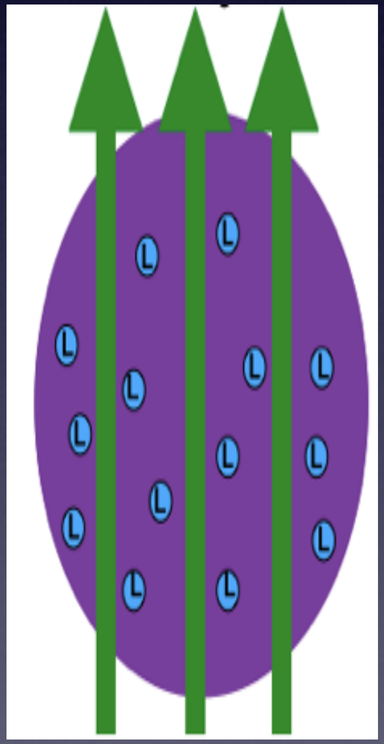
# Correlations far from equilibrium in strong magnetic fields

HoloQuark2018, Santiago de Compostela, Spain

July 3rd, 2018



Matthias Kaminski
in collaboration with Casey Cartwright
*University of Alabama*

# Motivation

- Einstein-Maxwell(-Chern-Simons) Theory with a magnetic field **in equilibrium has a universal magnetoresponse** variable, which agrees *well* with its QCD equivalent

  *[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]*

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  *[Cartwright, Kaminski; to appear]*

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# Scale invariance in LQCD with magnetic field

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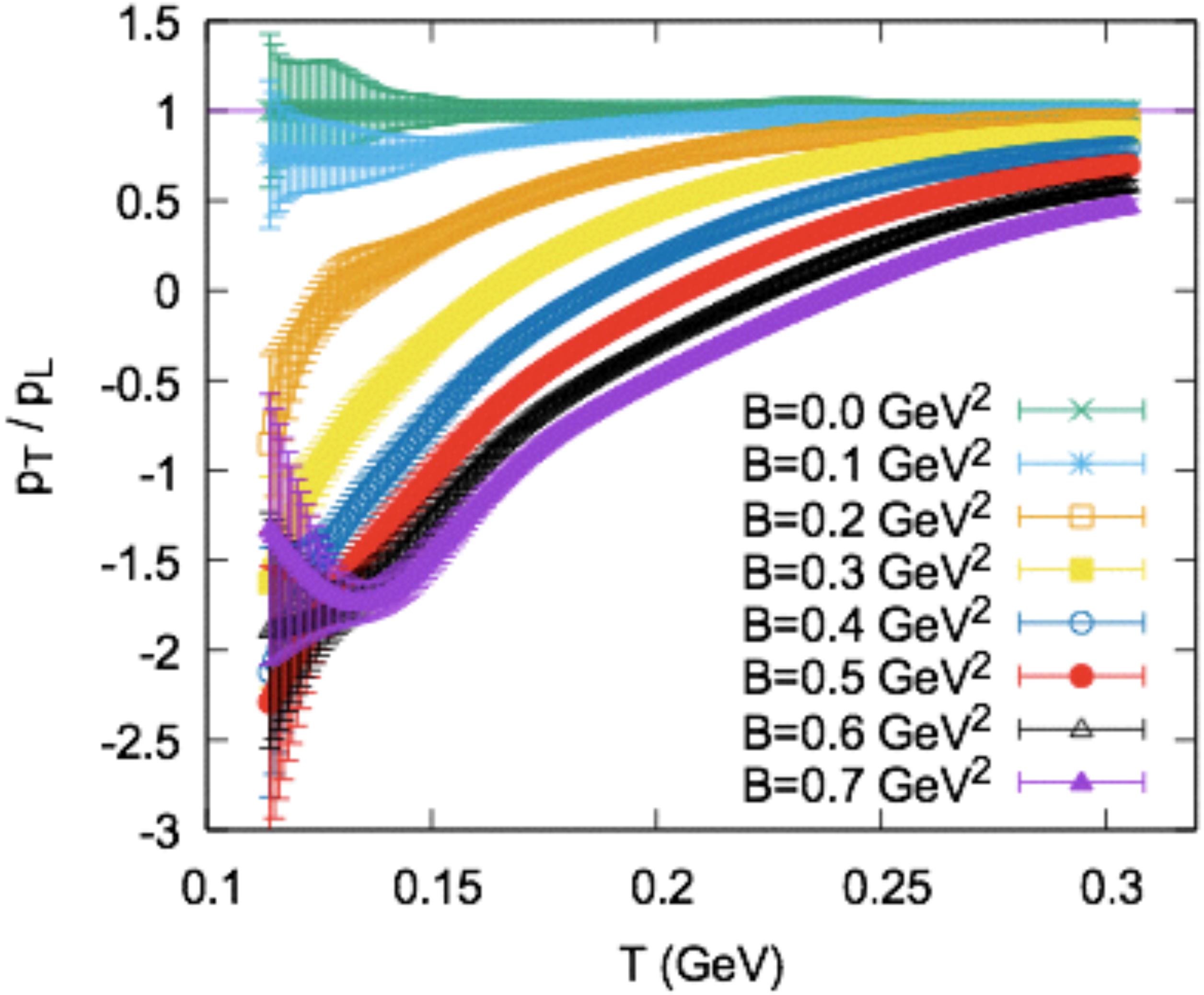


Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

*transverse pressure:* $p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$

*longitudinal pressure:* $p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$

$F_{\mathrm{QCD}}$ ... *free energy*

$L_{\mathrm{T}}$ ... *transverse system size*

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$V$ ... *system volume*

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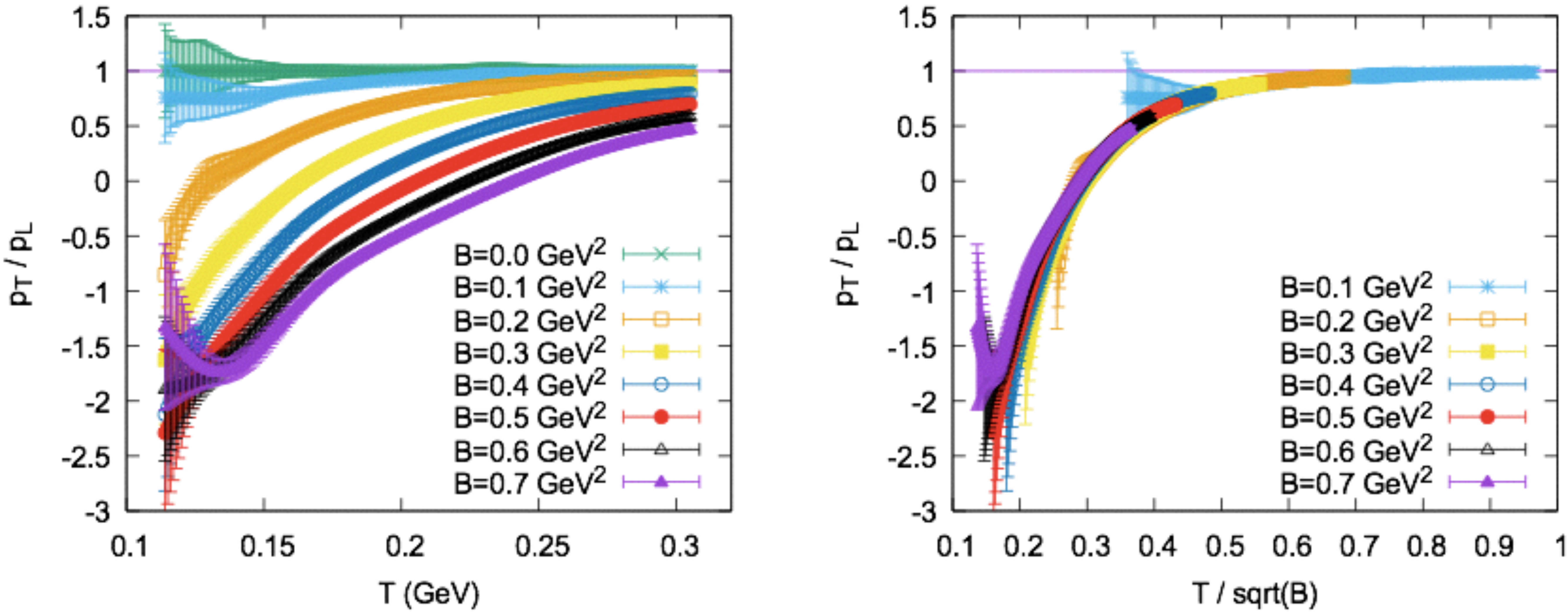


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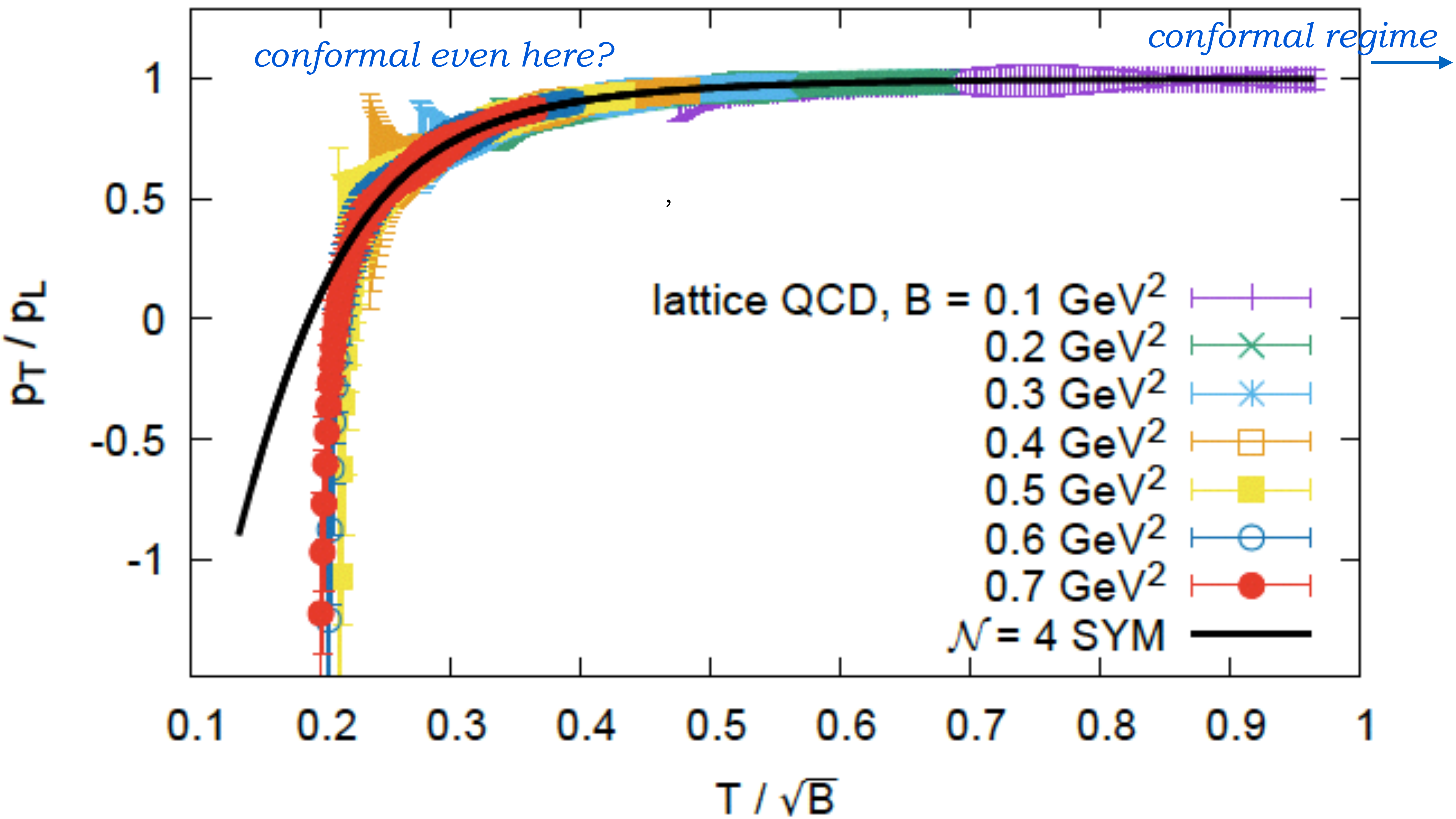
# Good agreement with N=4 Super-Yang-Mills (from holography)

*[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]*

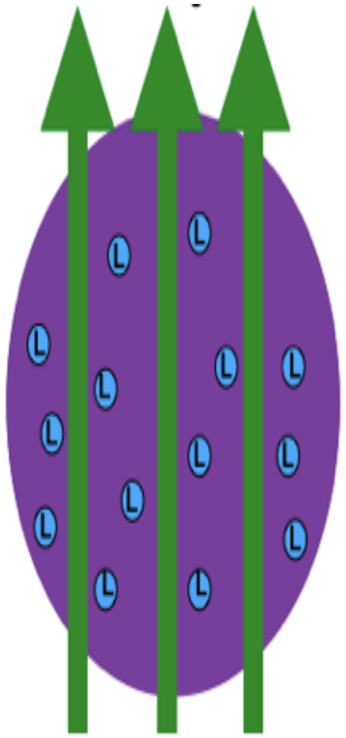


# Setup

Einstein-Maxwell-Chern-Simons action:

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}(R - 2\Lambda + F_{\mu\nu}F^{\mu\nu}) + \gamma\epsilon^{\alpha\beta\gamma\delta\eta}A_\alpha F_{\beta\gamma}F_{\delta\eta}$$

*neglect in this work*



Metric ansatz:

$$\mathrm{d}s^2 = -A(r,t)\mathrm{d}t^2 + 2\mathrm{d}r\mathrm{d}t + S(t,r)^2(e^{B(r,t)}(\mathrm{d}x^2 + \mathrm{d}y^2) + e^{-2B(r,t)}\mathrm{d}z^2)$$

# Setup

Einstein-Maxwell-Chern-Simons action:

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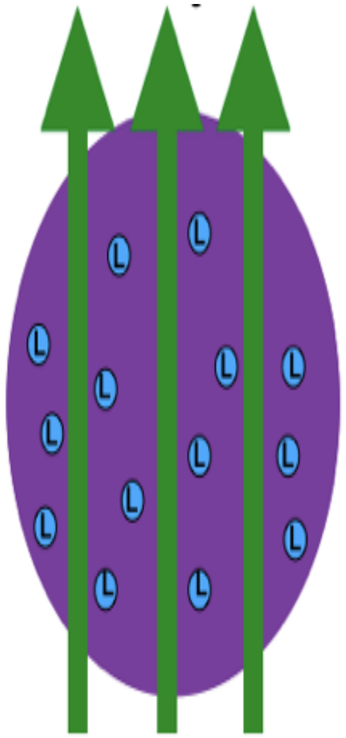

Metric ansatz:

$$ds^2 = -A(r,t)dt^2 + 2drdt + S(t,r)^2(e^{B(r,t)}(dx^2+dy^2) + e^{-2B(r,t)}dz^2)$$

