Semi-holographic non-Fermi Liquids

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BASED ON

A. Mukhopadhyay and G. Policastro, PRL 111 (2013), no. 22, 221603 [arXiv: 1306.3941[hep-th]]

B. Doucot, C. Ecker, AM and G. Policastro, Phys. Rev. D 96, no. 10, 106011 (2017) [arXiv: 1706.04975[hep-th]]

Ongoing works will be mentioned

Introduction

Quantum matter at finite density has seemingly endless complex phases.

Strong correlations which prevent independent motion of electrons is the generic feature (eg FQHE, strange metals, etc.)

A possible way to explore: Do not start from the microscopic description but follow Landau with a new twist

Look for a rather simple solvable non-perturbative set-up (instead of free electrons of Drude model) and ask

(i) if the energies of the fundamental excitations at the Fermi surface and

(ii) also their decay widths retain their forms at small ω and $k \approx k_F$ for *almost all arbitrary deformations*

Unlike Landau's Fermi liquids, however we will not require that the decay widths near Fermi surface to be smaller compared to the energies of the excitations

We will still have *generalized quasiparticles* in the sense that self-energy corrections preserve their qualitative nature under most deformations

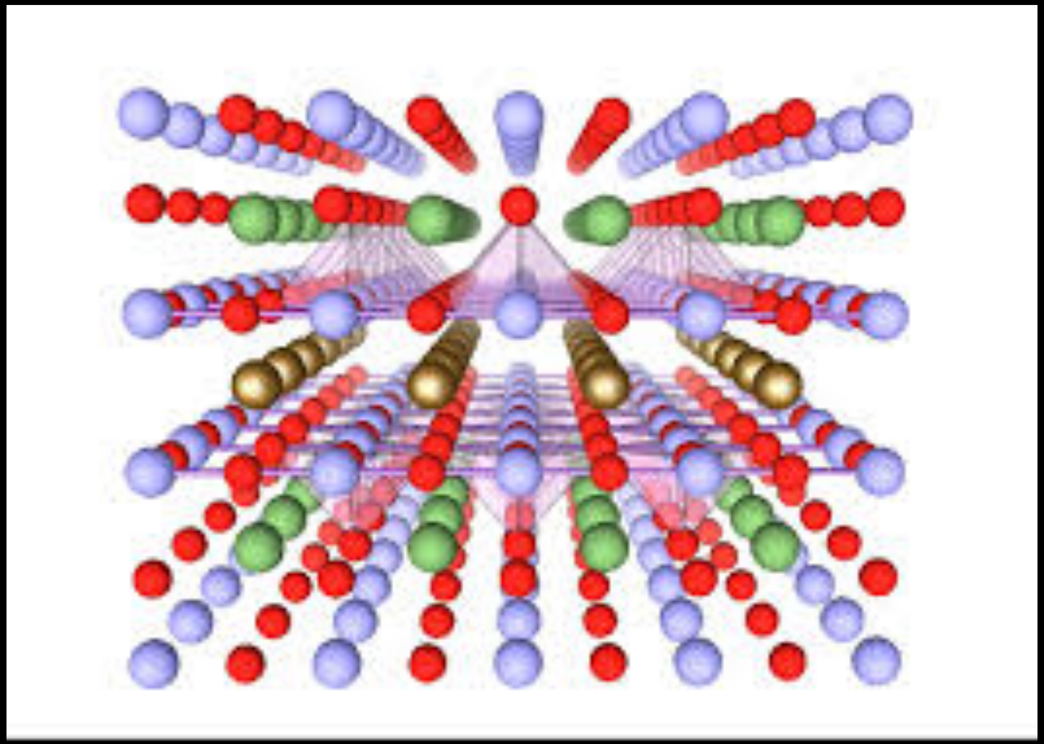
In modern language, such a scenario constitutes a stable IR fixed point with emergent scaling (but not conformal) symmetry since all deformations will be *irrelevant* in the IR

The low energy properties should be characterized by some simple parameters (as Landau parameters characterize metals)

Experimental motivation: Understanding the mechanism of unconventional superconductivity and also unconventional insulating phases (arising from instabilities of parent strange metallic state).

Broad question: Can we provide a necessary and sufficient condition for emergence of (certain forms of) unconventional superconductivity etc based on low energy physics at Fermi surface?

It will directly tell us about possible material architecture of high-Tc superconductors (eg why not $La_{4-x}Ba_{1+x}Cu_5O_{13}$ but $La_{2-x}Sr_xCuO_4$?)



The construction

A. Mukhopadhyay and G. Policastro, PRL 111 (2013), no. 22, 221603 [arXiv: 1306.3941[hep-th]]

HYBRID SYSTEM OF WEAKLY INTERACTING FERMIONS AND CRITICAL FERMIONS OF A QUANTUM CRITICAL SYSTEM

χ **Weakly interacting fermion (conduction band electrons)**

Ψ **Fermionic operator of a quantum critical D (spatial) dimensional system dual to a Dirac field in $AdS_2 \times R^D$ with a const E-field**

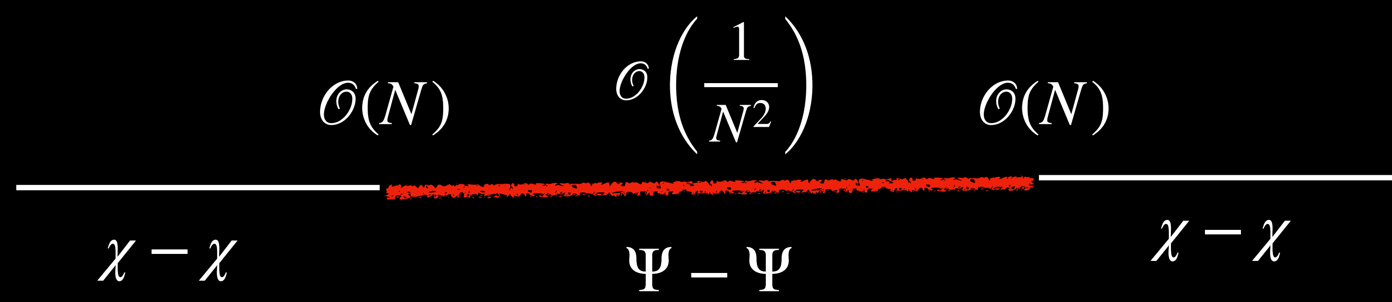
$$S = \int dt \Bigg[\sum_k \Big(\chi^\dagger_{\mathbf{k}} (i\partial_t - \epsilon_{\mathbf{k}} + \mu) \chi_{\mathbf{k}} + N^2 S_{\mathrm{CFT}}$$
$$+ N \sum_k (g_{\mathbf{k}} \chi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + c.c.\Big) + \frac{1}{2} \sum_{k,k_1,q} \chi^\dagger_{\mathbf{k}} \chi_{\mathbf{k}-\mathbf{q}} V(\mathbf{q}) \chi^\dagger_{\mathbf{k}_1} \chi_{\mathbf{k}_1-\mathbf{q}}$$
$$+ \sum_{k,k_1,q} \lambda_{\mathbf{k},\mathbf{k}_1,\mathbf{q}} \chi^\dagger_{\mathbf{k}} \chi_{\mathbf{k}-\mathbf{q}} \chi^\dagger_{\mathbf{k}_1} \chi_{\mathbf{k}_1-\mathbf{q}} + N \sum_{k,k',q} \eta_{\mathbf{k},\mathbf{k}'} \chi^\dagger_{\mathbf{k}} \chi_{\mathbf{k}'} \phi_{\mathbf{k}-\mathbf{k}'}$$
$$+ N \sum_{k,k_1,k_2} \Big(\tilde{g}_{\mathbf{k},\mathbf{k}_1,\mathbf{k}_2} \chi^\dagger_{\mathbf{k}} \chi_{\mathbf{k}_1} \chi^\dagger_{\mathbf{k}_2} \psi_{\mathbf{k}-\mathbf{k}_1+\mathbf{k}_2} + c.c. \Big) \Bigg].$$

$\chi - \chi : \mathcal{O}(1)$

$\chi - \Psi : \mathcal{O}(N)$

$\Psi - \Psi : \mathcal{O}(N^2)$

The large-N scaling ensures that backreaction on the CFT is suppressed, however the CFT critical fermion alters the boundary fermions profoundly



Quadratic terms, i.e. kinetic terms of boundary fermion and holographic Dirac field with their linear coupling is just the POLCHINSKI-FAULKNER (PF) model (2010). Essentially a generalized free theory!

$$\Sigma \approx \zeta \omega^{\nu(k_F)} \ll \omega, \quad \zeta = |g|^2 c(\nu(k_F))$$

The original motivation of Polchinski and Faulkner was to substitute UV holographic physics with ordinary fermions which indeed dominate UV behavior for appropriate range of scaling dimension of the CFT operator.

