Anomalous Transport from Holography: Review and Outlook

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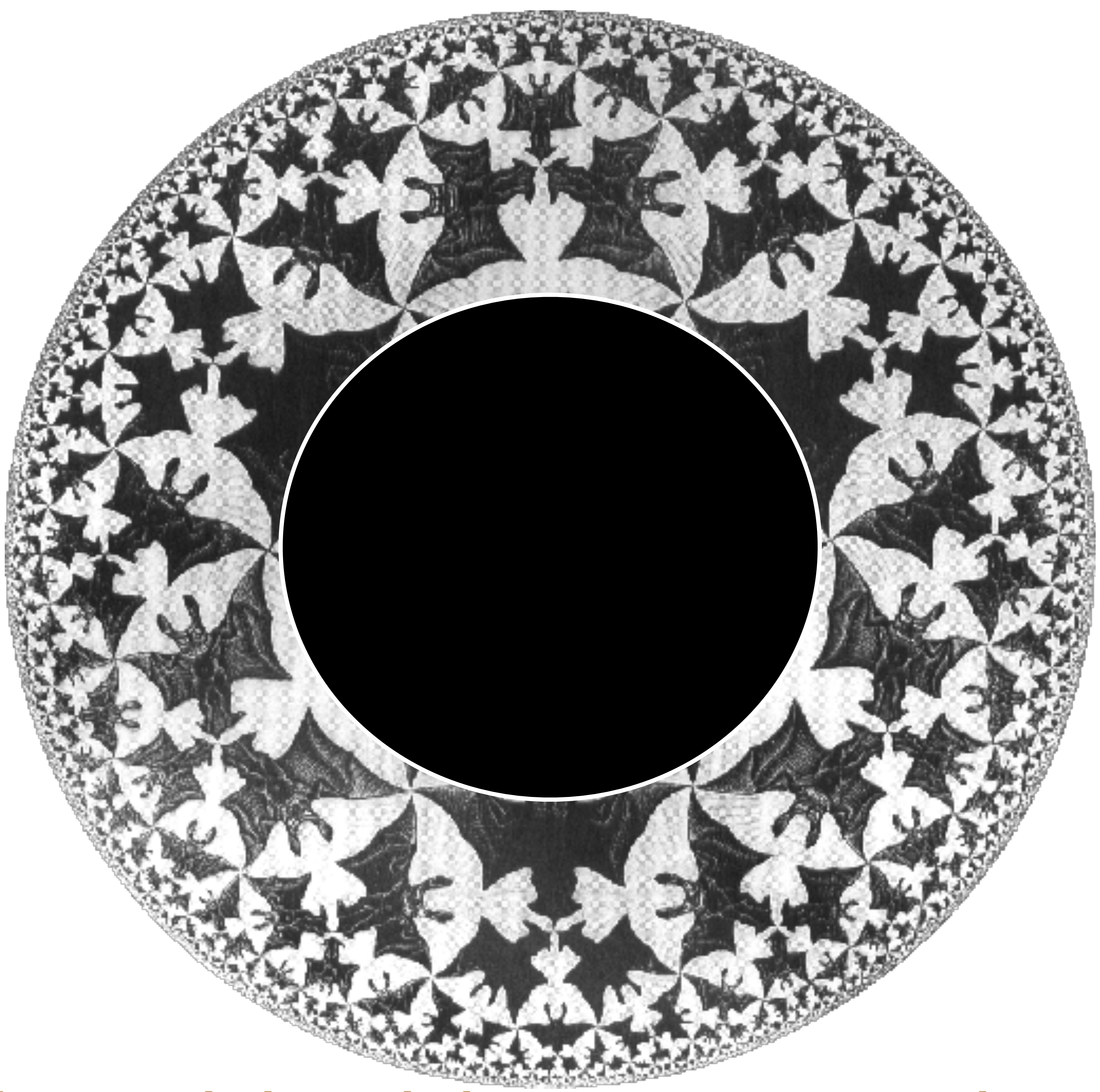
Outline

- Prehistory
- History
- 3 Examples
  - Symmetry breaking
  - Impurities (quenched disorder)
  - Qenching
- Cond-Mat applications
- Outlook

