Hydrodynamization and Attractors at Intermediate Coupling

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Motivation & Questions

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This amounts to studying the Anisotropy $R(w)$ as a function of the dimensionless parameter $w = \tau T$.

$$R(w) = \frac{P_T - P_L}{P_{\text{Tot}}}. \quad (1)$$

Cartoon of Heavy Ion Collision: Scenario 1

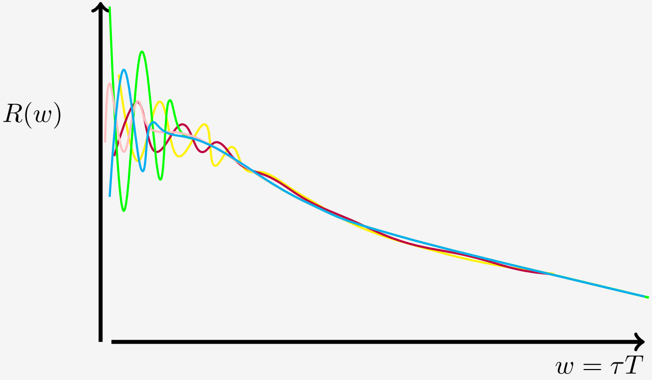

Figure: Early system dynamics has a microscopic description that relaxes to a common curve, called the Attractor. The late time dynamics should be described by Hydrodynamics when gradients are small $(w = \tau T > 1)$.

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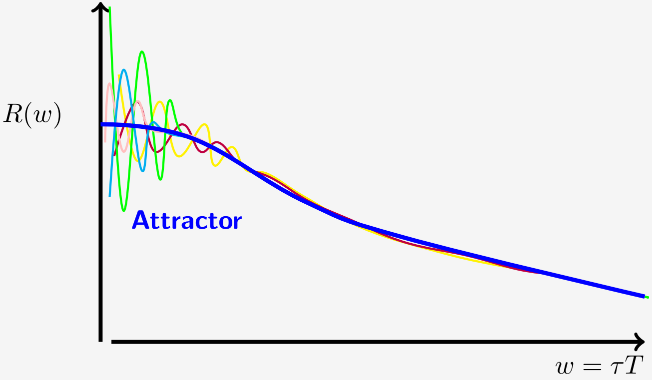

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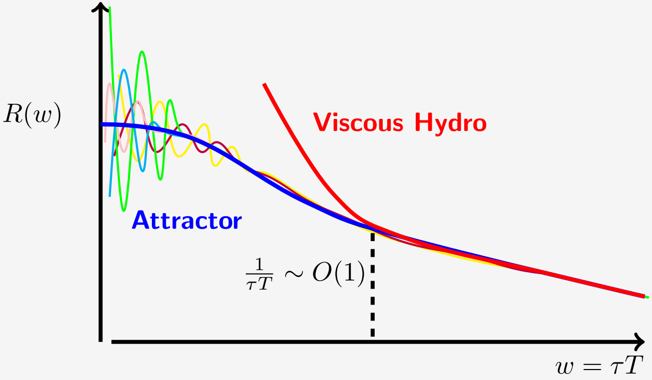

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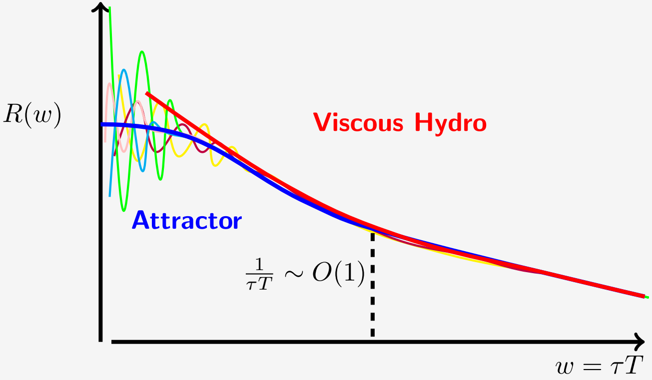
Cartoon of Heavy Ion Collision: Scenario 2



