Holography For Colour Superconductivity

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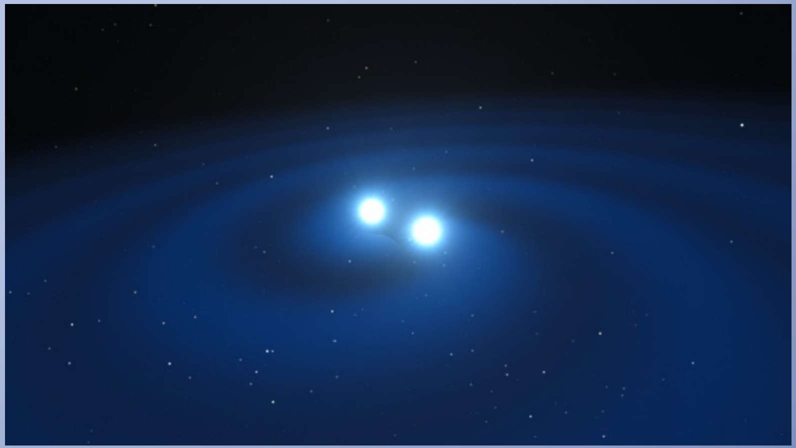
SPOLIER – we're not going to solve the problem you're thinking of – just dodge to a phenomenological description!

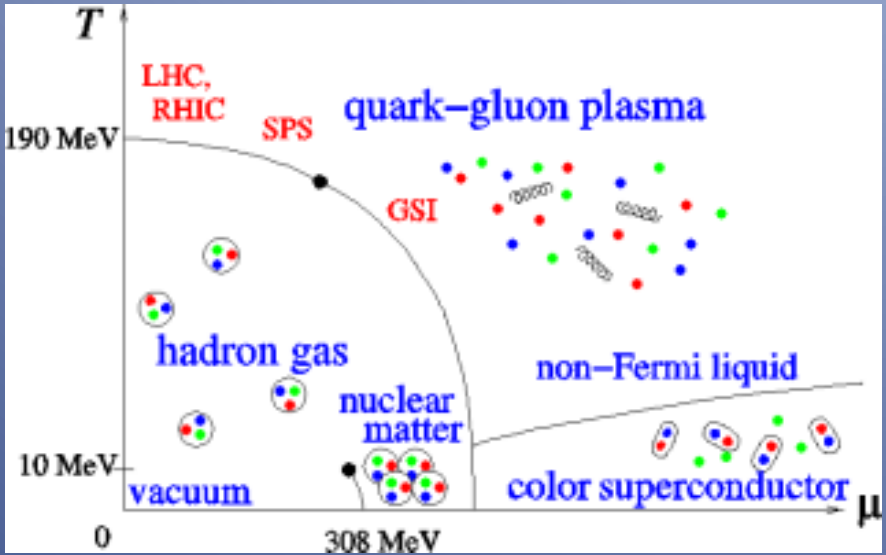
Neutron star collisions are now observable gravitationally…

Tidal deformability is beginning to allow study of the equation of state of nuclear matter…

Can holography shed light?



E. Annala, C. Ecker, C, Hoyos, N. Jokela, D. Rodriguez-Fernandez, A. Vuorinen
1603.02943, 1711.06244



CSC phase may play a role but how do we describe it?

Superconducting Instability

In the presence of a Fermi surface any attractive interaction will cause Cooper pair formation

$$\mu \bar{q} \gamma^0 q$$

A chemical potential sources fermion density

$$\langle A^0 \rangle \bar{q} \gamma^0 q$$

A vev for a temporal U(1) “number” gauge field plays the same role

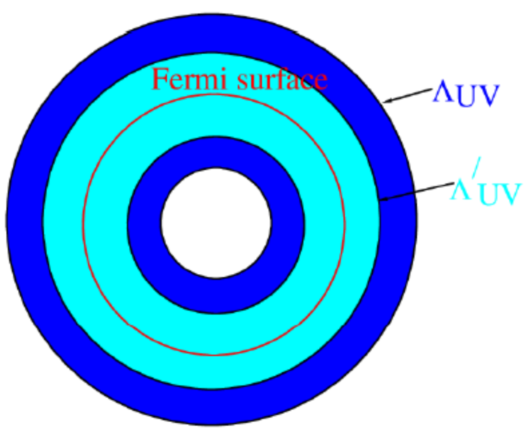
The Renormalization Group Approach

Benfatto and Gallavotti, Shankar, Polchinski, Weinberg (90-93)

Study effective field theory as flow to the fermi surface.

