Holographic Quark Matter and Color Superconductivity

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arXiv:1607.07773, arXiv:1611.05808, arXiv:1707.06989 and work in progress
in collaboration with Antón Faedo, David Mateos, Christiana Pantelidou

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Context of this talk

- Study strongly coupled field theories is a hard task
- But may be of interest in astrophysical setups or condensed matter models
- The purpose of this talk is to start studying the characteristics of a particular type of phase: those with spontaneous breaking of the gauge group (CSC)
- To do this in a *first principles* manner, I will resort to string theory, in particular holography

Interest on the phase diagram in astrophysical setups or condensed matter models

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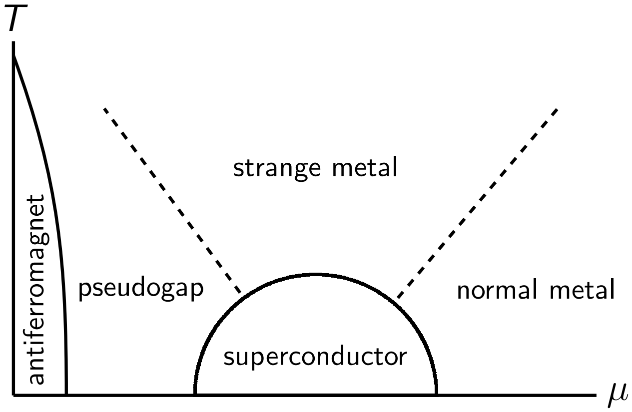
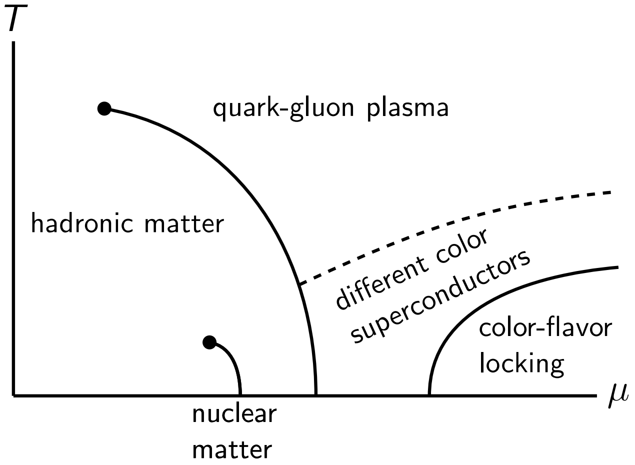
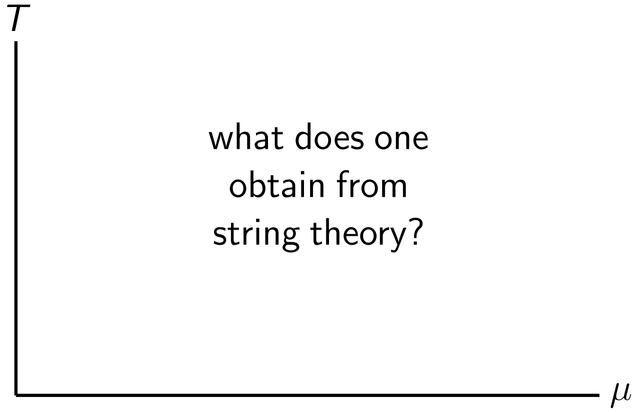


Interest on the phase diagram in astrophysical setups or condensed matter models



I will resort to string theory, in particular holography



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Some words about the setup

- I describe results from top-down models, where we extremize type IIB SUGRA, DBI and WZ actions

- D3/D7 system as the dual of $\mathcal{N}=4$ SYM with charged matter in the fundamental (not QCD)

$$1 \ll N_f \ll N_c^{1/3}$$

- D7-branes are smeared

$$U(N_f) \to U(1)^{N_f}$$

Backreaction of the flavor branes

- Consider the RR form the flavor brane sources

$$S_{IIB+D7} \supset \frac{1}{2}\int \mathrm{d}C_8 \wedge *\mathrm{d}C_8 + \int C_8 \wedge \underbrace{(\delta(f_1)\delta(f_2)\mathrm{d}f_1 \wedge \mathrm{d}f_2)}_{\Xi_2}$$

  which implies the Bianchi identity for a sourced RR form

$$\mathrm{d}F_1 = -\Xi_2$$

- The number of flavor branes is given by Gauss law

$$\int F_1 \sim N_f$$

Backreaction with smearing (D3/D7) [hep-th/0612118]