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  where λ_{GB} is a parameter of the theory we can tune to pick out our preferred coupling.
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- We calculate the hydrodynamic expansion for the Anisotropy $R(w)$, where powers of $w^{-1} = \frac{1}{\tau T}$ corresponds to orders in gradients:

$$R(w) = \underbrace{r_1 w^{-1}}_{\text{1^{st} order viscous hydro}} + \underbrace{r_2 w^{-2} + r_3 w^{-3} + \ldots}_{\text{Further gradient corrections}} \tag{3}$$

Gravity Solution & Hydrodynamics

The Action for Gauss-Bonnet Gravity is given by,

$$S = \int d^5x\,\sqrt{-g}\left(R + 12 + \frac{\lambda_{GB}}{2}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\right).$$

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the solution takes the form of a power series in inverse proper time $\tau^{-2/3}$:

$$ds^2 = -r^2 A(r,\tau) d\tau^2 + 2 d\tau dr + (1 + r\tau)^2 e^b dy^2 + r^2 e^{d - \frac{1}{2} b} dx_\perp^2 \tag{4}$$

where

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From which we can find the Energy Density

$$\epsilon(u) = u^2 \left(\epsilon_0 + \epsilon_1 u + \epsilon_2 u^2 + \epsilon_3 u^3 + ... \right)$$

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From which we can find the Energy Density → Anisotropy

$$\epsilon(u) \to R(w) = r_1 w + r_2 w^2 + r_3 w^3 + \ldots \quad , \quad w = \frac{1}{\tau T}$$

Resurgence

The coefficients r_n calculated from our gravity solution shows that each Hydrodynamic series does not converge.

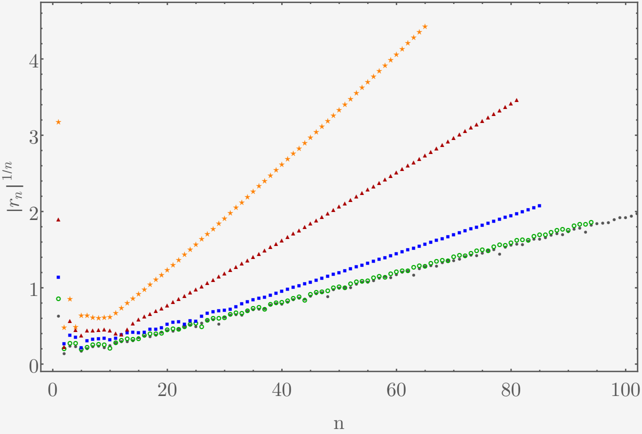

Figure: Anisotropy coefficients showing $r_n \sim n!$ as a function of order n. r_n are displayed for $\lambda_{GB} = 0$, -0.1, -0.2, -0.5, and -1.

Resurgence

We rewrite the divergent series a Laplace Transform,

$$\begin{aligned} R(w) &= r_1 w^{-1} + r_2 w^{-2} + r_3 w^{-3} + \ldots \\ &= w \int_0^{\infty e^{i\theta}} du\, e^{-uw} \underbrace{\left(\frac{r_1}{1!} w^{-1} + \frac{r_2}{2!} w^{-2} + \frac{r_3}{3!} w^{-3} + \ldots \right)}_{\text{The Borel Transform } R_B(u)} \end{aligned}$$

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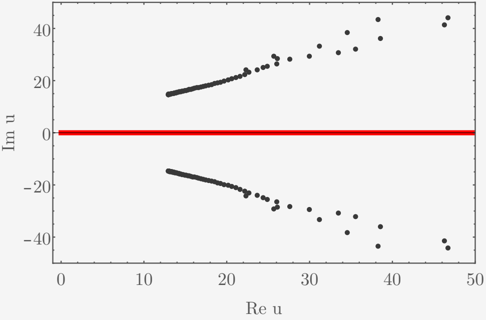
Any Laplace transform of the Pade Approximant of $R_B(w)$ is meant to be a valid solution, which means that to the linearized level the difference between two contours will also be a solution.