$p^\mu = (k_0, \vec{k} + \vec{l})$ $\quad |k_0|, |\vec{l}| < \Lambda$

Take $\Lambda \to 0$



μ Generates a Fermi surface – fermion states filled out to momentum μ

Interactions between fermions at k and –k become very strong and bind, then condense, Cooper pairs…

NE,Hsu, Schwetz hep-ph/9808444

The Simplest Holographic Superconductor

Sean Hartnoll, Chris Herzog, Gary Horowitz, arXiv:0803.3295 [hep-th]; NE, Michela Petrini, hep-th/0108052

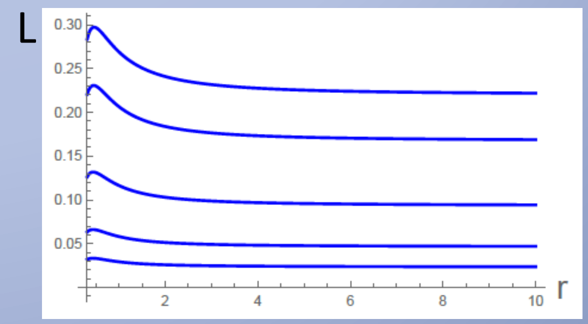
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + |\partial\psi - iBA\psi|^2 + 3\psi^2$$

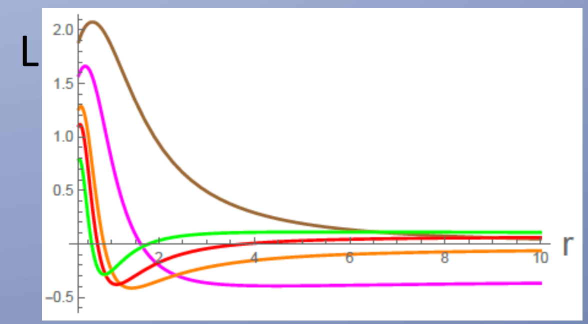
An A vev is a negative mass squared for the scalar L - describing some dimension 1/3 J & <O>…

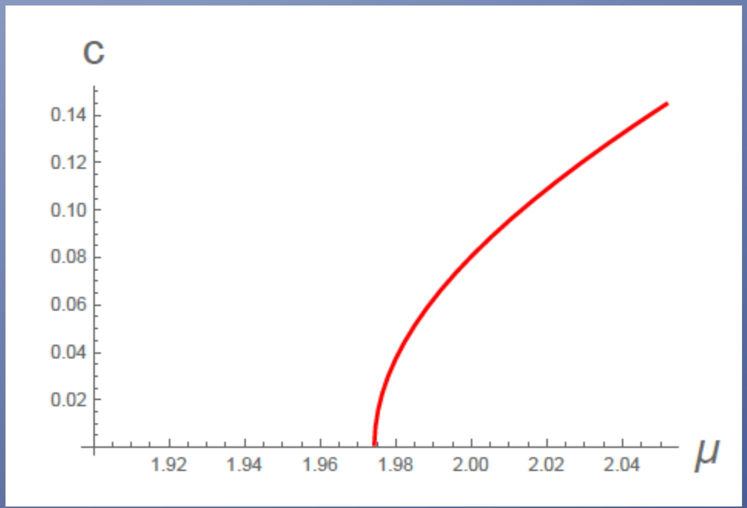
If it gets big enough it violates the BF bound and causes condensation…

The final state of the instability is unclear at T=0

If introduce a temperature by a black hole horizon in AdS a stable ground state does exist