Adding realistic complications (lattice & spin-orbit coupling) is possible so phenomenological flexibility is gained

ν 2 times the scaling dimension of the CFT fermion

$0 < \nu < 1$ for the CFT fermion and boundary fermion to dominate IR and UV respectively

Quadratic (PF) terms invariant under $\{\omega, \mathbf{k}_\perp, \mathbf{k}_{||}\} \rightarrow \{\lambda^{2/\nu}\omega, \lambda^2\mathbf{k}_\perp, \lambda\mathbf{k}_{||}\}$

Crucial point: With our large-N scaling, all interactions are irrelevant at the Fermi surface (so the effective parameter ζ is not renormalized)

eg. $[g\chi^4] = 3 - D - \dfrac{2}{\nu}$ irrelevant for $D \geq 1$

The Polchinski-Faulkner model therefore describes generalized quasi-particle (self-energy corrections irrelevant at low energy and near Fermi surface). We can thus generalize Landau's Fermi liquid theory!

CAVEAT I: The picture changes if $\mathcal{O}(N^2)$ lattice degrees of freedom (*valence band lattice electrons*) interacts (hybridizes) with CFT degrees of freedom.

In this case, one needs to proceed along the lines of semi-holographic QCD models (see talks by A. Rebhan and C Ecker) to solve self-consistently.

(Aside: Our NFL model can also be formulated using *democratic couplings* and an appropriate action with auxiliary fields — see A. Rebhan's talk.)

OPEN QUESTION: Do we get new types of generalized quasiparticles? Self-consistently determine ζ and ν ?

A very important question but beyond the scope of this talk.

CAVEAT II: We have assumed that V(q) is screened self-consistently. We will see that it is most likely true for

$$1/2 < \nu < 1$$

Generalized quasi-particle at low frequency and near Fermi surface

$$G_R(\omega, \mathbf{k}) = \frac{1}{\zeta\omega^\nu - \epsilon_{\mathbf{k}}}, \quad \epsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m} - \frac{k_F^2}{2m}, \quad 0 < \nu < 1.$$

Spectral function = - 2 Im G_R

$$\mathrm{Im}G_R(\omega, \mathbf{k}) = -\frac{\zeta_I \omega^\nu}{(\zeta_R \omega^\nu - \epsilon_{\mathbf{k}})^2 + \zeta_I^2 \omega^{2\nu}}\theta(\omega) - \frac{\tilde{\zeta}_I |\omega|^\nu}{(\tilde{\zeta}_R |\omega|^\nu - \epsilon_{\mathbf{k}})^2 + \tilde{\zeta}_I^2 |\omega|^{2\nu}}\theta(-\omega),$$

$$\tilde{\zeta} = \zeta e^{i\pi\nu}.$$

Particle-hole asymmetry is built in!

Positivity of spectral function implies $0 < \phi < \pi(1-\nu).$ $\phi := \arg(\zeta)$

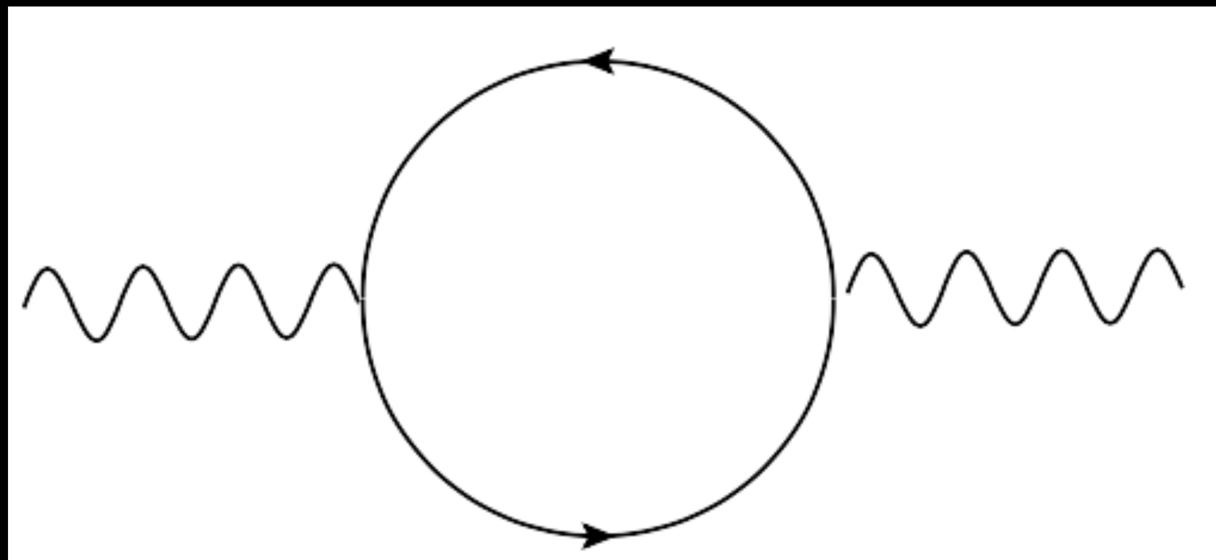
Holography implies $\arg(\zeta) = \arg(\Gamma(-\nu)(e^{-i\pi\nu} - e^{-2\pi q}))$

Small q implies ν **is closer to the upper bound**

Lindhard Function

B. Doucot, C. Ecker, AM and G. Policastro, Phys. Rev. D 96, no. 10, 106011 (2017) [arXiv: 1706.04975[hep-th]]

$$\omega + \frac{\Omega}{2}, \mathbf{k} + \frac{\mathbf{q}}{2}$$



$$\Omega, \mathbf{q}$$

$$\omega - \frac{\Omega}{2}, \mathbf{k} - \frac{\mathbf{q}}{2}$$

Lindhard Function: Density-density response function

At zero temperature:

$$\mathcal{L}(\Omega, \mathbf{q}) = -2i \int_{\mathbf{k}} \int_{\omega} G_F\left(\omega_+, \mathbf{k}_+\right) G_F\left(\omega_-, \mathbf{k}_-\right),$$

$$\omega_\pm = \omega \pm \frac{\Omega}{2}, \quad \mathbf{k}_\pm = \mathbf{k} \pm \frac{\mathbf{q}}{2}.$$

A fundamental object: Gives direct information about the degrees of freedom (low energy excitations) of the system

In a Fermi liquid the Lindhard function is determined completely by the long-lived particle-hole excitations at the Fermi surface

In our calculation (at zero temperature):

$$G_R(\omega, \mathbf{k}) = \frac{1}{\zeta\omega^\nu - \epsilon_{\mathbf{k}}}$$

$$\mathrm{Re}G_F(\omega, \mathbf{k}) = \mathrm{Re}G_R(\omega, \mathbf{k}), \quad \mathrm{Im}G_F(\omega, \mathbf{k}) = \mathrm{Im}G_R(\omega, \mathbf{k})\mathrm{sgn}(\omega).$$

Not an easy calculation to do analytically or numerically (particularly if the phase is close to upper bound). <u>The frequency loop integral should be done first with a cut-off.</u>

The imaginary part of the Lindhard function is cut-off independent if:

$\Omega < \omega_c$

The real part is cut-off dependent (ω_c) dependent but Kramers-Kronig relation is satisfied up to terms that are powers of Ω/ω_c PROVIDED $1/2 < \nu < 1$

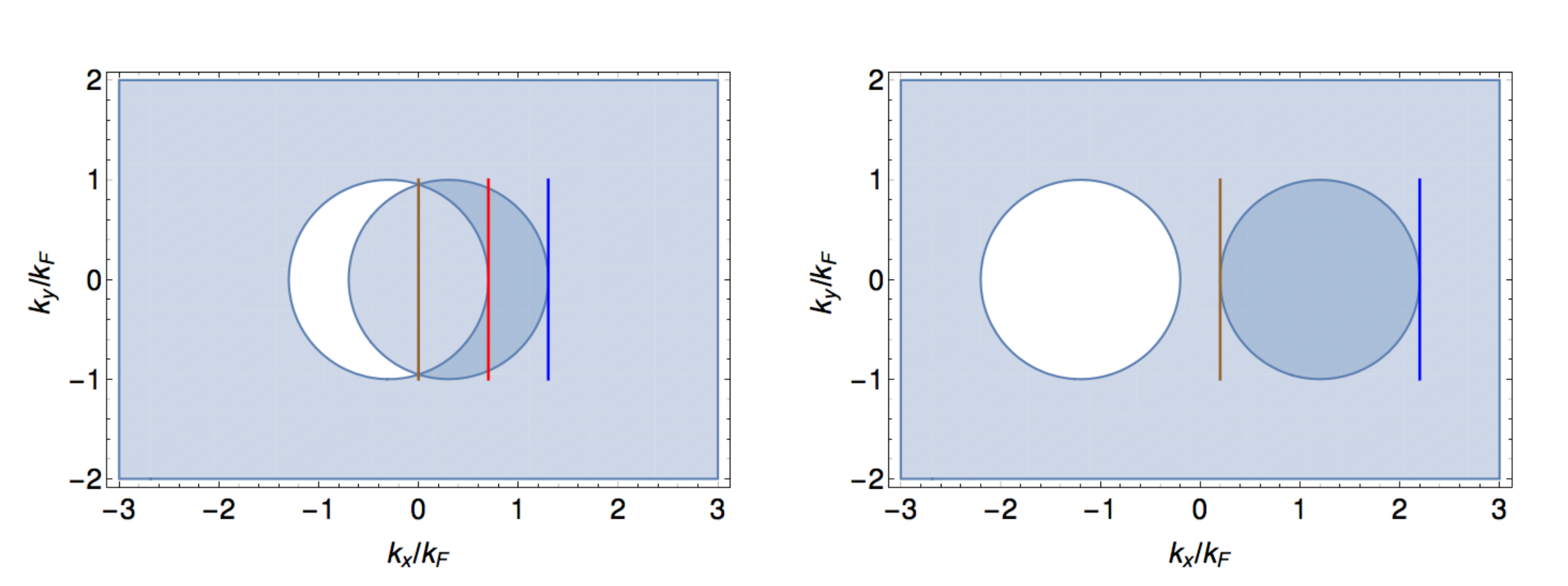
Radiative instability (via sensitivity to UV completion) possible if $0 < \nu \leq 1/2$