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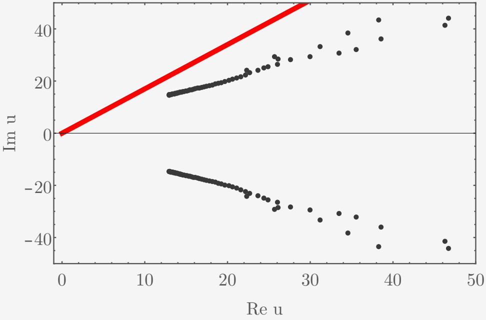

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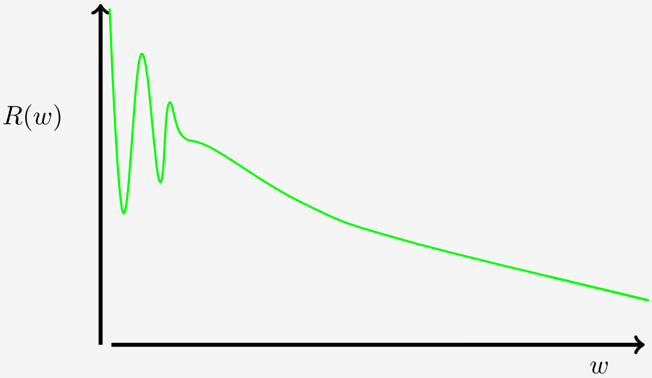

Figure: Choosing one contour leads to one particular evolution. Including linear combinations of the non-hydrodynamic solutions leads to a characteristic spread of solutions. These different choices amount to assigning different initial data to the system.

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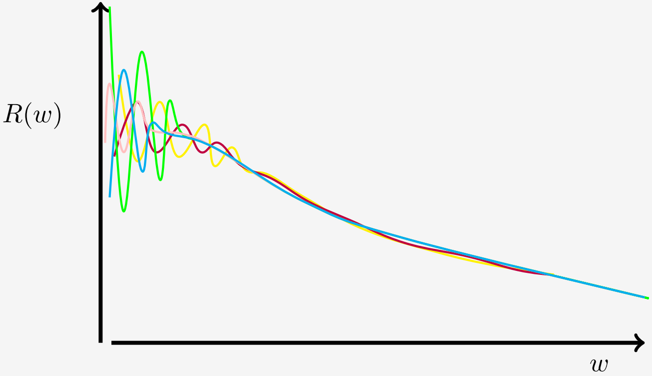

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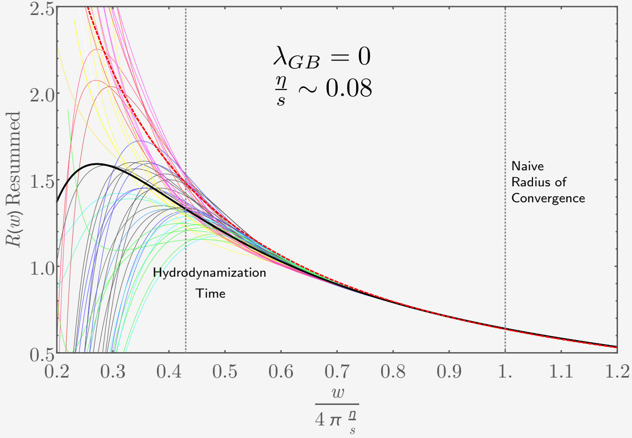

Figure: Resummed $R(w)$ plus non-hydrodynamic solutions with varied initial conditions. We define the Hydrodynamization Time as the w where $R(w)$ deviates from it's first order truncation by 10%. 1^{st} order hydro is given by the red dashed curve.

What are these Transient Modes?

