HOW DOES RELATIVISTIC KINETIC THEORY REMEMBER ABOUT INITIAL CONDITIONS?

MICHAL P. HELLER (AEI / NCBJ)

1609.04803 W/ KURKELA, SPALINSKI, SVENSSON

1802.08225 W/ SVENSSON

SEE ALSO 1707.02282 W/ FLORKOWSKI, SPALINSKI

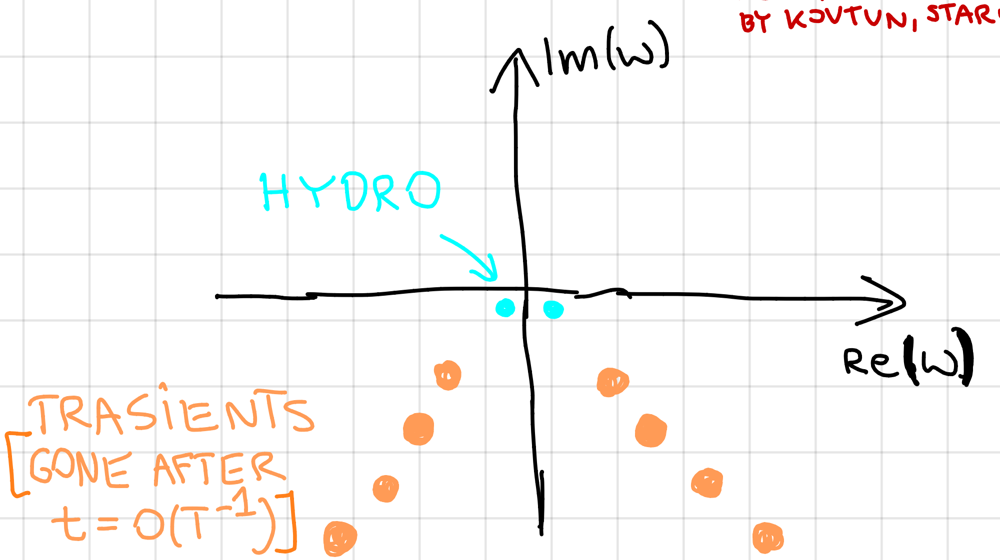
THE QUESTION

FOCUS: N-EQ PHYSICS AT "WEAK COUPLING" USING HOLO INSIGHTS

THE MODE PARADIGM: SING. IN $\omega \in \mathbb{C}$ OF $G^2_R(\omega, \underset{\text{FIXED}}{k})\big|_{T \leftarrow \text{EQUILIBRIUM}}$ GOVERN $\delta\langle T^{\mu\nu}\rangle$ $\longrightarrow$:

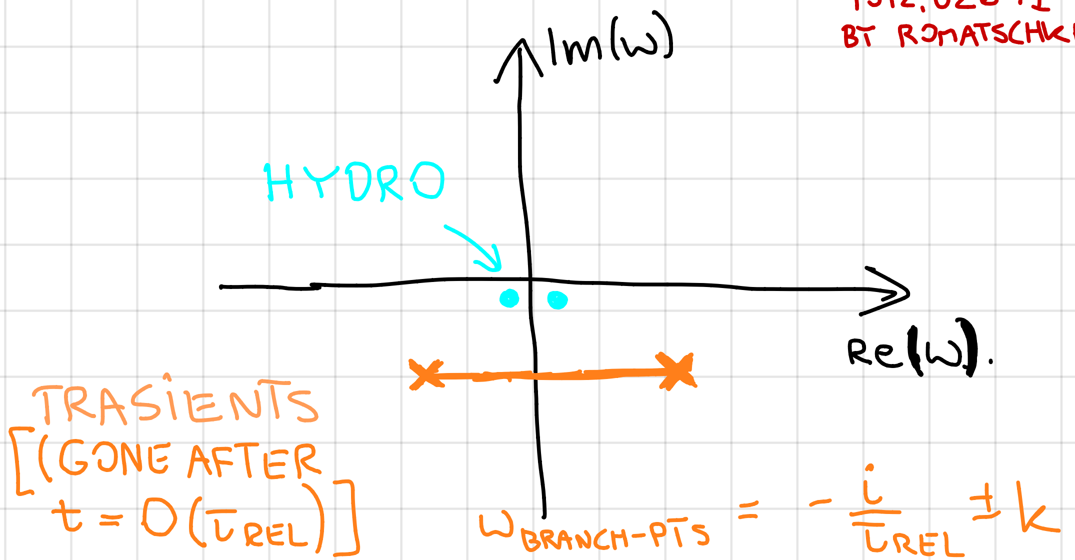
$\mathcal{N}=4$ SYM (SUGRA)