Maxwell equations are solved by:

$$\mathscr{A}(r,t) = (0, \phi(r,t), -\frac{1}{2}y\mathscr{B}, \frac{1}{2}x\mathscr{B}, 0)$$

$$-\partial_r\phi(r,t) = \mathscr{E}(r,t) = \frac{\rho(r,t)}{S(t,r)^3}$$

Einstein equations are nested:

$$S''(t,r) = -\frac{1}{2}B'(t,r)^2 S(t,r)$$

$$\dot{f} = \partial_t f + \frac{1}{2}A\partial_r f.$$

$$\dot{S}'(t,r) = \frac{\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^3} - \frac{2S'(t,r)\dot{S}(t,r)}{S(t,r)} + \frac{\rho^2}{3S(t,r)^5} + 2S(t,r)$$

$$\dot{B}'(t,r) = -\frac{3\dot{B}(t,r)S'(t,r)}{2S(t,r)} - \frac{3B'(t,r)\dot{S}(t,r)}{2S(t,r)} + \frac{2\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^4}$$

$$A''(t,r) = -3B'(t,r)\dot{B}(t,r) - \frac{10\mathscr{B}^2 e^{-2B(t,r)}}{3S(t,r)^4} + \frac{12S'(t,r)\dot{S}(t,r)}{S(t,r)^2} - \frac{14\rho^2}{3S(t,r)^6} - 4$$

$$\ddot{S}(t,r) = \frac{1}{2}A'(t,r)\dot{S}(t,r) - \frac{1}{2}\dot{B}(t,r)^2 S(t,r).$$

# Background

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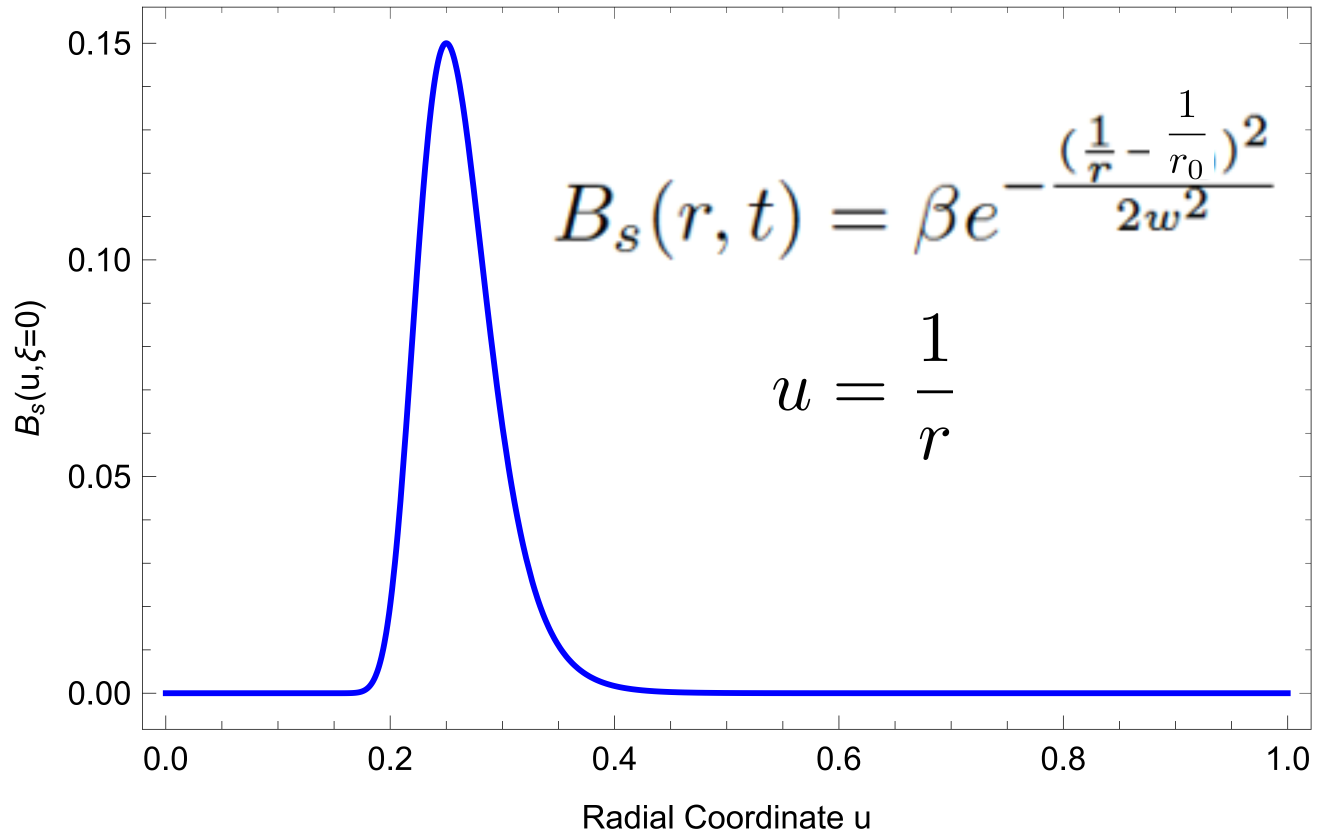


Numerical implementation- characteristic formulation

*[Chesler, Yaffe; PRL (2009)]*

- use (pseudo)spectral methods with Cardinal Function basis to solve ODEs in *r* at initial time for $S$, $\dot{S}$, $\dot{B}$, $A$ on Gauss-Lobatto grid
- time step forward using 4th order Runge-Kutta on first 4 time steps, and subsequently Adams-Bashforth
- boundary expand and solve for subtracted and scaled functions
- radial diffeomorphism used to keep horizon fixed

# Background - One Point Functions

*[Fuini, Yaffe; JHEP (2015)]*

**previous results:**

- time course of one point functions basically insensitive to charge **or** magnetic field

-pressure anisotropy is a linear functional of the initial anisotropy pulse profile

**raises two questions:**

* two point functions insensitive too?

* what happens at nonzero charge **and** magnetic field?

***dual field theory:***

$\mathcal{N}$=4 SYM in 3+1 dimensions,
minimally coupled to external U(1) gauge field,

with trace anomaly $T_\mu{}^\mu = -\frac{1}{2}\gamma\mathcal{B}^2$

in presence of charge density $\rho$

or/and external magnetic field $\mathcal{B}$

# Background - One Point Functions

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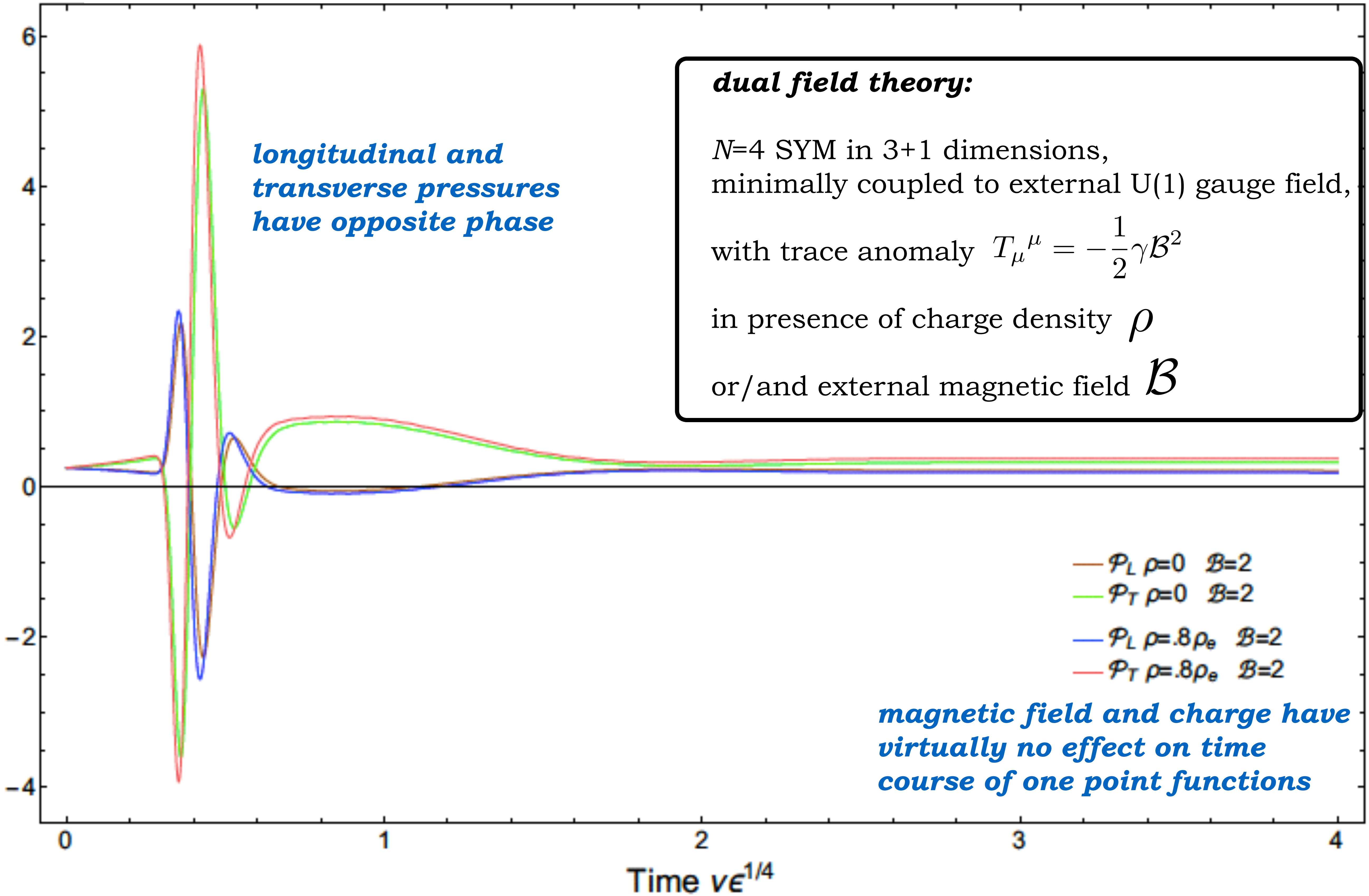


***longitudinal and transverse pressures have opposite phase***

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in presence of charge density $\rho$

or/and external magnetic field $\mathcal{B}$

***magnetic field and charge have virtually no effect on time course of one point functions***

# Correlations - geodesic approximation

*[Balasubramanian, Ross; PRD(2000)]*

Correlator as a sum over geodesics: $\Delta L = L - L_{\text{thermalized}}$

$$\langle \mathcal{O}(t,\vec{x}_1)\mathcal{O}(t,\vec{x}_2)\rangle = \int \mathcal{D}\mathcal{P} e^{i\Delta\mathcal{L}(\mathcal{P})} \approx \sum_{\text{geodesics}} e^{-\Delta L} \approx e^{-\Delta L}$$

Geodesic length (Lagrangian):

$$L = \int \mathrm{d}\lambda \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \quad \Rightarrow \quad \frac{d^2x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = 0$$

***geodesic equation***

( $L = m\int \mathrm{d}\lambda \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + \frac{q}{m}A_\mu \dot{x}^\mu$ charged probe particle )

***Lorentz force term***

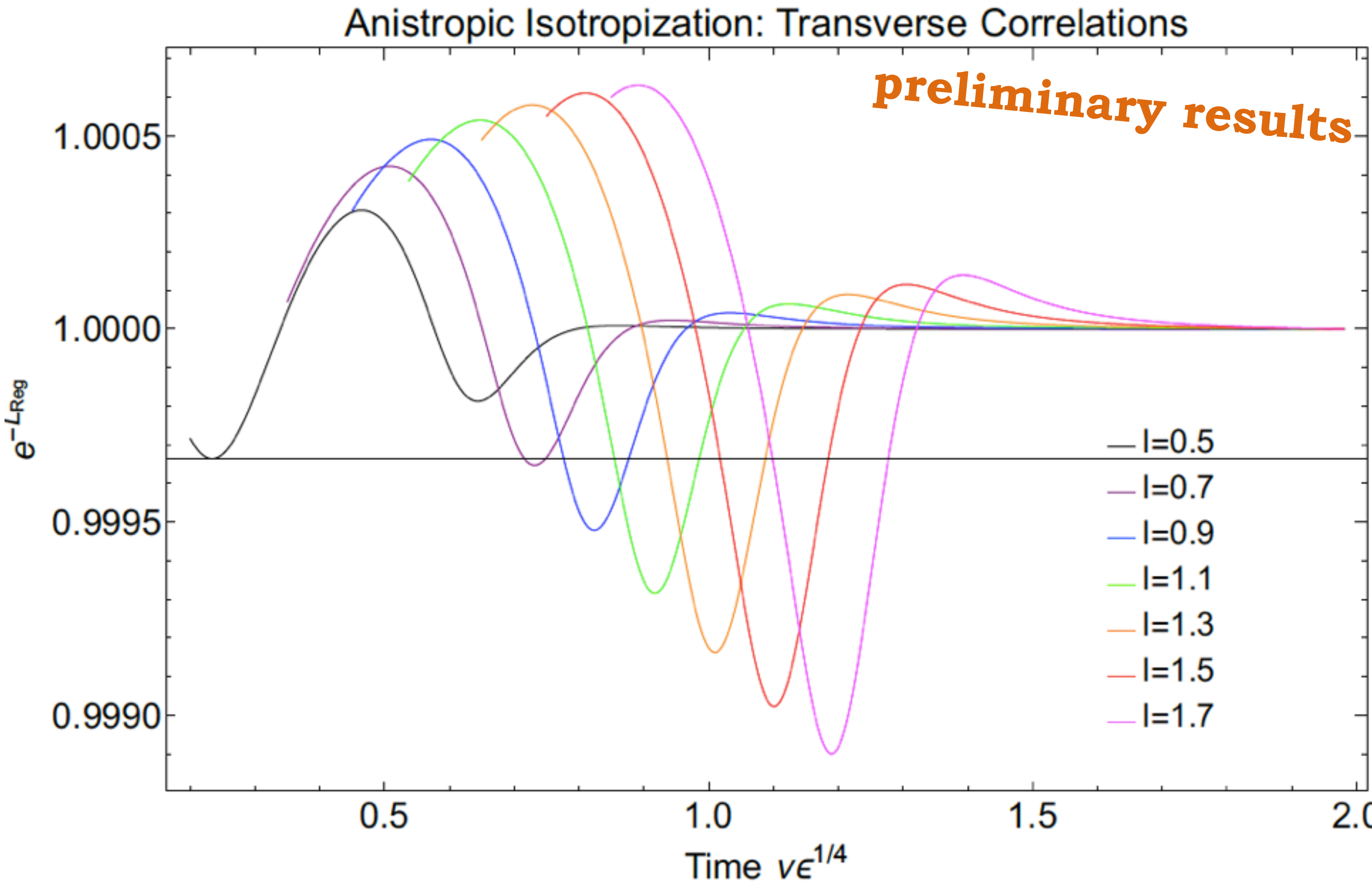
Numerical implementation - relaxation method:

*[Ecker, Grumiller, Stricker; JHEP (2015)]*

1. Generate the dynamic background
2. Generate interpolations of the metric functions
3. Discretize the geodesic equations using a relaxation scheme
4. Approximate the proper length using a Riemann sum