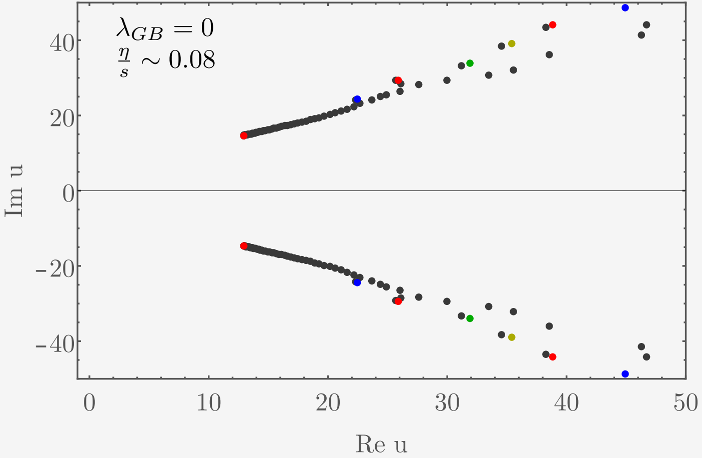

Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = 0$, M. Heller, R. Janik, P. Witaszczyk. 2013. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by A. Starinets. 2002.

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These poles show up in the Borel Plane because the true solution has the form,

$$A(\tau, r) = \sum_{i=0} u^i A_i^{\mathbf{0}}(s) + \sum_{\boldsymbol{n}\in\mathbb{N}^\infty} \Omega_{\boldsymbol{n}}(u) \sum_{i=0} u^i A_i^{\boldsymbol{n}}(s)\,,$$

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(See talk by **Jakub Jankowski** where we've calculated the "Non-hydrodynamic" gradient expansion for some sectors in collaboration with Inês Aniceto & Michał Spalińksi).

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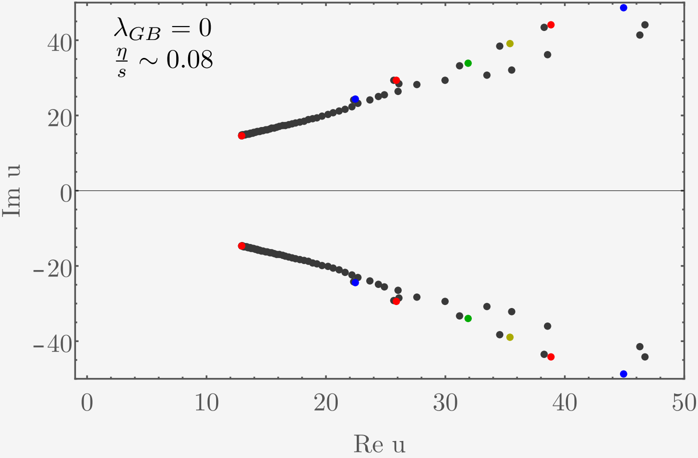

Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = 0$, M. Heller, R. Janik, P. Witaszczyk. 2013. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by A. Starinets. 2002.

Intermediate Coupling Effects

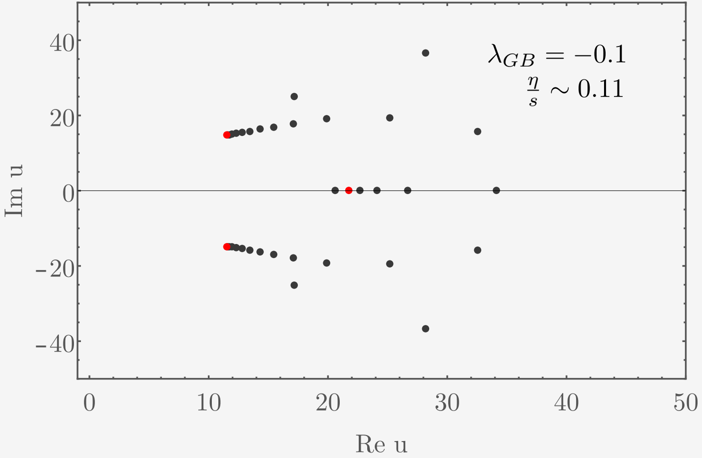

Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = -0.1$. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by S. Grozdanov, N. Kaplis, A. O. Starinets. 2016.