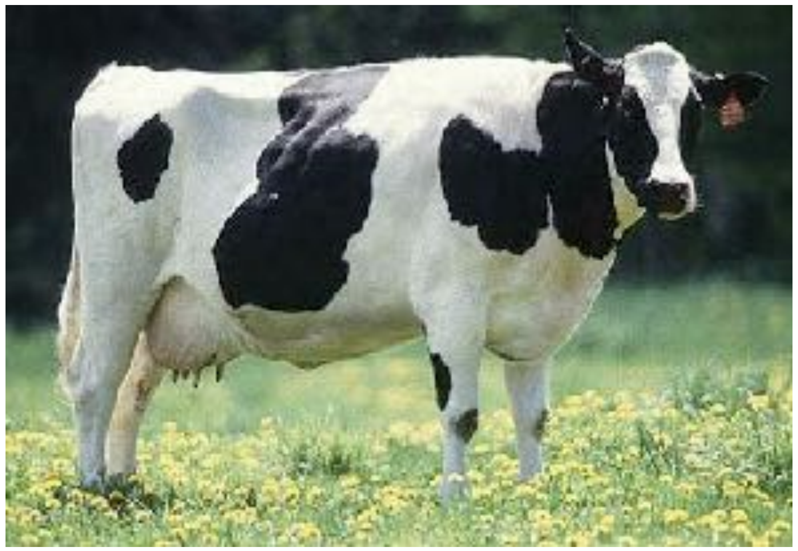
Holography:

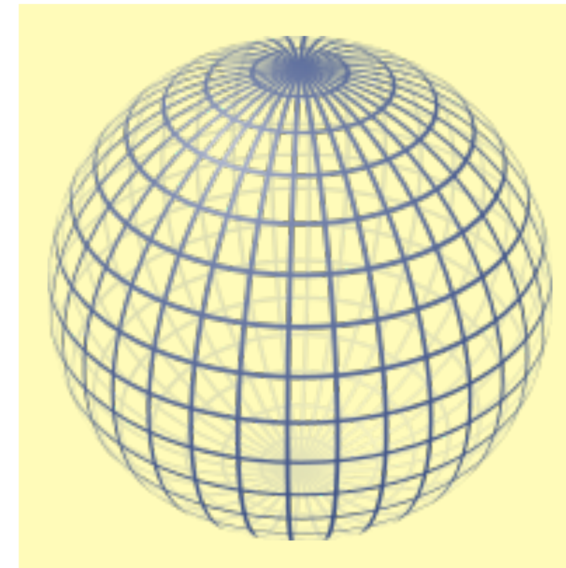
$$ds^2 = r^2(-f(r)dt^2 + d\vec{x}^2) + \frac{dr^2}{f(r)r^2}$$