The continuum is the region of $\Omega - q$ plane in which on-shell particle-hole pairs can exist

$$\Omega = \epsilon_{\mathbf{k}+\mathbf{q}/2} - \epsilon_{\mathbf{k}-\mathbf{q}/2} = \frac{\mathbf{q}\cdot\mathbf{k}}{m} \text{ with } k \in \{\epsilon_{\mathbf{k}-\mathbf{q}/2} < 0 \,\&\, \epsilon_{\mathbf{k}+\mathbf{q}/2} > 0\}.$$

$$\epsilon_{\mathbf{k}} = \frac{k^2}{2m} - \epsilon_F, \quad \epsilon_F = \frac{k_F^2}{2m}.$$

$q < 2k_F$ $\qquad$ $q > 2k_F$

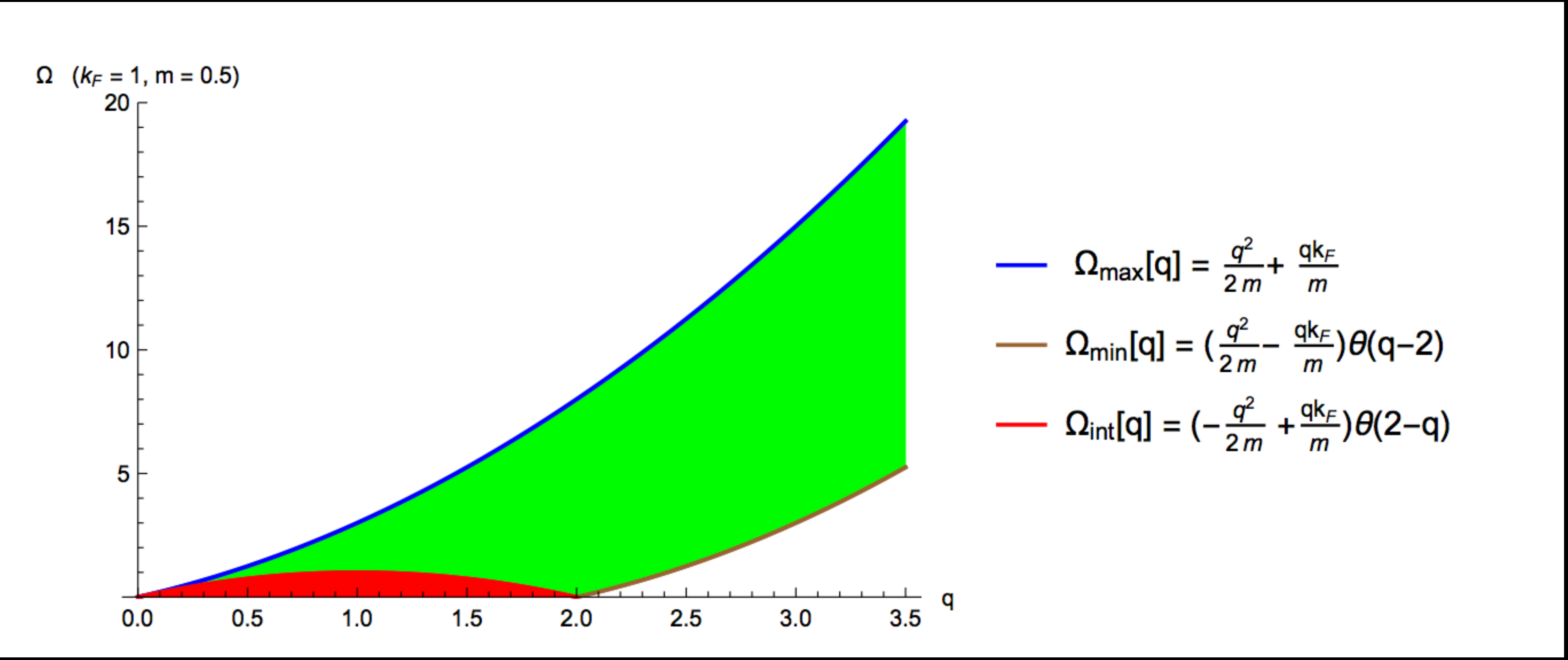


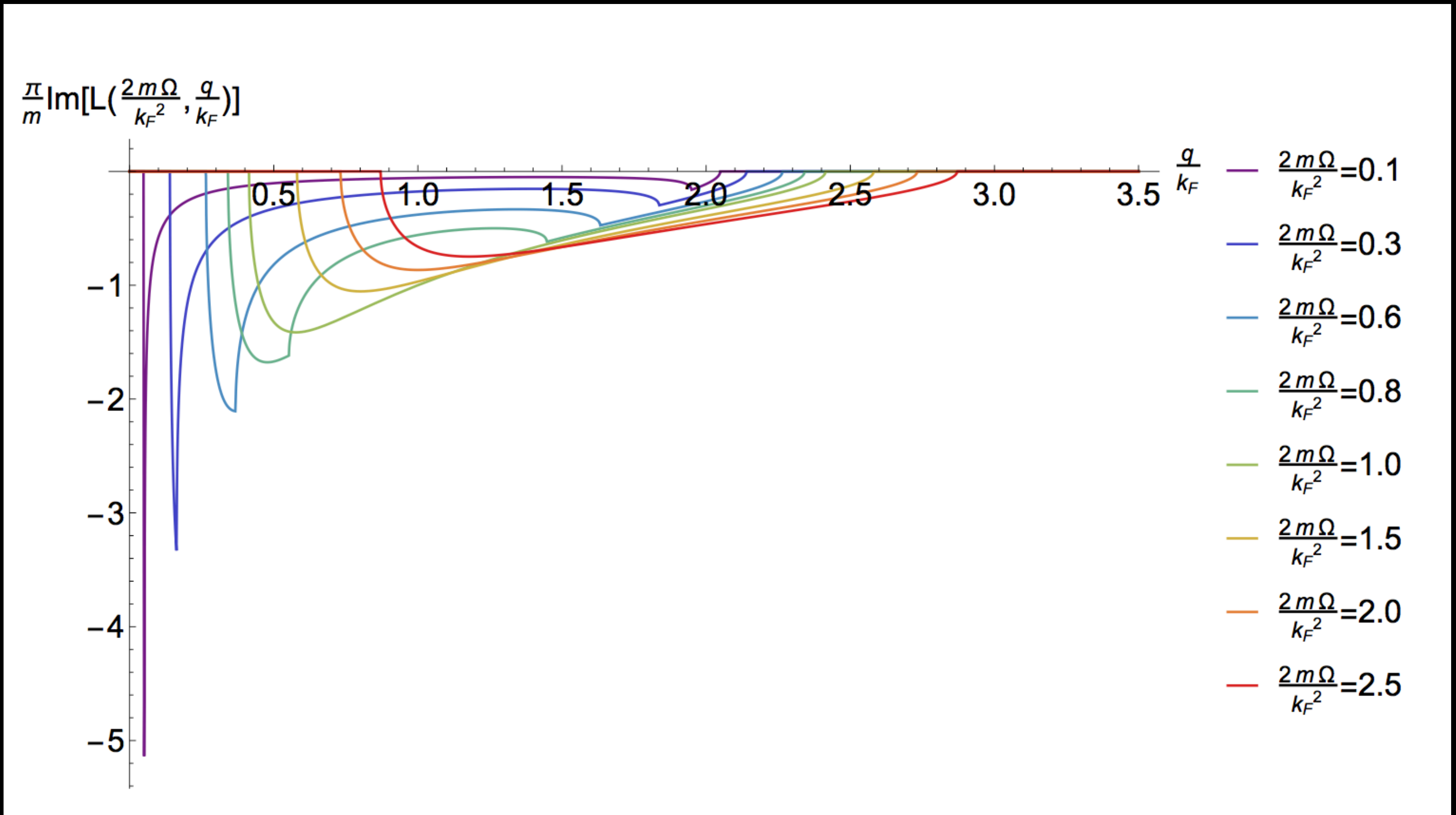
The vertical lines represent fixed values of Omega (with q along x-axis).