# Correlations - zero charge, zero B

*reproducing results similar to [Ecker, Grumiller, Stricker; JHEP (2015)]*



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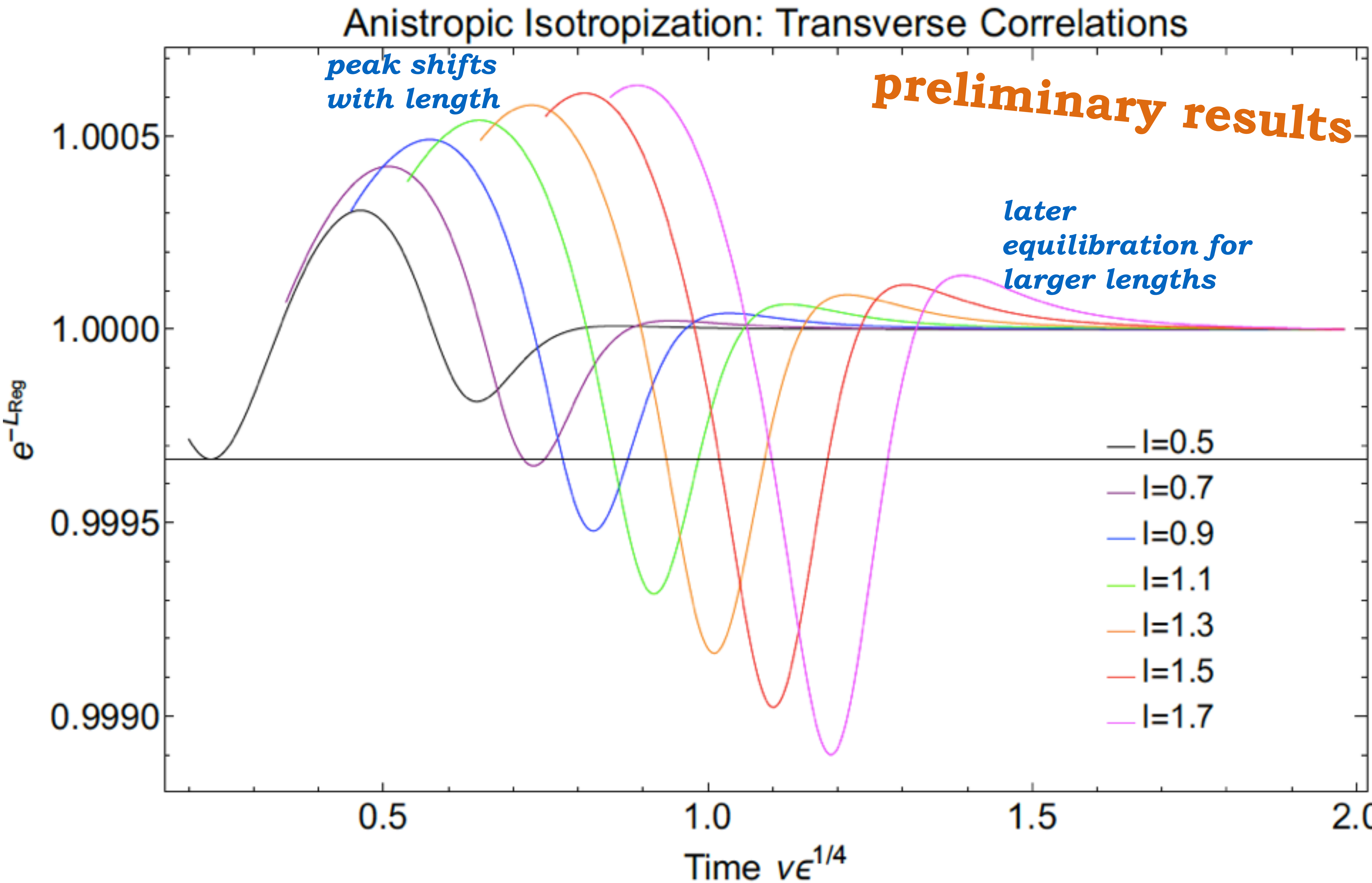

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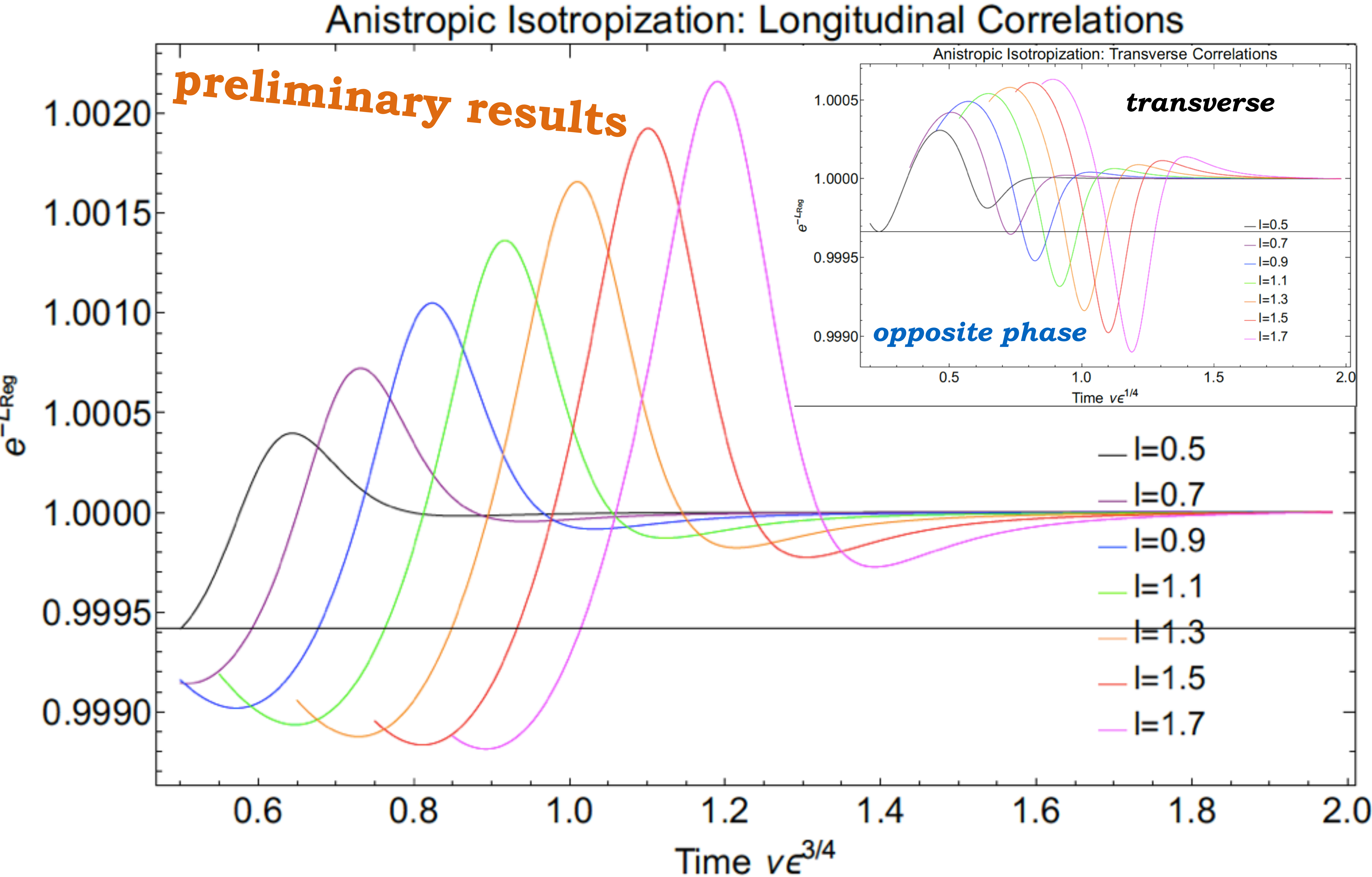

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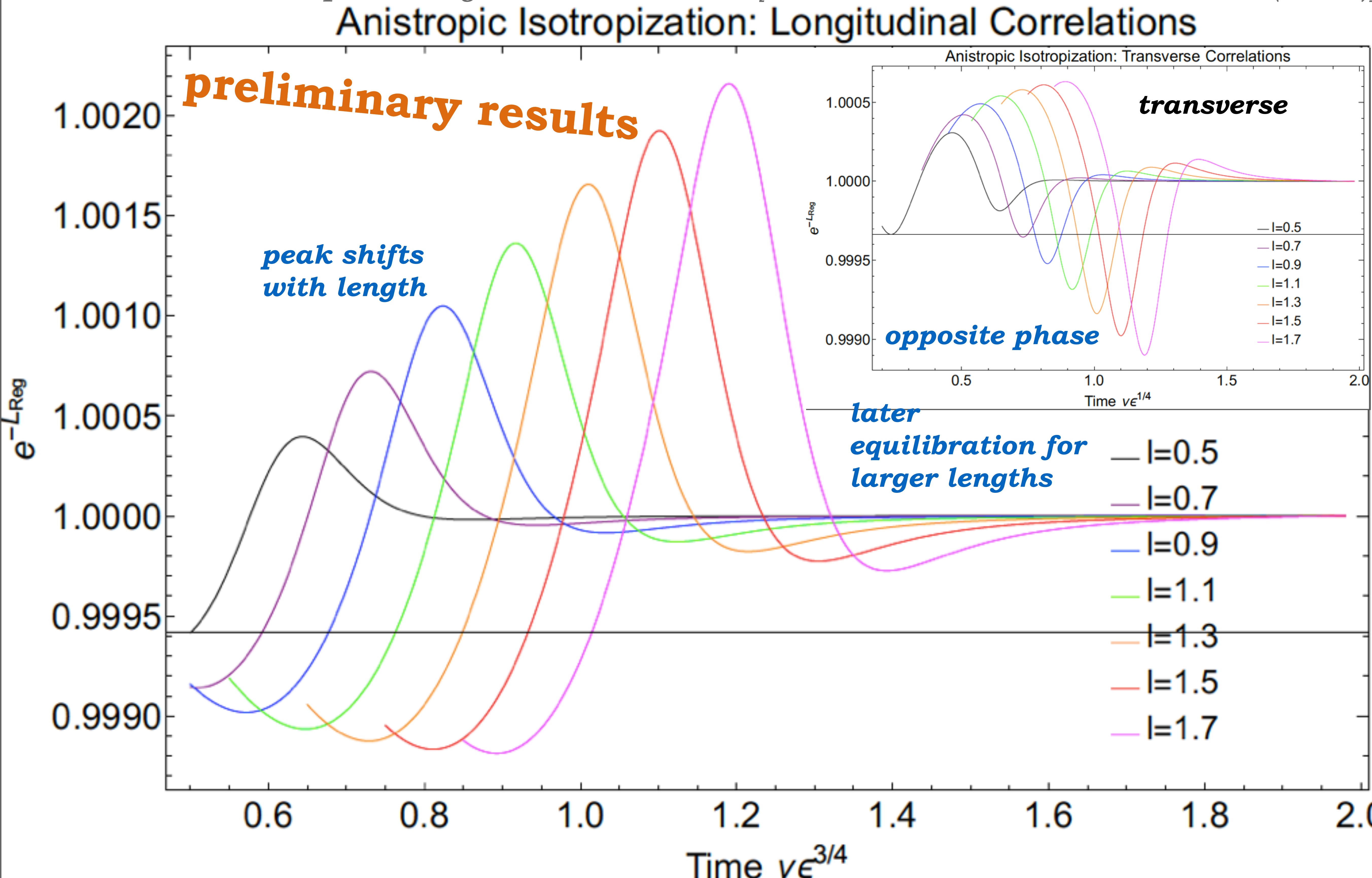

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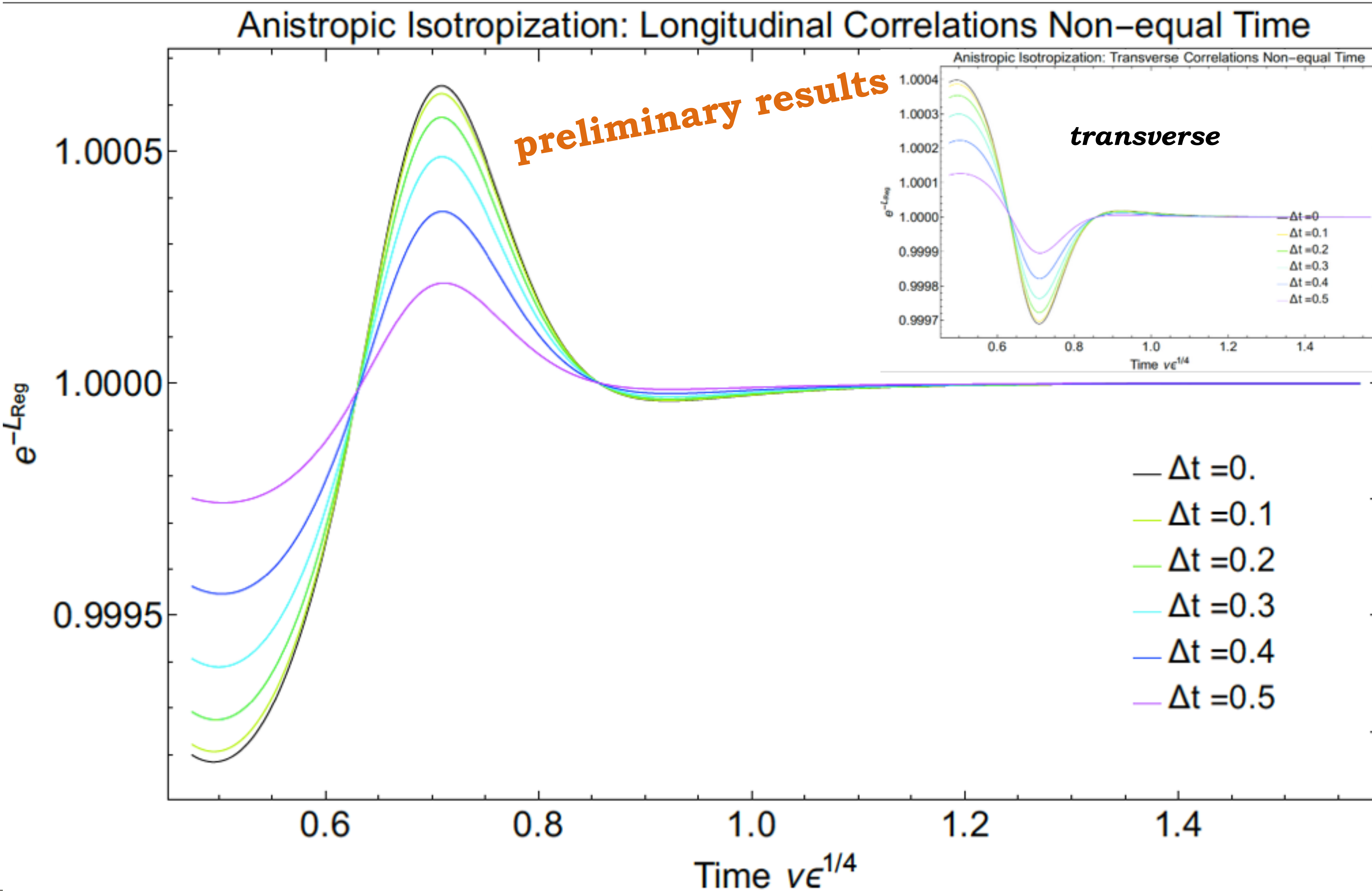

