Time Evolution of a Semiholographic Glasma

Based on work with Ayan Mukhopadhyay, Florian Preis, Anton Rebhan, and Alexander Soloviev (1806.01850)

Christian Ecker
(TU-Wien)

Santiago de Compostela
July 2, 2018

FWF

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

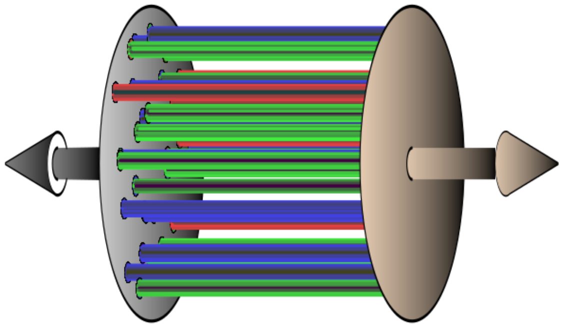
DOKTORATSKOLLEG PI
$\int dk\,\Pi$
Particles and Interactions

INSTITUTE FOR THEORETICAL PHYSICS

Outline

- Introduction
  - Combining Glasma and AdS/CFT
- Semiholographic Framework
  - Semiholography Action and Equations of Motion
  - Conserved Energy-Momentum Tensor
  - Solution Procedure: Self-Consistency Loop
- Example
  - (2+1)-dimensional YM + AdS4/CFT3
- Summary & Outlook

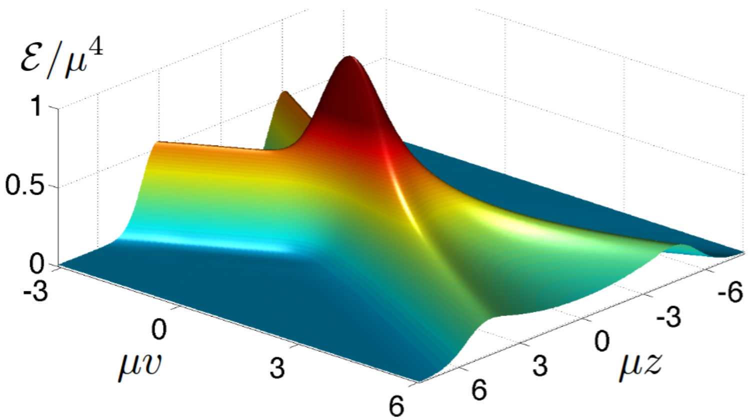
Early Time-Evolution of the QGP



Glasma approach

- Early times ($\tau \lesssim 0.1 fm/c$) semi-hard (Q_s) gluons dominate.
- Coupling is weak ($\alpha_s(Q_s) \ll 1$) but high occ. number $\sim 1/\alpha_s$.
- Effectively described by classical Yang-Mills fields (glasma).

[picture from: Gelis,Iancu,Jalilian-Marian and Venugopulan (1002.0333)]



Holographic approach

- HICs from colliding grav. shock waves on AdS5.
- N=4 SYM theory at infinite coupling, not QCD.
- Fast hydrodynamization.
- Initial conditions?

[picture from: Chesler and Yaffe (1011.3562)]

Semi-holographic approach

- Self-consistent coupling between Yang-Mills ("glasma") fields and a strongly coupled AdS/CFT sector.

[proposed by: Iancu and Mukhopadhyay (1410.6448)
developed further by: Mukhopadhyay,Preis,Rebhan and Stricker (1512.06445)]

Semiholography Action

$$S = -\frac{1}{4g_{YM}^2}\int d^4x(1+\chi(x))F^a_{\mu\nu}F^{a\mu\nu} + W_{1PI}[h(x)] - \frac{Q_s^4}{\boxed{\beta}}\int d^4x h(x)\chi(x)$$

dimensionless coupling β is the tunable parameter of the model

$$\chi(x) = \frac{\beta}{Q_s^4}\mathcal{H}\,, \quad \mathcal{H} := \frac{\delta W_{1PI}[h(x)]}{\delta h(x)}\,, \quad h(x) = -\frac{\beta}{4Q_s^4 g_{YM}^2}F^a_{\mu\nu}F^{a\mu\nu}$$