- Recall $S_{D7} = T_7 \int \left(-\mathrm{d}^8x\, e^{-\phi}\sqrt{-G} + C_8\right) \wedge \Xi_2$ with Ξ_2 exact
- For compact part write a U(1) fibration over KE manifold. This accommodates SUSY and implies existence of

$$\mathrm{d}\eta_{KE} = 2J_{KE}\ , \qquad vol(SE) = \frac{1}{2} J_{KE} \wedge J_{KE} \wedge \eta_{KE}$$

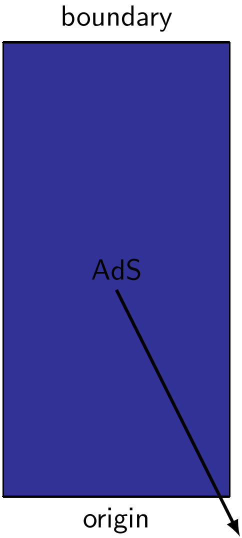
- Idea: to identify $\Xi_2 \sim J_{KE}$ and use the SU(2)-structure to write a *consistent radial ansatz* for the IIB+sources action

$$F_1 \sim N_f\, \eta_{KE} \quad \Rightarrow \quad \mathrm{d}F_1 \sim N_f\, J_{KE}$$

Qualitative description of the solution [1101.3560]

$$
\begin{aligned}
L = e^{-2\phi}\left[R * 1 - \frac{1}{2} H \wedge * H - \frac{1}{2} \mathrm{d}\phi \wedge * \mathrm{d}\phi\right] \\
- \frac{1}{2} F_1 \wedge * F_1 - \frac{1}{2} F_3 \wedge * F_3 - \frac{1}{4} F_5 \wedge * F_5 \\
- \frac{1}{2} C_4 \wedge H \wedge F_3 \\
- \frac{N_f}{N_c} \lambda\, e^{-\phi} \sqrt{-|G + \mathrm{d}A + B|} \wedge \Xi_2 \\
+ \frac{N_f}{N_c} \lambda\, e^{\mathrm{d}A + B} \left[C_8 - C_6 + C_4 - C_2\right] \wedge \Xi_2
\end{aligned}
$$

Qualitative description of the solution



$$L = e^{-2\phi}\left[R*1 - \frac{1}{2}H\wedge *H - \frac{1}{2}\mathrm{d}\phi\wedge *\mathrm{d}\phi\right]$$
$$-\frac{1}{2}F_1\wedge *F_1 - \frac{1}{2}F_3\wedge *F_3 - \frac{1}{4}F_5\wedge *F_5$$
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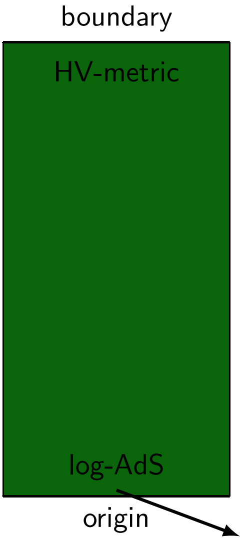
$$\mathrm{d}s^2 = -r^2\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2 + r^{-2}\mathrm{d}r^2$$

Qualitative description of the solution [hep-th/0612118][1611.05808]



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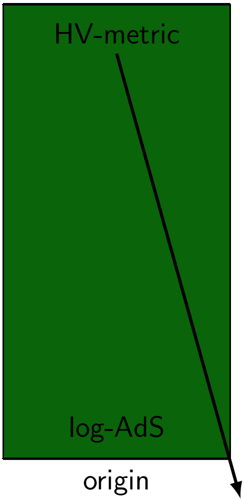
\end{aligned}
$$

$$
\mathrm{d}s^2 = \log r^{1/3} \left(-r^2 \mathrm{d}t^2 + r^2 \mathrm{d}\vec{x}^2\right) + r^{-2} \mathrm{d}r^2
$$

Qualitative description of the solution [1611.05808]