Circles with radius kF and centers $\pm\frac{q}{2}$ **: Particle & hole Fermi surfaces**

The dark blue region forms the continuum

The imaginary part of the Lindhard function in Fermi liquids is supported only in the continuum and is proportional to the phase space volume

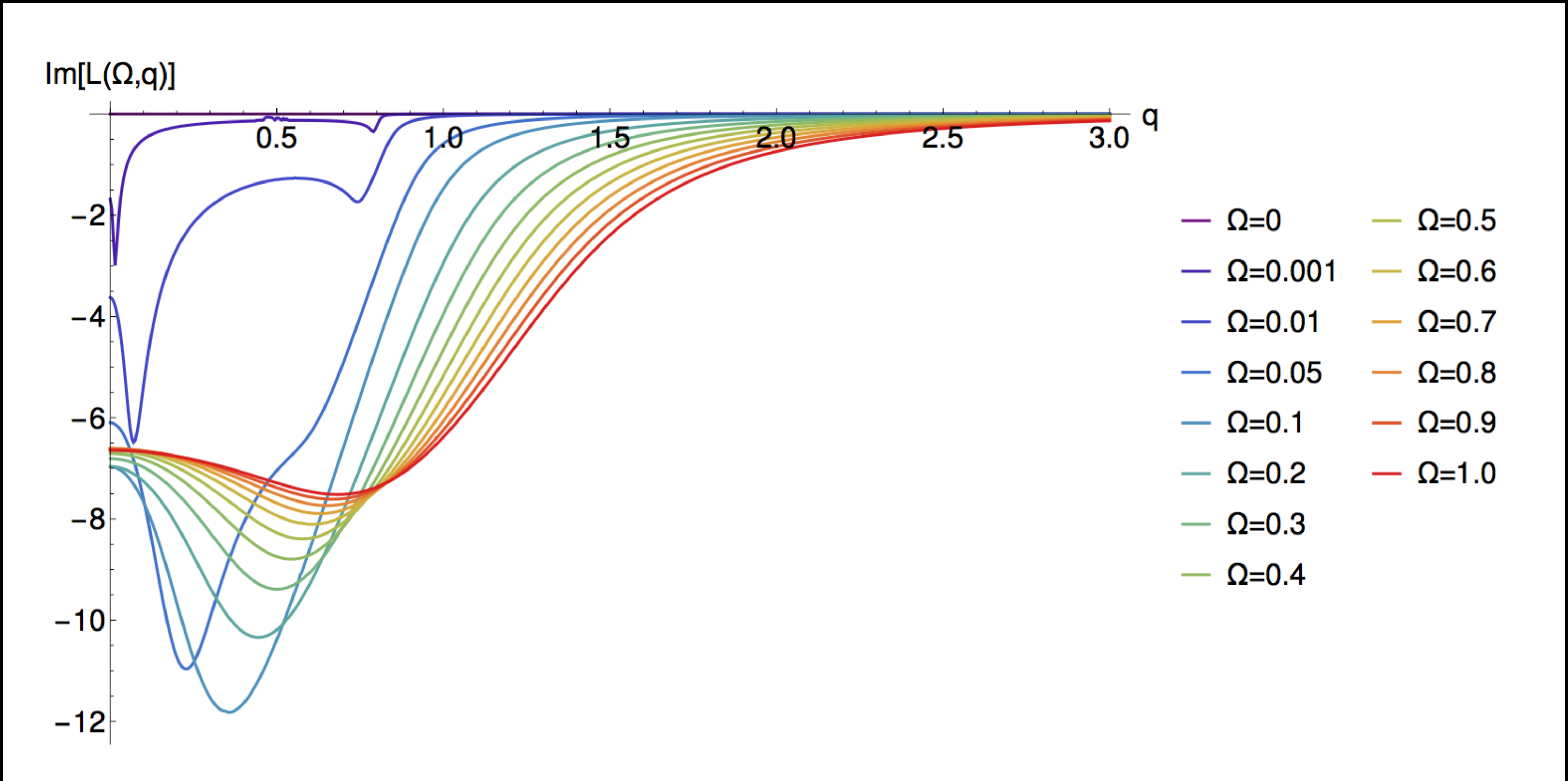




The red region is the INNER CORE of the continuum whose border includes particle Fermi surface