Equations of motion:

$$D_\mu\left[\left(1+\frac{\beta}{Q_s^4}\mathcal{H}\right)F^{a\mu\nu}\right] = 0\,,$$

$$R_{MN} - \tfrac{1}{2}RG_{MN} - 3G_{MN} = \kappa(\nabla_M\phi\nabla_N\phi - \tfrac{1}{2}G_{MN}(\nabla\phi)^2)\,,$$

$$G^{MN}\nabla_M\nabla_N\phi = 0\,,$$

with boundary source: $\phi_{(0)} = h(x) = -\frac{\beta}{4Q_s^4 g_{YM}^2}F^a_{\mu\nu}F^{a\mu\nu}$

Total Energy-Momentum Tensor

The energy-momentum tensor of the combined system contains contributions form the classical Yang-Mills sector, the holographic sector and an exchange-part

$$T^{\mu\nu} = t^{\mu\nu}_{YM} + \mathcal{T}^{\mu\nu}_{hol} + t^{\mu\nu}_{xc}$$

$$= \frac{1}{g^2_{YM}}\left(1 + \frac{\beta}{Q_s^4 \mathcal{H}}\right)\left(F^{a\mu\alpha}F^{a\nu}_{\alpha} - \frac{1}{4}\eta^{\mu\nu F^a_{\alpha\beta}}F^{a\alpha\beta}\right) + \mathcal{T}^{\mu\nu}_{hol} - h\mathcal{H}\eta^{\mu\nu}$$

Ward identities of the YM and holographic sector imply conservation of the total EMT

$$\nabla_\mu t^{\mu\nu}_{YM} = \frac{Q_s^4}{\beta} h\partial^\nu \left(1 + \frac{\beta}{Q_s^4}\mathcal{H}\right), \qquad \partial_\mu \mathcal{T}^{\mu\nu}_{hol} = \mathcal{H}\partial^\nu h\,,$$

$$\Rightarrow \boxed{\partial_\mu T^{\mu\nu} = 0\,.}$$

[for proof see: Mukhopadhyay,Preis,Rebhan,Stricker (1512.06445)]

Example

Yang-Mills sector

SU(2) Yang-Mills theory in 2+1 dimensions, homogeneous, isotropic, temporal gauge ($A_0^a = 0$) and color-space locking ($A_i^a(t) \propto \delta_i^a$).

=> only two (equivalent) non-vanishing components: $A_1^1 = A_2^2 = f(t)$

$$f''(t) + f(t)^3 = f'(t)\frac{\beta \mathcal{H}'}{1 + \frac{\beta}{Q_s^3}\mathcal{H}}\,, \quad h(t) = \frac{\beta}{2Q_s^3 g_{YM}^2}(2(f'(t))^2 - f(t)^4)$$

holographic sector

Homogeneous and isotropic AdS4 black brane + minimally coupled massless scalar.

$$ds^2 = -A(r,v)dv^2 + 2drdv + S(r,v)(dx_1^2 + dx_2^2)\,, \;\; \phi = \phi(r,v)$$

$$S'' = -\tfrac{\kappa}{4}S(\phi')^2\,, \qquad A'' = \tfrac{4\dot{S}S'}{S^2} - \frac{\kappa\dot{\phi}^2 S}{4}\,,$$

$$\dot{S}' = \tfrac{3S}{2} - \tfrac{\dot{S}S'}{S}\,, \qquad \ddot{S} = \tfrac{\dot{S}A'}{2} - \tfrac{\kappa\dot{\phi}^2 S}{4}\,. \qquad \mathcal{H} = \tfrac{3}{\kappa}(\phi_3 + \tfrac{1}{3}\phi_0''' - \tfrac{1}{4}(\phi_0')^3)$$