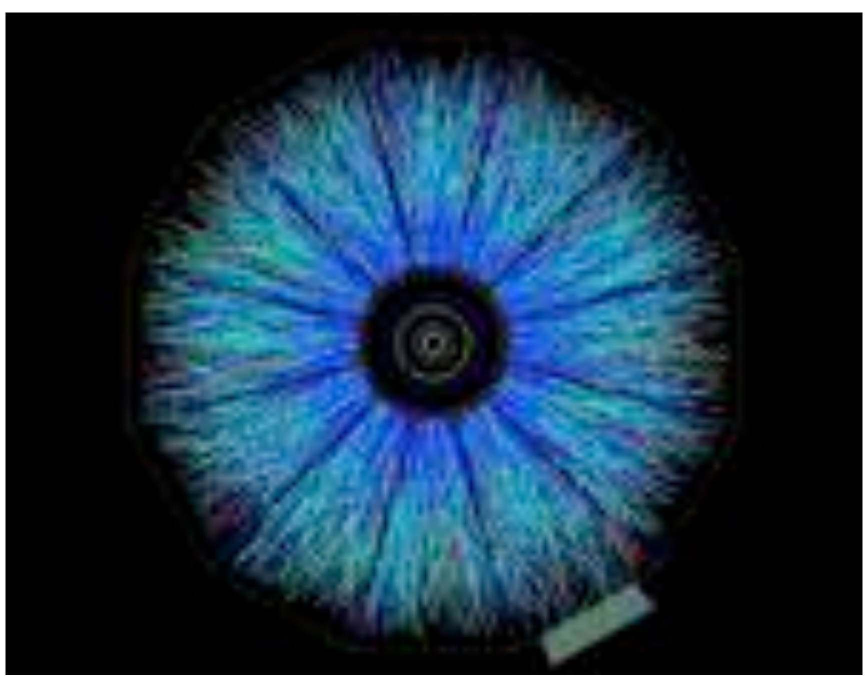


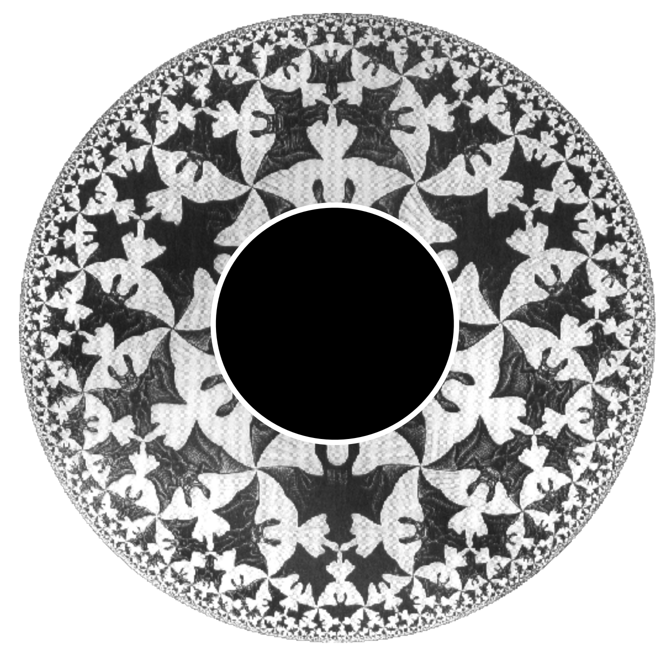
[Maldacena] [Witten] [Gubser, Klebanov, Polyakov]