It's distinctly visible in the imaginary part of the FL Lindhard function

$$|\zeta| = 1,\ \arg(\zeta) = \pi/4, \nu = 2/3, k_F = 0.4, m = 0.5$$

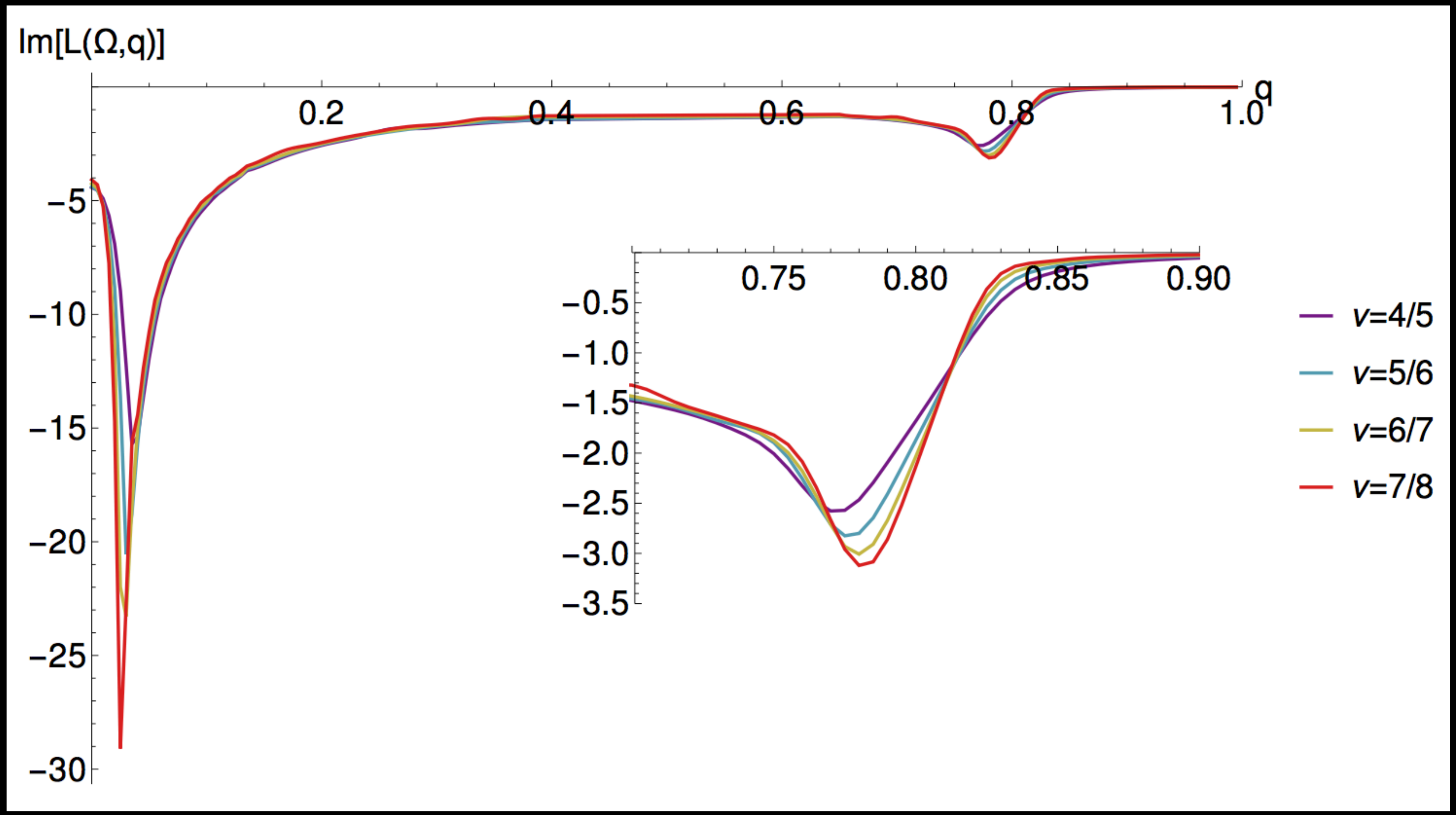


The boundaries of the continuum are blurred out — long tails emerge

<u>Remarkably the inner core of the continuum is still distinctly visible</u>

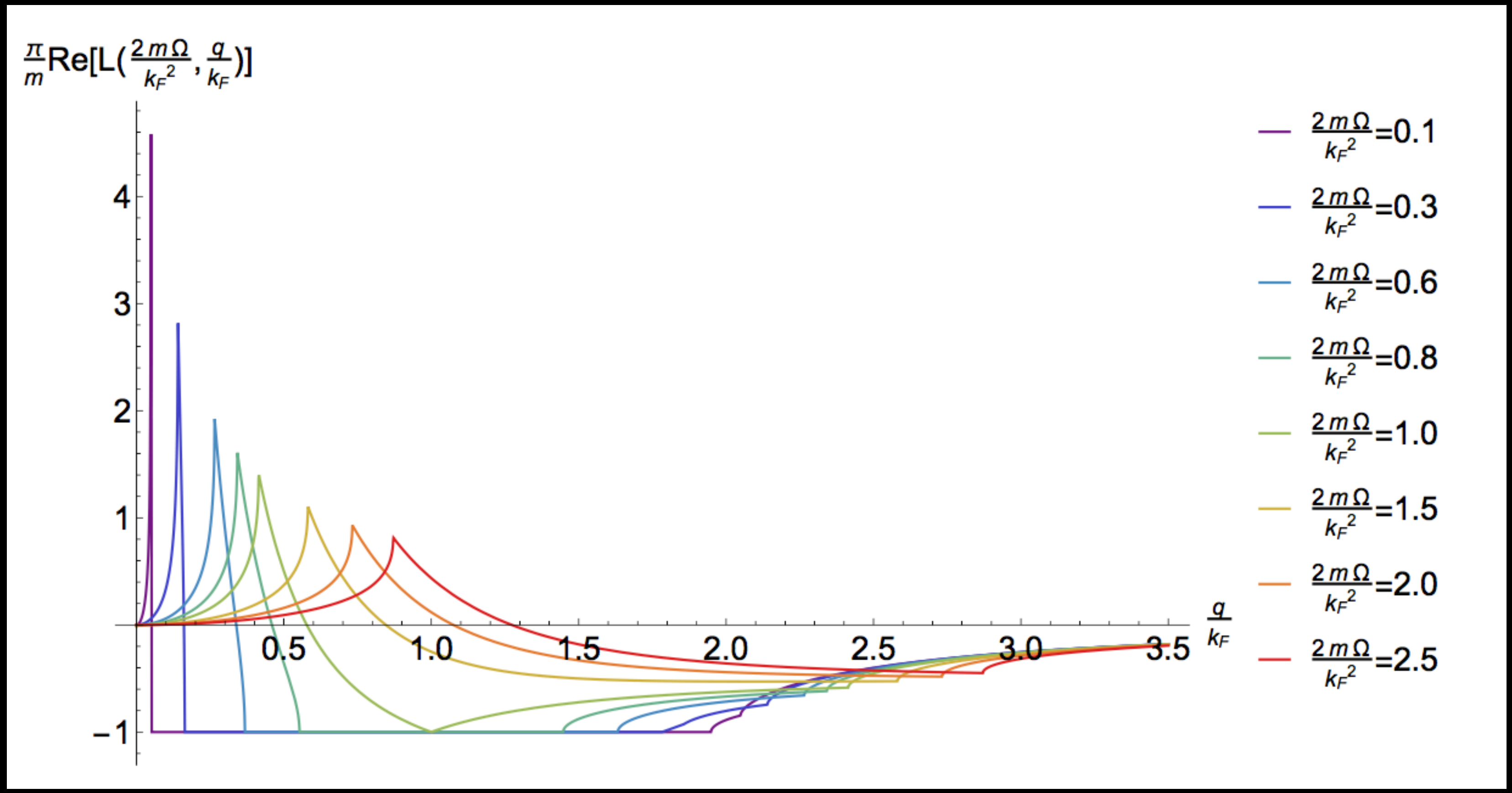
We have hybrid behavior of incoherent and coherent particle-hole excitations co-existing!

$$|\zeta| = 1,\ \arg(\zeta) = \pi/(n+2), \nu = n/(n+1), k_F = 0.4, m = 0.5$$
$$n = 4,5,6,7$$

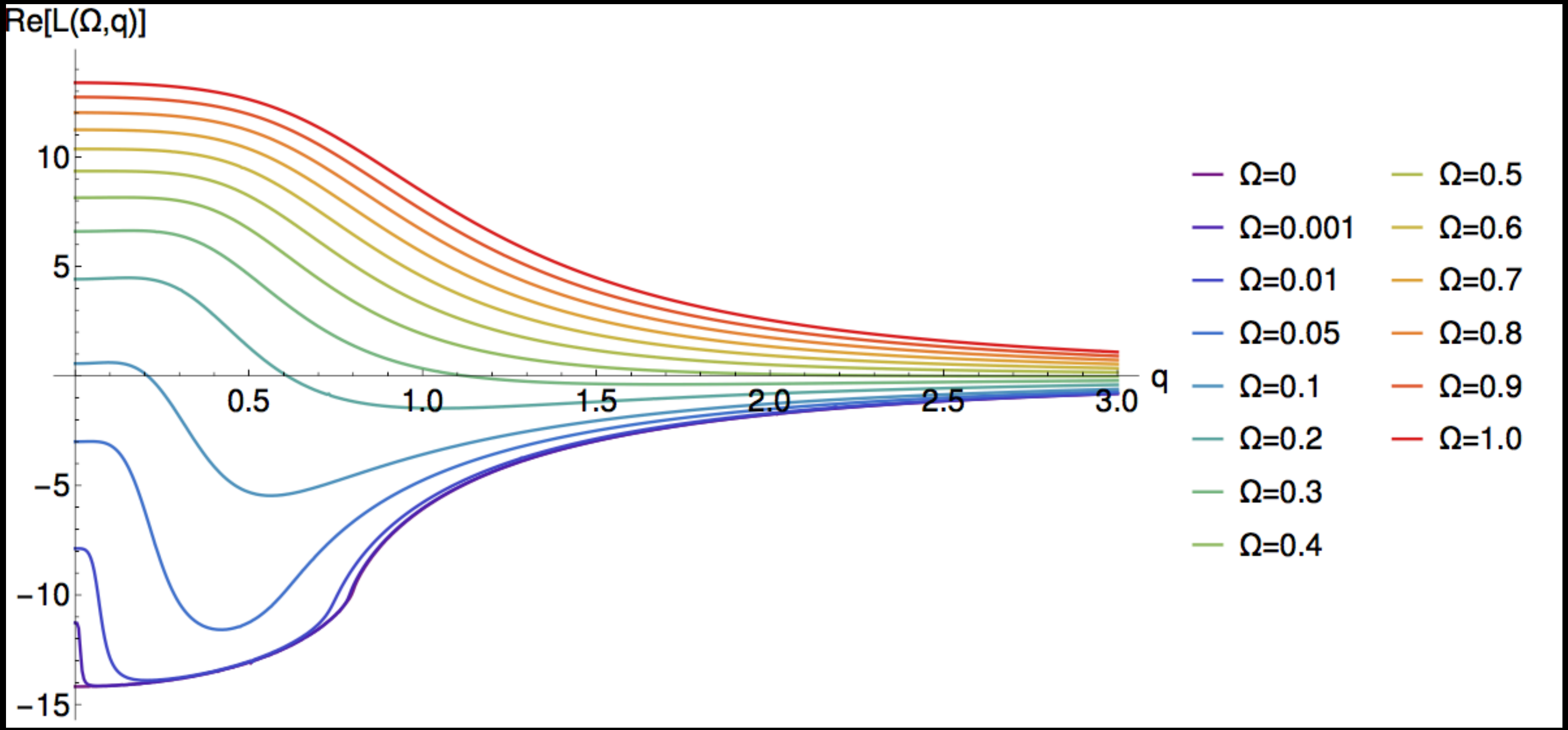


As ν moves closer to one, the behavior tends to become more Fermi-liquid like.