Interpreting the Simplest Holographic Superconductor

The holographic description emerges for gauge invariant gaugino condensates in

large Nc

N=4 SYMs

At finite U(1)R density

Hartnoll et al took a leap of faith to

electron Cooper pairs

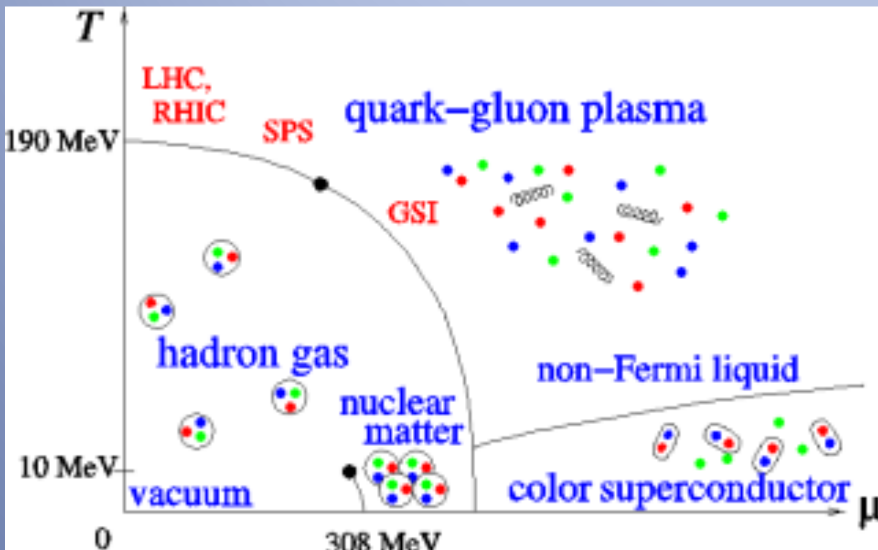
phonon interactions

At finite U(1)Lep density

Nc=1,   Nf=1

-> AdS/CM

The QCD Phase Diagram



In QCD – anti-symmetry requires

spin $\quad 2 \times 2 = \bar{1}_{AS} + 3_S$

attractive colour channel is

$$3 \times 3 = \bar{3}_{AS} + 6_S$$

so in flavour Nf=2 -> 1 AS

Nf =3 -> 3 AS

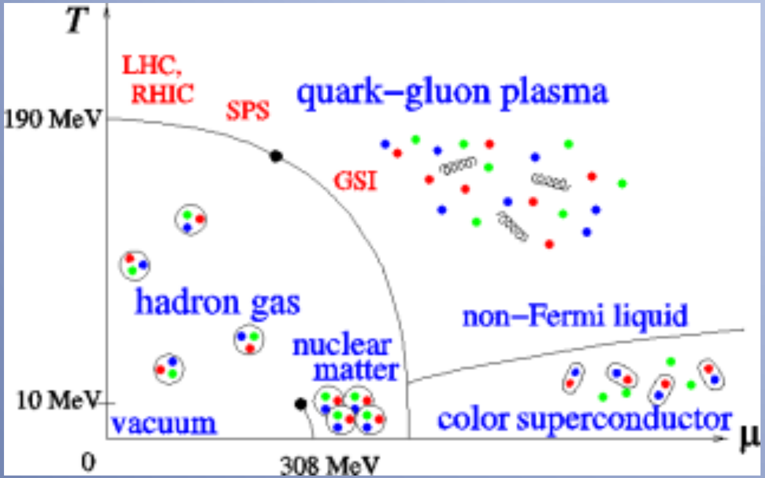
$$\begin{array}{ll|ccc|} & & \bar{R} & \bar{G} & \bar{B} \\ & & BG - GB & BR - RB & RG - GR \\ \bar{u} & sd - ds & 1 & & \\ \bar{d} & su - us & & 1 & \\ \bar{s} & ud - du & & & 1 \end{array}$$

Holographically Colour Superconductivity is beyond us though (?)…

Not there at large Nc (no clear limit either)…

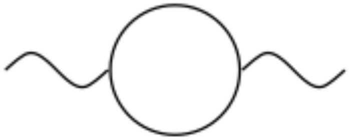
Not clear how to describe with only gauge invariant operators either…

An Argument For A New Approach to QCD Phase



Folk Lore is that gauge coupling relevant to a plasma is $g(\mu)$. Asymptotic freedom then implies at high density theory is weakly coupled (hence the deconfinement phase transition).

Perturbation theory in a plasma was studied by Freedman and McLerran (77)



A_0 acquires a mass $g\mu$

A_i is only Landau damped $g^2\mu^2|q_0|/q$

HOWEVER, close to the confined phase the QCD vacuum contains effective magnetic scalar degrees of freedom… these will gap magnetic fields also… so should we replace gluons by FOUR FERMION INTERACTIONS?

Gauge and flavour indices now all become global symmetries that can be described in the bulk…. (note although condensation breaks colour it's a very small part of gluon mass at *strong* coupling)

The holographic superconductor is already describing such a situation… CSC becomes imaginable….

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\partial\psi - iGBA\psi|^2 + 3/L^2\psi^2$$

Psi represents the vev of the di-quark condensing operator….

Only the 3bar channel has attraction…

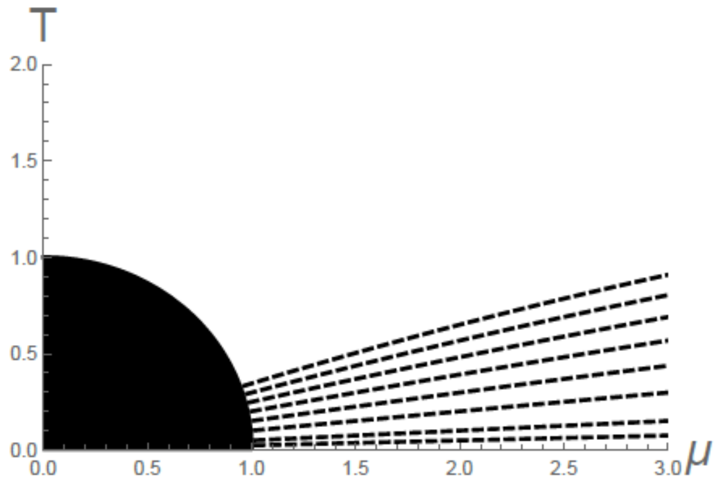


FIG. 5: Plot of the superconducting phase boundary at different $G = 0.5, 1, 2, 3, 4, 5, 6, 7$ from bottom to top in the T - μ plane. The black region is expected to be the chirally symmetric phase below a scale of $\mu^2 + T^2 = 1$.

$$G^2 = \frac{\kappa}{b\ln(T^2+\mu^2)/\Lambda_c^2}, \qquad b = 11N_c/3 - 2N_f/3,$$