AdS = spherical (hyperbolic) cow of sQGP



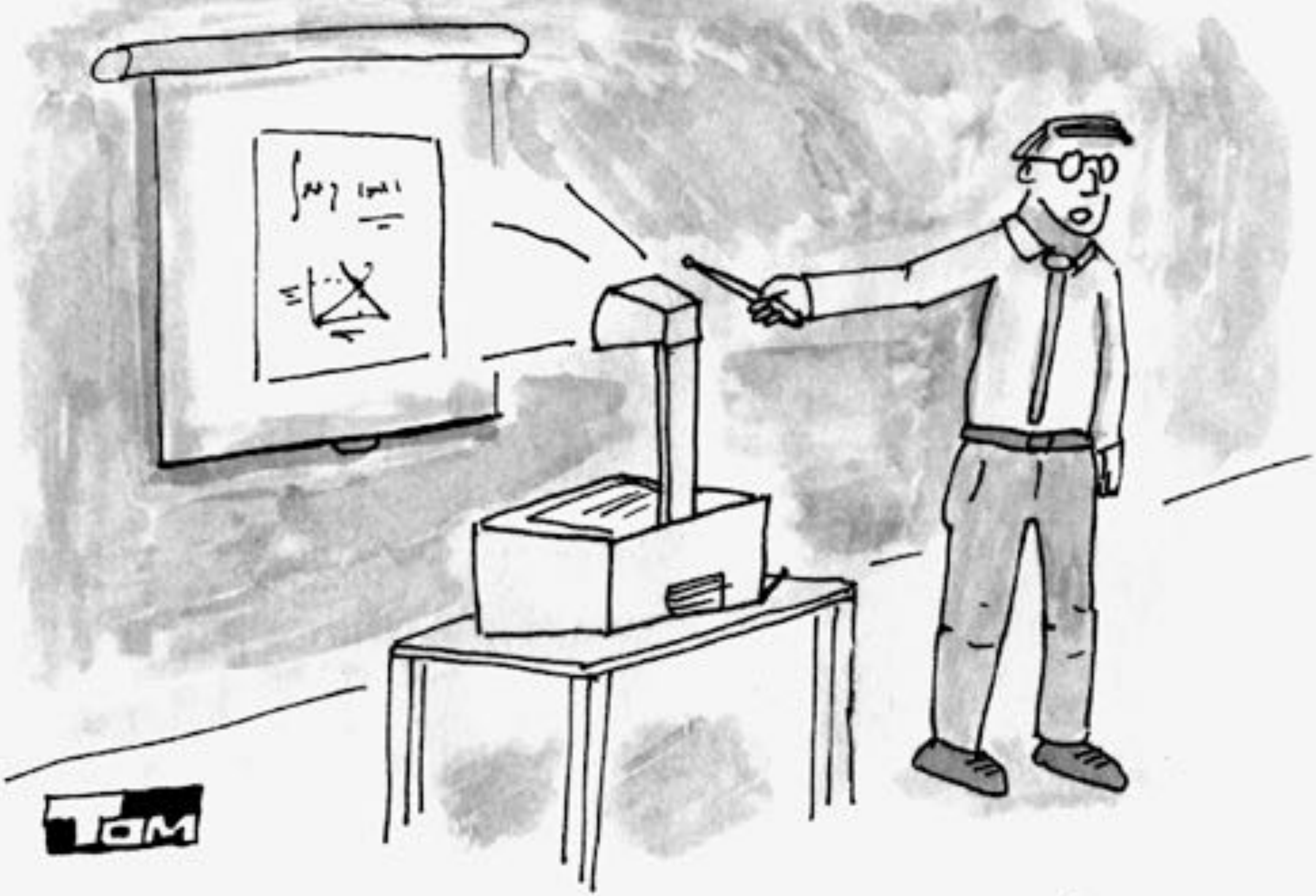






$$\frac{\eta}{s} = \frac{1}{4\pi}$$

[Policastro, Son, Starinets]
[Kovtun, Son, Starinets]



ACTUALLY, THAT ASSUMPTION ISN'T REALLY NECESSARY. WE CAN SEE HERE THAT THE POINT-COW APPROXIMATION WORKS EQUALLY WELL.

Pre-History

- [Vilenkin] 1979-80: Chiral fermions in magnetic field and rotation

$$\vec{J} = \frac{\mu}{4\pi^2}\vec{B} + \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\Omega}$$

- CME and CVE
- Free fermions in equilibrium
- No (clear) relation to anomalies
- Famous physicist: IMPOSSIBLE, currents vanish in equilibrium

  (artefact of free theory, washed out by interactions …???)
- Results were basically forgotten: 7 citation1980-2011, currently **249** !!

Pre-History

- Partial re-discoveries now and then :
  - Astro/cosmo
    - [Giovannini, Shaposhnikov] 1997
  - Cond-mat
    - [Alekseev, Cheianov, Froehlich] 1998
  - QCD matter
    - [Son, Zhitnitsky] 2004
    - [Metliski, Zhitnisky] 2005
    - [Kharzeev, Zhitnisky] 2007
  - Non of these papers cites Vilenkin, all have relation to anomaly

Pre-History

- 1980's: steep development of theory of anomalies
- [Bardeen, Zumino] 1984: Covariant vs. Consistent form of anomaly
- Wess-Zumino consistency condition $s\Gamma = \mathcal{A}. \qquad s\mathcal{A} = 0$

$$\mathcal{J} = \frac{\delta\Gamma}{\delta A}$$

- Covariant current $sJ = [c, J]$

$$\mathcal{J} = J + \mathrm{Chern\,Simons}$$

- Beautiful math, physics ?

History

- Precurser 2006: [Newman] "Anomalous Hydrodynamics"

- Sep. 2008: enters Holography

[Erdmenger, Haack, Kaminski, Yarom]

[Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surowka]

- Study hydrodynamics via fluid/gravity
- Find Chiral Vortical Effect is due to anomaly

  (Newman: CME due to anomaly)
- Strongly interacting system!

$$S_{5D} = \frac{\kappa}{3} \int A \wedge F \wedge F \qquad \vec{J} = 8\kappa\mu^2\vec{\Omega} = \frac{N_f N_c}{4\pi^2}\mu^2\vec{\Omega}$$