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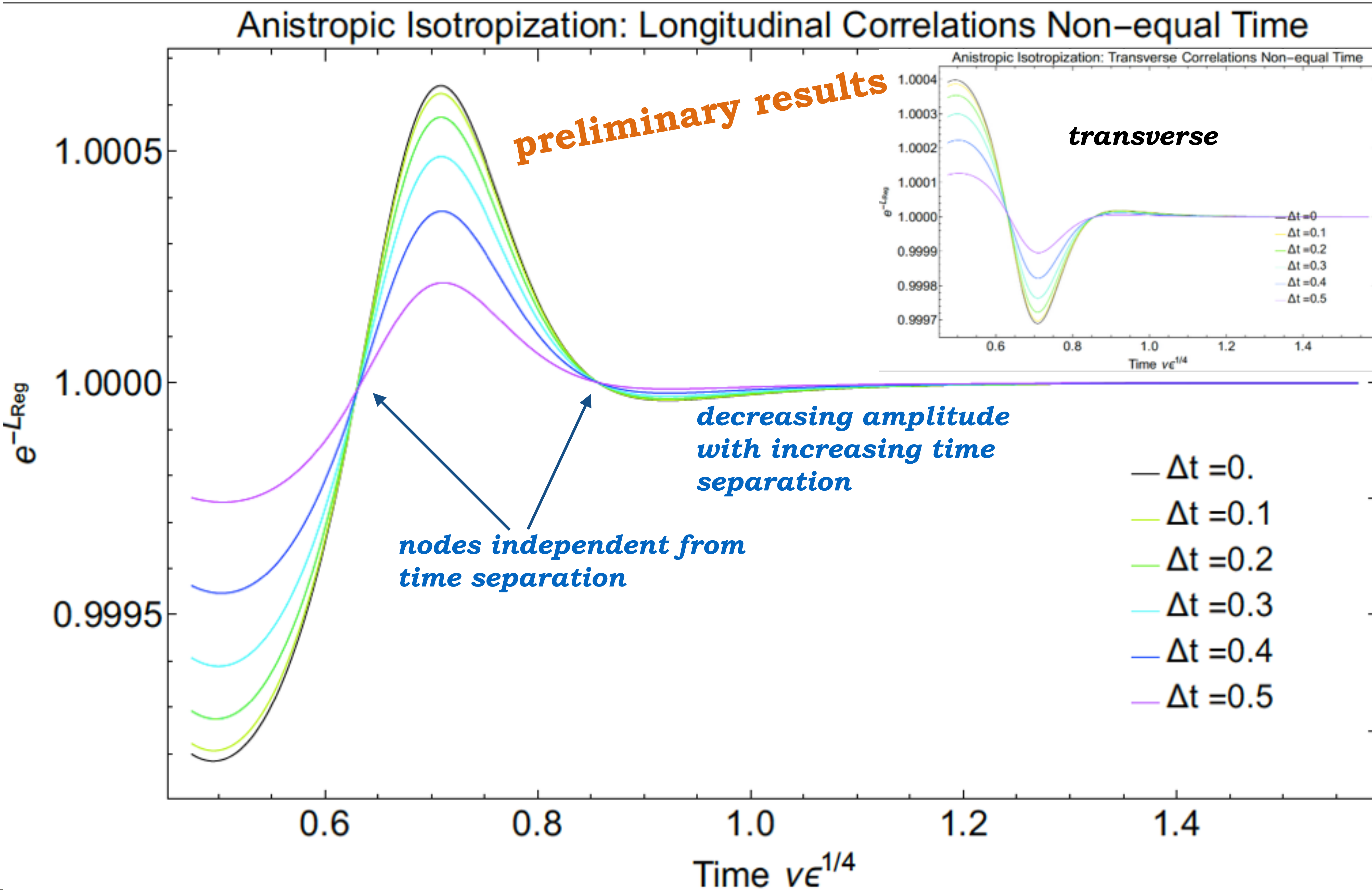

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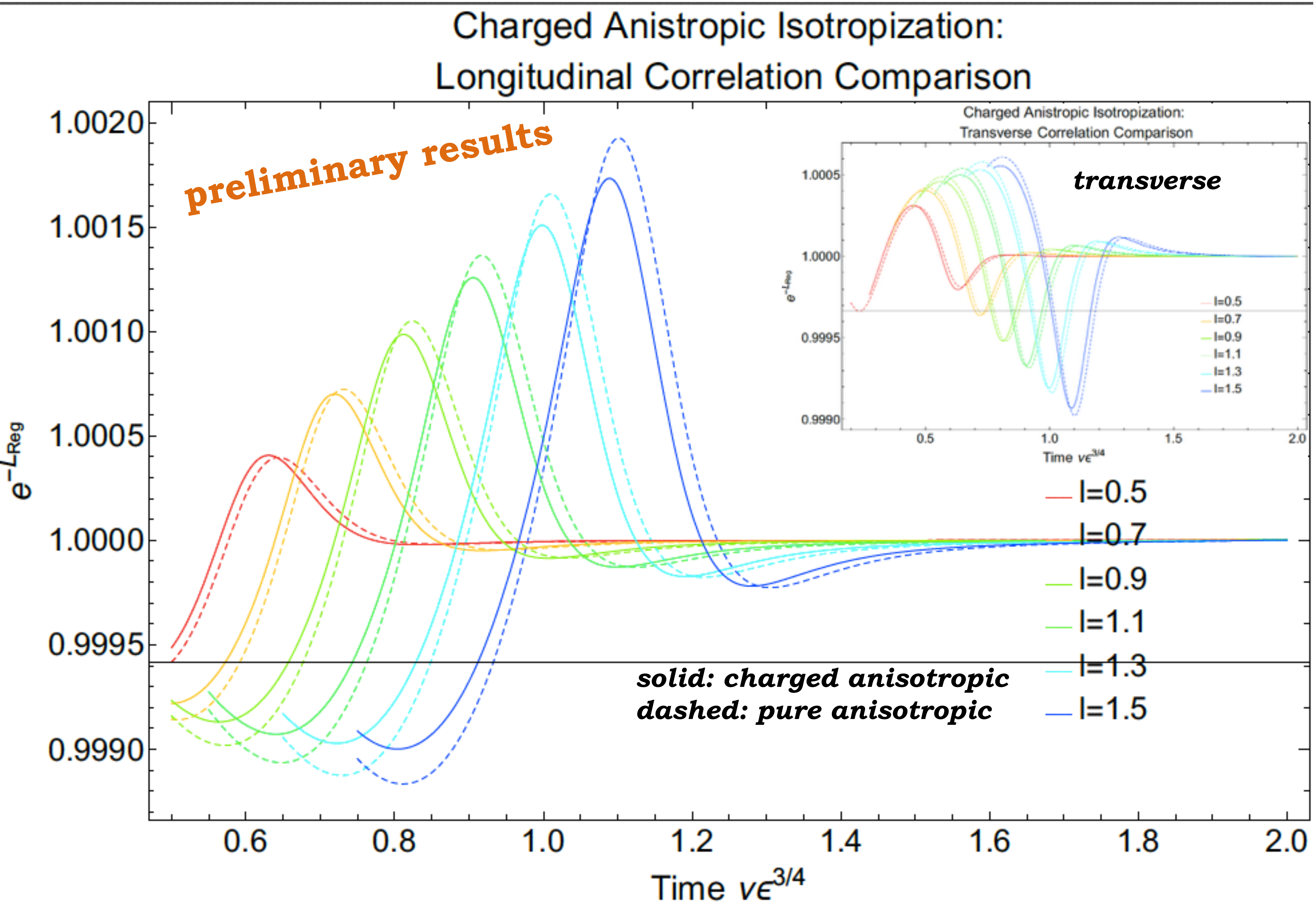

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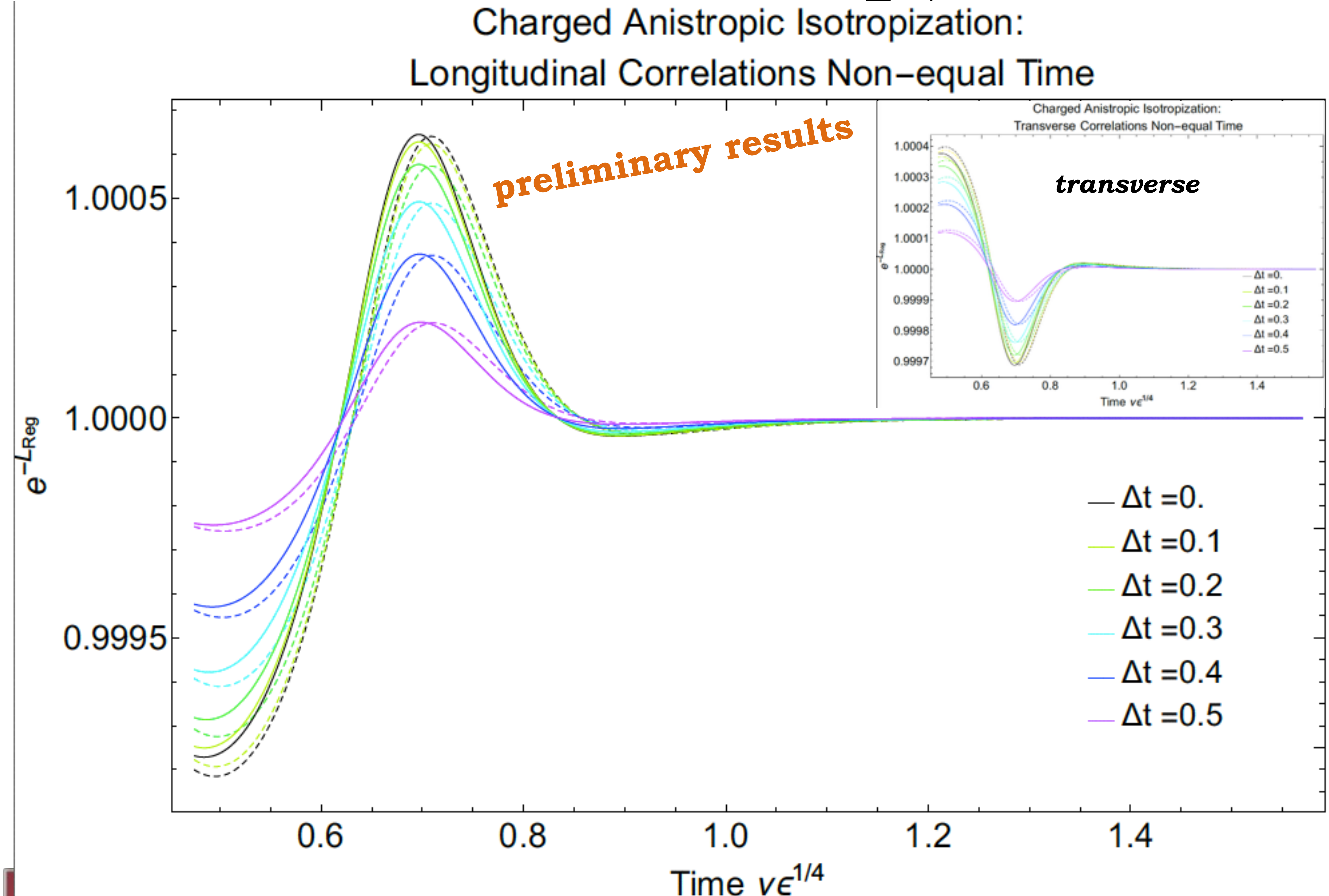

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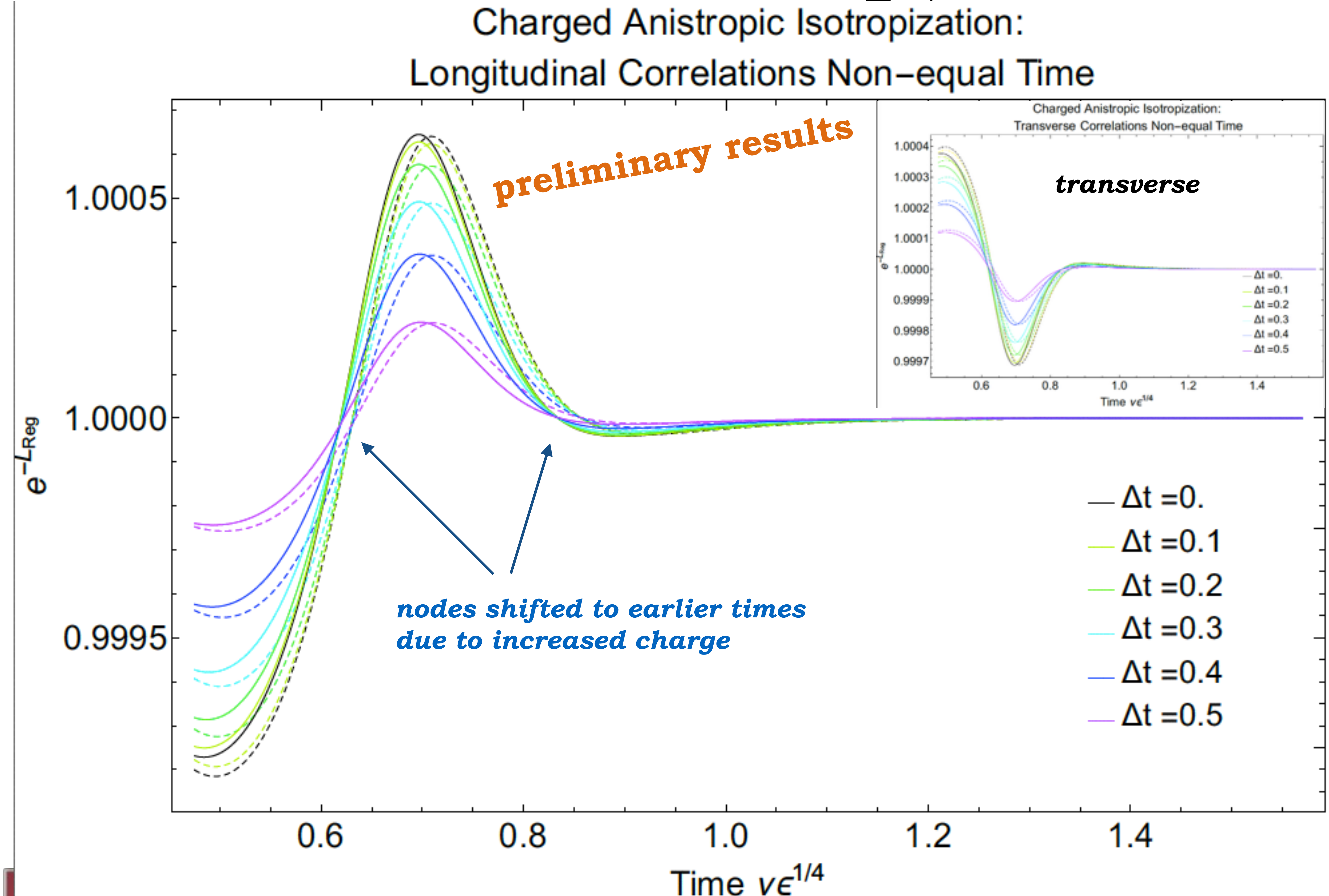

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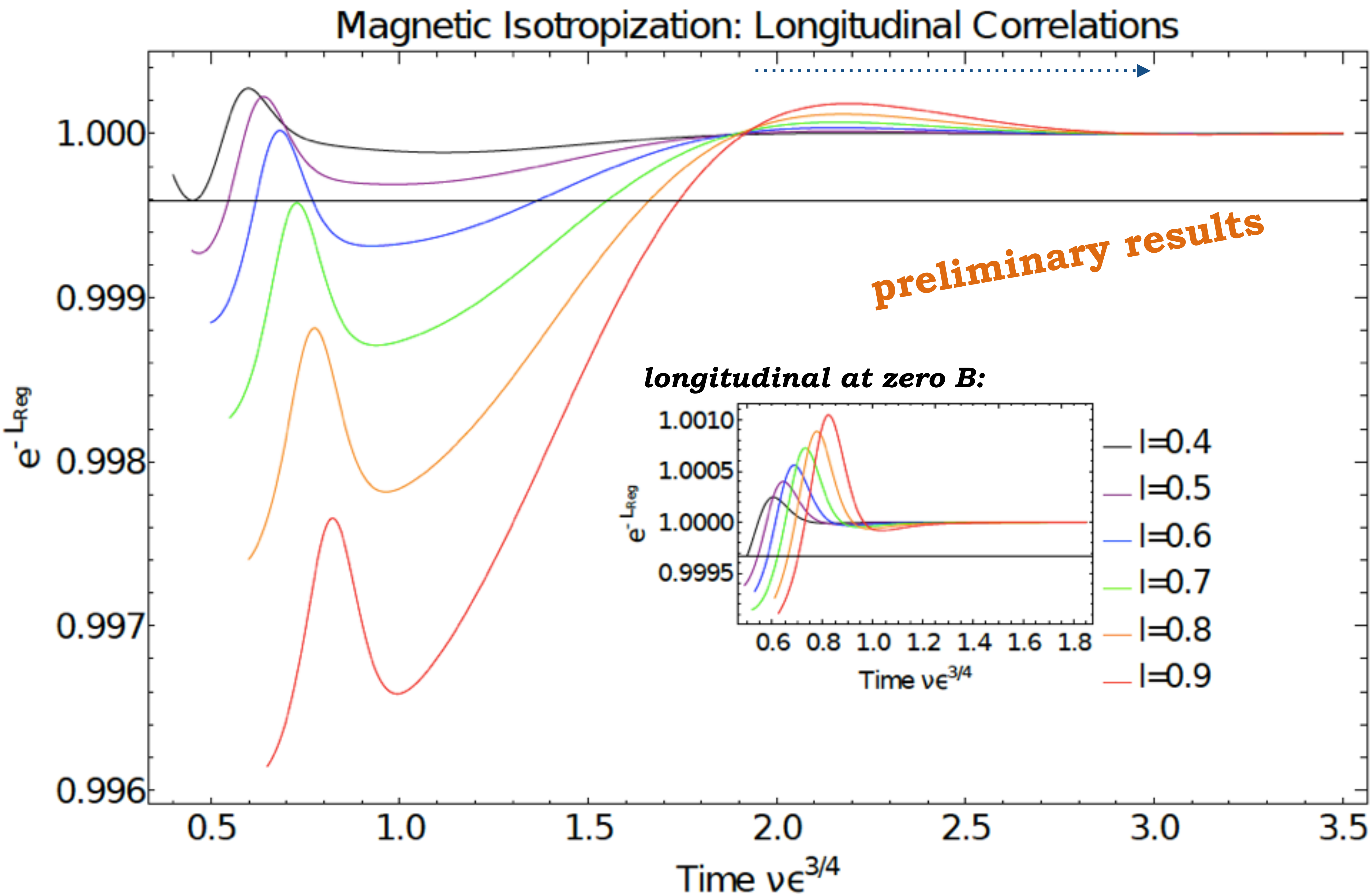