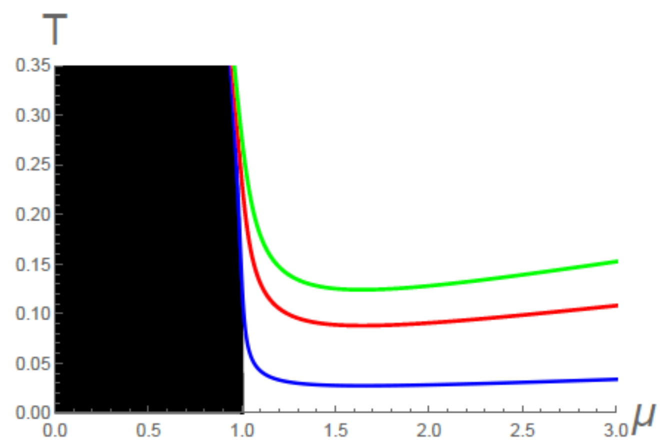


FIG. 6: QCD phase diagram: the blacked out area is below Λ_c where chiral symmetry breaking is expected. The remaining phase edges shows where the CFL phase is present for the choices of $\kappa = 1, 10, 20$ from bottom to top.

Quark Mass

$$\chi = m/r + \dots$$

$$|\chi|^2|\psi|^2$$

$$\begin{array}{ll|ccc|} & & \bar{R} & \bar{G} & \bar{B} \\ & & BG-GB & BR-RB & RG-GR \\ \hline \bar{u} & sd-ds & 1 & & \\ \bar{d} & su-us & & 1 & \\ \bar{s} & ud-du & & & 1 \end{array}$$

We're neglecting the physics of the chiral condensate…. Note now 3 distinct psis

$$\psi'' + \left(\frac{f'}{f} + \frac{5}{r}\right)\psi' + \frac{G^2B^2}{r^4 f^2}A_t^2\psi + \frac{1}{r^2 f}\left(3 - \frac{m^2}{r^2}\right)\psi = 0.$$

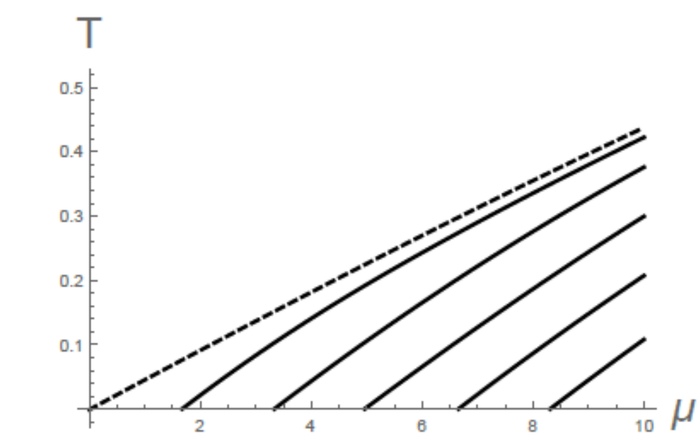


FIG. 7: Phase diagram for the model at fixed $G = 0.9$ and for quark mass $m = 0, 1.3, 2, 3, 4, 5$ from left to right

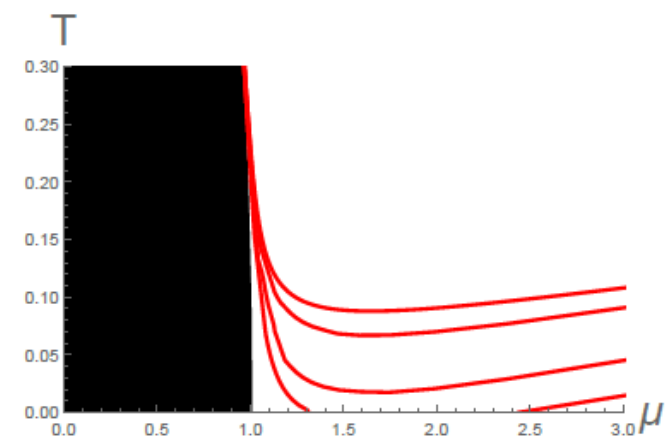


FIG. 8: QCD phase diagram with quark mass m: in the blacked out area chiral symmetry breaking is expected. The remaining phase edges shows where the CFL phase is present for the choices of $\kappa = 10$ and $m = 0, 0.5, 1.0, 1.3$ from top to bottom.

We got distracted by 4-fermion Operators (so you have to too!)

In the field theory:

$$\Delta\mathcal{L} = -\frac{g^2}{\Lambda^2}\mathcal{O}\mathcal{O},$$

$$J \simeq \frac{g^2}{\Lambda^2}\mathcal{O}.$$

$$\Delta\mathcal{L} = -\frac{\Lambda^2 J^2}{g^2}$$

Holographically Witten tells us to add a surface term at the scale L

$$\Delta\mathcal{L} = -\frac{\Lambda^4\psi^2}{g^2}$$

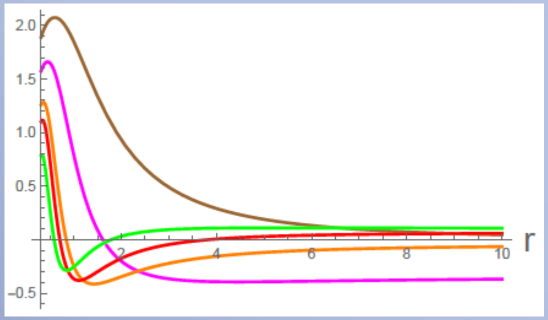
since

$$\psi \sim J/\Lambda + ..$$

This implies one imposes on the E-L solutions at L:

$$J \simeq \frac{g^2}{\Lambda^2}\mathcal{O}.$$

r psi



So we plot the regular solutions in the O vs J plane…

And re- find spiral structures..