HEP-TH/0506184
BY KOVTUN, STARINETS



RTA KINETIC THEORY WITH $\tau_{REL} \sim \frac{1}{T}$

1512.02641
BY ROMATSCHKE

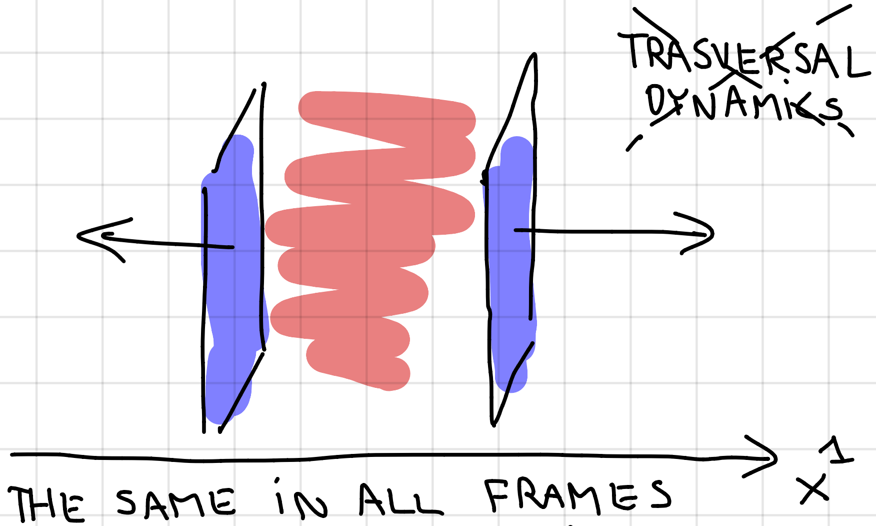


THE HYDRODYNAMIZATION PARADIGM $\rightarrow$ TRANSIENTS ON TOP OF vHYDRO
(SEE ALSO TALKS BY BUCHEL, JANKOWSKI, MEIRING AND SPALINSKI)

HOW DOES ×——× LOOK LIKE IN SUCH A SETTING?

BOOST-INVARIANT FLOW

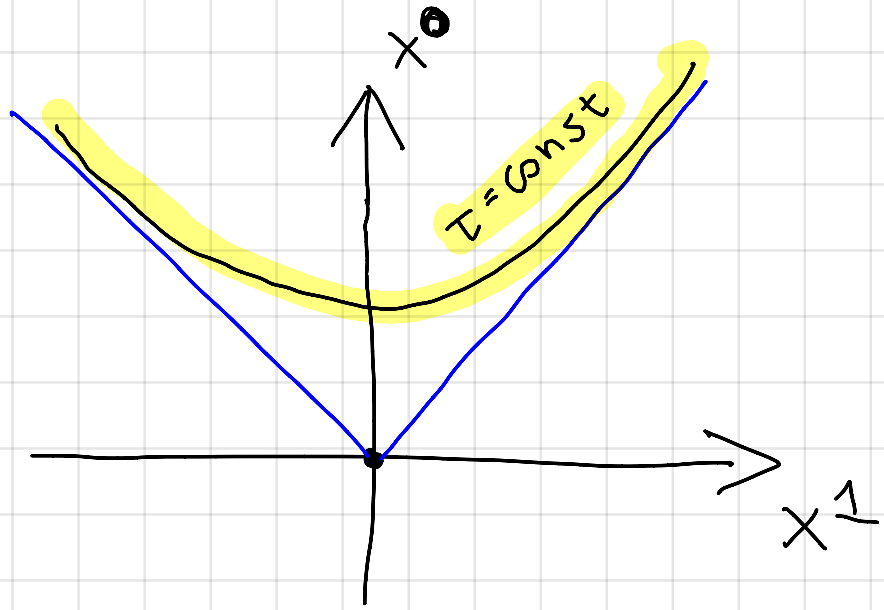
1982 BJORKEN



THE SAME IN ALL FRAMES
BOOSTED ALONG x^1

↓

PHYSICS [$\tau = \sqrt{(x^0)^2 - (x^1)^2}$]



$$\langle T^\mu{}_\nu \rangle_{\text{OF INTEREST}} = \mathrm{DIAG}\Big\{-\varepsilon \,,\; \overbrace{-\varepsilon - \tau\dot{\varepsilon}}^{P_L} \,,\; \overbrace{\varepsilon + \tfrac{1}{2}\tau\dot{\varepsilon}}^{P_T} \,,\; \overbrace{\varepsilon + \tfrac{1}{2}\tau\dot{\varepsilon}}\Big\}^\mu{}_\nu$$

BOOST-INVARIANT HYDRO: $u^\mu \partial_\mu = \partial_\tau \,, \quad \nabla u \sim \frac{1}{\tau}$

RELATIVISTIC KINETIC THEORY IN RTA

1954 BHATNAGAR, GROSS, KROOK; 1974 ANDERSON, WITTING

IN KT THE FUNDAMENTAL VAR. IS $f(x,p)$, WHICH SOLVES THE BOLTZMANN EQ. $p^\mu \partial_\mu f = C[f]$ ← INTERACTIONS

IN KT $\langle T^{\mu\nu} \rangle (x) = \int dP \; f(p,x) \; p^\mu p^\nu$

RTA: $C[f] \sim -\frac{1}{\tau_{REL}} \left(f - f_{EQ}[f] \right)$

WE TAKE $m=0$ AND $\tau_{REL} \sim \frac{1}{T^\Delta} \sim \frac{1}{\varepsilon^{\Delta/4}}$

$\langle T^\mu{}_\mu \rangle = 0$

$\Delta=1$: CONFORMAL; $\Delta=0$: SIMPLE; ...