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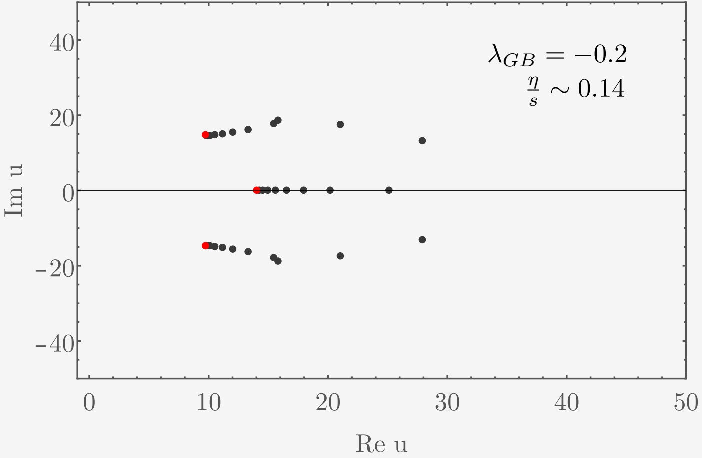

Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = -0.2$. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by S. Grozdanov, N. Kaplis, A. O. Starinets. 2016.

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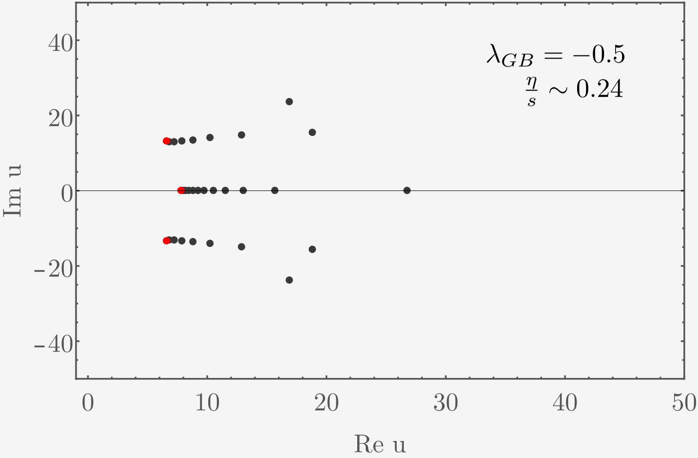

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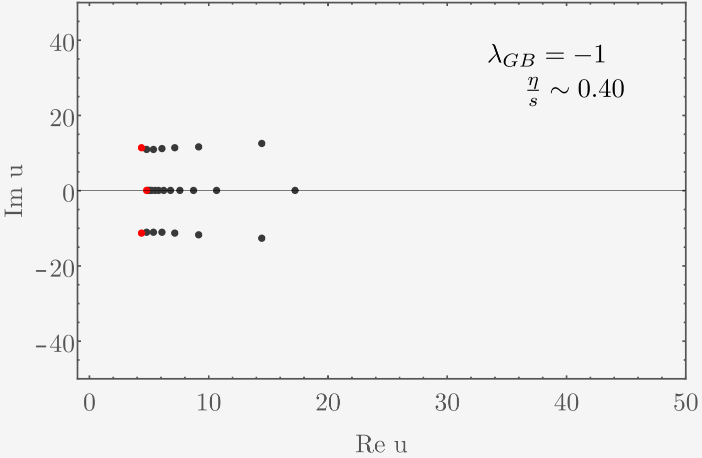

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Intermediate Coupling Effects

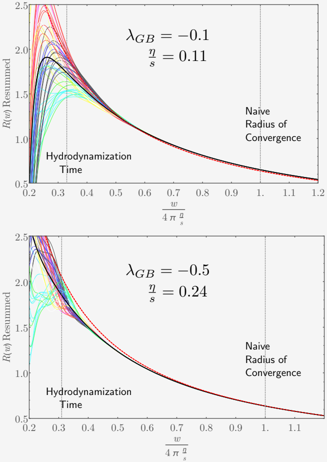

Figure: Applying similar methodology in the case of finite coupling, we can estimate the characteristic spread of solutions as they decay to the attractor. In all cases we study the result is well approximated by 1^{st} order hydrodynamics (red dashed line).