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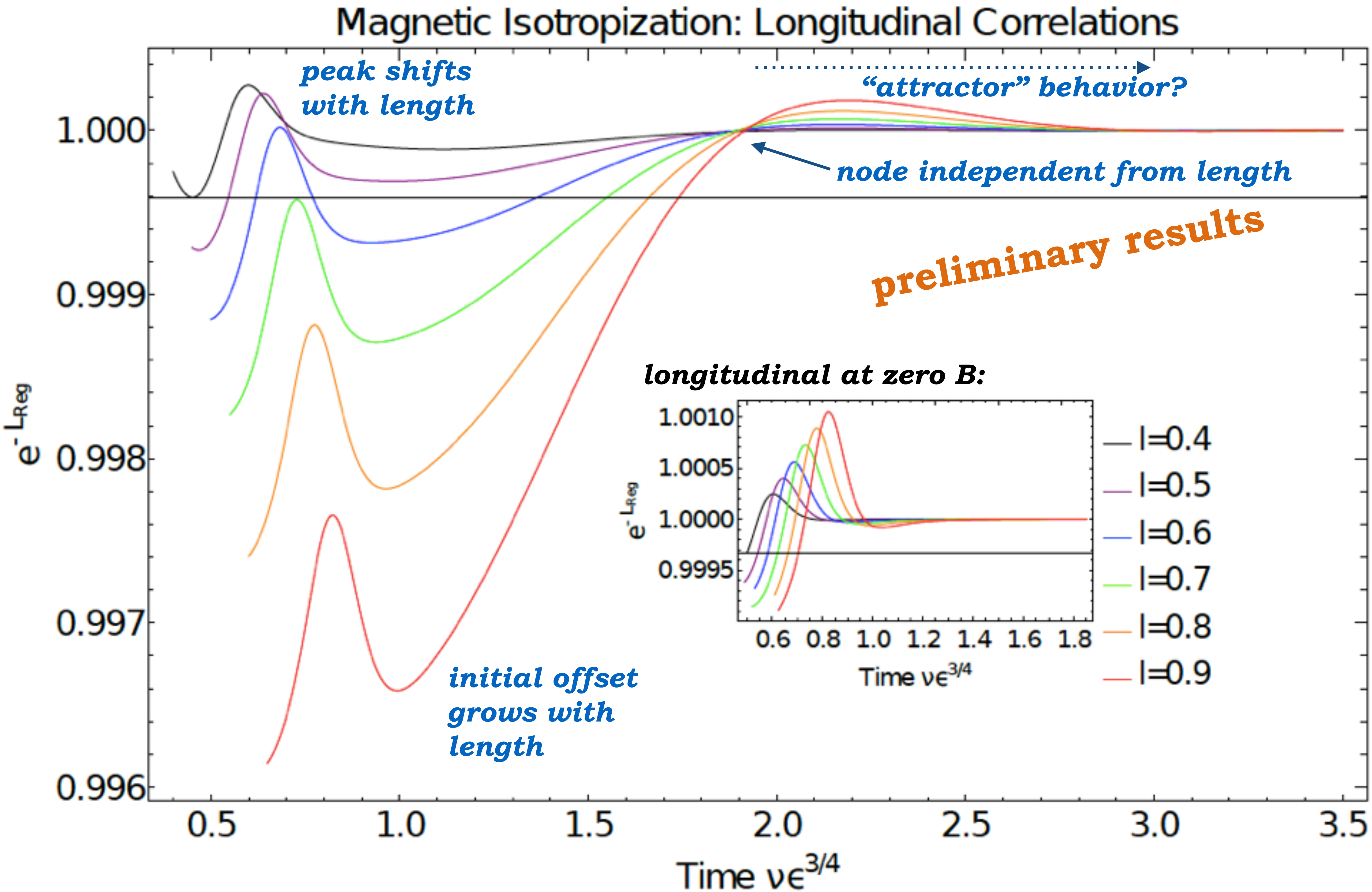

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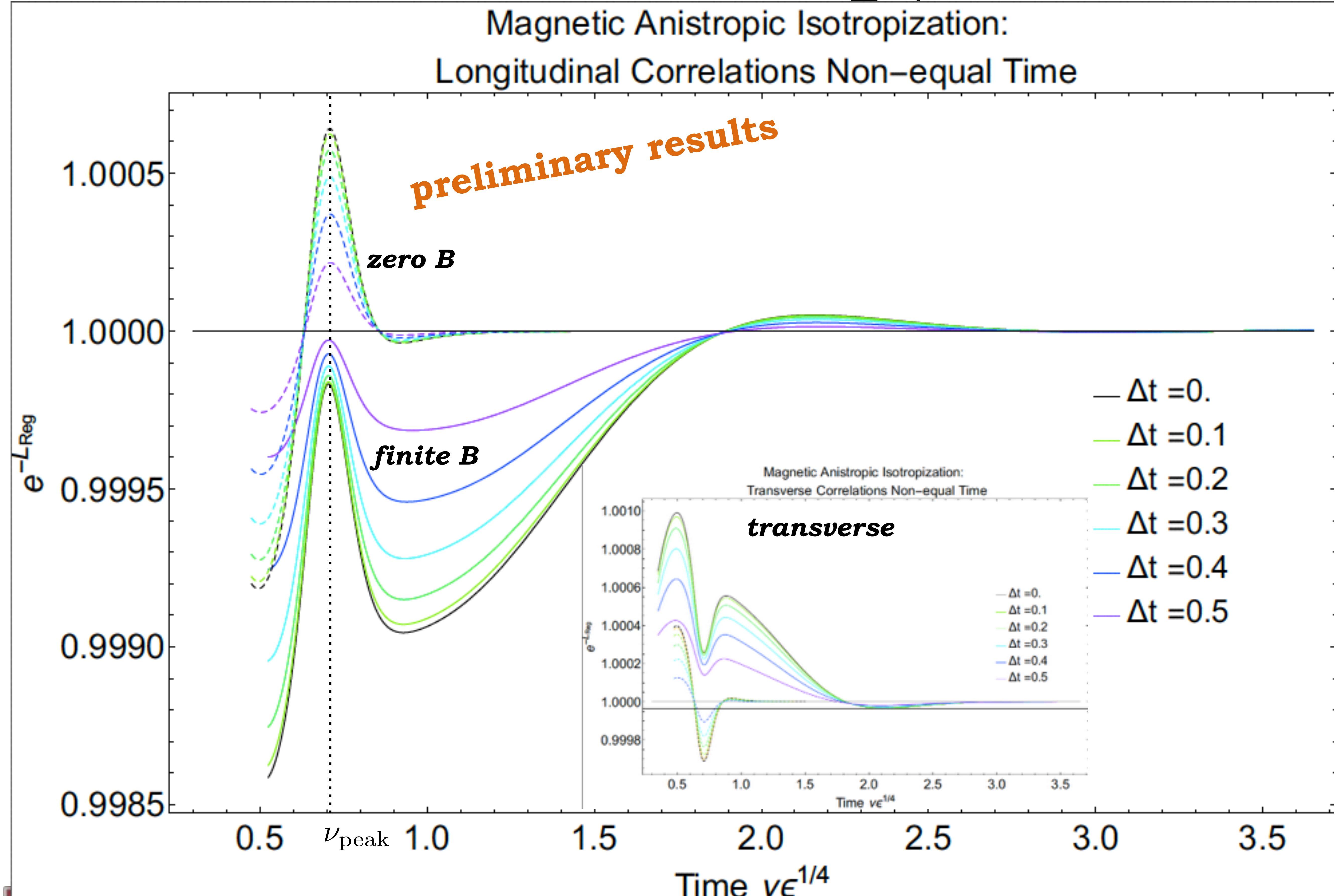

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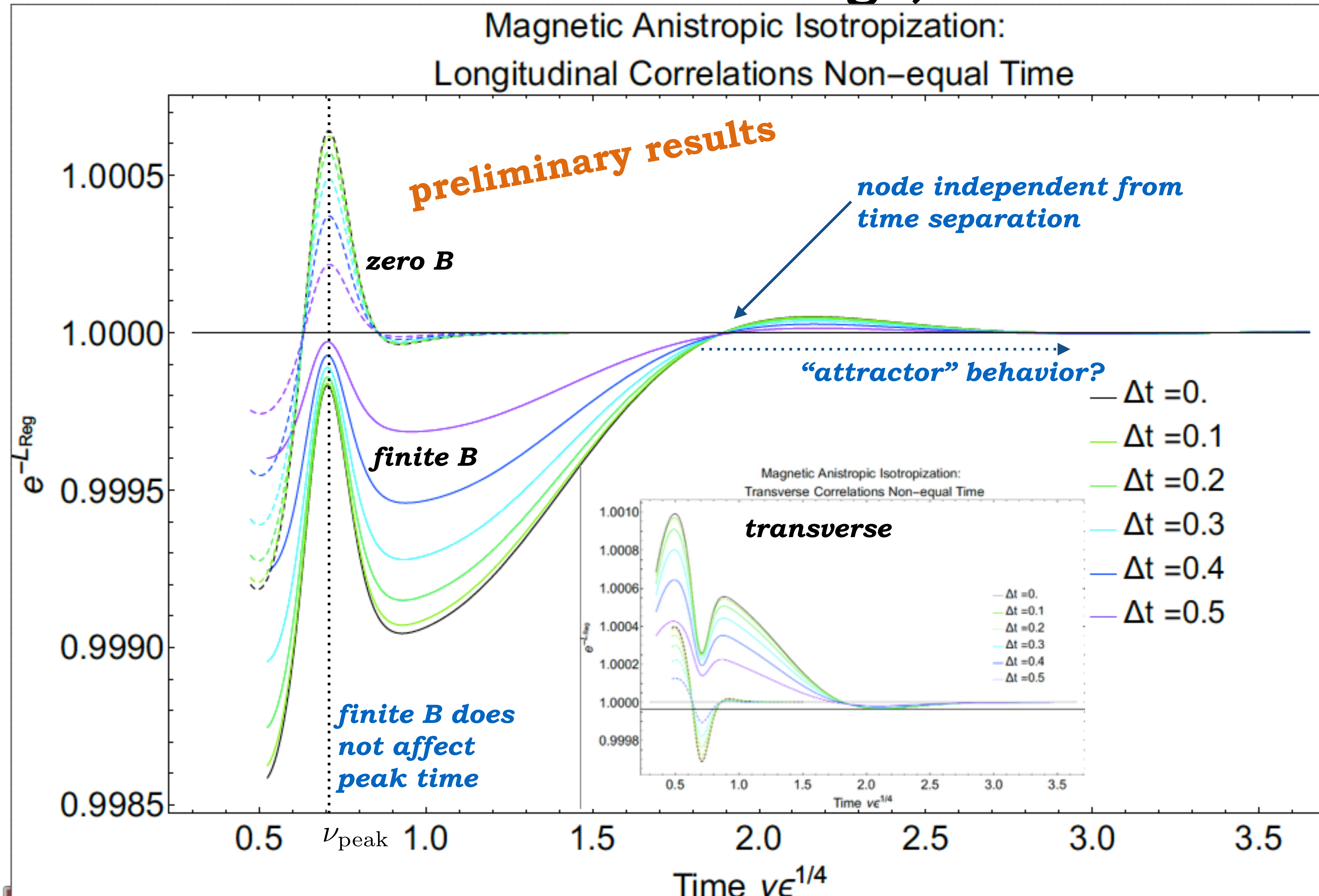

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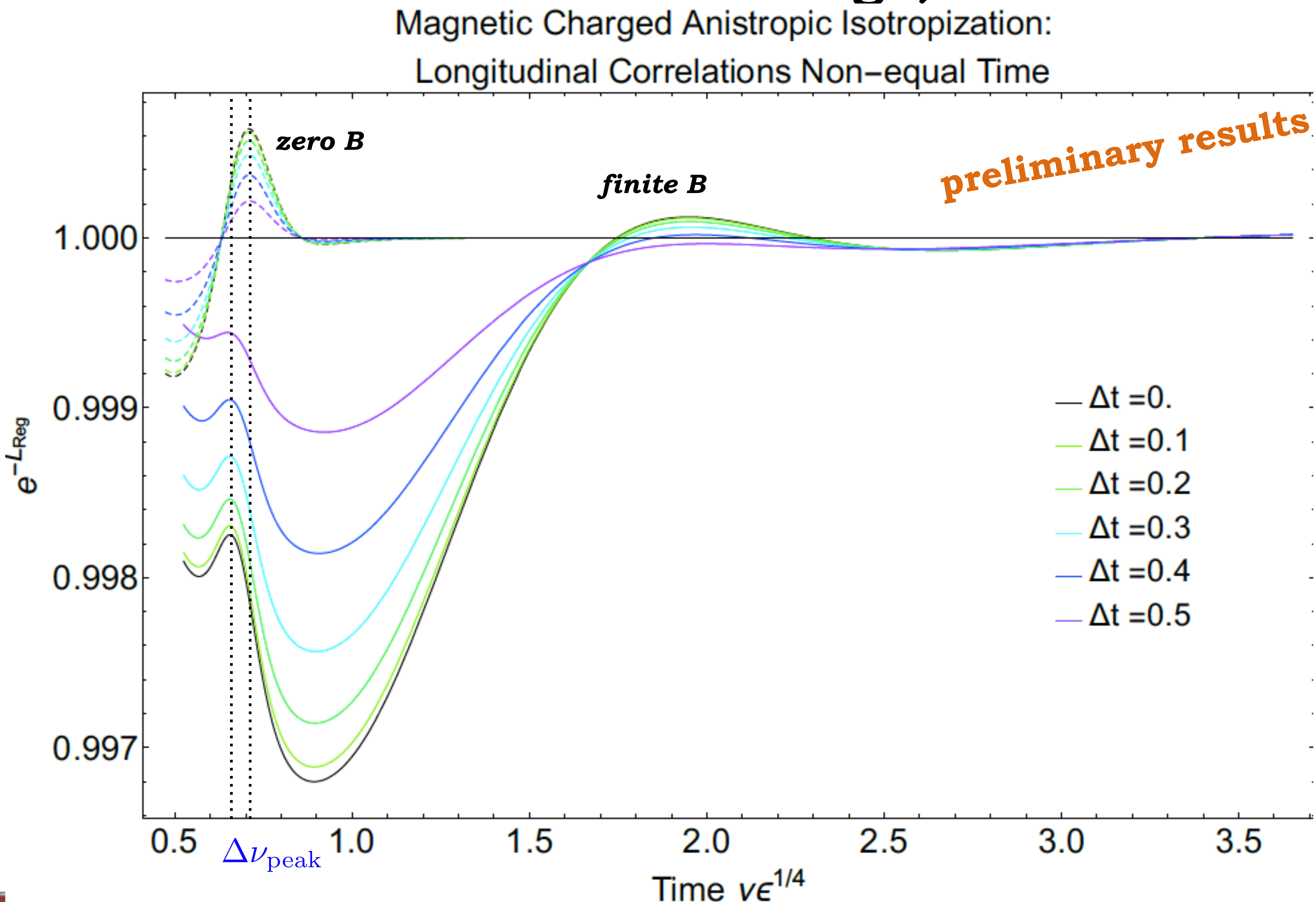