HYDRODYNAMICS AT LARGE ORDERS

TEXTBOOK HYDRO:

- A CLASS OF $\langle T^{\mu\nu}\rangle = \mathcal{E}\, u^\mu u^\nu + P(\mathcal{E})\{g^{\mu\nu} + u^\mu u^\nu\} + \pi^{\mu\nu}[\mathcal{E}, u^\alpha]$
- WITH $\pi^{\mu\nu}[\mathcal{E}, u^\alpha] = \underbrace{-\eta\, \sigma^{\mu\nu}}_{\sim \partial} + \underbrace{\eta\, \tau_\pi D \sigma^{\mu\nu} + \lambda_1 \sigma^{\langle\mu}{}_\gamma\, \sigma^{\nu\rangle\gamma}}_{\sim \partial^2} + \ldots$

BOOST INVARIANCE: $\Pi_{TT} - \Pi_{LL} = \frac{\mathcal{E}}{3}\left\{ a_1 \underbrace{\frac{\tau_{REL}}{\tau}}_{\equiv \frac{1}{w}} + a_2 \underbrace{\left(\frac{\tau_{REL}}{\tau}\right)^2}_{\equiv \frac{1}{w^2}} + \ldots \right\}$

WHAT ONE FINDS IS THAT $a_1 = \frac{8}{5}$, $a_2 = \frac{88}{105} - \frac{8}{15}\Delta$ AND $a_n \sim n!\big|_{n\to\infty}$

WE KNOW HOW TO MAKE SENSE OUT OF IT

$\downarrow$ SEE TALKS BY MEIRING, JANKOWSKI, SPALINSKI AND 1707.02282 FOR A REVIEW

$A(w) = \frac{\Pi_{TT} - \Pi_{LL}}{\mathcal{E}/3}$ CAN BE REPRESENTED AS A TRANS-SERIES

THE PUZZLE

1609.04803v1 W/ KURKELA, SPALINSKI

TRANS-SERIES:

TRANSIENTS ON vHYDRO

POSSIBLY NONLINEAR TERMS

$$A(w) = \frac{a_1}{w} + \frac{a_2}{w^2} + \dots + \sum_j \sigma_j \, e^{-\Omega_j w} \, w^{\beta_j}\left(1 + \frac{b_{j,1}}{w} + \dots\right) + \dots$$

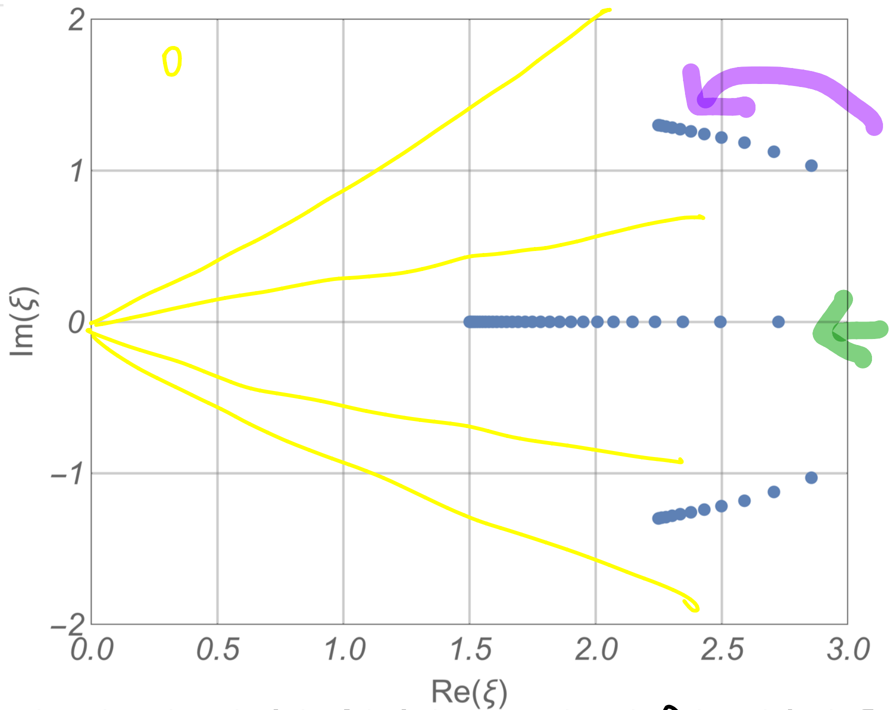
BOREL TRAFO

$a_n \to a_n/n!$

$$B A_H(\xi)$$

δ BOREL^{-1} TRAFO:

$$\delta\left[\frac{1}{w}\int_{C_0^\infty} d\xi \, e^{-\xi \cdot w} \, B A_H(\xi)\right]$$



WHAT IS THIS ???