Eg Johnson et al D3/D7 + B field

Eg Kiritsis & Jarvinen holography of the conformal window

Eg Iqbal, Liu, Mezei - condensed matter models

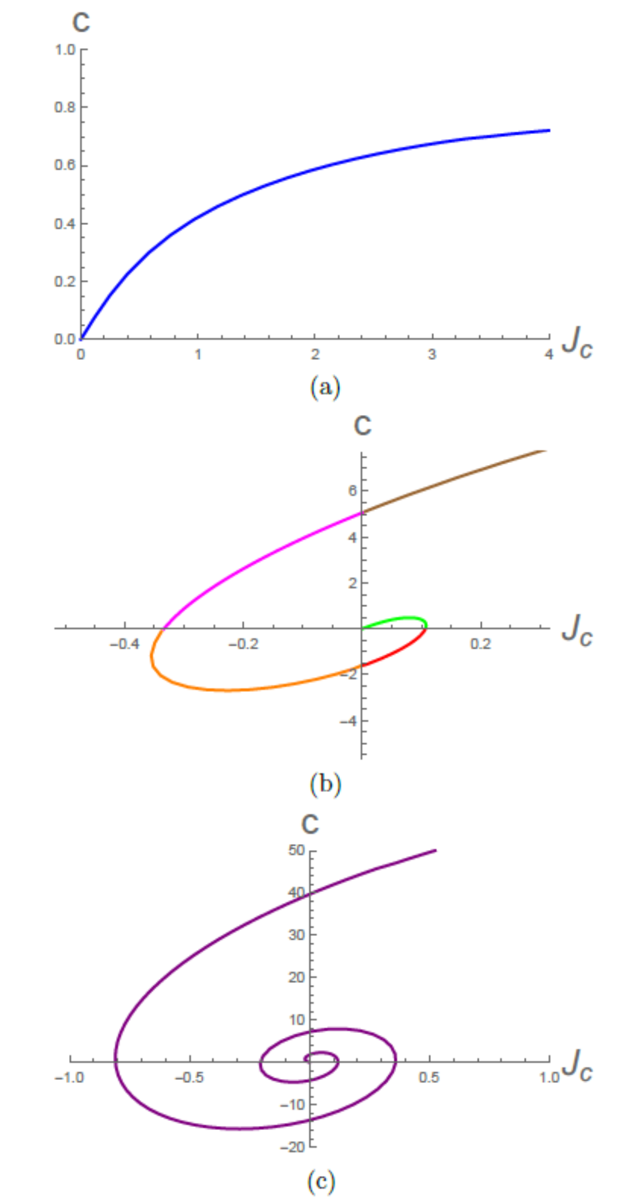
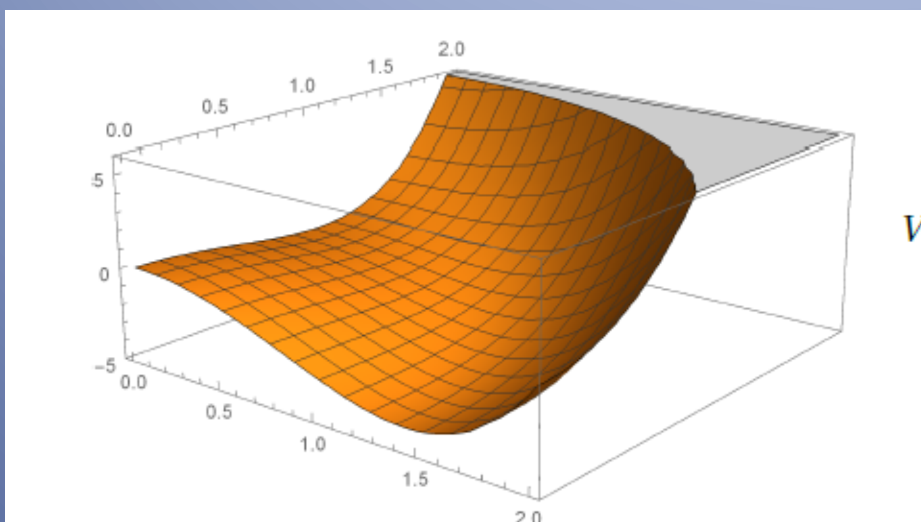


FIG. 3: c vs J_c where $T = 0.1$. (a) Unbroken phase where $\mu = 1.0$. (b): Broken phase where $\mu = 5.0$. (c) Broken phase where $\mu = 10.0$.

What's going on here?