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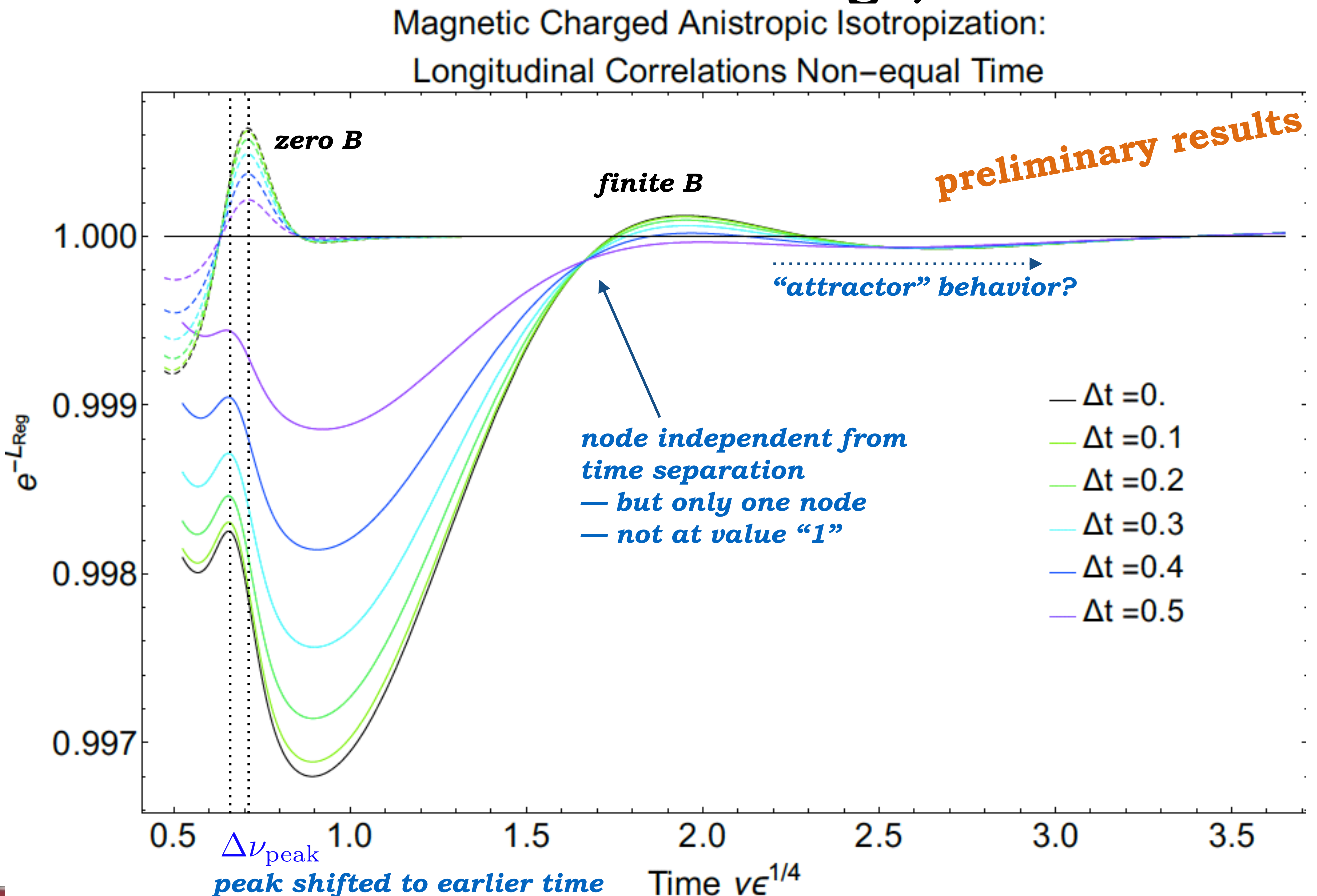


# Correlations - Summary

In **magnetic (nonzero *B*)** case we observe:
- significant changes in 2-point functions as $B$ changes
- longer equilibration time of 2-point functions
- "nodes" (and attractor-like behavior) for equal time
- growing initial offset
- charged operator non-equal time correlators (at finite charge density): peaks shifted to earlier time and "node" shifted away from value 1

At **B=0**, we confirm previous observations
- longitudinal/transverse correlators are in opposite phase
- correlators take longer to thermalize than pressures

*[Balasubramanian,Bernamonti,de Boer,Copland,Craps,Keski-Vakkuri,Muller,Schafer,Shigemori,Staessens; PRD (2011)]*

*[Ecker, Grumiller, Stricker; JHEP (2015)]*

*[Ecker, Grumiller, Stanzer, Stricker, van der Schee; JHEP (2016)]*

Play further with: initial anisotropy vs. n-point functions

*[Fuini, Yaffe; JHEP (2015)]*

# Discussion

- comparison to near-equilibrium at **finite *B*** *(see appendix)*

*[Ammon, Kaminski et al.; JHEP (2017)]*

*[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; work in progress]*

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^z J^z}(\omega, \mathbf{k}{=}0) = \sigma_{\|} + \cdots$$

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^x}(\omega, \mathbf{k}{=}0) = \omega^2 \rho_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}$$

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^y}(\omega, \mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2 \tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}\, \mathrm{sign}(B_0)$$

$$\langle J^x_{cons}(\mathbf{k}) J^y_{cons}(-\mathbf{k})\rangle = -ik_z(\xi_B - \tfrac{1}{3} C\mu)$$

**preliminary results**

$w_0 = \epsilon_0 + P_{||}$

$M_{5,\mu}$ derivative of "vortical susceptibility" w.r.t. chemical potential

… so, correlators receive altered physical interpretation

- analytic solutions for time-dependent backgrounds?

*cf. [Horowitz, Iqbal, Santos; PRD (2013)]*

- chiral transport far from equilibrium

# Collaborators

**Friedrich-Schiller University of Jena, Germany**

Prof. Dr. Martin Ammon

Dr. Julian Leiber

Sebastian Grieninger

**Regensburg University, Germany**

Prof. Dr. Andreas Schäfer

**Goethe University, Frankfurt, Germany**

Dr. Gergely Endrödi

**University of Washington, Seattle, USA**

Prof. Dr. Laurence Yaffe

**University of Alabama, Tuscaloosa, USA**

Dr. Jackson Wu

Roshan Koirala

Casey Cartwright

***special thanks to Wilke van der Schee and Larry Yaffe**

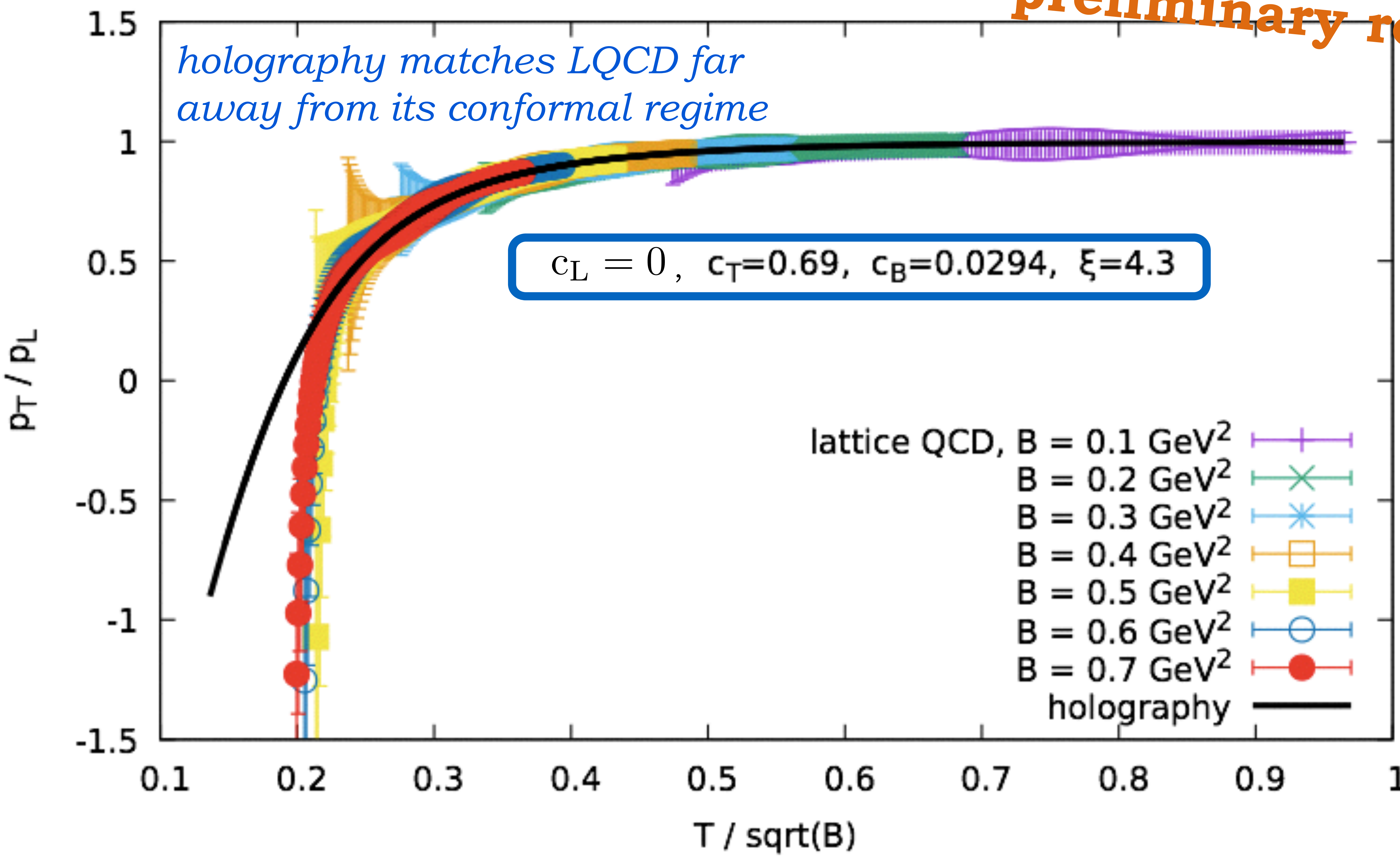
**Thank you!**

# APPENDIX

# Good agreement with N=4 Super-Yang-Mills (from holography)

*[Endrödi, Kaminski, Schäfer, Wu, Yaffe; work in progress]*

**preliminary results**




***"ideal" renormalization scale:***

- ***maximum overlap in LQCD data***
- ***maximum overlap with holography***
- ***minimum number of "fit parameters"***

$$\mu = \sqrt{c_T T^2 + c_L \Lambda_H^2 + c_B |B|}$$

# How does the renormalization scale enter?

[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]
[Fuini, Yaffe, JHEP (2015)]

Total action: $$S = S_{\text{QCD}}(e, B) + S_{\text{EM}}(e, B)$$

*QCD action coupled to external magnetic field (through covariant derivative)*

*action for external magnetic field; not included in code (not part of the dynamics)*

Electric charge is renormalization scale dependent: $e^2(\mu) = Z_e(\mu)\, e_0^2, \qquad Z_e(\mu) = 1 + 2b_1 e^2 \log\frac{\mu}{\Lambda}, \qquad \mu = \sqrt{c_T T^2 + c_L \Lambda_{\text{H}}^2 + c_B |B|}$

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Free energy: $$F = -T \log \mathcal{Z}[S]$$

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Transverse pressure: $$p_{\rm T} = -\frac{L_{\rm T}}{V}\frac{\partial F_{\rm QCD}(e,B)}{\partial L_{\rm T}}$$

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*action for external magnetic field; not included in code (not part of the dynamics)*

Free energy: $F = -T \log \mathcal{Z}[S]$

$$= F_{\mathrm{QCD}}(e, B) + F_{\mathrm{EM}}(e, B) \qquad F_{\mathrm{EM}}(e, B) = -V \frac{B^2}{2e^2}$$

Transverse pressure: $p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V} \frac{\partial F_{\mathrm{QCD}}(e, B)}{\partial L_{\mathrm{T}}}$

***this free energy is renormalization scale dependent***

Electric charge is renormalization scale dependent: $e^2(\mu) = Z_e(\mu)\, e_0^2, \qquad Z_e(\mu) = 1 + 2b_1 e^2 \log \frac{\mu}{\Lambda}, \qquad \mu = \sqrt{c_T T^2 + c_L \Lambda_{\mathrm{H}}^2 + c_B |B|}$

# How does the renormalization scale enter?

*[Bali, Bruckmann, Endrödi, Katz, Schäfer; JHEP (2014)]*
*[Fuini, Yaffe, JHEP (2015)]*

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***hence this pressure is renormalization scale dependent***

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# How to compare QCD to Super-Yang-Mills

SYM action: $$S = S_{\rm SYM}(e, \mathcal{B}) + S_{\rm EM}(e, \mathcal{B})$$

SYM field content: fermions, scalar particles, vector field

SYM properties: conformal symmetry, supersymmetry, ...

**SYM appears to be entirely different from QCD!**

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SYM properties: conformal symmetry, supersymmetry, ...