$$A'' = \tfrac{4\dot{S}S'}{S^2} - \kappa\dot{\phi}\phi'\,,$$

Initial Conditions

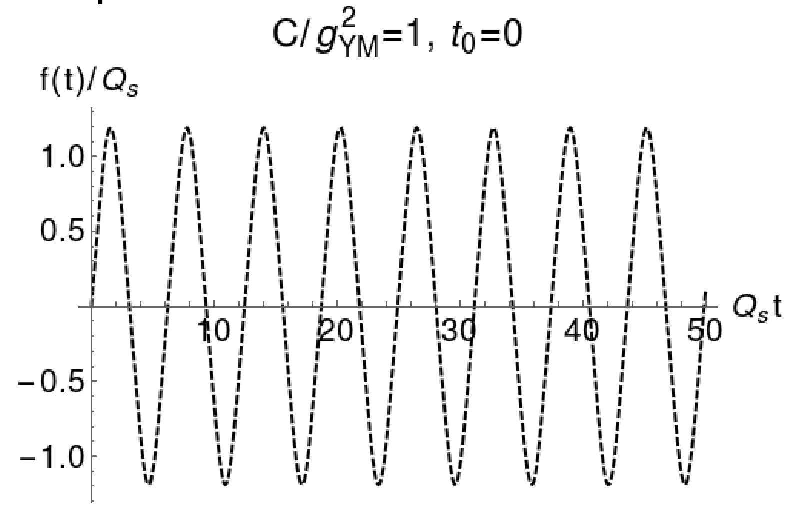
The solution procedure is initialized with a solution of the non-coupled ($\beta = 0$) Yang-Mills equation which is given by the Jacobi elliptic function.

$$f''(t) + 2f(t)^3 = 0\,,$$

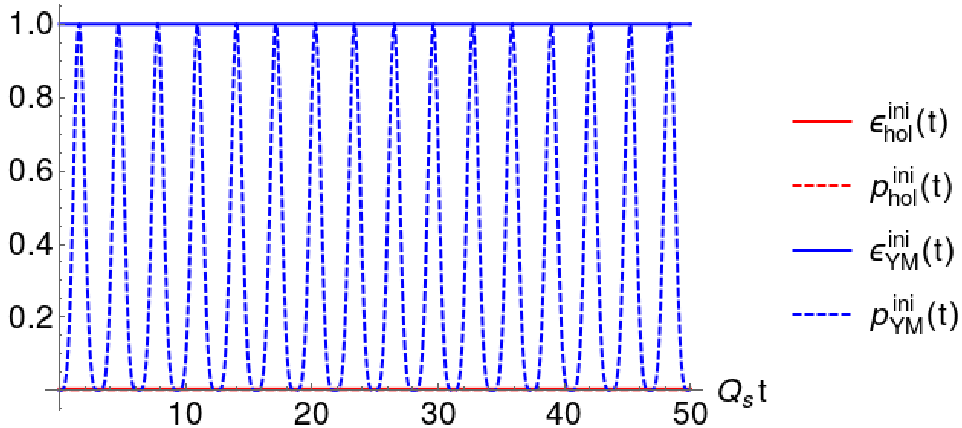
$$f(t) = \pm\sqrt[4]{2C}\,\mathrm{sn}\left(\sqrt[4]{\tfrac{C}{2}}(t - t_0)||-1\right),$$

$$\epsilon^{ini}_{YM} = \frac{1}{2g^2_{YM}}(2(f')^2 + f^4)\,, \quad p^{ini}_{YM} = \frac{1}{2g^2_{YM}}f^4$$

Solution has constant energy: $C/g^2_{YM} = \epsilon^{ini}_{YM}$



On the holography side we choose a small initial energy $a_3(t=0) = -\epsilon^{ini}_{hol}/2$ and the radial profile of the scalar field $\phi(z, t=0)$ on the initial time-slice in the bulk.