Fermi liquid case

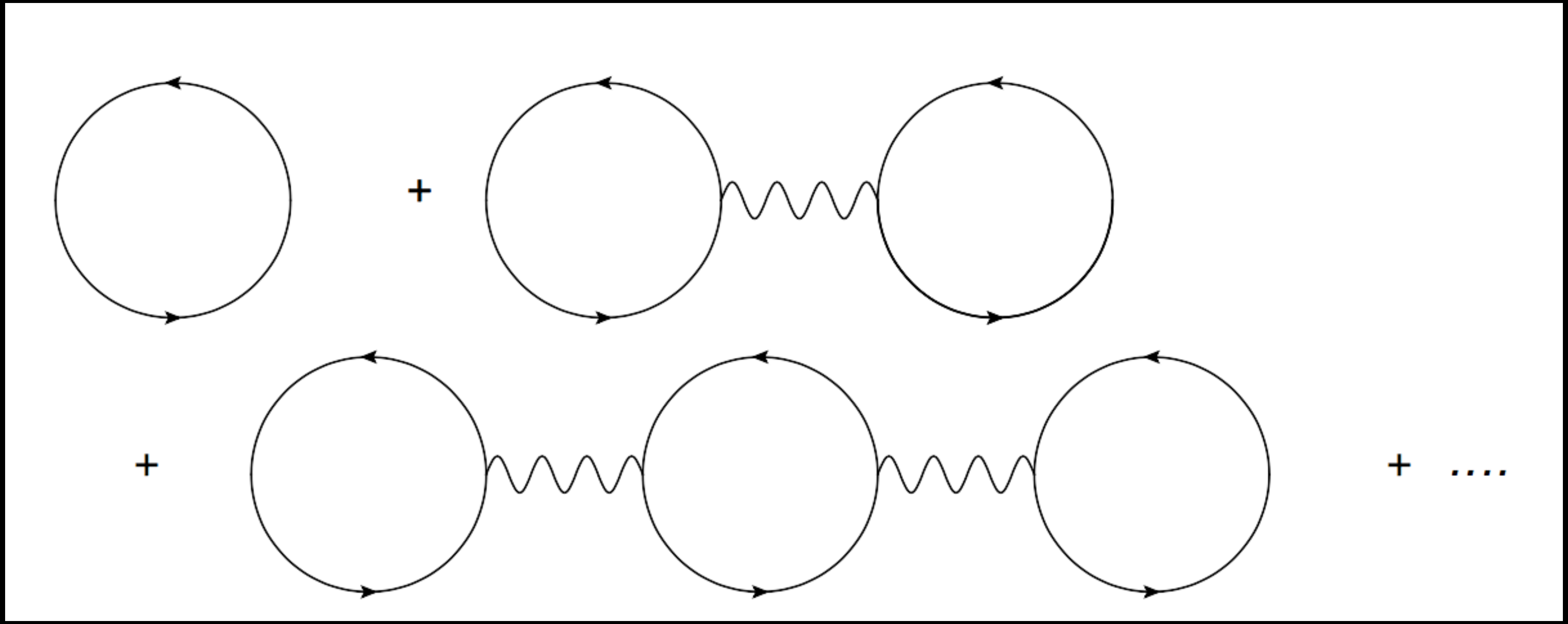


Semi-holographic case



The real part of the Lindhard function also exhibits parallels of Fermi liquid behaviour in the inner core region.

Resummation and collective excitations



Ring resummations of Lindhard function aka Random Phase Approximation

RPA of Lindhard function and dynamical screened potential

$$V(q) = \frac{e^2}{2\epsilon_b q}$$

Poles determining collective excitations

$$\mathcal{L}^{\mathrm{imp}}(q,\Omega) = \frac{\mathcal{L}(q,\Omega)}{1 - V(q)\mathcal{L}(q,\Omega)},$$

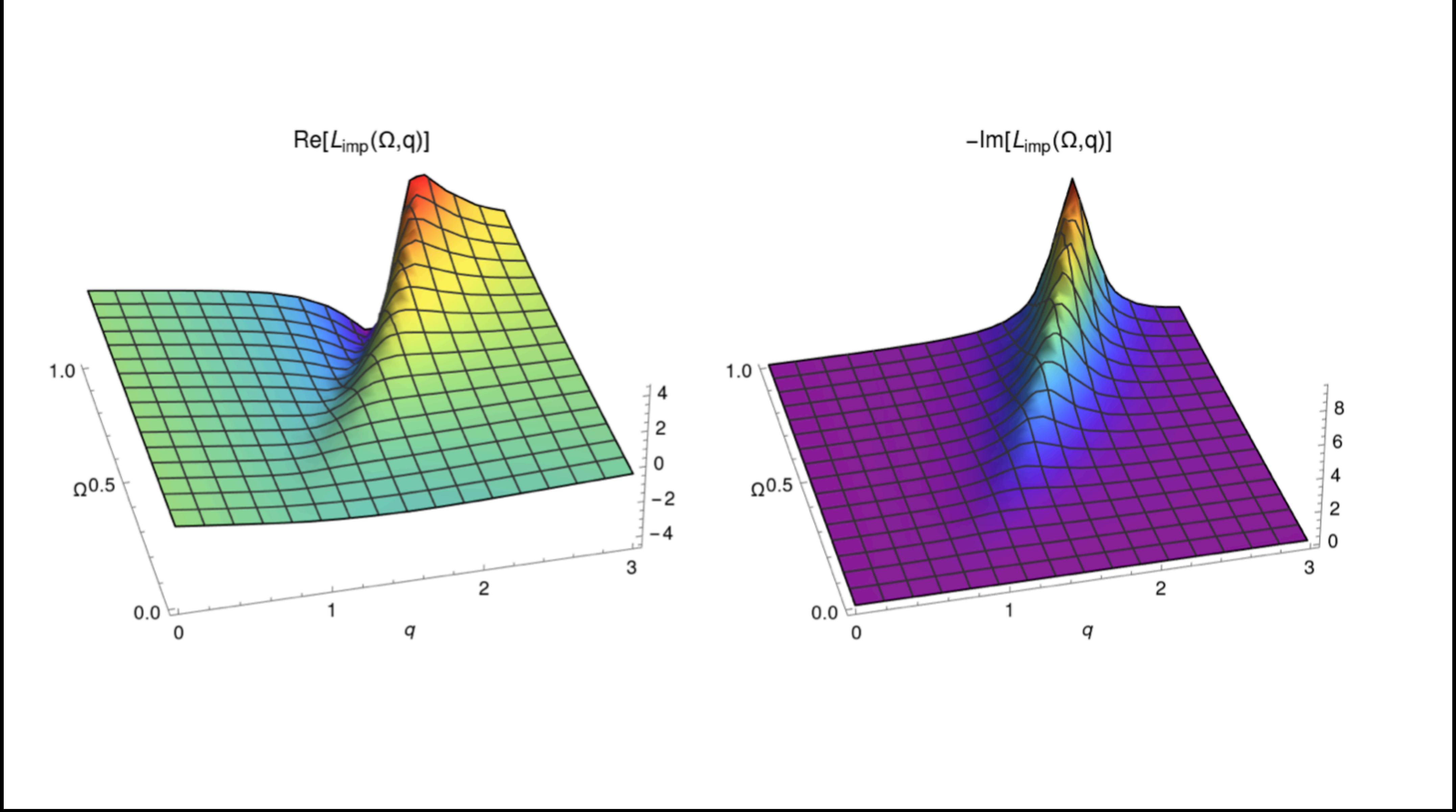
$$V_s(q,\Omega) = \frac{V(q)}{1 - V(q)\mathcal{L}(q,\Omega)}.$$

$$\Omega(q) = \Omega_q - i\gamma_q$$

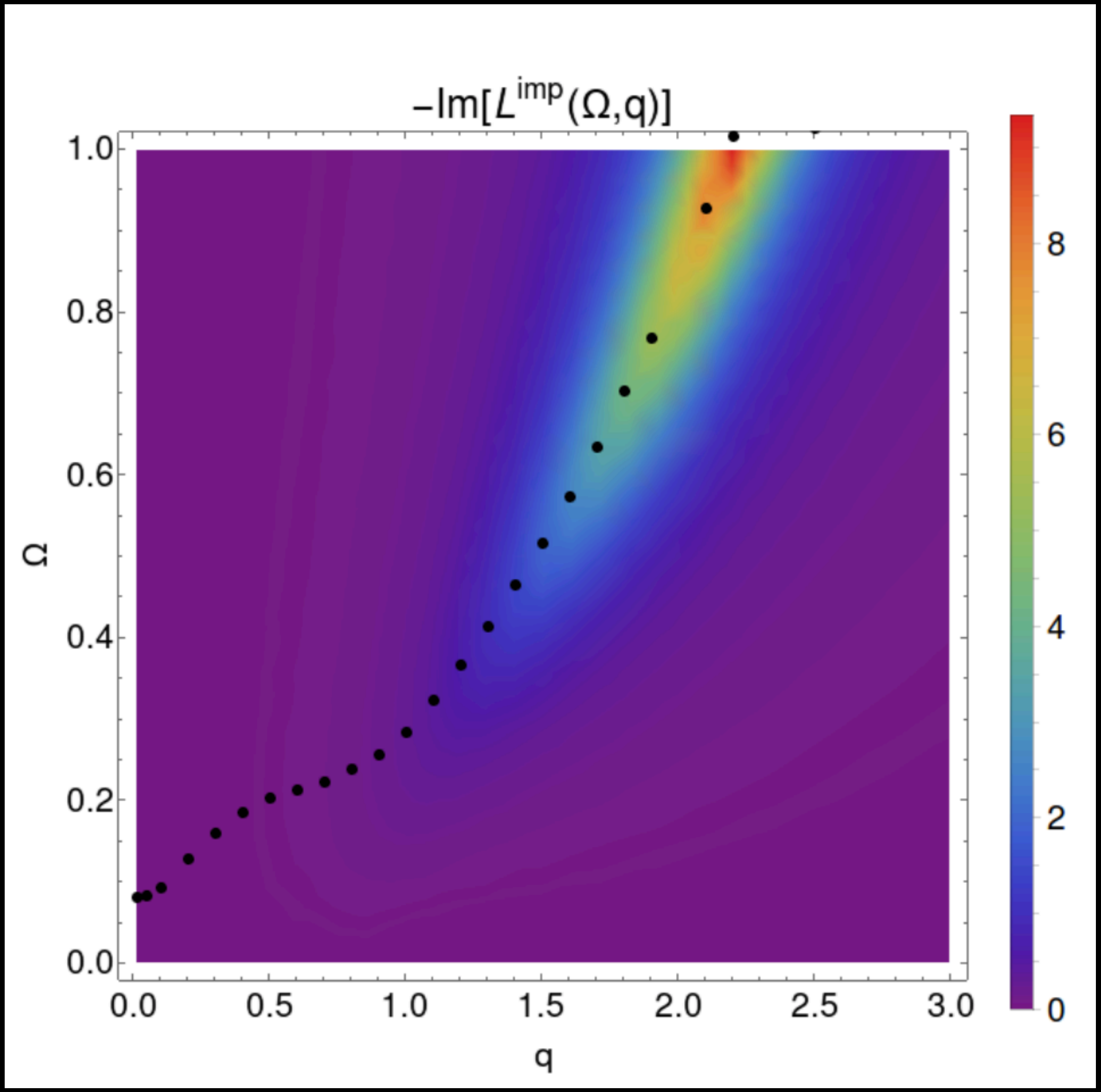
$$1 = V(q)\,\mathcal{L}_R(q,\Omega_q - i\gamma_q).$$

In Fermi Liquids sharp collective excitations can live only OUTSIDE the continuum

In semi-holographic non-Fermi liquids, sharp collective excitations exist INSIDE the continuum but outside the INNER CORE (can be proved generally).