Fluid vorticity: $\vec{\Omega} = \frac{1}{2}\vec{\nabla} \times \vec{v}$

History

- Objections raised:
  - Violation of 2nd law?
  - Not in Landau-Lifshitz

History

- [Son, Surowka] 2009 "Hydrodynamics with triangle anomalies"
- Combine 2nd law with anomalies
- [Neimann, Oz] 2010 generalisation to non-abelian

$$J^\mu = \xi_B B^\mu + \xi_V \omega^\mu$$

$$\partial_\mu J^\mu_S \geq 0$$

$$\xi_B = C_{abc}\mu^c + \frac{n_a}{\epsilon + p}\left(\frac{1}{2}C_{abc}\mu^c\mu^d + \beta_b T^2\right)$$

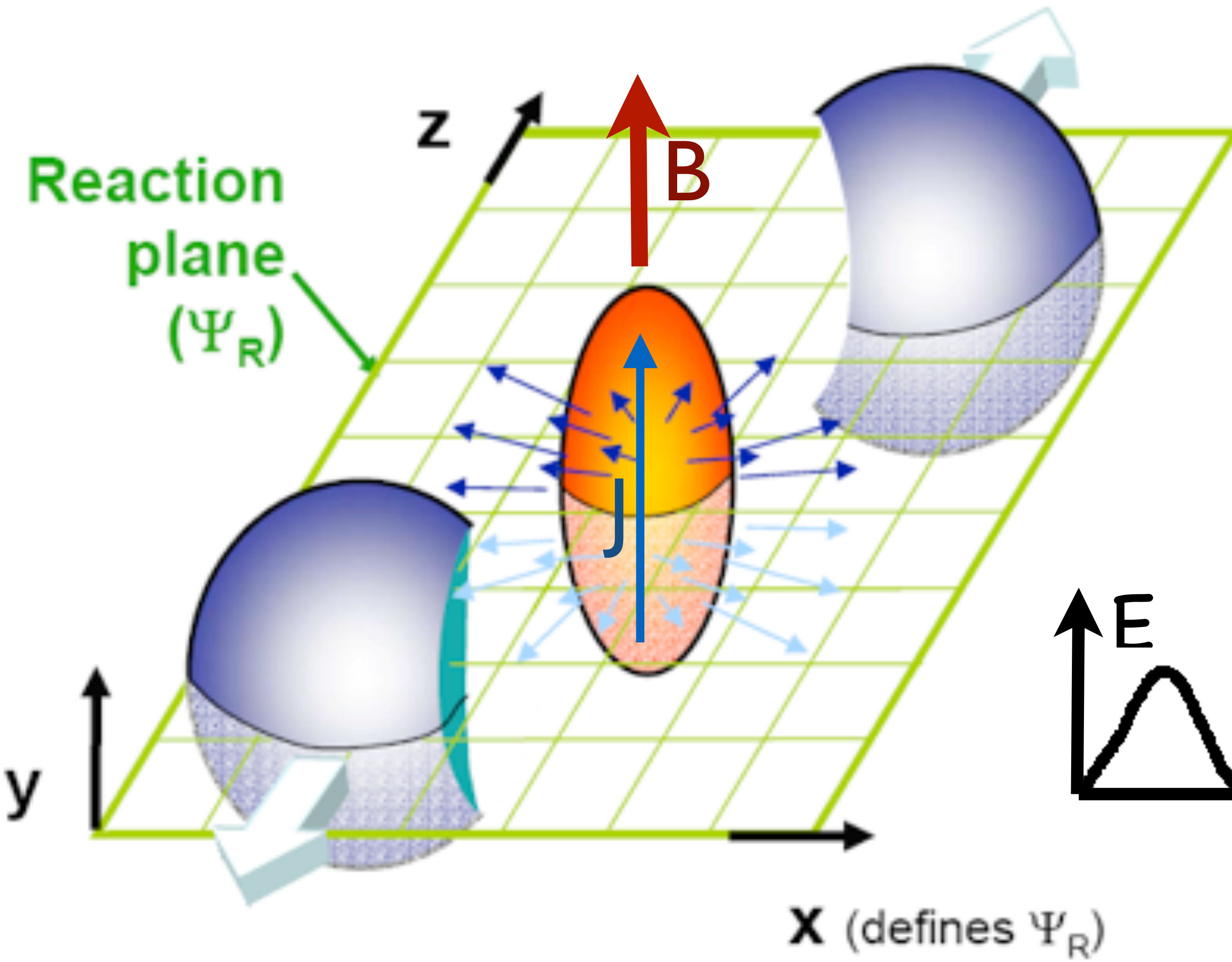
$$(D_\mu J^\mu)_a = \frac{1}{8}C_{abc}\epsilon^{\mu\nu\rho\lambda}F^b_{\mu\nu}F^c_{\rho\lambda}$$