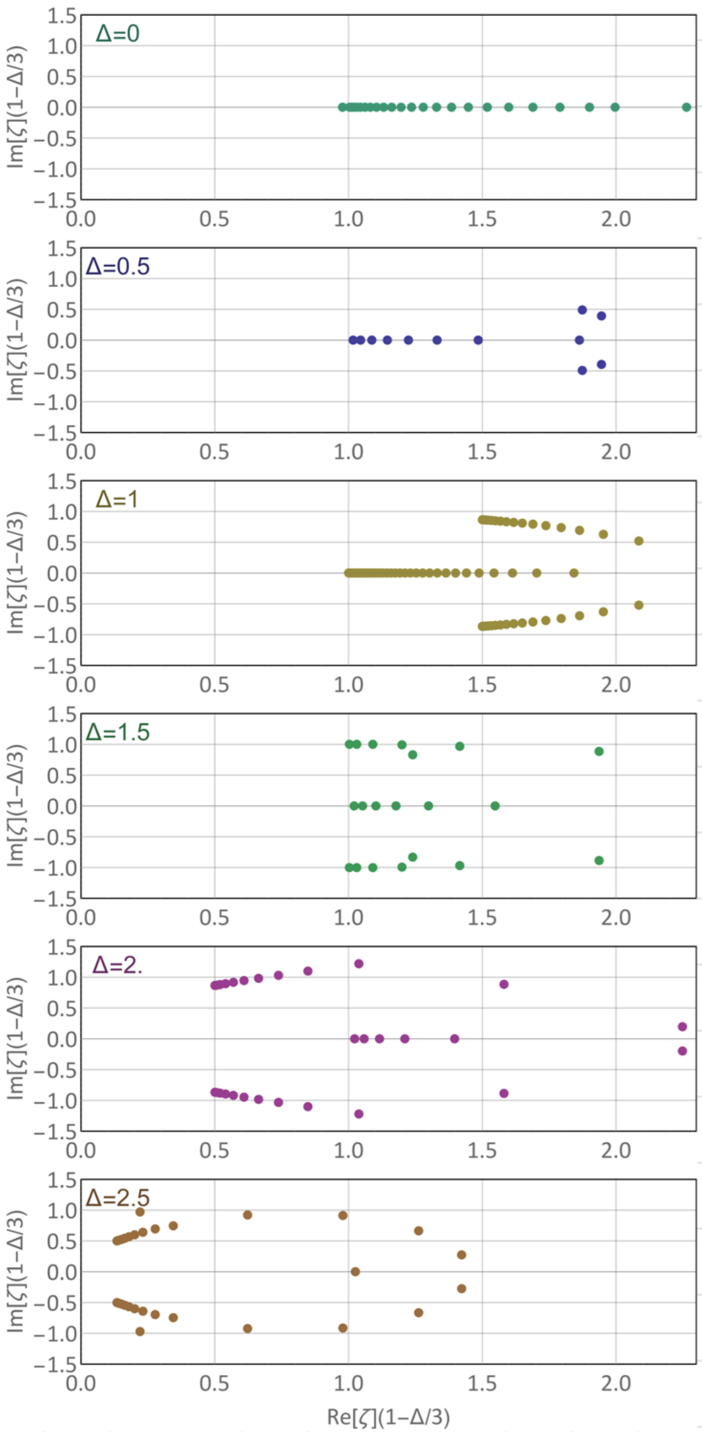
PURELY* DECAYING MODE* WITH $\exp\left[-\int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{REL}(\tau')}\right]$

$\Delta = 1$, $BA_H(\xi)$ FROM 200 ORDERS

IS IT PHYSICAL?

1802.08225 w/ SVENSSON

PREVIOUS SLIDE



← NO TRACE OF IT FOR $\Delta = 0$

← $\Delta > 1.5$: IT WOULD BE THE DOMINANT TRANSIENT, BUT WE COULD NOT SEE IT IN NUMERICS IN $A_a(w) - A_b(w)$

↓

IT MUST ENTER $A(w)$ WITH THE SAME COEFF. FOR ALL $A(w)$

$\Delta > 2$: IT NEVERTHELESS DOMINATES THE HYDRO GRADIENT EXPANSION AT LARGE ORDERS

EXPLANATION

1609.04803v2 (FEB 2018) w/ KURKELA, SPALINSKI + SVENSSON

THERE IS AN ALTERNATIV FORMULATION OF THE RTA KINETIC THEORY (1984 BAYM)

$$\mathcal{E}(\tau)\cdot \exp\left[\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{REL}(\tau')}\right] = \mathcal{E}_0(\tau) + \int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{REL}(\tau')}\, H\left(\frac{\tau'}{\tau}\right)\, \mathcal{E}(\tau')\, \exp\left[\int_{\tau_0}^{\tau'}\frac{d\tau'}{\tau_{REL}(\tau')}\right]$$

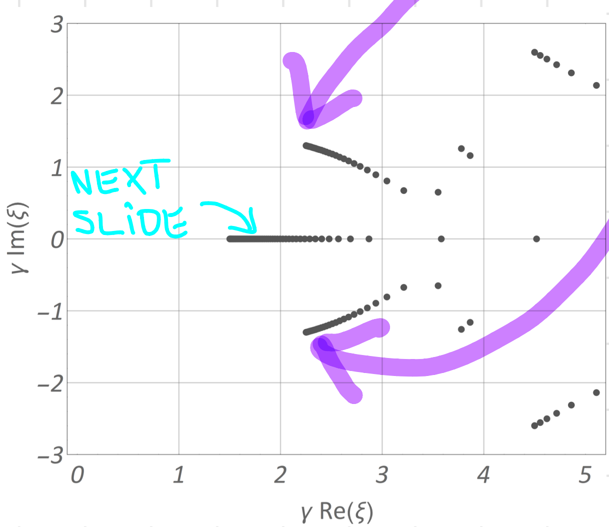
$$H\left(\frac{\tau'}{\tau}\right) = \frac{\tau'^2}{2\tau^2} + \frac{\arctan\sqrt{\frac{\tau^2}{\tau'^2}-1}}{2\sqrt{\frac{\tau^2}{\tau'^2}-1}}$$

TAKING ARCTAN IN NON-PRINCIP. BRANCH AND $\dfrac{1}{\sqrt{\frac{\tau^2}{\tau'^2}-1}}$ FOR $\Delta \neq 0$ LEADS

TO CONTOUR DEFORMATIONS THAT BRING THE MYSTERIOUS EXPONENTS:

$\Delta = 1$

NOW 426 TERMS



HOWEVER, THE PHYSICAL CONTOUR IS HOMOLOGOUS TO $\mathbb{R}$!!!