$arg(\zeta) = \pi/4, |\zeta| = 1, \nu = 2/3$ $e^2/2\epsilon_b = 1$ $k_F = 0.4, m = 0.5$



The collective excitations have energies and widths prop to q

They are a intermediate frequency but short wavelength phenomena

$$q > k_F, \quad \Omega_q, \gamma_q < E_F$$

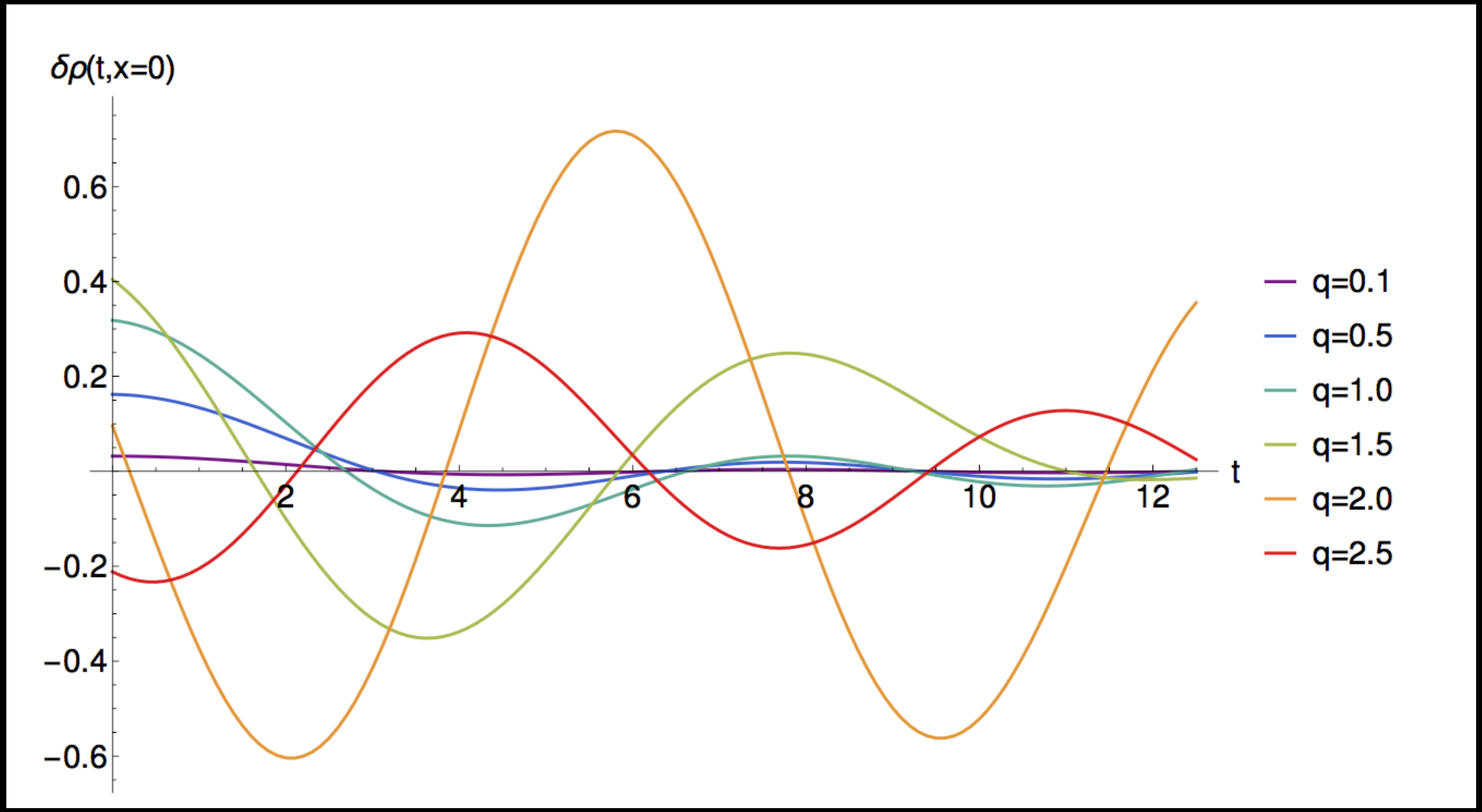
PLASMA OSCILLAITONS

One way to see this is to kick the system with a potential:

$$\phi^{\text{ext}}(\mathbf{x}, t) = \phi_0 \cos(\mathbf{q} \cdot \mathbf{x}) \delta(t)$$

This will induce charge oscillations

$$\delta\rho(\mathbf{x} = 0, t) = -2e\phi_0 (2\pi)^{-1} \int_0^\infty \mathrm{d}\Omega \, \left(\mathrm{Re}\mathcal{L}^{\mathrm{imp}}(q, \Omega) \cos(\Omega t) - \mathrm{Im}\mathcal{L}^{\mathrm{imp}}(q, \Omega) \sin(\Omega t)\right)$$



$q < 1.5$ **Oscillations decay fast and periods are almost independent of q**

$q > 1.5$ **Oscillations decay very slowly and periods are proportional to q**

The mid-infrared scenario for superconductivity and semi-holography

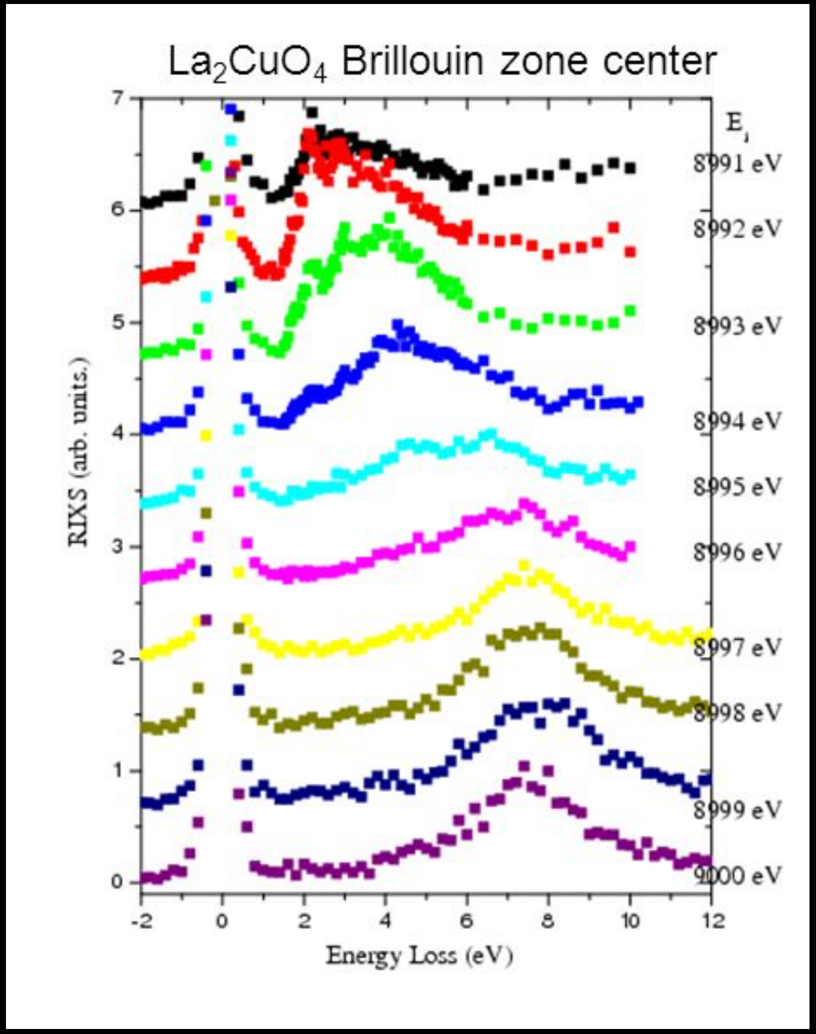
Leggett's MID-INFRARED Scenario (Nature Physics, 1999)

The binding energy of pair formation in unconventional superconductivity comes from "saving Coulomb energy"