**SYM appears to be entirely different from QCD!**

**Strategy:**

- compare thermodynamic quantities (macroscopic / effective); e.g. pressure
- match divergencies in the two theories, i.e. match beta functions
- measure magnetic fields in "same units"
- compare two theories at same renormalization scale

SYM magnetic field $\mathcal{B}$ vs. QCD magnetic field $B$: $$B = \xi \mathcal{B}$$

# Background

$$ds^2 = -A(r,t)dt^2 + 2drdt + S(t,r)^2(e^{B(r,t)}(dx^2+dy^2) + e^{-2B(r,t)}dz^2)$$

Near boundary expansion of metric functions:

$$S(r,t) = r + \xi + \mathcal{O}(r^{-7})$$

$$B(r,t) = \log(r)\left(-\frac{20\mathcal{B}^2\xi(t)^3}{3r^7} + \frac{10\mathcal{B}^2\xi(t)^2}{3r^6} - \frac{4\mathcal{B}^2\xi(t)}{3r^5} + \frac{\mathcal{B}^2}{3r^4}\right) + \frac{b_4(t)}{r^4} + \mathcal{O}(r^{-8})$$

$$A(r,t) = (r+\xi(t))^2 - 2\xi'(t) + \frac{a_4(t)}{r^2} + \log(r)\left(\frac{8\mathcal{B}^2\xi(t)^3}{3r^5} - \frac{2\mathcal{B}^2\xi(t)^2}{r^4} + \frac{4\mathcal{B}^2\xi(t)}{3r^3} - \frac{2\mathcal{B}^2}{3r^2}\right) +$$

Subtracted and rescaled metric functions:

$$S(r,t) = r + \xi + \frac{1}{r^4}S_s(r,t)$$

$$B(r,t) = \log(r)\left(\frac{10\mathcal{B}^2\xi(t)^2}{3r^6} - \frac{4\mathcal{B}^2\xi(t)}{3r^5} + \frac{\mathcal{B}^2}{3r^4}\right) + \frac{1}{r^4}B_s(r,t)$$

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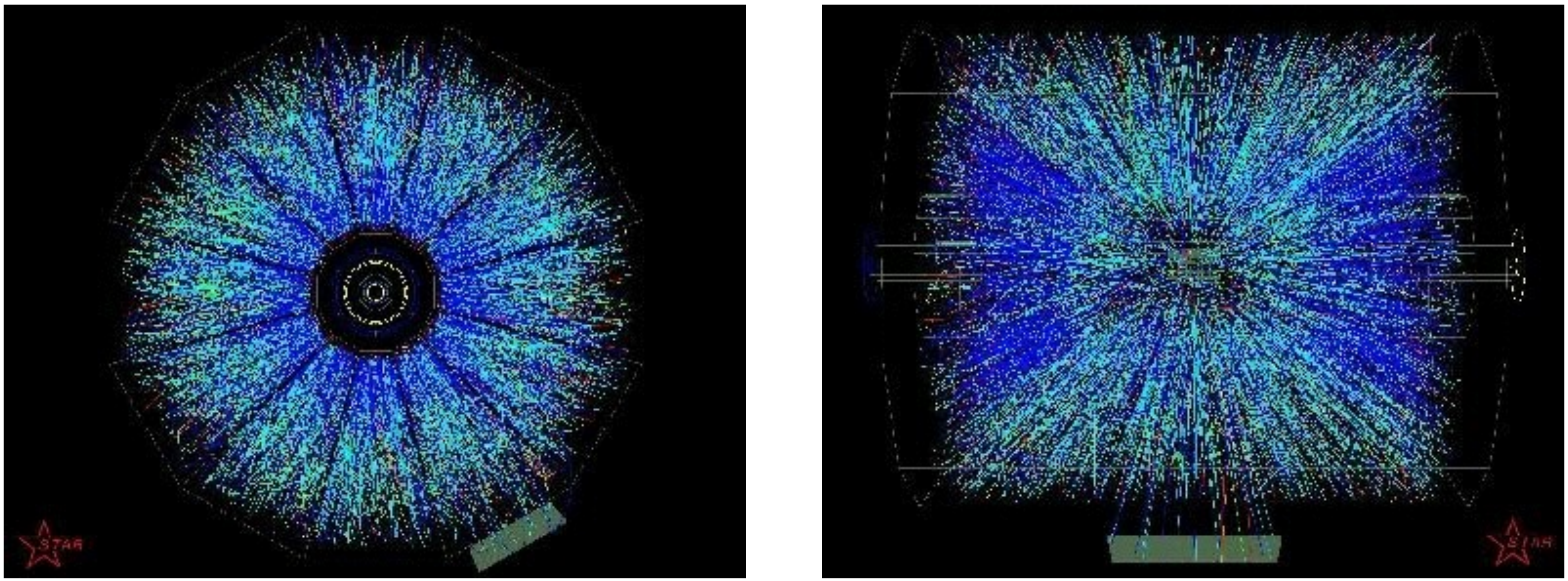
# Motivation: Chiral transport effects in charged anisotropic plasma within a magnetic field at strong coupling



Front and side view of collision between gold ions at Brookhaven National Lab's Relativistic Heavy Ion Collider, captured by the Solenoidal Tracker at RHIC (STAR detector).

***Method: use effective field theory (EFT) and holography in parallel (as effective descriptions)***

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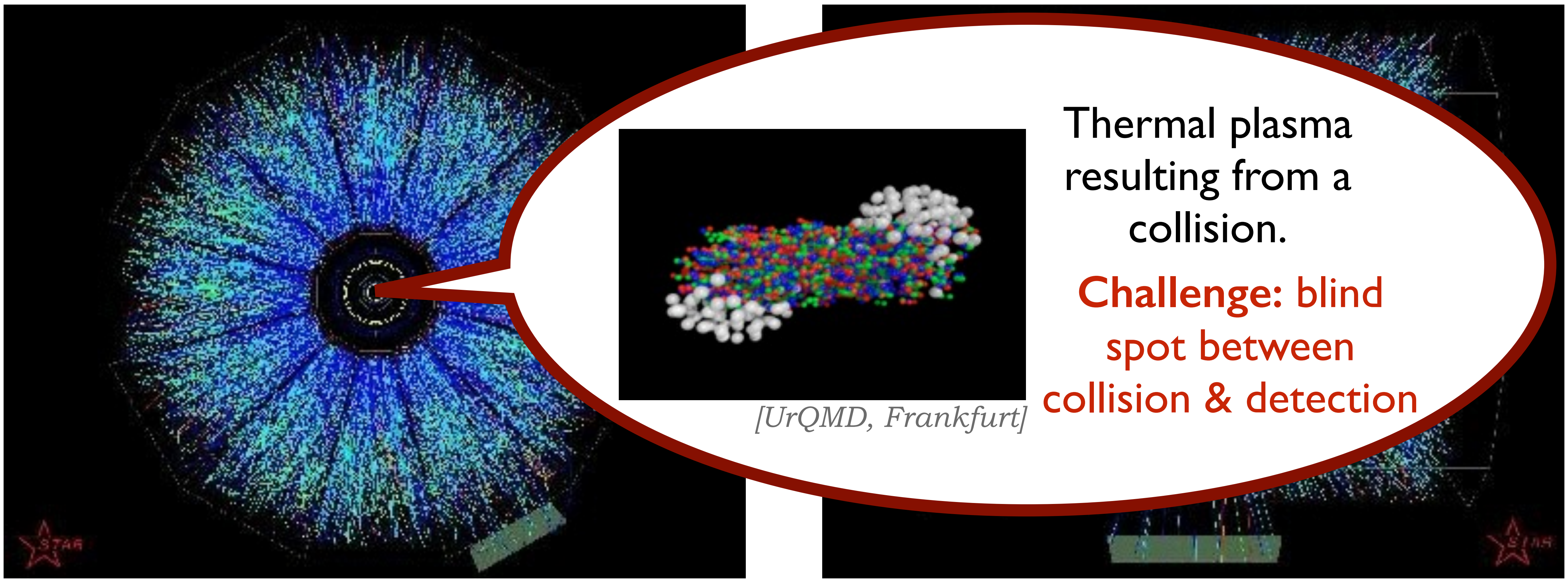


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# Method summary: holography & hydrodynamics

Assume we have a hard problem that is difficult to solve in a given theory,
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Simple problem in a particular gravitational theory

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- ➡ Holography is good at predictions that are **qualitative** or **universal**.
- ➡ **Compare** holographic result to hydrodynamics of model theory.
- ➡ **Compare** hydrodynamics of original theory to hydrodynamics of model.
- ➡ **Understand holography as an effective description.**

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**CONSISTENCY CHECK**

Simple problem …
gravitational the…

Solve problems in effective field theory (EFT), e.g.:

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**REALITY CHECK: MODEL APPROPRIATE?**

- hydrodynamic approximation of model theory

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The same way, we choose a hydrodynamic model:

- match symmetries (and anomalies)
- include interesting operators

*depends on the physical question*

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Einstein-Maxwell-Chern-Simons gravity has dual with:

*cf. talk by K. Landsteiner*

- chiral anomaly, breaking a U(1) axial symmetry
- axial current and energy momentum tensor

*chiral magnetic transport*

- thermodynamics match well (in external B field)

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

dual to $\mathcal{N}$=4 Super-Yang-Mills theory coupled to U(1)

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**Successful example: holographic model discovering a chiral vortical effect (2008)**

*[Erdmenger, Haack, Kaminski, Yarom; JHEP (2009)]*

*[Banerjee et al; JHEP (2011)]*

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# EFT: Hydrodynamics - definitions

*[Landau, Lifshitz]*

universal **effective field theory (EFT)**, expansion in derivatives of temperature, chemical potential and velocity

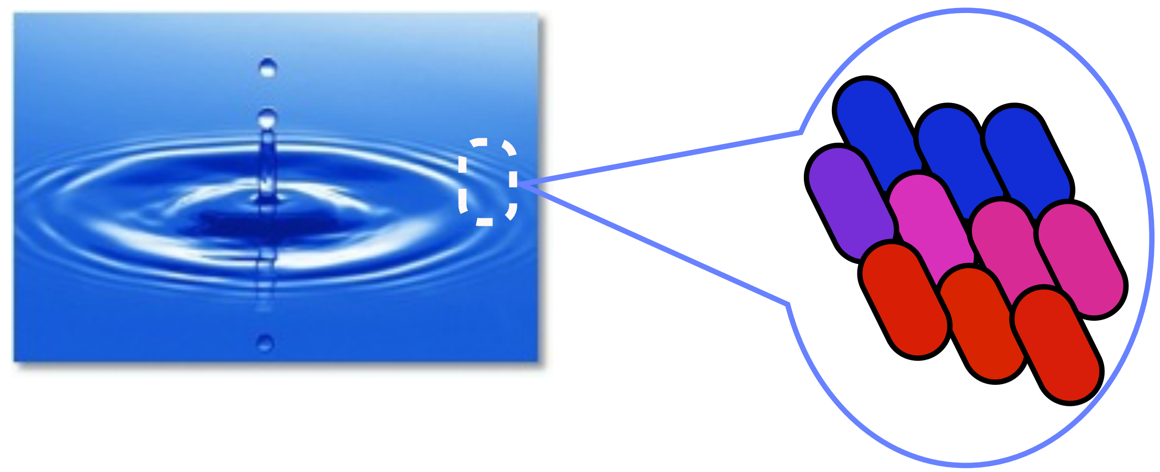
- fields $T(x),\ \mu(x),\ u^{\nu}(x)$

  *temperature* *chemical potential* *fluid velocity*

- conservation equations

$$\nabla_{\nu} J^{\nu} = 0$$

$$\nabla_{\mu} T^{\mu\nu} = 0$$



- constitutive equations

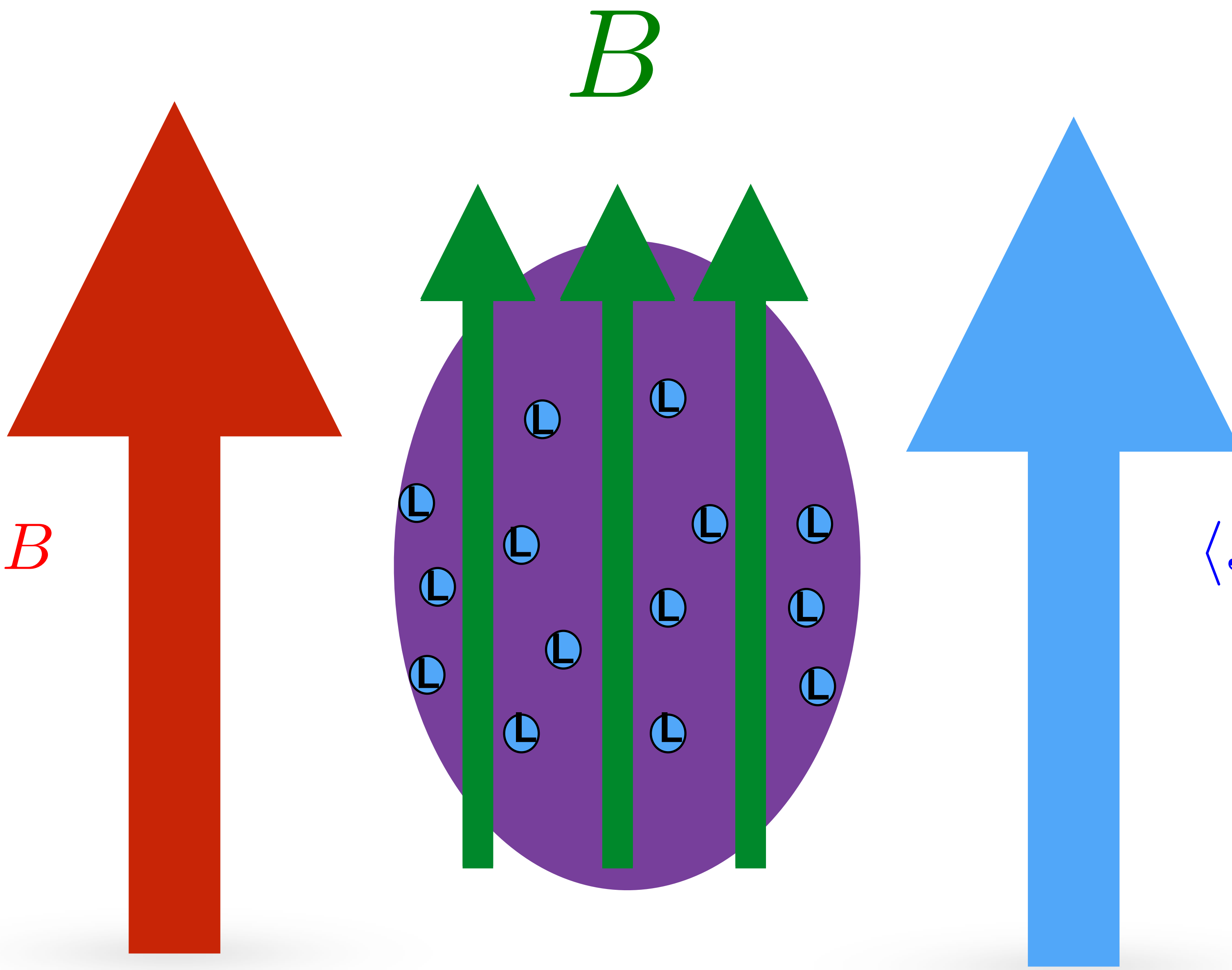
*Conserved current* $$\langle J^{\mu} \rangle = n u^{\mu} + \nu^{\mu}$$

*charge density*

*Energy momentum* $$\langle T_{\mu\nu} \rangle = \epsilon u_{\mu} u_{\nu} + P(\overbrace{g_{\mu\nu} + u_{\mu} u_{\nu}}^{\Delta_{\mu\nu}}) + \dots$$

**Physical question:**
**What is the equilibrium state of a theory with <u>chiral anomaly</u> + external magnetic field ?**

# Currents in equilibrium




$\langle T^{0z} \rangle = \xi_V^{(0)} B$

***heat current***

$\langle J^z \rangle = \xi_B^{(0)} B$

***axial current***

# EFT result I: strong B thermodynamics

*[Ammon, Kaminski et al.; JHEP (2017)]*
*[Ammon, Leiber, Macedo; JHEP (2016)]*

Strong B thermodynamics with anomaly in **thermodynamic frame:**

Energy momentum tensor:

$$B \sim \mathcal{O}(1)$$

$$\langle T^{\mu\nu}_{\rm EFT}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

Axial current:

$$\langle J^{\mu}_{\rm EFT}\rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B}\right) + \mathcal{O}(\partial)$$

based on previous work:

*[Kovtun; JHEP (2016)]*

*[Jensen, Loganayagam, Yarom; JHEP (2014)]*

*[Israel; Gen.Rel.Grav. (1978)]*

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Energy momentum tensor:

*equilibrium heat current*

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*"magnetic pressure shift"*

Axial current:

$$\langle J^{\mu}_{\mathrm{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

*equilibrium charge current*

➡ **new contributions to thermodynamic equilibrium observables**

based on previous work:

*[Kovtun; JHEP (2016)]*

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*[Israel; Gen.Rel.Grav. (1978)]*

# Holographic result: thermodynamics

*[Ammon, Kaminski et al.; JHEP (2017)]*

Background solution: charged magnetic black branes

*[D'Hoker, Kraus; JHEP (2009)]*

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- **external magnetic field**
- **charged plasma**
- anisotropic plasma

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- **external magnetic field**
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Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^{\mu}\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

$$\langle T^{\mu\nu}_{\text{EFT}}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB} B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB} B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^{\mu}_{\text{EFT}}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