- Landau frame
- Covariant anomaly
- Undetermined integration constant T^2

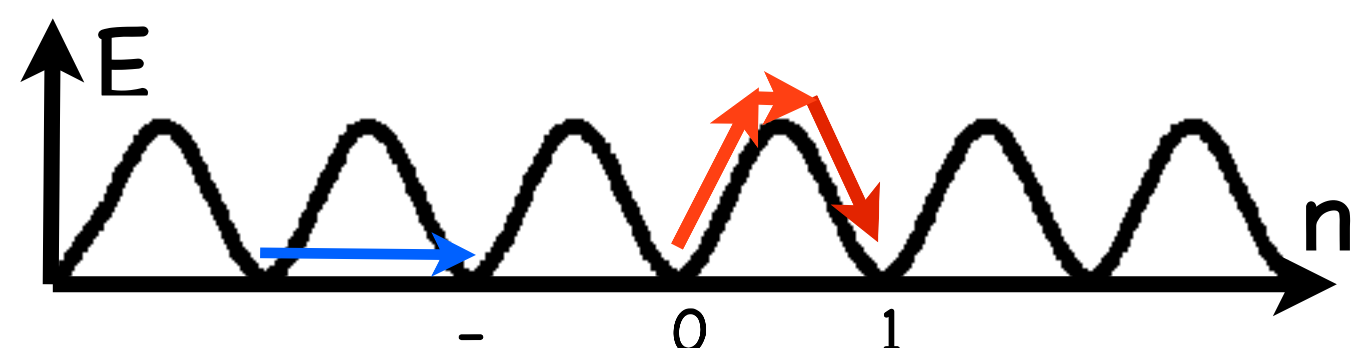
History

- Aug. 2008: "*The Chiral Magnetic Effect*" [Fukushima, Kharzeev, Warringa]

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$



- Topology of QCD fields
- CME induces charge separation
- First observable of anomalous transport



CME in Holography

- Anomalies = Chern-Simons terms

$$S = \int d^5x\sqrt{-g}\frac{1}{4}\left(F_R^2 + F_L^2\right) + \frac{\alpha}{3}\left[A_R \wedge F_R \wedge F_R - A_L \wedge F_L \wedge F_L\right]$$

- Change to V-A basis

$$S = \int \frac{\alpha}{3}\left(A \wedge F_A \wedge F_A + A \wedge F_V \wedge F_V + V \wedge F_A \wedge F_V\right)$$

- Eliminate explicit V-dependence by adding boundary term

$$S_{ct} = c_1 \int_\partial V \wedge A \wedge F_V$$

[Sakai, Sugimoto]

- Chose coefficient such that

V-A ≠ R-L

$$S_{CS} = \int \frac{\alpha}{3} A \wedge \left[F_A \wedge F_A + 3F_V \wedge F_V\right]$$

[Rebhan, Stricker, Schmitt]
[Gynther, K.L., Pena-Benitez, Rebhan]

CME in Holography

Currents

$$\mathcal{J}^\mu_V = \sqrt{-g}F^{\mu r}_V + 4\alpha\epsilon^{\mu\nu\rho\lambda}A_\nu F^V_{\rho\lambda}$$
$$\mathcal{J}^\mu_A = \sqrt{-g}F^{\mu r}_A + 4\frac{\alpha}{3}\epsilon^{\mu\nu\rho\lambda}A_\nu F^A_{\rho\lambda}$$

$$J^\mu_V = \sqrt{-g}F^{\mu r}_V$$
$$J^\mu_A = \sqrt{-g}F^{\mu r}_A$$

Anomalies

$$\partial_\mu \mathcal{J}^\mu_V = 0$$
$$\partial_\mu \mathcal{J}^\mu_A = -\frac{\alpha}{3}\epsilon^{\mu\nu\rho\lambda}\left(F^A_{\mu\nu}F^A_{\rho\lambda} + 3F^V_{\mu\nu}F^V_{\rho\lambda}\right)$$