The holographic field that describes σ also describes σ* σ ** σ***…

If the σ is unstable to condensation so are the rest of them… but the energy of these vacua increases with the number of nodes… there is no local maxima, local minima interchange.. The s is tachyonic in all but the lowest energy state

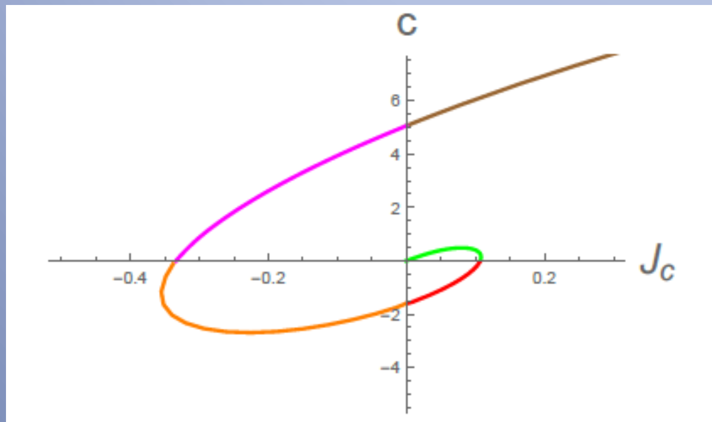


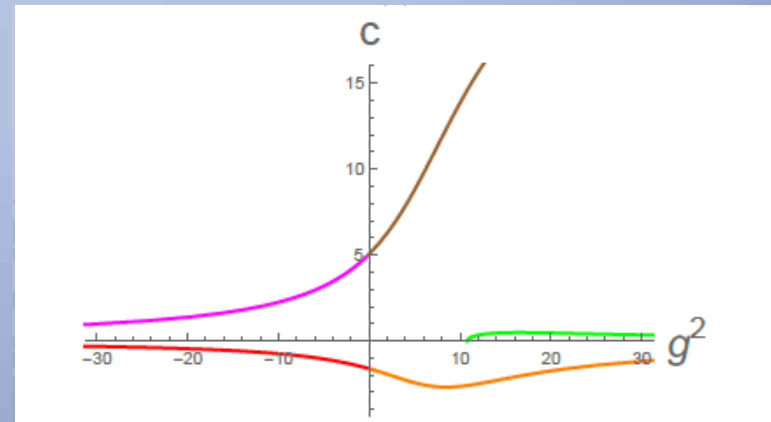
$$V_{eff} = -m_1^2|\sigma|^2 - m_2^2|\sigma_*|^2 + \lambda(|\sigma|^2 + |\sigma_*|^2)^2 + \ldots$$

This seems to be fairly generic in symmetry breaking set ups in holography…

Can anyone engineer these states to be metastable??

Impose $$J \simeq \frac{g^2}{\Lambda^2}\mathcal{O}$$





An attractive 4-Fermi term increases the condensate…

A repulsive one decreases condensation but an infinitely repulsive force is needed to switch off the condensation… actually here we are seeing the interaction at a local maxima of the potential so it's a bit unclear…

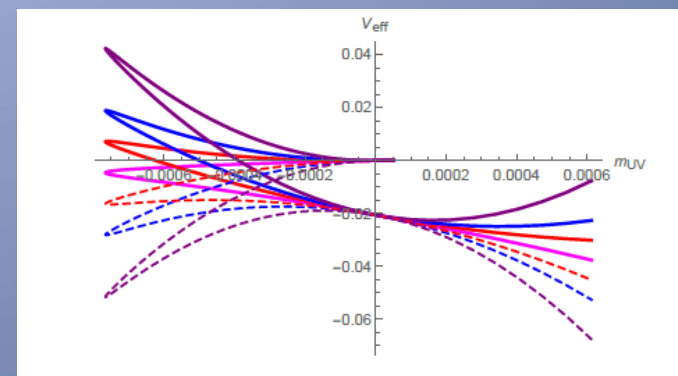
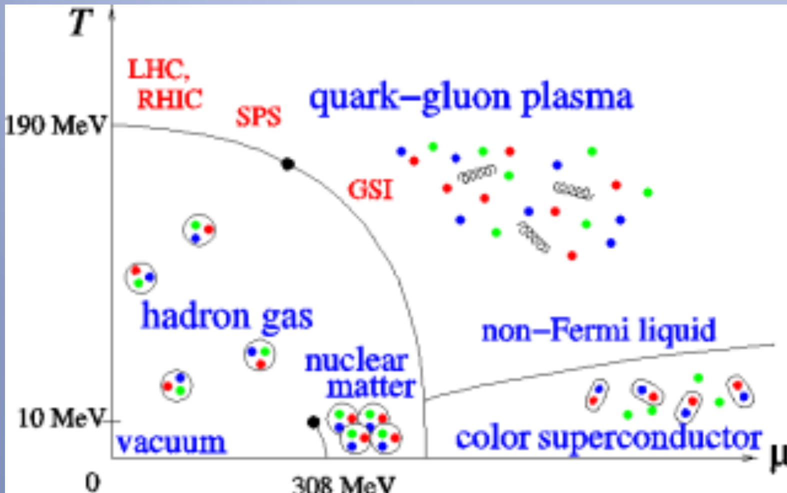


FIG. 6: Effective potential with attractive and repulsive NJL.