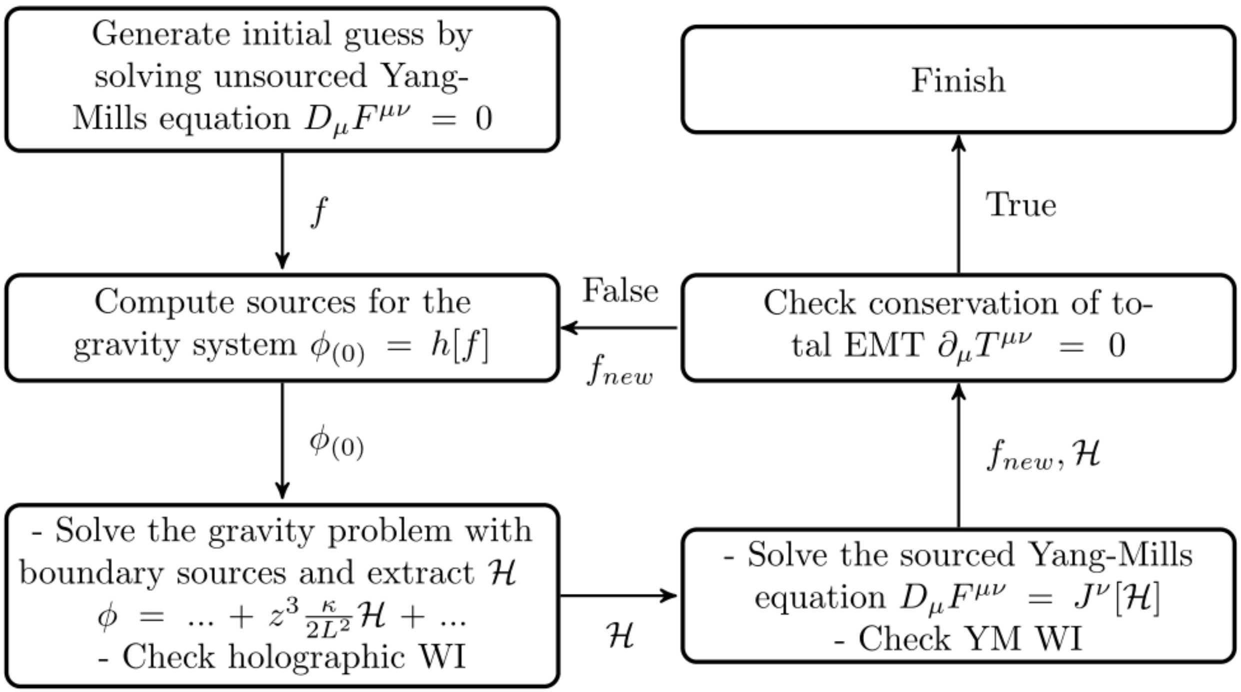
All time-derivatives of the non-coupled solution vanish at t=0.

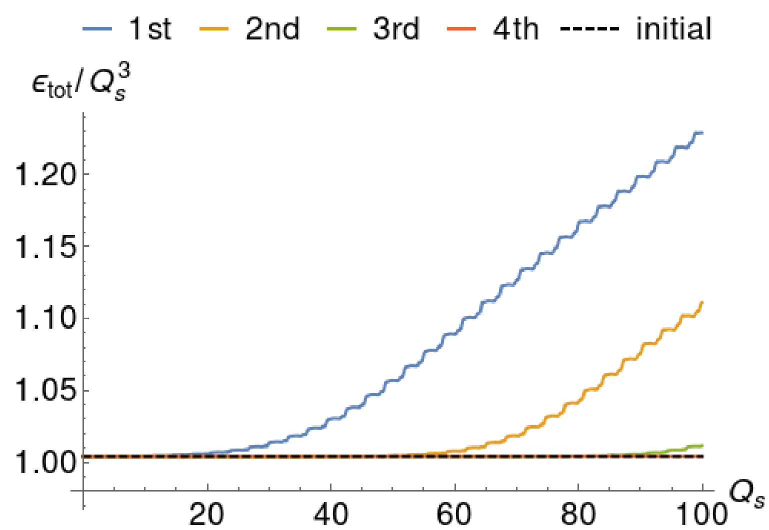
Ward identity fixes $\mathcal{H}(t=0) = 0$.

The initial radial profile is constant

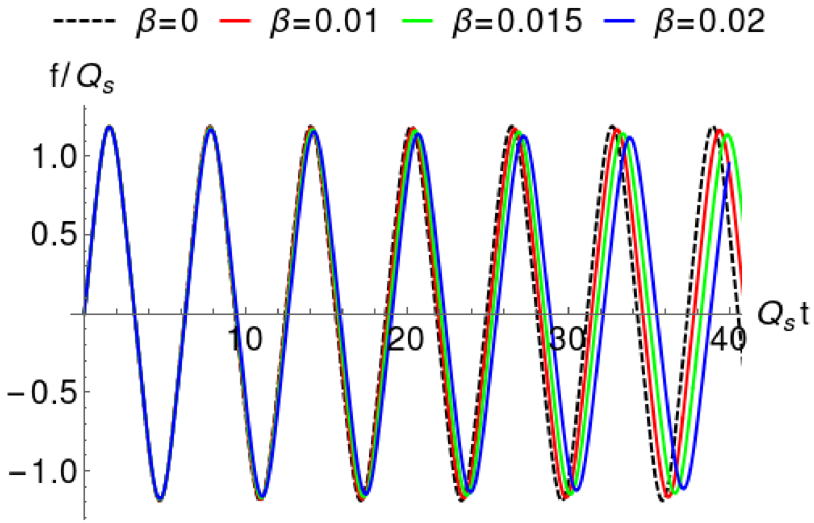
$$\phi(z, t=0) = \phi_{(0)} = -\beta\epsilon^{ini}_{YM}$$