How else can we study Bjorken Flow in $\mathcal{N} = 4$ SYM?

Do we have any other expansion parameters we can use to solve this system? [Ongoing work with Jorge Casalderrey-Solana & Chris Herzog]

$$ds_5^2 = -A dt^2 + 2 dr d\tau + S^2 \left(e^{-2B} dy^2 + e^{B} dx_\perp^2\right), \qquad (8)$$

where A, B and S are functions of r and τ.

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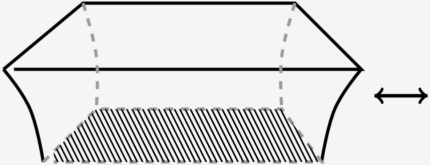
We can study Bjorken Flow in n dimensions, and use $1/n$ as an expansion parameter when n is large.

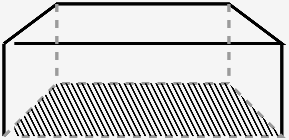
$$ds^2 = -A\,d\tau^2 + 2d\tau dr + S^2\left(e^{-(n-2)B}dy^2 + e^B dx_\perp^2\right)\ , \tag{9}$$

Bjorken Flow in large dimensions $d = n + 1$

In the large dimension limit, gravity backgrounds look flat with small corrections.



Black-brane in $d = 5$



Black-brane in $d = \infty$

Bjorken Flow in large dimensions $d = n + 1$

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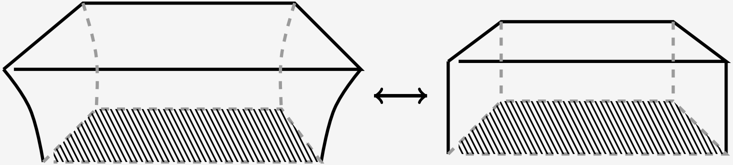

Black-brane in $d = 5$ **Black-brane in $d = \infty$**

We fix $R = r^n$ and use an Ansatz given on the right hand side. The functions factored out from the sum give the flat space solution in n dimensions, and the power series' in $1/n$ gives finite n corrections.

$$A = r^2 \sum_{i=0}^{\infty} \frac{1}{n^i} A_i(R, \tau)\,,$$

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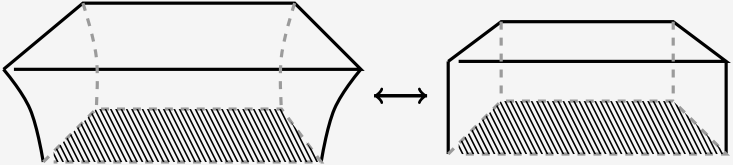

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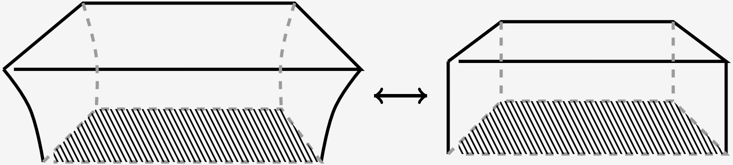

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Bjorken Flow in large dimensions $d = n + 1$

We have calculated these corrections to order $O(n^{-3})$. At leading and subleading order they are given by,

$$S_0 = 1\,, \qquad S_1 = 0\,,$$

$$A_0 = 1 - \frac{\epsilon}{(1+\tau)}\frac{1}{R}\,, \qquad A_1 = \frac{\epsilon \log(1+\tau)}{(1+\tau)}\frac{1}{R} - \frac{\epsilon}{(1+\tau)^2}\frac{\log R}{R}\,,$$

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with $R = r^n$, and ϵ a constant of integration that turns out to be the energy density.

