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Top-down holography and the Chern-Simons diffusion rate

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based on
FB, A.L. Cotrone, F. Porri, 1804.09942

Plan

- The Chern-Simons diffusion rate
- Holographic derivation
- Anomaly induced effects

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- The Chern-Simons diffusion rate
- Holographic derivation
- Incuding the anomaly

A hard to compute real-time observable relevant e.g. for the chiral magnetic effect

The Chern-Simons diffusion rate

$$\Gamma_{CS} = \frac{\langle (\Delta N_{CS})^2 \rangle}{Vt} = \int d^4x \langle Q(x) Q(0) \rangle_W$$

- Wightman two point function (real time)
- $Q = \frac{1}{16\pi^2} \mathrm{Tr} F \tilde{F}$ topological charge density in Yang-Mills
- $\Delta N_{CS} = \int d^4x \, Q(x)$ change in Chern-Simons number

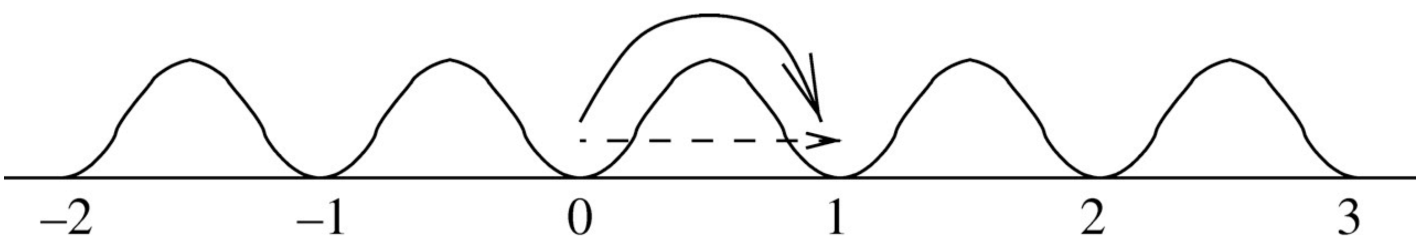
The Chern-Simons diffusion rate

- In equilibrium state, temperature T, fluctuation-dissipation thm:

$$\Gamma_{CS} = -\lim_{\omega \to 0} \frac{2T}{\omega} \mathrm{Im} G_R(\omega, \vec{k} = 0)$$

- Kubo formula for Γ_{CS} : compute retarded correlator of Q

- In QCD plasma thermal fluctuations can excite sphalerons. Sphaleron decay induces local variation of Chern-Simons number

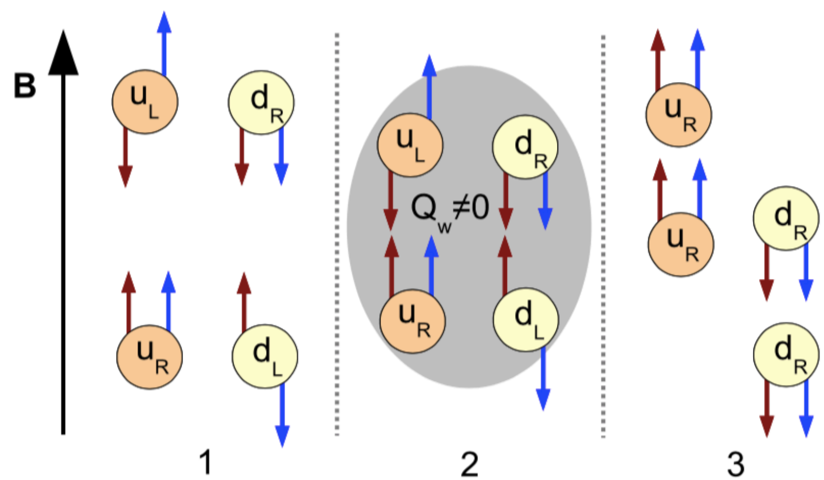


The Chern-Simons diffusion rate

- Why is Chern-Simons diffusion rate interesting?

a) Bariogenesis in Standard Model: sphaleron transitions cause $\Delta(B+L)\neq 0$

b) Chiral magnetic effect in the Quark-Gluon Plasma



[Kharzeev 1312.3348]

$eB >> p^2$. Stage1: $N_L=N_R$ in lowest Landau level. Red: momentum; blue: spin

The Chern-Simons diffusion rate

- Chiral magnetic effect [Kharzeev, Mc Lerran, Warringa 2007; Fukushima, Kharzeev, Warringa 2008]