- Variation of effective action
- Wess-Zumino consistency condition
- Not invariant under axial gauge trafo
- *"Consistent"*

$$\partial_\mu J^\mu_V = -2\alpha\,\epsilon^{\mu\nu\rho\lambda}F^A_{\mu\nu}F^V_{\rho\lambda}$$
$$\partial_\mu J^\mu_A = -\alpha\,\epsilon^{\mu\nu\rho\lambda}\left(F^A_{\mu\nu}F^A_{\rho\lambda} + F^V_{\mu\nu}F^V_{\rho\lambda}\right)$$

- Not Variation of effective action
- Invariant under axial gauge taro
- *"Covariant"*

AdS has rediscovered theory of *covariant* and *consistent* anomaly [Bardeen, Zumino]

CME in Holography

CME and CSE

- Argument of famous guy is OK
- "Bloch theorem"
- But: valid only for exactly conserved current

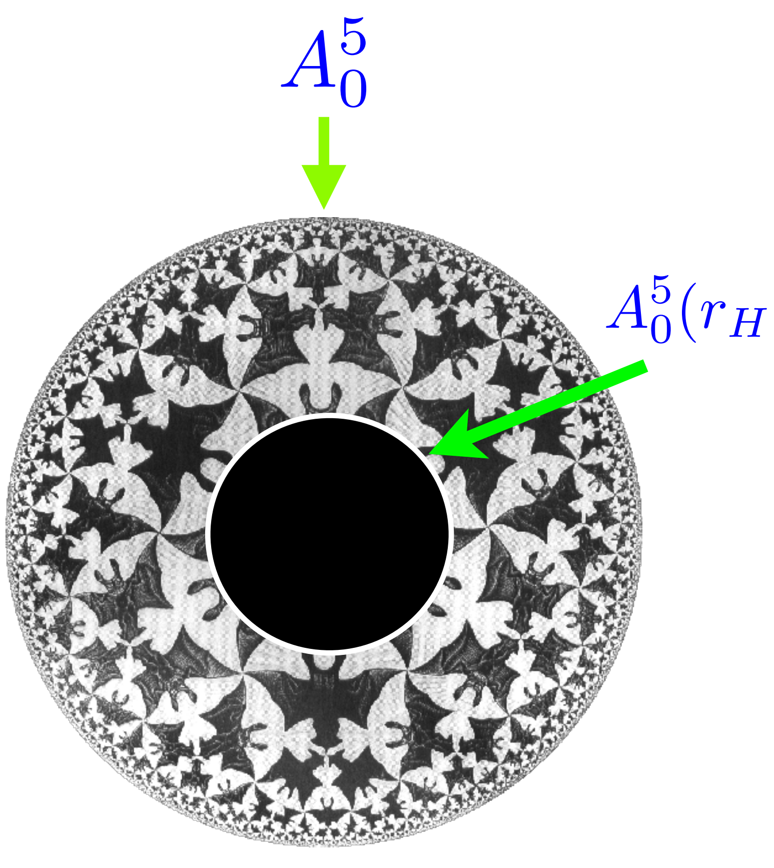
$$\delta E = \int \delta \vec{A} \cdot \langle \vec{J} \rangle$$ [Yamamoto]

$$\vec{\mathcal{J}} = \frac{\mu_5 - A_0^5}{2\pi^2} \vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu}{2\pi^2} \vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu_5 - \frac{A_0^5}{3}}{2\pi^2} \vec{B}_5$$

- Lower energy by gauge transformation



- Equilibrium: regular Euclidean section

$$H \to H - \mu_5 Q_5 - \mu Q$$

$$A_0^5 = \mu_5$$

$$V_0 = \mu$$

Gravitational anomaly

- Objection: 4-th order in derivatives, too high

$$D_\mu J^\mu = \frac{1}{768\pi^2}\epsilon^{\mu\nu\rho\lambda} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda}$$