EXPS UNPHYSICAL AS MODES

$\boxed{7/12}$

MODES IN THE RTA KINETIC THEORY

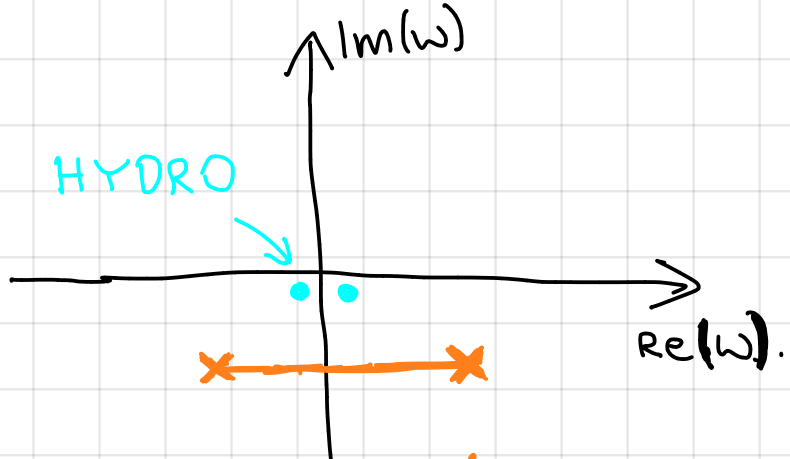
1802.08225 W/ SVENSSON

$$\mathcal{E}(\tau)\cdot \exp\left[\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{REL}(\tau')}\right] = \mathcal{E}_0(\tau) + \int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{REL}(\tau')}\, H\left(\frac{\tau'}{\tau}\right)\, \mathcal{E}(\tau')\, \exp\left[\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{REL}(\tau')}\right]$$

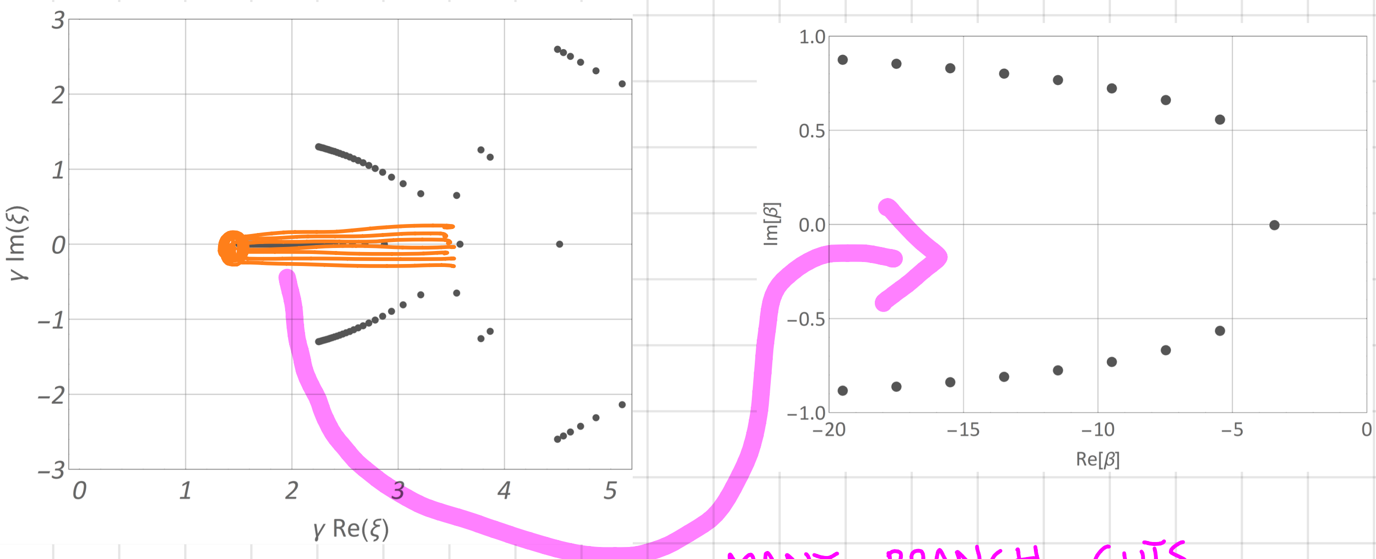
LINEARIZATION $\mathcal{E}(\tau)-\mathcal{E}_H(\tau) \sim \exp\left[-\hat{\alpha}\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{REL}(\tau')}\right] w^{\beta} + \ldots$

$\hat{\alpha} = 1$: EXP DECAY OVER τ_{REL} WITH β's FIXED BY $0=\int_0^1 dz\, H(z)\, z^{\beta(1-\Delta/3)-\Delta/3}$