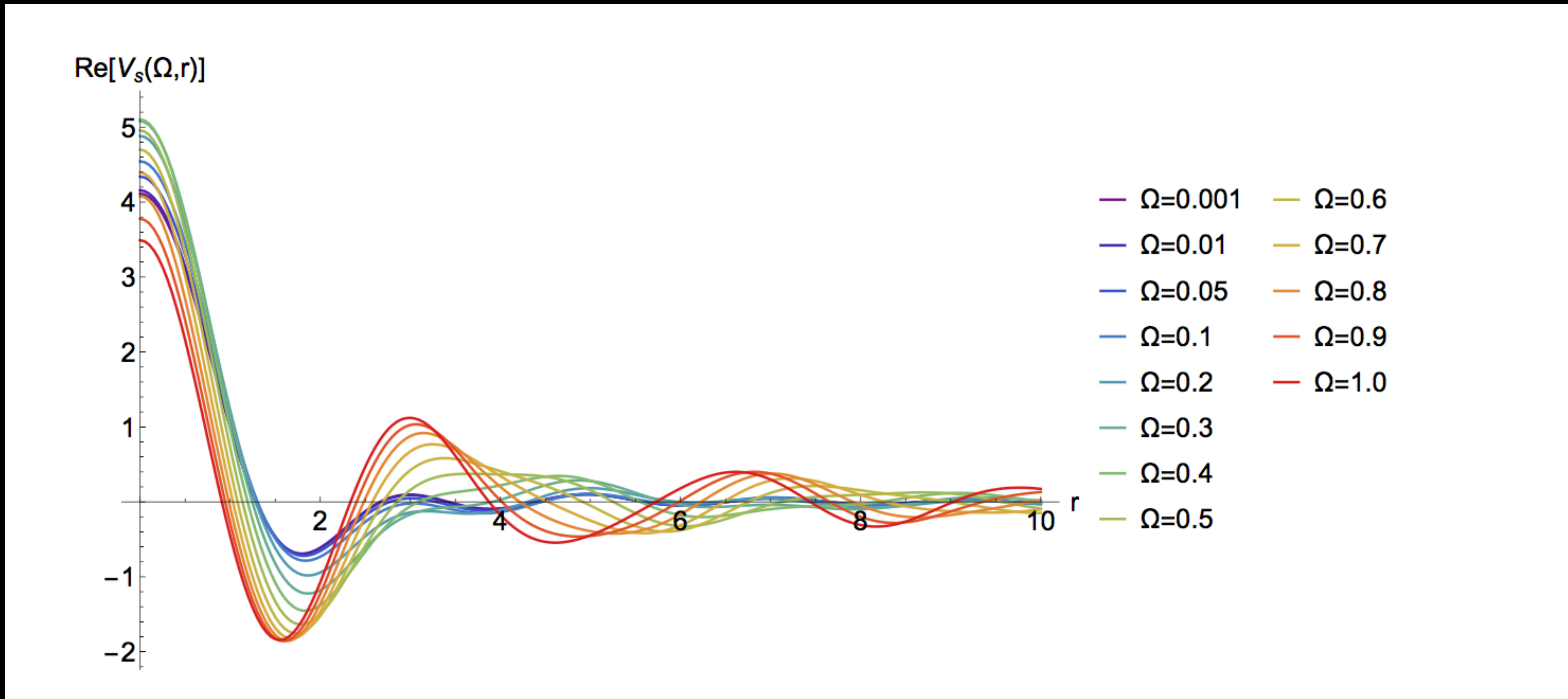
Necessary condition: Collective excitation within the continuum at intermediate energy scales (from sum rules)!

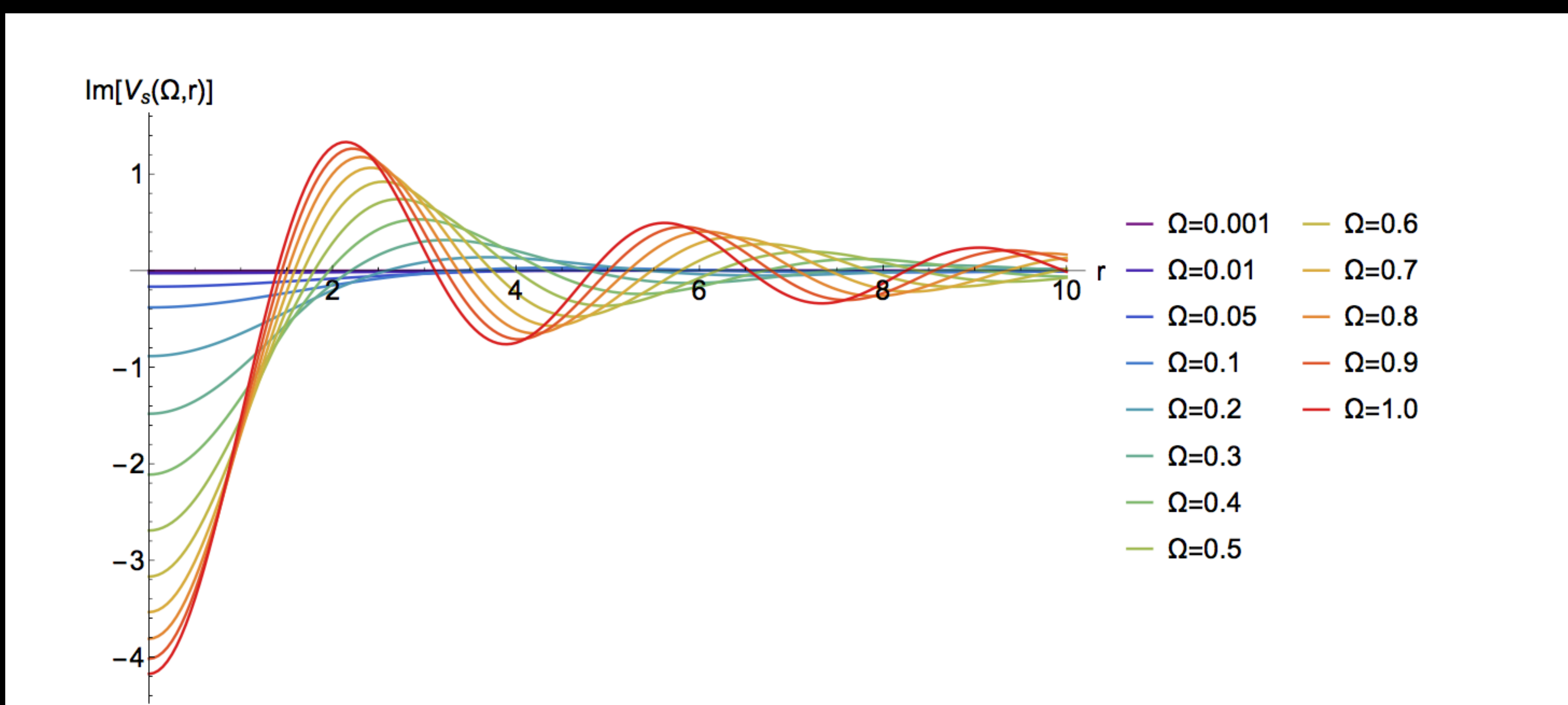
This argument does not give a pairing mechanism itself but a *necessary* (*not sufficient*) condition for non-BCS superconductivity *driven by electronic interactions alone* to exist

Such mid-infrared peaks can be seem by electron energy loss spectroscopy (EELS) that directly measures resummed Lindhard function or by resonant inelastic X-ray scattering (RIXS)



Abbamonte et. al. 1999





Indeed the dynamical screening potential shows ATTRACTIVE STABLE WELLS

Semi-holographic non-Fermi liquids could be the first system to realize “plasmonic pairing mechanism”

Open question and Conclusions

OPEN QUESTION: DOES PAIRING INDEED HAPPEN?

Hard question but can be addressed systematically by a generalization of Midgal-Eliashberg theory. Onging work (calculating pair susceptibility) along with a very bright postdoc and also PhD students

It could be that we need to incorporate realistic features for pairing to happen — spin and therefore spin fluctuations, non-spherical Fermi surface, etc.

Although the plasmonic MIR (mid infrared) peak is *necessary*, it is not *sufficient.* Counterexamples include layered materials such as $La_{4-x}Ba_{1+x}Cu_5O_{13}$

Aside: Leggett claims that the MIR scenario rules out 2D Hubbard t-J models as viable since these cannot lead to the MIR peak. He insists that the starting point should be a model of the normal state that shows MIR plasmonic peak.

Ref: https://uwaterloo.ca/institute-for-quantum-computing/sites/ca.institute-for-quantum-computing/files/uploads/files/lecture-12.pdf (2010)

CONCLUSIONS

The pairing should be *dynamical* and not a static phenomenon.

It should utilize

(i) the attractive wells of the dynamic screened potential at non-zero frequency and

(ii) the fact that our generalized quasi-particles have sufficient spectral weight at non-vanishing frequencies

Semi-holography is indeed a concrete way to explore what conditions can make the MIR plasmonic peak *sufficient* for superconductivity

THANKS FOR YOUR ATTENTION