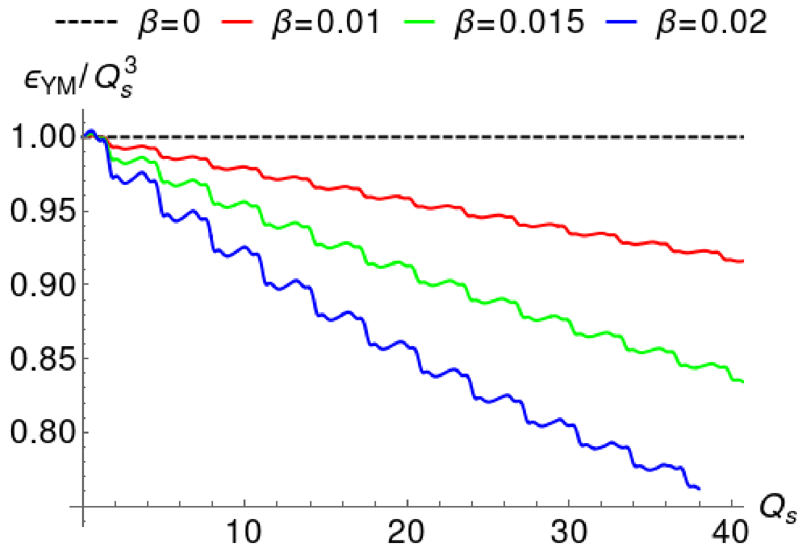
Solution Procedure: Self-Consistency Loop