LINEAR RESPONSE THEORY | BOREL PLANE | MODES ON vHYDRO



$\omega_{BRANCH\text{-}PTS} = -\frac{i}{\tau_{REL}} \pm k$

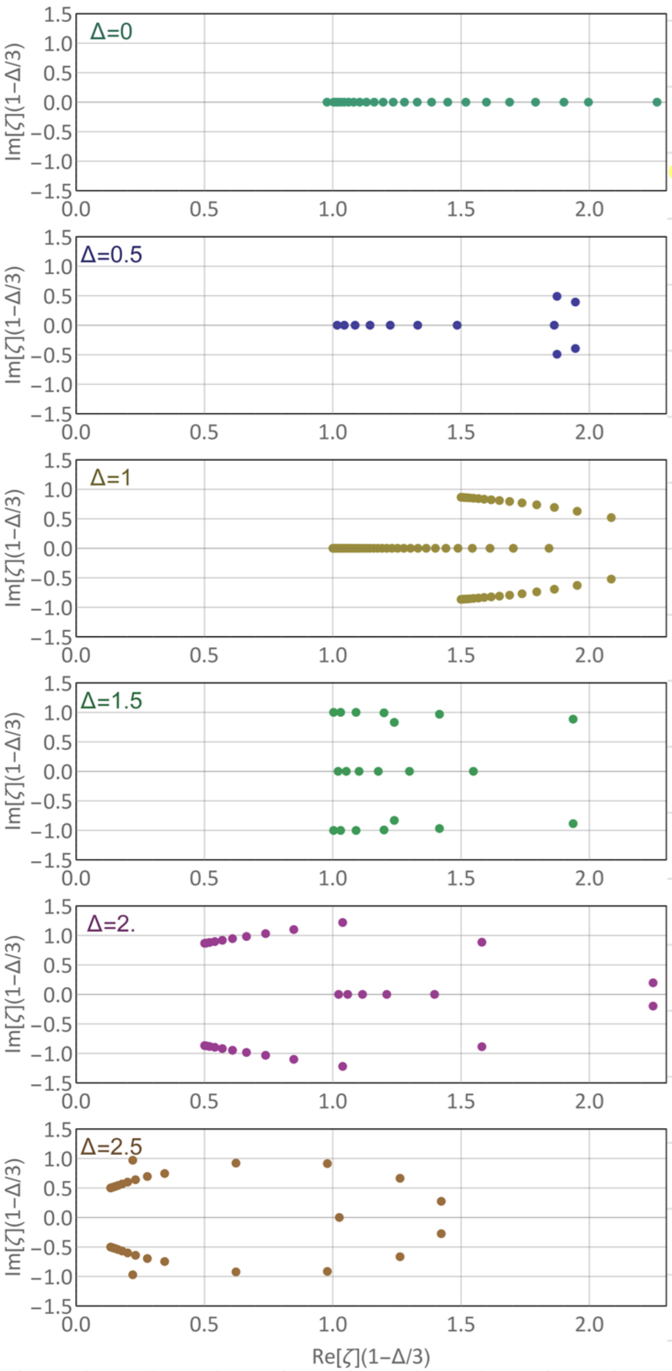


MANY BRANCH CUTS STACKED ON EACH OTHER

$(\xi-\xi_0)^{\#\sim\beta}$

NUMERICAL VERIFICATION – SETUP

1802.08225 W/ SVENSSON



WE WANT TO VERIFY THE EXISTENCE OF MODES / β's

← WE CAN DO THIS FOR THE SIMPLEST CASE $\Delta = 0$

WE CAN SOLVE THE INT. EQ. BY FINDING FIXED POINT FOR EACH INI. COND.