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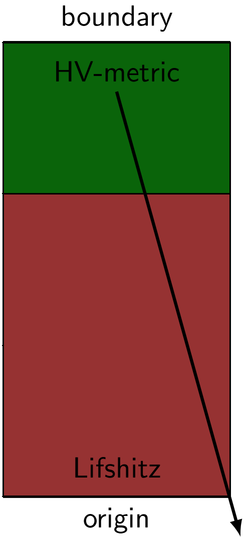
$$\mathrm{d}s^2 = r^{-7/3}\left(-r^2\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2 + r^{-2}\mathrm{d}r^2\right)$$

Qualitative description of the solution [1707.06989]



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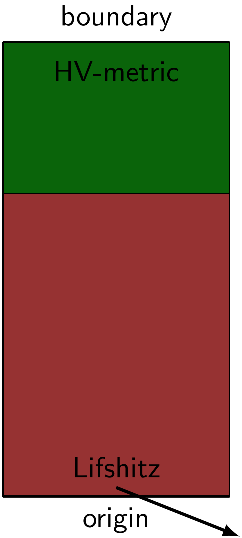

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$$\mathrm{d}s^2 = -r^{14}\mathrm{d}t^2 + r^2\mathrm{d}\vec{x}^2 + r^{-2}\mathrm{d}r^2$$

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Main idea behind the holographic CSC mechanism

- Recall how the gauge group is encoded geometrically by number of D3-branes

$$S_{D3} = -N_c \int \sqrt{-g}\, \mathrm{d}t \mathrm{d}^3 x + N_c \int_{t,\vec{x}} C_4$$

- It turns out D7-branes also carry D3-brane charge!

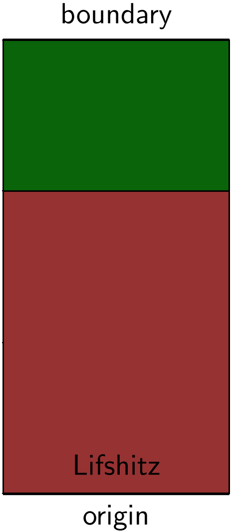
$$S_{D7} \supset \frac{N_f}{8\pi^2} \int_{t,\vec{x},r,S^3} C_4 \wedge F \wedge F = N_f \int_{t,\vec{x}} C_4 \wedge \left(\frac{1}{8\pi^2} \int_{r,S^3} F \wedge F \right)$$

- If the appropriate F is turned on then we vary the rank of the gauge group

$$N_c \to N_c + N_f \psi$$

for Higgsing in the probe see e.g. [hep-th/0504151, hep-th/0703094]

Instability towards color superconduction work in progress



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$$A = A_t(r)\mathrm{d}t$$

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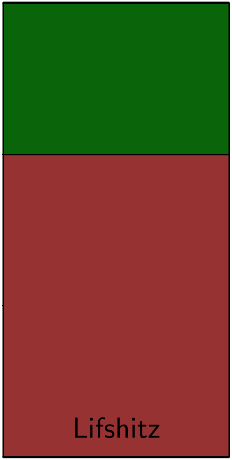

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$$A = A_t(r)\mathrm{d}t + \psi(r)\, w^3(\theta s)$$

Instability towards color superconduction work in progress

- The BF bound in Lifshitz is

$$m^2 \geq -\frac{(p+z-\theta)^2}{4} = -25$$

- The new field has mass below the BF bound, so it needs to condense to avoid dynamic instability

- Backreaction of the mode in the supergravity fields affects the Gauss law for the D3-branes

$$\int_{S^5} F_5 \sim N_c - \frac{1}{2} N_f \psi(r)^2$$

the color branes are separated: ~~$SU(N)$~~. Since color symmetry is broken we have *color superconductivity*

Instability towards color superconduction work in progress

- But turning on that field is technically difficult (20-ish fields to turn on)
- One way out is to promote to a non-abelian configuration
  - Pros: Only two extra fields, susy solution exists
  - Con: Non-abelian DBI
- NADBI/YM-Higgs offers a different path towards CSC: an instantonic configuration with massive quarks and isospin density! [hep-th/9907014]

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Summary and conclusions

- We have identified an IR phase of cold YM theories with charge density, given in the gravity side by a Lifshitz metric
- The IR is dynamically unstable towards condensation of $\mathcal{O}^I \sim Q^\dagger \sigma^I Q$
- Condensation of the dual scalar field gives rise to a color superconductor phase
- We have identified other ways to break the gauge group. This involves the presence of an isospin chemical potential, rising the possibilities of having as well FFLO (crystalline) phases

Thank you