- Axial anomaly: $\partial_\mu J^\mu_A = -2N_f Q$

- ΔN_{CS} generates a $\Delta_{chirality}$ ➔ $\mu_A \neq 0$ (axial chemical potential)

- Non central collisions in QGP have large magnetic field B

- $\Delta_{chirality}$ + B generate electric current

$$\vec{J} = \sigma_{CME}\vec{B} \qquad \sigma_{CME} = \frac{e^2}{2\pi}\mu_A$$

- Currently under experimental search at RHIC and LHC

The Chern-Simons diffusion rate

- Magnitude of Γ_{CS} in the QGP?
- Real-time non-perturbative physics: no reliable computational methods in QCD
- Effective theory result [Moore, Tassler 2010]

$$\Gamma_{CS} \sim c\lambda^5 T^4\,, \quad \lambda = g_{YM}^2 N_c$$

- Notes: c is non-perturbative; result valid at α_s<<1.
- $\Gamma_{CS} \sim \mathcal{O}(N_c^0)$

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Holographic derivation

The Chern-Simons diffusion rate in $N = 4$ SYM [Son, Starinets 2002]

- Backgound is BH AdS_5 x S^5 generated by Nc D3-branes
- D3-brane action contains

$$\int C Tr F \wedge F$$

- ➔ gravity field dual to Q is the RR 0-form potential C

Holographic derivation

- Action for C in 5d

$$\int d^5x\sqrt{-g_5}\left[-\frac{1}{2}\partial_M C\partial^M C\right]$$

- Solve eq. of motion for C with incoming b.c. at horizon
- ➔ Retarded correlator G_R
- Use Kubo formula. Result:

$$\Gamma_{CS} = \frac{\lambda^2}{256\pi^3}T^4$$

Holographic derivation

Non-universal-looking expressions (unlike $\eta/s = 1/4\pi$)

$$\Gamma_{CS} = \frac{\lambda^2}{256\pi^3} T^4$$

N=4 SYM

[Son, Starinets 2002]

$$\Gamma_{CS} = \frac{g_{YM}^4}{2^7\pi^5} \frac{N^2}{2\pi} \frac{T}{v\sqrt{w}}$$

N=4 SYM, finite B

[Basar,Kharzeev 2012]

$$\Gamma_{CS} = \frac{\lambda^2}{256\pi^3} f(a) T^4$$

Mateos-Trancanelli anisotropic N=4 SYM

[Bu 2014]

$$\Gamma_{CS} = \frac{1}{2\pi} \frac{\lambda^3}{3^6\pi^2} \frac{T^6}{M_{KK}^2}$$

Witten's YM model

[Craps, Hoyos et al 2012]

Holographic derivation

Non-universal-looking expressions (unlike $\eta/s = 1/4\pi$)

Sure??

$$\Gamma_{CS} = \frac{1}{2^7\pi^5}\left(\frac{\lambda}{N_c}\right)^2 s\,T$$

N=4 SYM

[Son, Starinets 2002]

$$\Gamma_{CS} = \frac{1}{2^7\pi^5}\left(\frac{\lambda}{N_c}\right)^2 s\,T$$

N=4 SYM, finite B

[Basar,Kharzeev 2012]

v = v (B), w = w(B)

$$\Gamma_{CS} = \frac{1}{2^7\pi^5}\left(\frac{\lambda}{N_c}\right)^2 s\,T$$

Maldacena-Nunez SYM

[FB, Cotrone, Porri 2018]

$$\Gamma_{CS} = \frac{1}{2^7\pi^5}\left(\frac{\lambda}{N_c}\right)^2 s\,T$$

Mateos-Trancanelli anisotropic N=4 SYM

[Bu 2014]

$$\Gamma_{CS} = \frac{1}{2^7\pi^5}\left(\frac{\lambda}{N_c}\right)^2 s\,T$$

Witten's YM model

[Craps, Hoyos et al 2012]

“Universality” of the result

- Wrapped D-brane models
- Wrap Dp-brane on p-3 cycle $\Omega_{p\text{-}3}$ ➔ 4d gauge theory in IR

- N=4 SYM included
- Other interesting models included (Witten-Sakai-Sugimoto; Maldacena-Nuñez etc)
- All computations of Γ_{CS} in the literature performed in this class of models

“Universality” of the result

- Expanding DBI+WZ D-brane action at low energies get

$$S_{YM} = -\int d^4x \left[\frac{1}{2g_{YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta_{YM}}{16\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