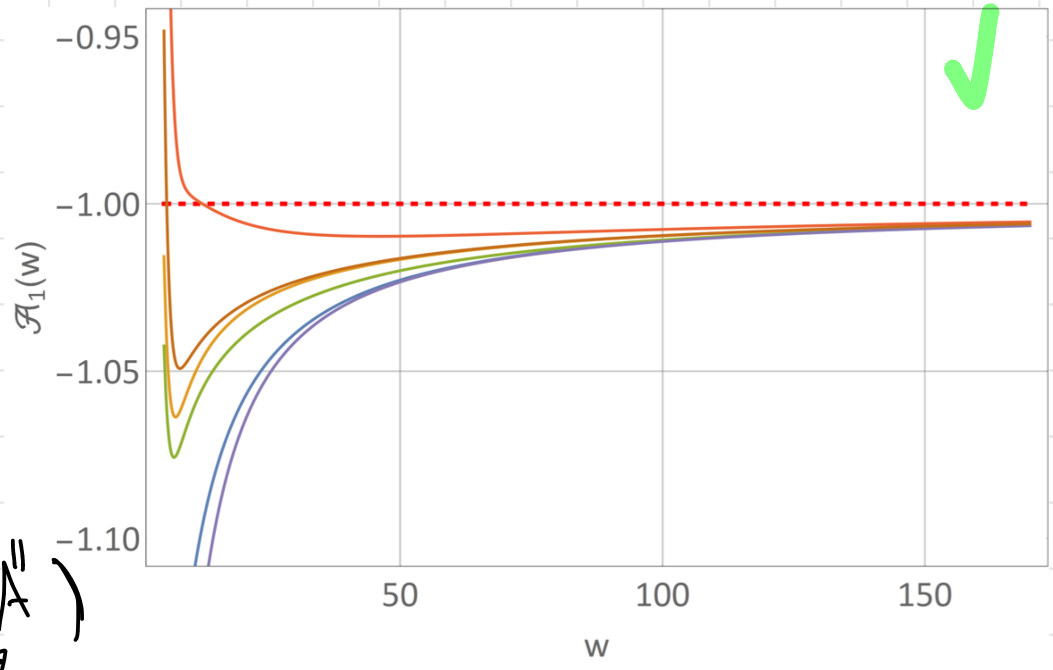
WE NEED LARGE ACCURACY TO RESOLVE β's

↓

PSEUDOSPECTRAL METHODS LOVED IN NUMHOL

NUMERICAL VERIFICATION ~ THE LEADING TRANSIENT

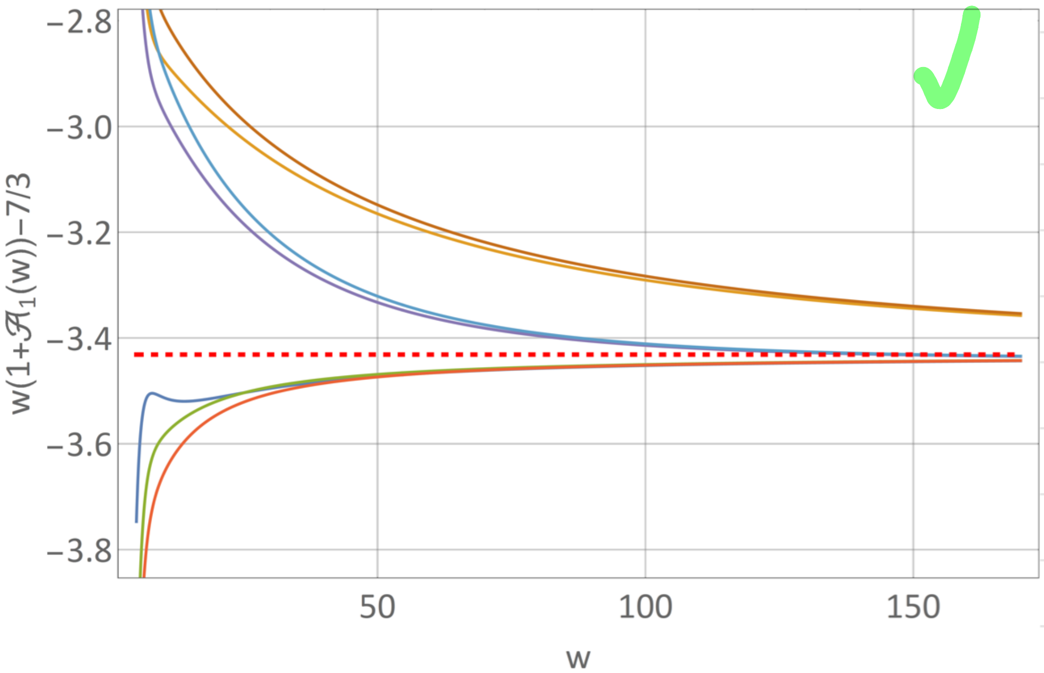
1802.08225 W/ SVENSSON

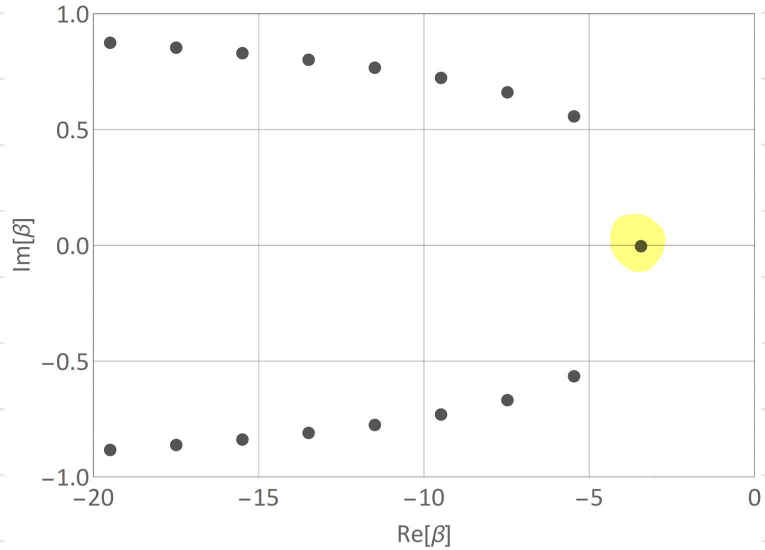


$$A' - A'' \sim \exp(-w)$$

$$A_1 \equiv \partial_w \log(A' - A'')$$

2 SOLUTIONS



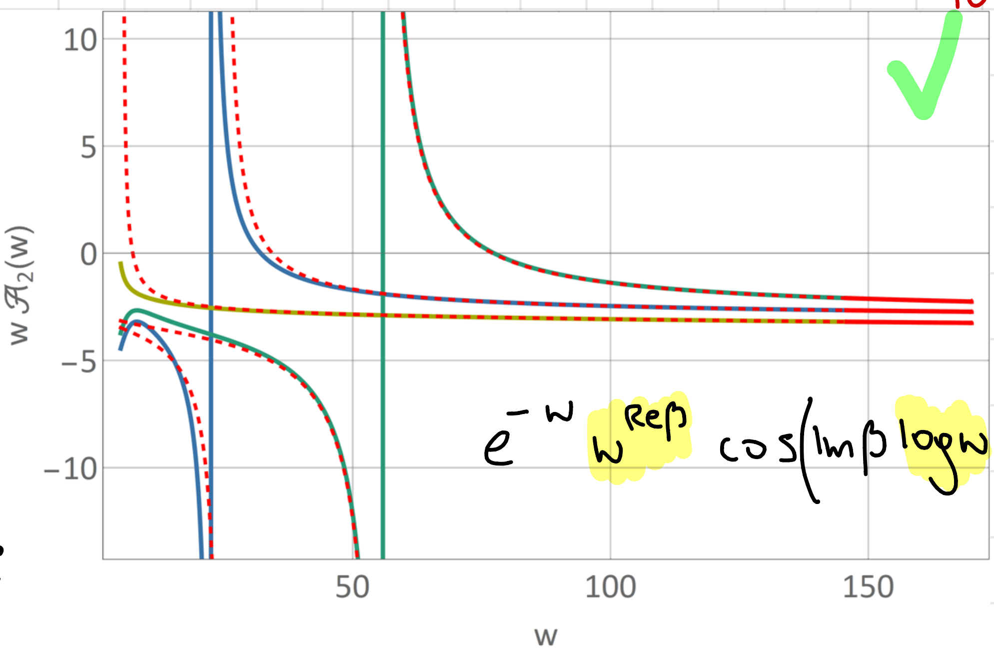


NUMERICAL VERIFICATION ~ THE SECOND TRANSIENT

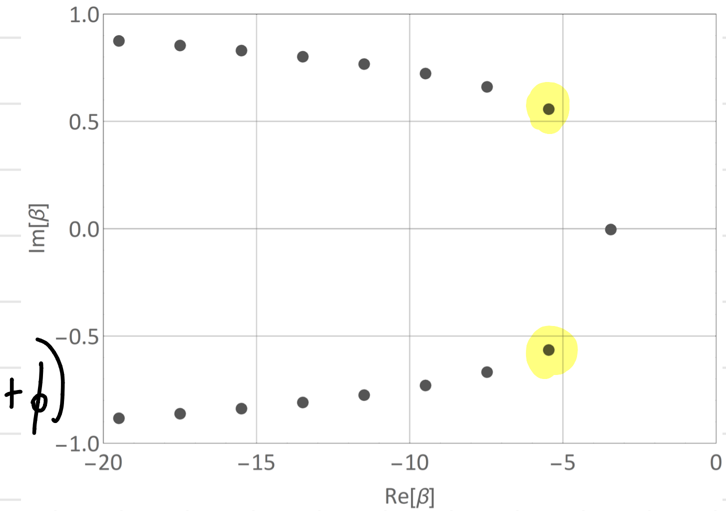
1802.08225 W/ SVENSSON

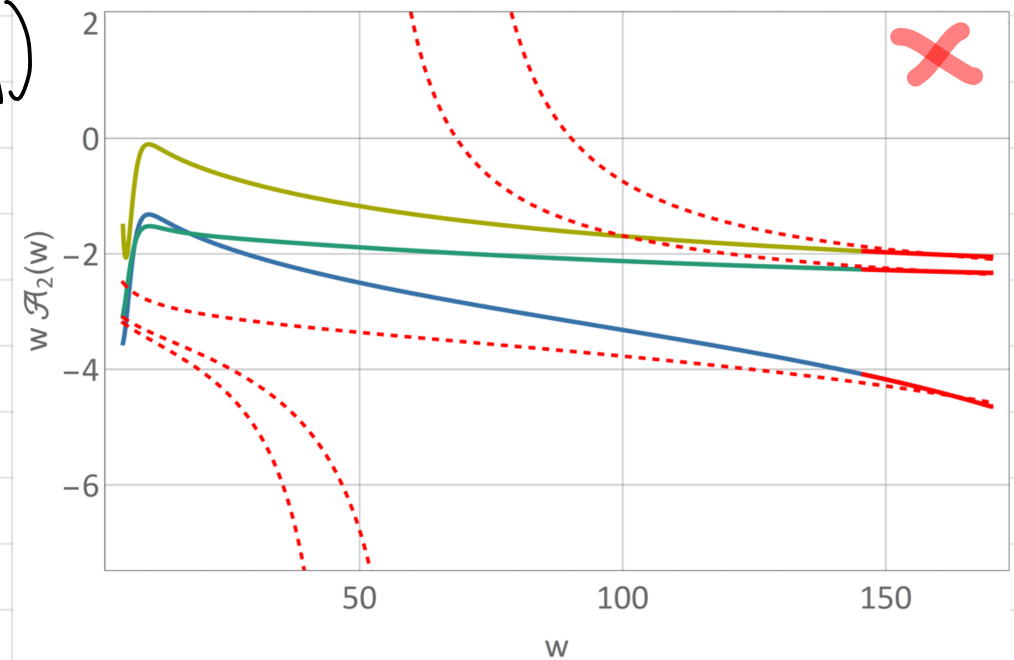
ONE MORE $\partial_w \log$:

$$A_2 = \partial_w \log\left(A_1' - A_1''\right)$$



