(8\pi)^n\Gamma(4-\Delta)}\; F_{\Delta,n}(z\equiv 1)$$

Note that

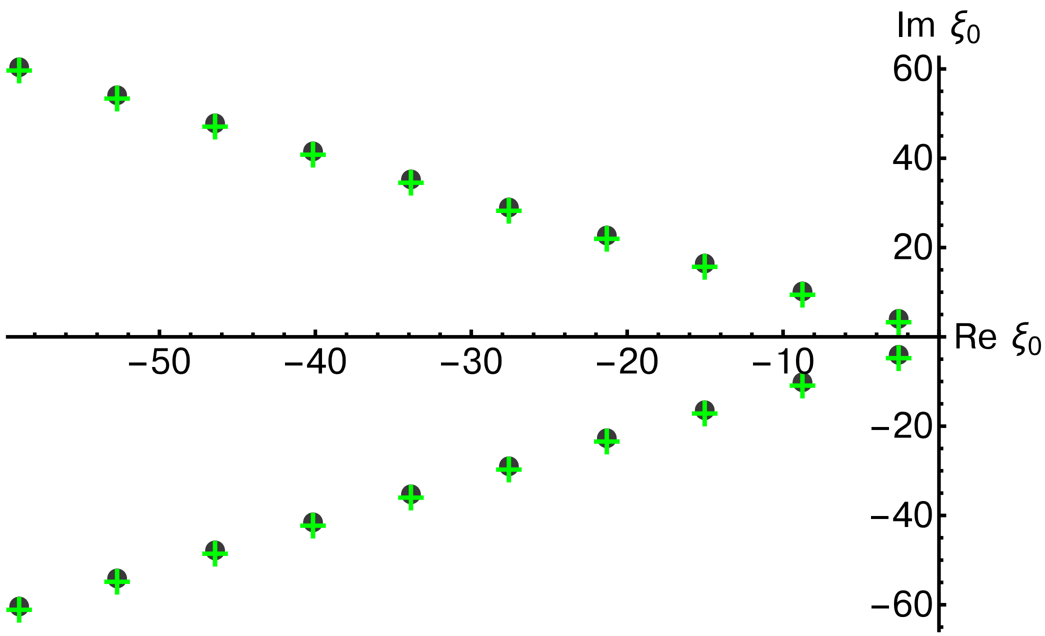
$$c_n \;\propto\; n!\; F_{\Delta,n}(1)$$

so unless $F_{\Delta,n}(1)$ dies off factorially fast (*it does not!*) hydrodynamic expansion is divergent



Comparison of the bulk viscosity coefficient ζ_Δ, extracted from the sound waves dispersion relation and the corresponding coefficient $\hat{\zeta}_\Delta$, extracted from the leading hydrodynamic contribution in the entropy production rate for the FLRW flow.

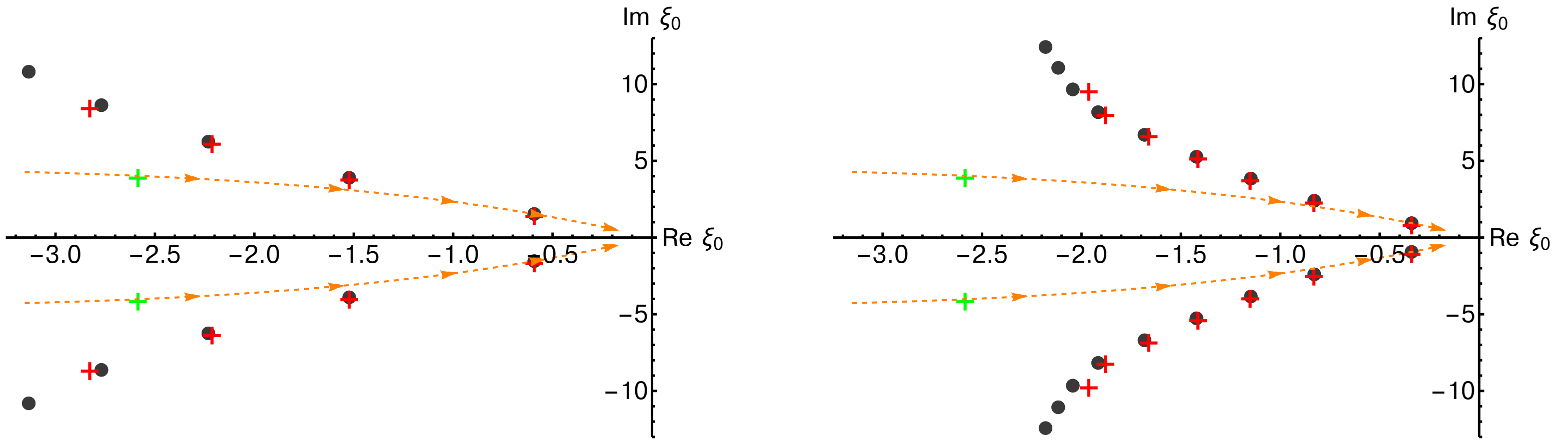
$\Longrightarrow$ Hydrodynamic expansion is Borel summable, and the Borel transform of $\Omega_\Delta(\xi \equiv \frac{H}{T}) \to \Omega_\Delta^B$ has poles at complex $\xi = \xi_0$:



QNMs and leading singularities on the Borel plane for the $\Delta = 2$ RG flow with $\beta_2 = 1$ (or $\lambda_{\rm GB} = 0$):

- filled circles — poles
- green crosses — QNMs (non-hydrodynamics modes in plasma)

What if $\lambda_{\rm GB} \neq 0$? and in particular outside causal regime?



QNMs and leading singularities on the Borel plane for the $\Delta = 2$ RG flow with $\beta_2 = 3$ (or $\lambda_{\mathrm{GB}} = -6$) (left panel) and $\beta_2 = 5$ (or $\lambda_{\mathrm{GB}} = -20$) (right panel):

- orange lines show the 'flow of QNMs' from $\lambda_{\mathrm{GB}} = 0$ to $\lambda_{\mathrm{GB}} \neq 0$ (corresponding QNMs red crosses)
- hydrodynamic expansion stays asymptotic, even when we are driven out of causal regime
- note the accumulation of poles as β_2 increases: poles $\to$ branch-cuts?

Conclusions and future directions

- please refer to the paper for some phenomenological application of the results

Work with Matteo Baggioli:

$\Longrightarrow$ from the last part of the talk:

- hydrodynamic expansion for fluids has zero radius of convergence
- the series in the derivative expansion can be Borel-resummed
- the poles in the Borel transform identify that the physical reason for the asymptotic character of the hydrodynamics are the
  non-hydrodynamic
  excitation in fluids (black brane QNMs in the dual holographic picture)

$\Longrightarrow$ this is an old story [**Michal Heller+Romuald Janik+**. . . , 2013]

$\Longrightarrow$ Now, an even older story [**Alex Buchel+Jim Sethna**, 1996]:

$\Longrightarrow$ Recall the Hooke's Law:

$$F = k\ x$$

where k is a spring constant

- Of course, if can not be a full story:

$$F = k\ x + k_2\ x^2 + k_3\ x^3 + \cdots$$

where k_i are non-linear elastic coefficients

$\Longrightarrow$ We argued that in brittle materials (those that can develop cracks under the stress), the Hooke's Law is the first term in otherwise asymptotic series, *i.e.*,

Elastic theory has zero radius of convergence

$\Longrightarrow$ Specifically,

- consider the fully non-linear in external pressure P expression for the bulk modulus K of a solid:

$$\frac{1}{K(P)} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = c_0 + c_1\ P + c_2\ P^2 + \cdots$$

- c_0 represents the Hooke's Law and $c_i\ , i \geq 1$ are higher-order coefficients
- as $n \to \infty$, for 2D elastic materials at temperature T, the crack surface tension α, Yong's modulus Y and the Poisson's ratio σ,

$$\frac{c_{n+1}}{c_n}\ \longrightarrow\ -n^{1/2}\ \left(\frac{\pi T(1-\sigma^2)}{8Y\alpha^2}\right)^{1/2}$$

or

$$c_n\ \propto\ \Gamma(\frac{n+1}{2})\ \sim\ (\frac{n}{2})!$$

$\Longrightarrow$ Elastic theory and hydrodynamics are **similar**:

- both have a well-defined effective description, akin to derivative expansion in EFT;
- both expansions are asymptotic series (gradient expansion in fluids, powers of strain expansion in solids)
- both have 'non-perturbative' effects responsible for zero radius of convergence of effective description

$\Longrightarrow$ Elastic theory and hydrodynamics are **different**:

- non-perturbative effects in hydrodynamics: non-hydro modes in plasma
- non-perturbative effects in theory of elasticity: cracks

$\Longrightarrow$ BUT solids and fluids are rather different:

- there is no shear in fluids; as a result the transverse long-wave length fluctuations are non-propagating, *i.e.*, purely dissipative:

$$\omega = -iD\, q^2$$

where D is the diffusive constant, $TD = \frac{\eta}{s}$

- on the contrary, in solids we have transverse sound waves:

$$\omega = c_\perp q\,, \qquad c_\perp^2 = \frac{\mu}{\epsilon + P}$$

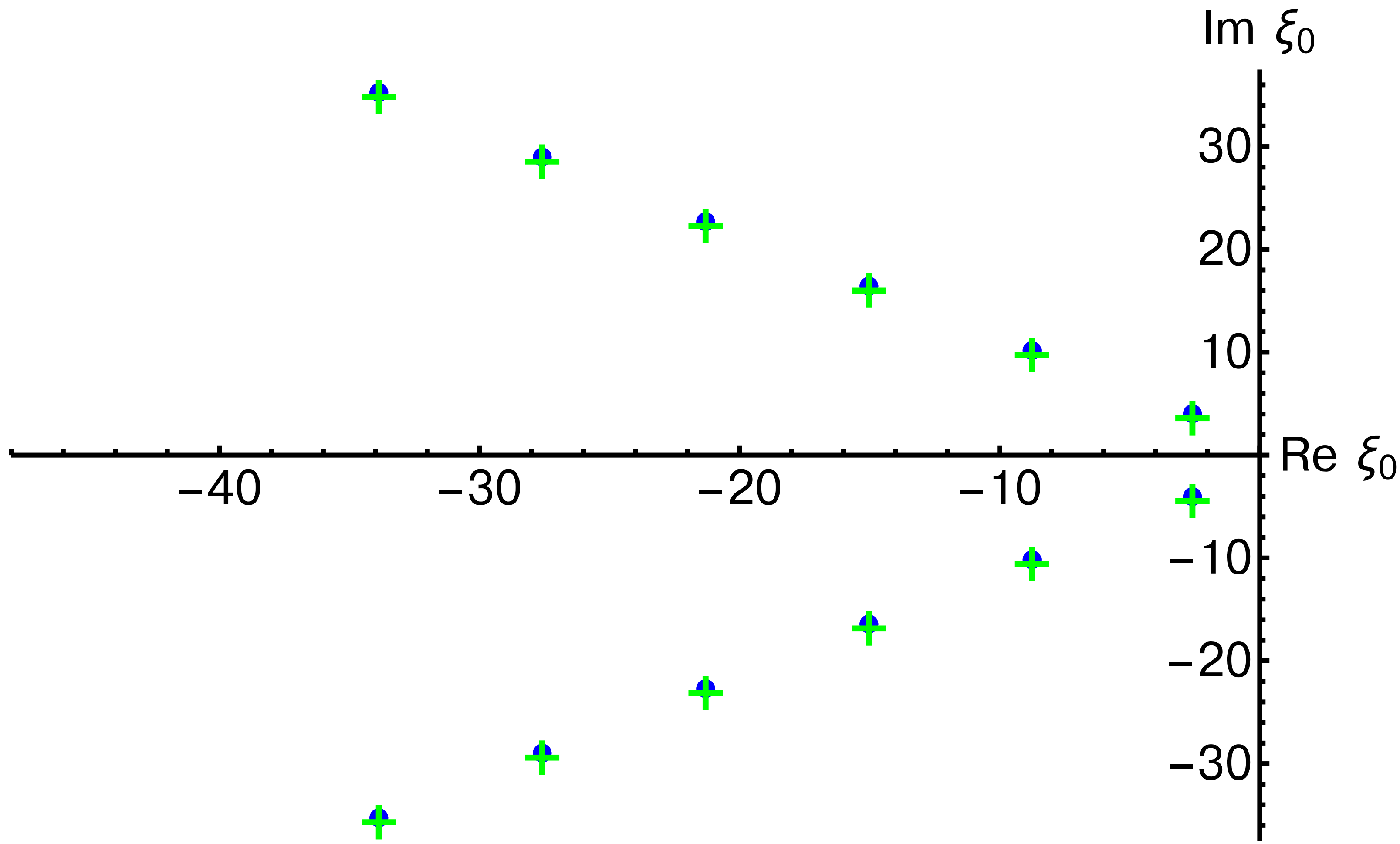
where μ is the shear elastic modulus

$\Longrightarrow$

$$\text{solids+fluids} = \text{viscoelastic materials}$$

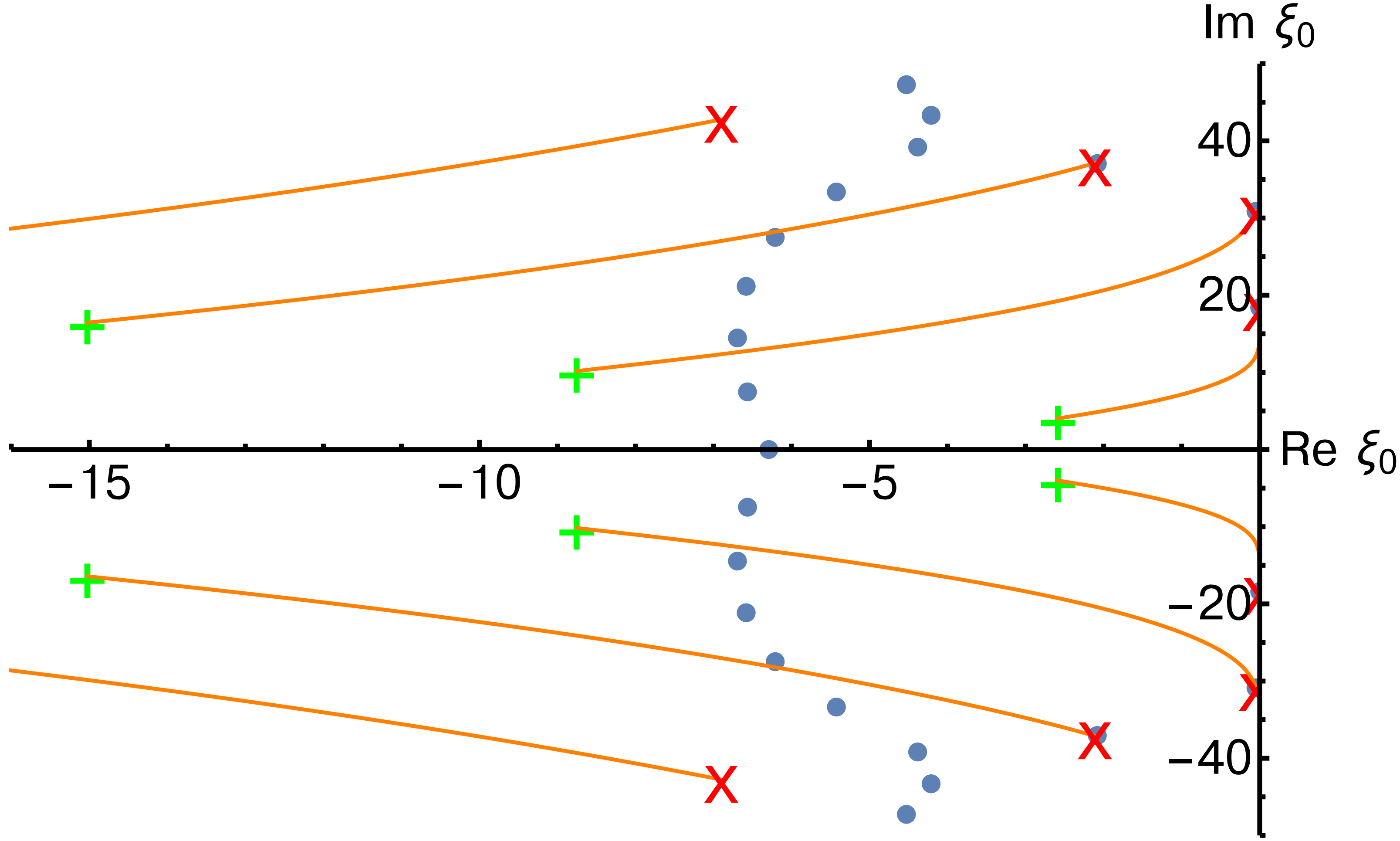
- Embed viscoelastic materials in holography
- Have a control parameter $k = \frac{1}{\text{lattice spacing}}$ that interpolates from more solid like—to—more fluid like
- study all-derivative viscoelastic hydrodynamics
- signature of holographic cracks?

$\Longrightarrow k = 0$ case (fluid)



- blue filled circles: poles of the (Pade approximation of the) Borel transform of $\Omega_{\Delta=2}$
- green crosses: Starinets-Nunez QNMs

$\Longrightarrow \frac{k}{T} = 100$ case (viscoelastic)



- red crosses: QNMs in the model at $\frac{k}{T} = 100$
- orange lines: spectral flows of QNMs from $\frac{k}{T} = 0$ to $\frac{k}{T} = 100$

$\Longrightarrow \frac{k}{T} = 1000$ case (solid)