- with

$$\frac{1}{g_{YM}^2} = \frac{\tau_p}{2}(2\pi\alpha')^2 \int_{\Omega_{p-3}} d^{p-3}x\, e^{-\phi}\sqrt{\det g}\,,$$

$$\theta_{YM} = (2\pi)^2\tau_p(2\pi\alpha')^2 \int_{\Omega_{p-3}} C_{p-3}\,.$$

- Thus, gravity field dual to Q is

$$C = \tau_p(2\pi)^2(2\pi\alpha')^2 \int_{\Omega_{p-3}} C_{p-3}$$

"Universality" of the result

- Derivation of the 5d action for C
- Action of $F_{p-2} = dC_{p-3}$ in 10d

$$\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}\, e^{\frac{7-p}{2}\phi}\left[-\frac{1}{2}F_{p-2}^2\right]$$

- Reduction ansatz: $ds_{10}^2 = e^f ds_5^2 + ds_{\rm int}^2$
- Reduction ansatz for F_{p-2}

$$F_{p-2}^2 = \partial_M \tilde{C}\partial^M \tilde{C}[\det(g_{E,\Omega'_{p-3}})]^{-1}e^{-f}$$

- where

$$\int_{\Omega_{p-3}}\sqrt{g_{E,\Omega_{p-3}}} = \mathrm{Vol}(\Omega_{p-3})\sqrt{\det(g_{E,\Omega'_{p-3}})}$$

- $C = \tau_p(2\pi)^2(2\pi\alpha')^2\mathrm{Vol}(\Omega_{p-3})\tilde{C}$

"Universality" of the result

- Final result

$$\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\,H\left[-\frac{1}{2}\partial_M C\partial^M C\right]$$

$$H = \frac{1}{(2\pi)^4}\left(\frac{1}{\tau_p(2\pi\alpha')^2\int_{\Omega_{p-3}} e^{\frac{p-7}{4}\phi}\sqrt{\det g_E}}\right)^2 = \left(\frac{g_{YM}^2}{8\pi^2}\right)^2$$

- Hence, using results in [Son, Starinets 2002; Gursoy,Iatrakis,Kiritsis, Nitti, O'Bannon 2013] we get that Chern-Simons diffusion rate has "universal" form

$$\boxed{\frac{\Gamma_{CS}}{s\,T} = \frac{\alpha_s^2(T)}{2^3\pi^3}}$$

“Universality” of the result

- Can use as a benchmark for QGP:

  - $\alpha_s(T_c) \approx 1/2$; s(T_c) $\approx 10\ T_c^3$ [Borsanyi et al 2013, Bazavov et al 2014]

$$\Gamma_{CS}(T_c) \sim 0.01\, T_c^4$$

- Can compute first $1/\lambda$ corrections: Γ_{CS} decreases [Bu 2014]

$$\frac{\Gamma_{CS}}{sT} = \frac{1}{2^7 \pi^5} \left(\frac{\lambda}{N_c}\right)^2 \left[1 - \frac{15}{2} \frac{\zeta(3)}{\lambda^{3/2}} + \cdots\right]$$

- Is holographic result an upper bound on Γ_{CS} ?

- Problem in extending result to other models: identification of coupling λ and of gravity field dual to Q

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Including the anomaly

- Axial anomaly

$$\partial_\mu J^\mu_A = -qQ$$

- Holographically reproduced by Stueckelberg action [Klebanov et al 2002; Landsteiner et al] formally given by

$$\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\left[-\frac{1}{2}(\partial_M C + qA_M)(\partial^M C + qA^M) - \frac{1}{4}F_{MN}F^{MN}\right]$$

- A^M gravity field dual to $J^\mu{}_A$; q : anomaly coefficient

- From dimensional reduction of many holographic models: N=4 SYM with flavors, Maldacena-Nunez, Klebanov-Strassler, Witten-Sakai-Sugimoto

Including the anomaly

- Define B = dC + q A so that dB = q F = F_B
- Get action for a massive vector

$$\frac{1}{2\kappa_5^2 q^2}\int d^5x\sqrt{-g_5}\left[-\frac{1}{4}F_{B,MN}F^{B,MN}-\frac{1}{2}q^2B^2\right]$$

$$B_\mu \leftrightarrow J^\mu_B = \frac{1}{q}J^\mu_A$$

$$\partial_\mu J^\mu_B + Q \simeq 0 \qquad \langle Q(x)\,Q(0)\rangle_R = \langle \partial_\mu J^\mu_B(x)\partial_\nu J^\nu_B(0)\rangle_R$$