➡ **agrees in form with strong *B* thermodynamics from EFT**

**Physical question:**
**What is the near-equilibrium transport behavior of a theory with chiral anomaly + external magnetic field ?**

# EFT result II: weak B hydrodynamics

*[Ammon, Kaminski et al.; JHEP (2017)]*
*[Abbasi et al.; PLB (2016)]*
*[Kalaydzhyan, Murchikova; NPB (2016)]*

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$ :

**spin 1 modes under SO(2) rotations around *B***

$$\omega = \quad - ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

*former momentum diffusion modes*

$\mathfrak{s}_0 = s_0/n_0$

$\tilde{c}_P = T_0 (\partial \mathfrak{s}/\partial T)_P$

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**spin 1 modes under SO(2) rotations around *B***

$$\omega = \mp\frac{Bn_0}{\epsilon_0 + P_0} - ik^2\frac{\eta}{\epsilon_0 + P_0} + k\frac{Bn_0\xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$$

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*former momentum diffusion modes*

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**spin 0 modes under SO(2) rotations around *B***

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

*former charge diffusion mode*

$$\omega_+ = v_+\, k - i \Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

*former sound modes*

# EFT result II: weak B hydrodynamics

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$ :

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**spin 1 modes under SO(2) rotations around *B***

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

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*former sound modes*

➡ **a chiral magnetic wave**
*[Kharzeev, Yee; PRD (2011)]*

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0} \left( \tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

# EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

*[Ammon, Kaminski et al.; JHEP (2017)]*

*[Abbasi et al.; PLB (2016)]*

*[Kalaydzhyan, Murchikova; NPB (2016)]*

**spin 0 modes under SO(2) rotations around *B***

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*former sound modes*

$$w_0 = \epsilon_0 + P_0$$

$$\mathfrak{s}_0 = s_0/n_0$$

$$\tilde{c}_P = T_0(\partial \mathfrak{s}/\partial T)_P$$

$$c_s^2 = (\partial P/\partial \epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6 w_0} + c_s^2 \frac{w_0\,\sigma}{2 n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\,\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3 C T_0 \mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3 C \mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\,\frac{1 - c_s^2}{w_0}\,\xi_V^{(0)}\,, \qquad v_0 = \frac{2 B\, T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3 C \mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

*known from entropy current argument*

*[Son,Surowka; PRL (2009)]*

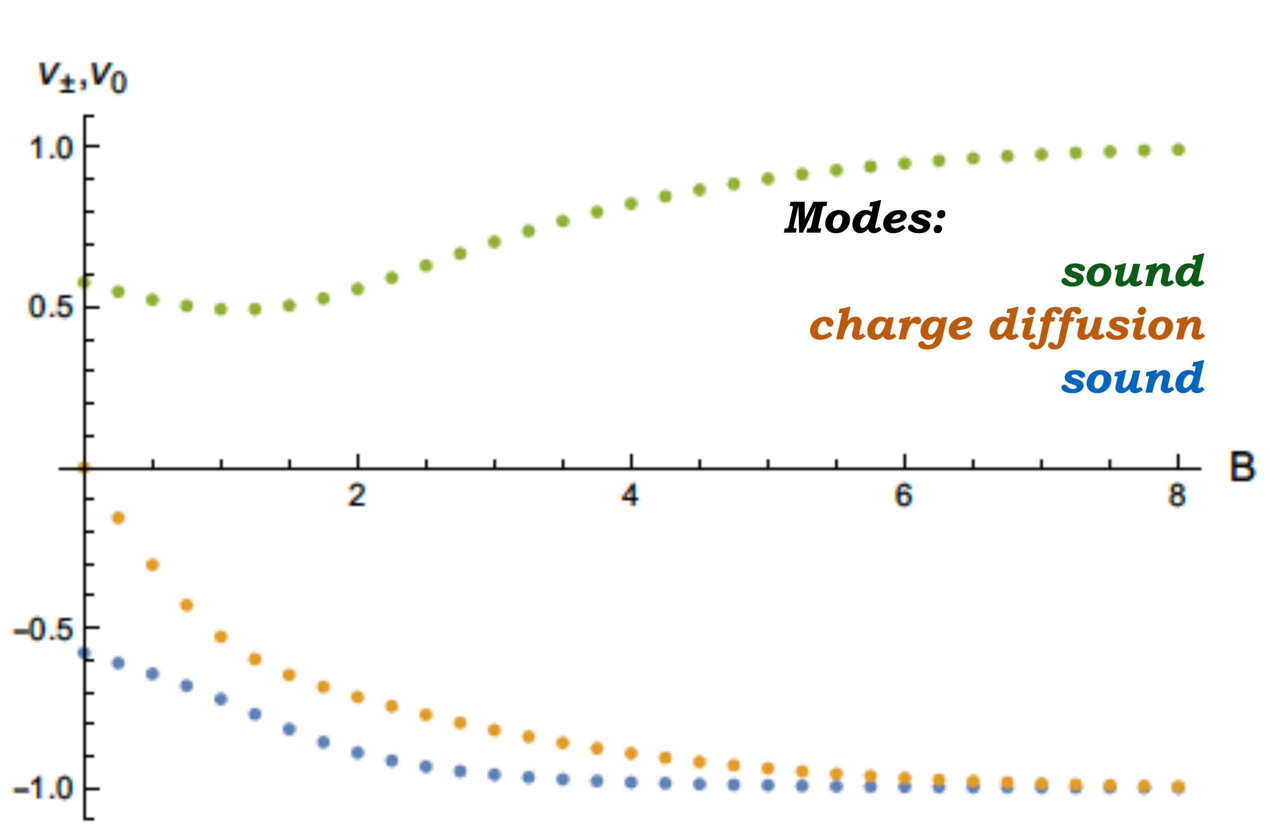
*[Neiman,Oz; JHEP (2010)]*

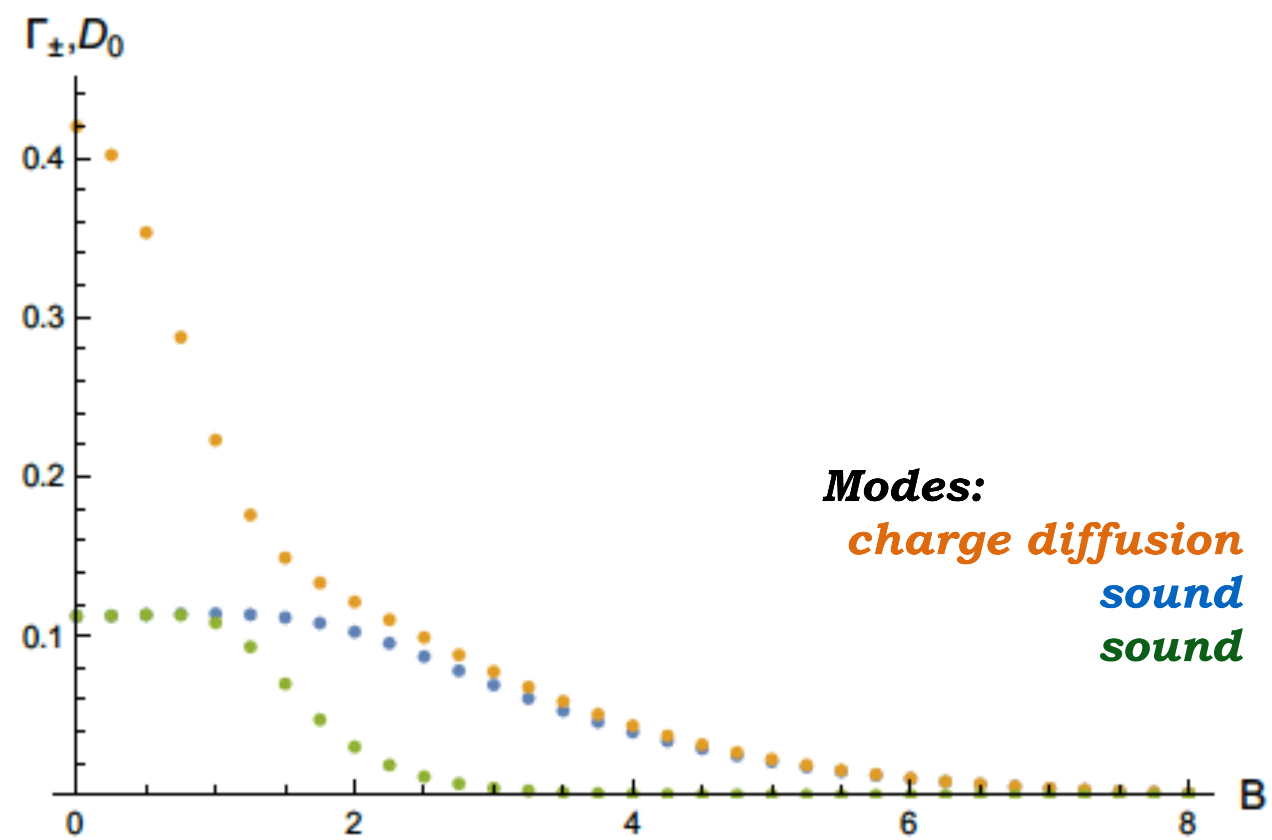
# Holographic result: hydrodynamic poles

Fluctuations around charged magnetic black branes *[Ammon, Kaminski et al.; JHEP (2017)]*

- Weak $B$: **holographic results are in "agreement" with hydrodynamics.**
- Strong $B$: holographic result in agreement with thermodynamics, and numerical result indicates that **chiral waves propagate at …**

**the speed of light** **and without attenuation**

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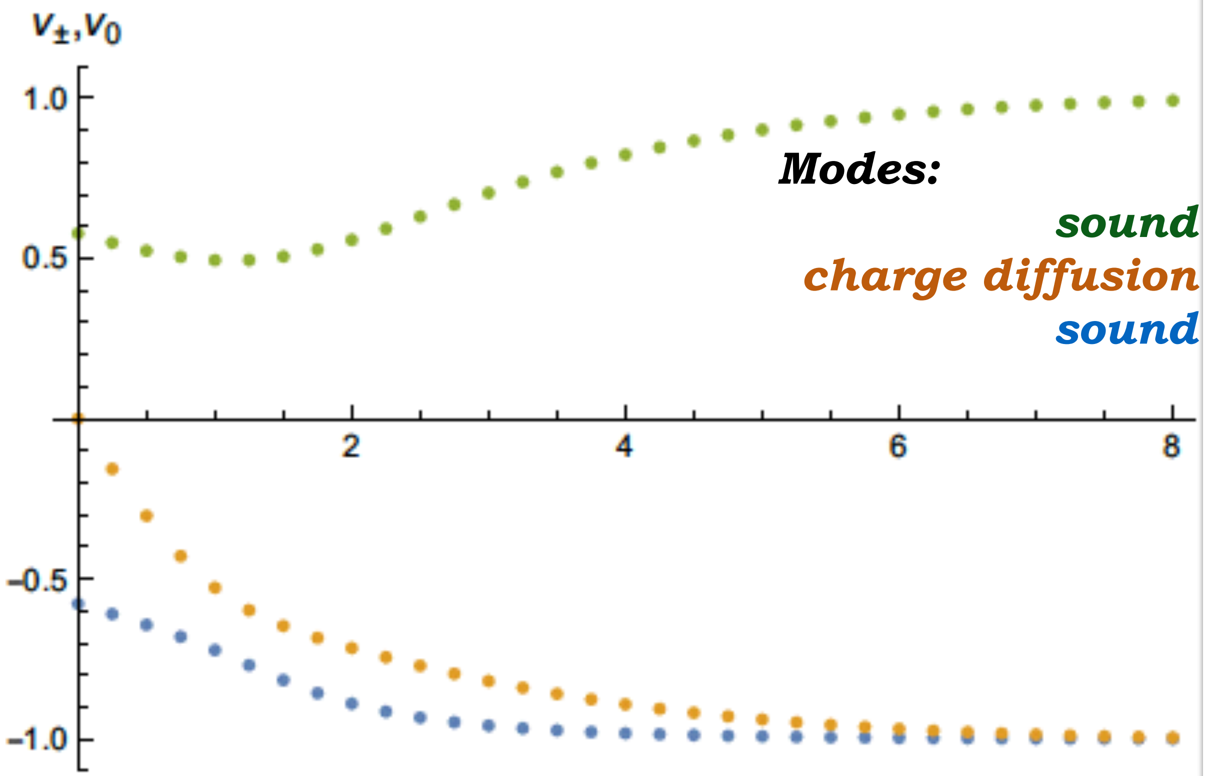


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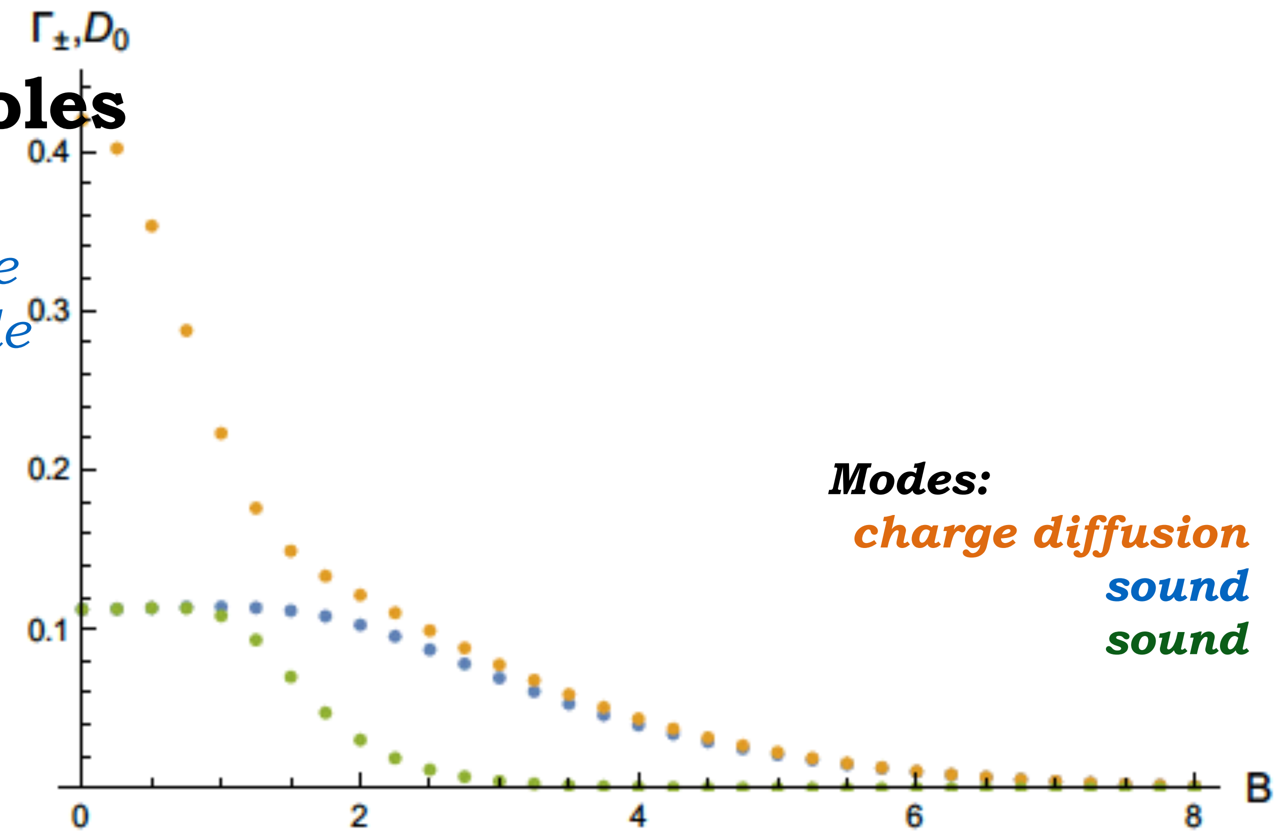


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