The holographic interaction is more complex? Anyway we abandon NJL operators…

Ongoing Work Towards Neutron Star Structure



We would like a full model of the chiral dynamics and the CSC instability…

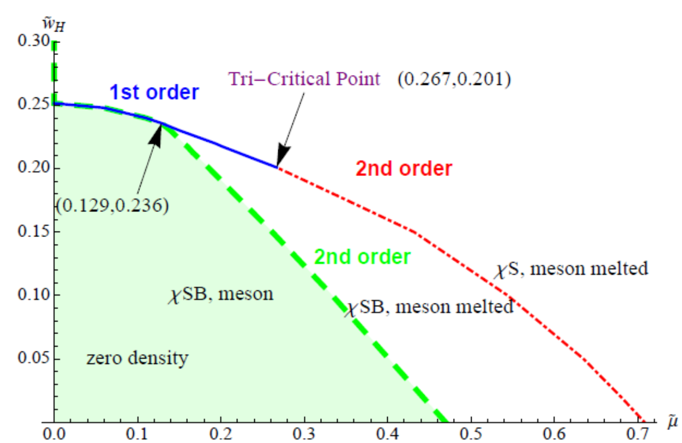
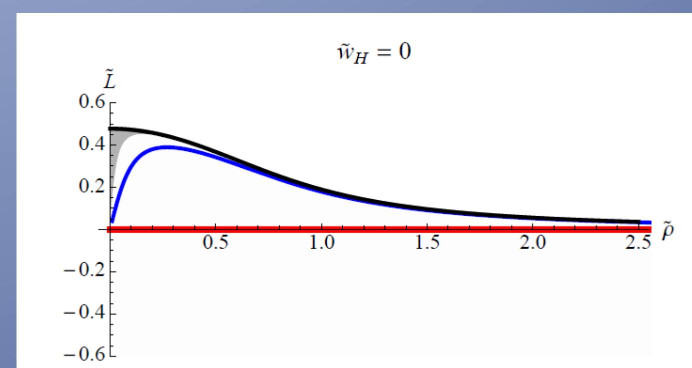


FIG. 5: The phase diagram of the $\mathcal{N} = 2$ gauge theory with a magnetic field. The temperature is controlled by the parameter $\tilde{w}_H$ and chemical potential by $\tilde{\mu}$. (Parameters are scaled or $B = 1/2R^2$ in terms of parameters without tilde.)

Nuclear matter..

Chirally broken quark matter?



NE, Gebauer, Kim,Magou 1002.1885 [hep-th]

TOV equations

$$P = -\mathfrak{F}, \qquad \mathcal{E} = \mu \frac{\partial P}{\partial \mu} - P$$

for M vs R relations
for neutron stars..

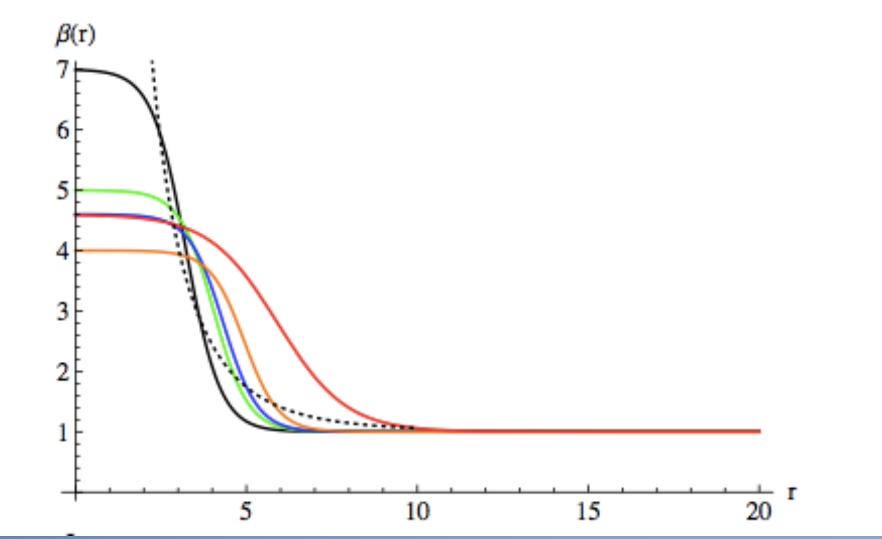
$$\frac{dP}{dR} = -G\,(\mathcal{E} + P)\,\frac{M + 4\pi R^3 P}{R(R - 2GM)},$$
$$\frac{dM}{dR} = \frac{M + 4\pi R^3 P}{R(R - 2GM)}$$