Including the anomaly

- Calculation of G_R on asymptotically AdS BH backgound (can be more generic).
- Near horizon

$$B_t \sim \left(\frac{r}{r_h}\right)^{\Delta}\left(1-\frac{r}{r_h}\right)^{-i\frac{\omega}{T}}\left[b_h^{(0)}+b_h^{(1)}\left(1-\frac{r}{r_h}\right)+\cdots\right]$$

- Get

$$\Gamma_{CS} = \Gamma_{CS}(q=0)|b_h^{(0)}(\omega\to 0)|^2$$

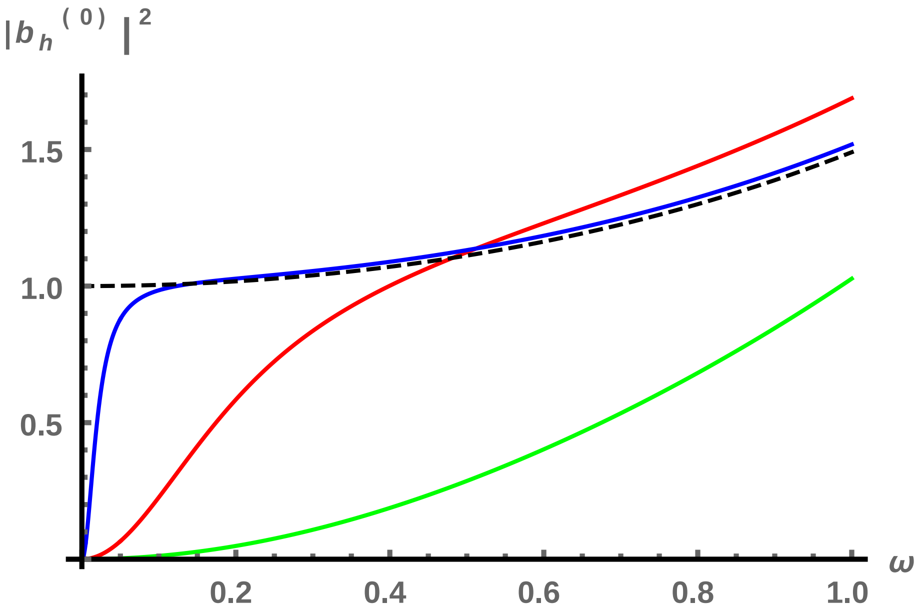
$$b_h^{(1)} = i\left(\frac{q^2}{\omega} + \text{regular in } \omega\right)\cdot b_h^{(0)}$$

- Two possibilities: q=0 (no anomaly), or

$$b_h^{(0)} \sim \omega^a \text{ with } a \geq 1 \text{ for } \omega \to 0 \;\Rightarrow\; \Gamma_{CS}(q\neq 0) = 0.$$

Including the anomaly

Numeric results on AdS_5 BH



Black dashed: q=0

Blue: q=0.04

Red: q=0.44

Green: q=3

Including the anomaly

- Expected?
- Anomaly: $\partial_\mu J^\mu_A = -qQ$ ➔ $\Gamma_{CS} \sim \langle QQ \rangle \sim \langle \partial J_A \partial J_A \rangle$
- $Q_A = \int d^3x J^t_A$ not conserved (anomaly) thus

$$\Gamma_{CS} \sim \langle Q_A(t \to \infty) Q_A(0) \rangle_R = 0$$

- In fact since Q_A not conserved, only gapped modes, expect

$$\langle Q_A(t) Q_A(0) \rangle_R \sim e^{-\frac{t}{\tau}}$$

- τ relaxation time

Including the anomaly

- Definition of Γ_{CS} makes sense only if there is a separation of time scales [Moore, Tassler 2010]

$$\Delta t < t_* \ll \tau$$

$$\Delta t = \text{(microscopic) time scale of CS number fluctuations}$$
$$t_* = \text{cut} - \text{off}$$
$$\tau = \text{relaxation time}$$

- Thus we can define

$$\Gamma_{CS} = \int^{t_*} dt \int d^3x \, \langle Q(t,x) Q(0) \rangle$$

- Note:
  - Can remove cut-off if $\tau \to \infty$.
  - Large N_c: $\tau \sim N_c^2/T \gg 1/T \sim$ microscopic time scale.