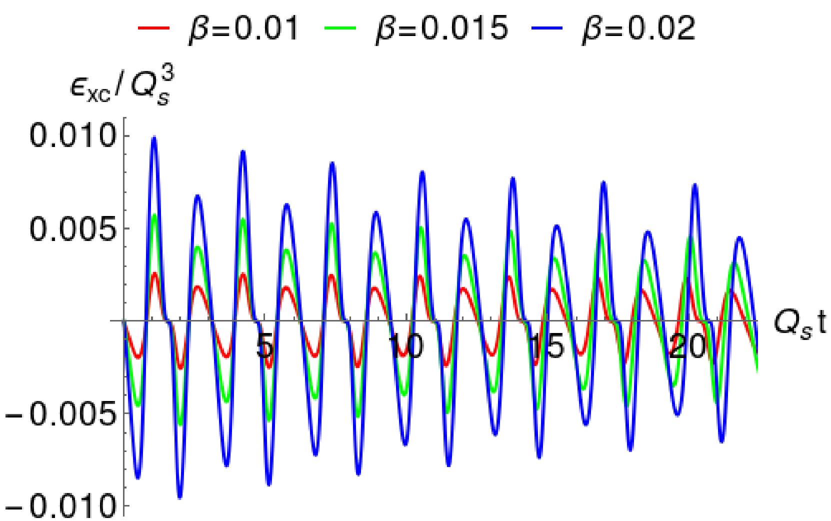


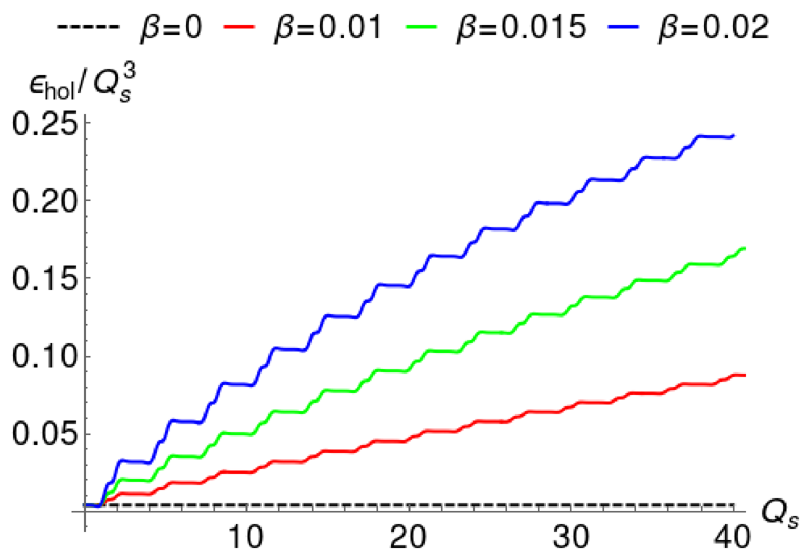


Results: Energy-Flow





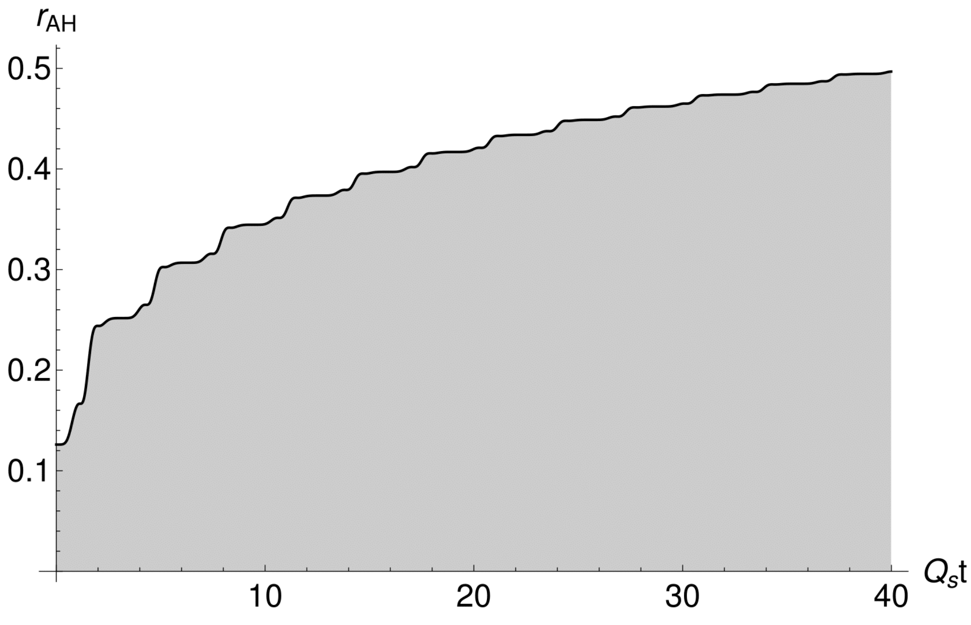


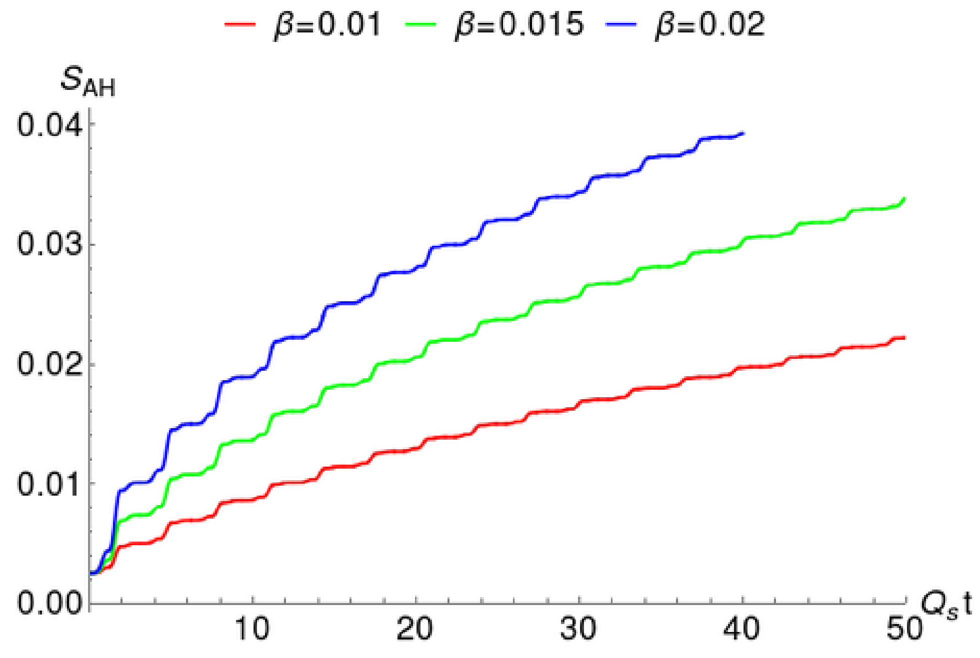


Results: Entropy

Total entropy of the combined system can be estimated by the effective horizon entropy density.