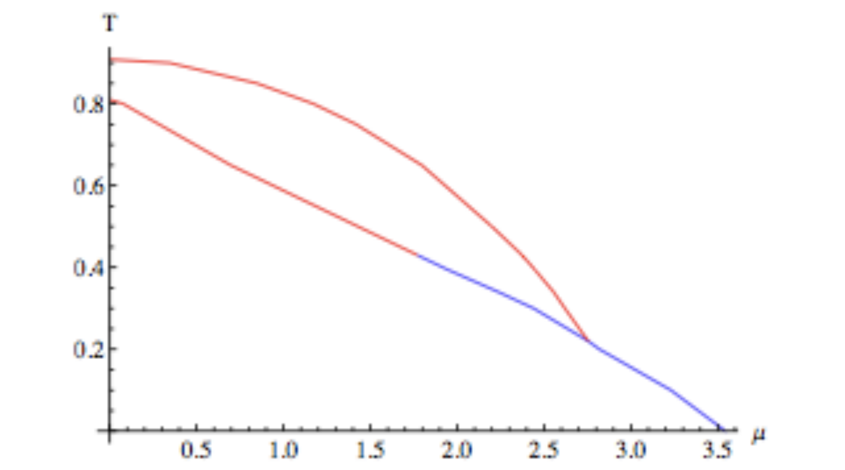
At T=0 ignore glue.. Quarks give…

$$\mathfrak{F} = a\mu^4 + b\mu^2 m^2 + c\mu^2\Sigma^2 + d\mu^2\Delta^2 + e$$

Interplay of first and second order transitions?….

$$S_{D7} = -T \int d^4x d\rho\, \rho^3 e^{\phi} \sqrt{1 + (\partial_\rho L)^2}$$





(c)$\alpha = 3, A = 3, \Gamma = 1, \lambda = 1.715$

A first order chiral restoration transition can be cooked in holography…

arXiv:1109.2633 [hep-th]

Quark Mass Re-cap

$$\chi = m/r + \ldots$$

$$|\chi|^2|\psi|^2$$

$$\begin{array}{ll|ccc|}
 & & \bar{R} & \bar{G} & \bar{B} \\
 & & BG-GB & BR-RB & RG-GR \\
\bar{u} & sd-ds & 1 & & \\
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\bar{s} & ud-du & & & 1
\end{array}$$

We're neglecting the physics of the chiral condensate…

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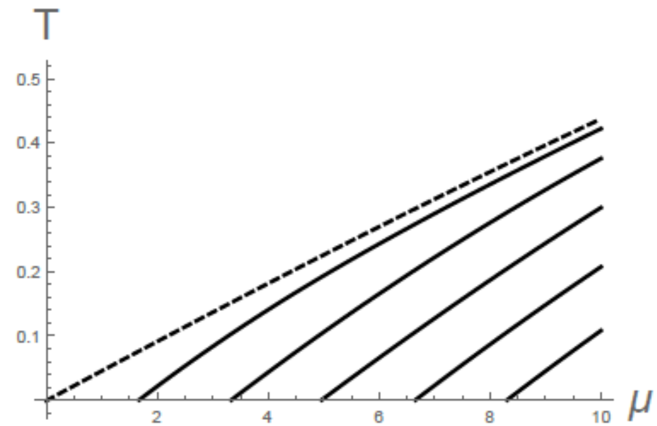

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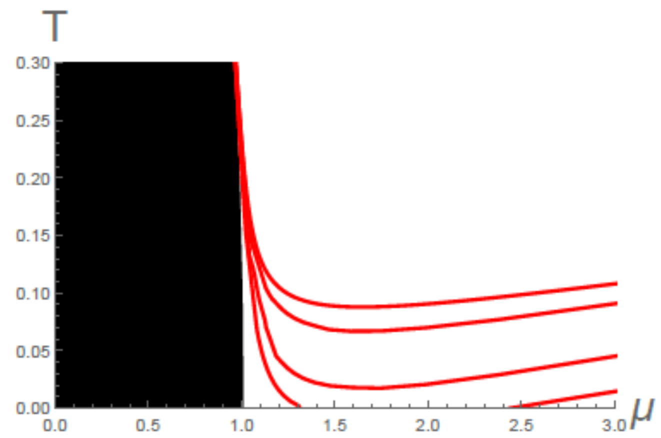

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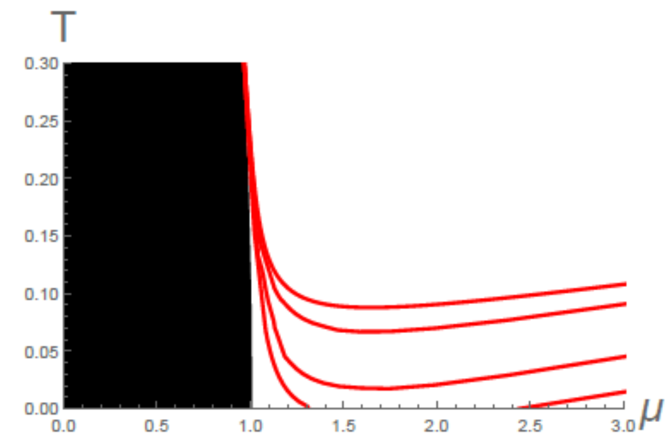
Frustration

The normal CSC descriptions say that when the s and d chemical potentials separate more than Δ then a first order transition will switch it off…

Can anyone describe that in holographic (non-abelian action)???



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Summary

There's a sensible holographic programme here.. Lets do better models eg with

back reaction,
non-abelian DBI,
transport computations…

What can we say about neutron star collision?