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We know how to read off the energy density corresponding to this solution, which gives ideal and viscous Hydrodynamics to 2^{nd} order.

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\epsilon(\tau) = \frac{\epsilon}{\tau} - \frac{\epsilon}{n}\left(\frac{\log\tau}{\tau} + \frac{2}{\tau^2} - \frac{1}{2}\frac{1}{\tau^3}\right) + O(n^{-2}) \tag{10}
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Conclusion

- Why does Hydrodynamics work outside it's regime of applicability?
  - Our estimate of this regime relies on the series converging, which it does not.
  - All you can do to justify hydrodynamics is to resum the series and compare to truncations.
- What qualitative changes do we observe as we interpolate between gauge theories at infinite and finite coupling?
  - At finite coupling our microscopic theory gains a dissipative mode, compatible with kinetic theory.
  - Comparing the full resummation to the truncated series, 1^{st} order viscous hydro works very well in all cases.
- Can the large dimension limit be used to study Bjorken Flow in $n = 4$?
  - We have an anayltic solution that seems to reproduce the first few terms of Hydrodynamics.

Back up slides

Hydrodynamics in 3+1 Dimensions

The equation of motion for Hydrodynamics is the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 \tag{13}$$

where $T^{\mu\nu} = T^{\mu\nu}(\epsilon, P, u^\mu)$ with ϵ the energy density, P the Pressure, and u^μ the fluid velocity.

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$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + c_1 \partial^\mu u^\nu + c_2 \partial^\nu u^\mu + c_3 \eta^{\mu\nu} \partial_\alpha u^\alpha + c_4 u^\mu u^\nu \partial_\alpha u^\alpha + \ldots \tag{15}$$

Hydrodynamics in 3+1 Dimensions

When ∂u is small we can order the series in derivatives of u^μ

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- This series is known as the Gradient Expansion.

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- This series is known as the Gradient Expansion.
- The coefficients c_i are known as transport coefficients and uniquely specify our theory.

The Fluid-Gravity correspondence

We can perform classical gravity calculations to find strongly coupled QFT results.

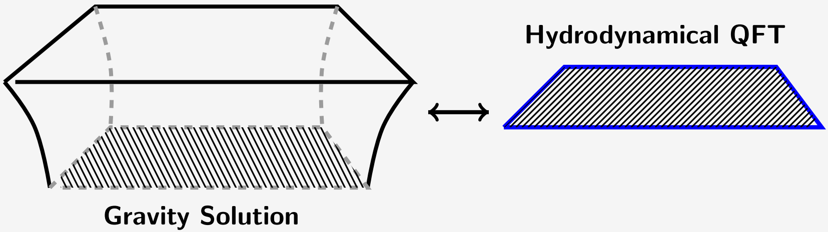

Figure: Some Gauge theories are Gravity theories are conjectured to be the same theory under a field redefinition.

The Fluid-Gravity correspondence

We can construct a dynamical gravity solution which will be dual to Bjorken Flow for $N = 4$ SYM:

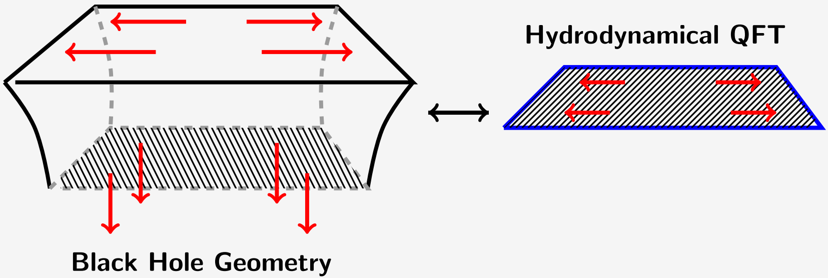

Figure: Some Gauge theories and Gravity theories are conjectured to be the same theory under a field redefinition.