$$e^{-w}\, w^{\mathrm{Re}\,\beta} \cos\left(\mathrm{Im}\,\beta \log w + \phi\right)$$





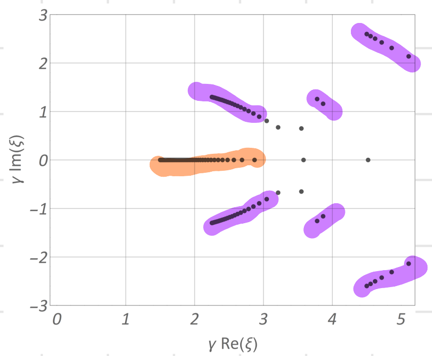
MOST LIKELY POLLUTION FROM OTHER MODES

OUTLOOK

1609.04803 W/ KURKELA, SPALINSKI, SVENSSON
1802.08225 W/ SVENSSON

IDEA: USE RESURGENCE IN HYDRO AS A WAY TO UNDERSTAND TRANSIENTS

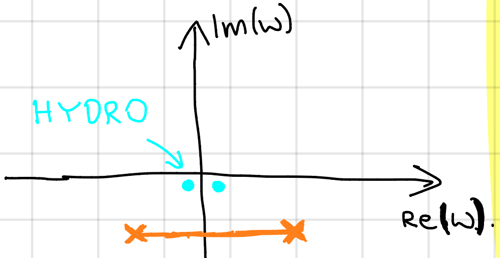
SUBTLE IN RTA KINETIC THEORY:



UNDERSTOOD ✓

SOME OPEN PROBLEMS:

ANALOGUE OF FOR MORE GENERAL C



$\omega_{BRANCH-PTS} = -\frac{i}{\tau_{REL}} \pm k$

SEE 1712.04376 BY KURKELA, WIEDEMANN

$\frac{\tau_R}{\tau}$ FOR $\tau_{REL} \sim \frac{1}{T^\Delta}$ WITH $\Delta > 3$

$\to \infty$: NO HYDRO AT LATE TIMES

ATTRACTOR ???