Including the anomaly

- Hydro model [Iatrakis, Lin, Yin, 2015]
- Anomaly $\partial_\mu J^\mu_A = -qQ$ ➔ $\langle (\Delta Q_A)^2 \rangle = q^2 \langle (\Delta N_{CS})^2 \rangle$
- For t<<τ

$$\langle (\Delta Q_A)^2 \rangle \sim \chi_A T \left[1 - e^{-\frac{2t}{\tau}} \right] V \approx \frac{2\chi_A T}{\tau} V t \equiv q^2 \Gamma_{CS} V t$$

where χ_A is the axial susceptibility.

- Hence

$$\frac{1}{\tau} = \frac{q^2 \Gamma_{CS}}{2\chi_A T}$$

- Makes sense if: $$\Gamma_{CS} \equiv \Gamma_{CS}(q=0)$$

(τ goes to infinity when q goes to zero)

Including the anomaly

An analogy:

- Axial relax time $$\frac{1}{\tau} = \frac{q^2\Gamma_{CS}}{2\chi_A T}$$ $$\Gamma_{CS} \equiv \Gamma_{CS}(q=0)$$

- Witten-Veneziano $$m_{WV}^2 = \frac{2N_f\chi_g}{f_\pi^2}$$ $$\chi_g \equiv \chi_g(N_f = 0)$$

Including the anomaly

In holography:

- τ from quasi-normal modes of gravity field A^M dual to $J^\mu{}_A$ on black hole background

- Expected behavior at small q from A^M equations of motion [Landsteiner et al 2014]

$$1/\tau \sim q^2$$

Conclusions

- Top-down holography seems to point towards a large coupling "universality" of Γ_{CS} in terms of s, T, α_s
- Can use result for estimates in the QGP
- Including anomaly:

  a) Naive Γ_{CS}=0 ➔ must use cut-off or $\Gamma_{CS} = \Gamma_{CS}(q=0)$
  b) Relaxation time goes as $1/\tau \sim q^2\Gamma_{CS}/\chi_a T$

Thank you!

$$G_R^{tt}(\omega) = \langle J_A^t J_A^t \rangle_R$$

$$G_R^{tt}(\omega) \sim \frac{\mathrm{i}\, R}{\omega + \frac{\mathrm{i}}{\tau}}$$

$$\langle Q_A(t) Q_A(0) \rangle_R \sim R\, \theta(t)\, \mathrm{e}^{-\frac{t}{\tau}}$$

$$\mathrm{Im} G_R^{tt}(\omega) \sim \omega \frac{R}{\omega^2 + \tau^{-2}}$$

$$\frac{1}{\omega} \mathrm{Im} G_R^{QQ} = \frac{\omega}{q^2} \mathrm{Im} G_R^{tt} \sim \frac{1}{q^2} \omega^2 \frac{R}{\omega^2 + \tau^{-2}}$$

Diffusion of axial current $\vec{J}_A = -D\vec{\nabla} J^t_A$; $J^x_A = -\,\mathrm{i}\, kDJ^t_A$

With a source $E_x = -\,\mathrm{i}\,\omega A_x$

linear response $\langle J^x_A \rangle = -G^{xx}_R A_x$ $G^{xx}_R(\omega, \vec{0}) = \mathrm{i}\,\omega\sigma$

$$\langle J^t_A \rangle = \frac{\langle J^x_A \rangle}{-\,\mathrm{i}\,kD} = \frac{G^{xx}_R}{\mathrm{i}\,kD} A_x$$

$\partial_\mu C + qA_\mu$ implies $\mathrm{i}\,kC = qA_x$ source for Q induced by A_x

$$\langle Q \rangle = -G^{QQ}_R C = -\frac{G^{QQ}_R}{\mathrm{i}\,k} qA_x = \frac{\Gamma_{CS}\,\omega}{2kT} qA_x \qquad G^{QQ}_R = -\,\mathrm{i}\,\frac{\Gamma_{CS}}{2T}\omega + \mathcal{O}(k^2)$$

$$\frac{\langle Q \rangle}{\langle J^t_A \rangle} = \frac{\mathrm{i}\,\omega\, q\,\Gamma_{CS} D}{2TG^{xx}_R} = \frac{q\,\Gamma_{CS}\, D}{2T\sigma} + \mathcal{O}(k) \equiv \frac{1}{q\tau} + \mathcal{O}(k) \qquad \frac{1}{\tau} = \frac{q^2\Gamma_{CS}}{2T\chi_A}$$