$$\dot{S}(r,t)|_{r=r_{AH}} = 0 \qquad S_{AH}(t) = \frac{1}{2\pi} S(r_{AH},t)^2$$



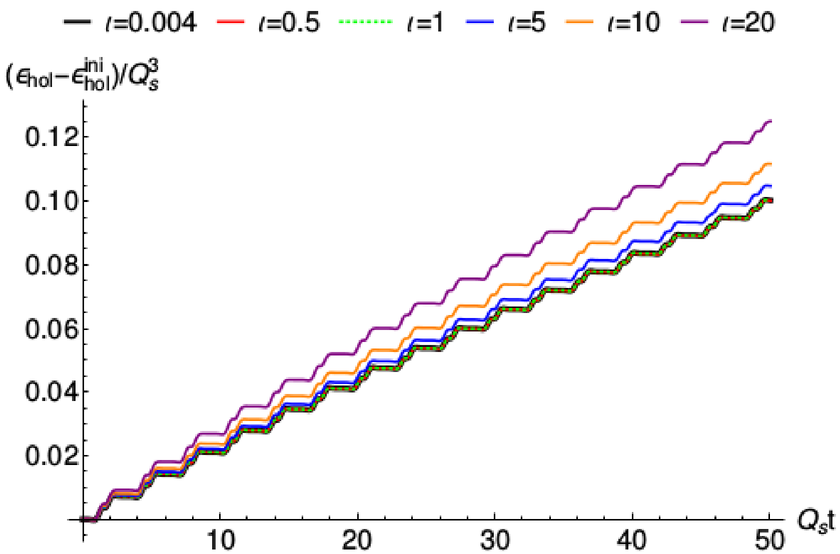


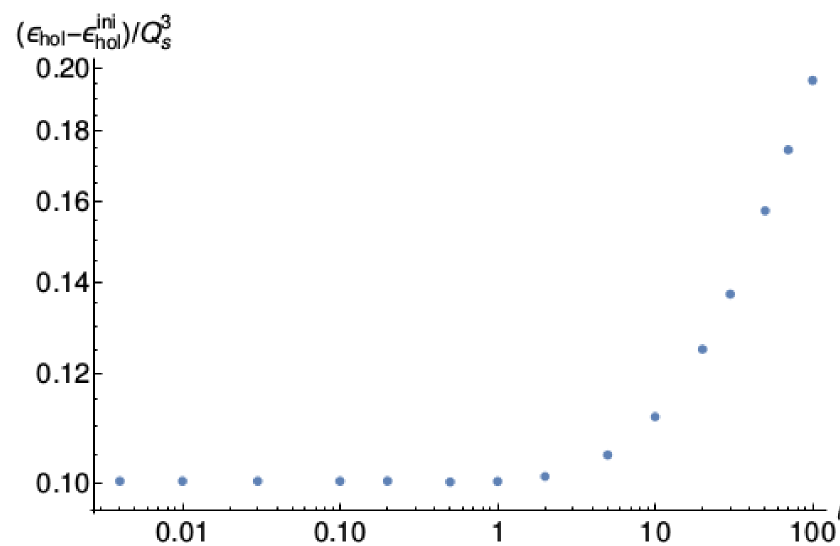
Note: The Yang-Mills sector has no entropy because it has only one degree of freedom f(t).

Sensitivity on Initial Black Hole

Simulations are not sensitive to the size of the seed black hole, as long as its energy is smaller than the initial YM-energy.

$$\iota \equiv \epsilon_{hol}^{ini} / \epsilon_{YM}^{ini}$$





Summary

- First semi-holographic simulations featuring energy exchange between classical Yang-Mills sector and a holographic CFT.
- Energy is transfered from the Yang-Mills sector to the strongly coupled CFT sector.
- Successful proof of principle: self-consistent numerical AdS/CFT simulation with back-reacted dynamical boundary source.

Ongoing work

- Improve numerics: make larger couplings accessible. Expect to find qualitatively different phases: under-damping, critical damping, over-damping, …?
- Simulations for 4 dimensional Yang-Mills coupled to AdS5/CFT4 (numerically harder).
- Include additional couplings: e.g. tensor coupling, axion coupling.
- More general: couple classical field theories to strongly coupled AdS/CFT sectors. e.g.: classical GR sourced by strongly coupled AdS/CFT matter

$$S = -\frac{1}{\kappa} \int d^4x \sqrt{g} R[g_{\mu\nu}] - W[g_{\mu\nu}]$$