- BUT: AdS has 5-th dimension

$$\int d^5x A \wedge \mathrm{tr}\,(R \wedge R) = \int d^5x A \wedge \mathrm{tr}\left(R^{(4)} \wedge R^{(4)} + A \wedge D(K \wedge DK)\right)$$

- Is there even in flat space
- Vanishes at the boundary
- Picks up a contribution from the horizon
- Suggest "low energy anomaly"
- even in flat space, 2 space time derivatives

$$K_{ij} \approx \frac{\partial}{\partial r} g_{ij}$$

$$D_\mu J^\mu = D(K \wedge DK)$$

- BUT: K is covariant tensor w.r.t. to 4-dim spacetime

Quantum Currents from 5D

- BUT: at the black hole horizon

$$ds^2 = r^2 f(r)(-dt^2 + 2\vec{A}d\vec{x}dt) + r^2 d\vec{x}^2 + \frac{dr^2}{r^2 f(r)}$$

$$D(K \wedge DK) \propto f'(r_h)^2 \frac{\partial \vec{A}}{\partial t}(\vec{\nabla} \times \vec{A})$$

- Following Hawking $D(K \wedge DK) \propto 64\pi^2 T^2 \vec{E}_g \cdot \vec{B}_g$
- Current $J^\mu = \sqrt{-g} F^{\mu r} - \frac{\lambda}{2\pi G} \epsilon^{\mu\nu\rho\lambda} K_\nu^\sigma D_\rho K_{\lambda\sigma}$

[C. Copetti, J. Fernandez-Pendas]

- Luttinger $\vec{E}_g = -\frac{\vec{\nabla} T}{T}$ $\vec{B}_g = \nabla \times \vec{v}$

$$\vec{J} = 64\pi^2 \lambda T^2 \vec{\omega}$$

Chiral Vortical Effect

Gravitational anomaly

Field theory perspective

- Mismatch of derivative counting
- Coefficient of CVE = grav. Anomaly coefficient [Megias, K.L., Pena-Benitez]
- Consistency with Euclidean vacuum (cone / flat space) [Jensen, Loganayagam, Yarom]
- Black hole has thermal reservoir [Stone, Kim]

- Seemingly fails for spin 3/2 gravitons (energy-momentum tensor? coupling to gravity?) [Loganayagam] [Chowdhury, David]

- Global gravitational anomaly
- Discrete phase
- Fixes CVC only up to multiples of 2 (24 chiral fermions have no global grav. Anomaly)
- Large N limit: theories with global gravitational anomalies and without differ by O(1) d.o.f. Sugra limit of AdS/CFT?

[Golkar, Sethi], [Chowdhury, David] [Glorioso, Liu, Rajagopal]

Summary

Anomalies

$$S = \int d^5x \epsilon^{MNPQR} A_M \left(\frac{\kappa}{3} F_{NP} F_{QR} + \lambda R^A{}_{BNP} R^B{}_{AQR} \right)$$

Imply CVE and CVC currents

$$J^i = 8\kappa\mu B^i + \left(8\kappa\mu^2 + 64\pi^2\lambda T^2\right)\Omega^i$$

$$T^{0i} = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) B^i + \left(\frac{8}{3}\kappa\mu^3 + 64\pi^2\lambda\mu T^2\right)\Omega^i$$

3 Examples

I. Breaking the axial symmetry (Mass)

II. Breaking translation symmetry (Impurities)

III. Quenching anomalous transport

Symmetry breaking - I

CME and CSE

$$\vec{\mathcal{J}} = \frac{\mu_5 - A_0^5}{2\pi^2}\vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu}{2\pi^2}\vec{B}$$

$$\vec{\mathcal{J}}_5 = \frac{\mu_5 - \frac{A_0^5}{3}}{2\pi^2}\vec{B}_5$$

- Holographic mass term
- Breaks axial symmetry at tree level
- Soft: only in IR
- Counterterms not affected
- Fate of CME et al.?

- In Holography charged scalar

$$S = \int d^5x\sqrt{-g}\left(|D_A\phi|^2 + m^2|\phi|^2\right) \quad , \quad \phi \approx \frac{M}{r} + \cdots$$

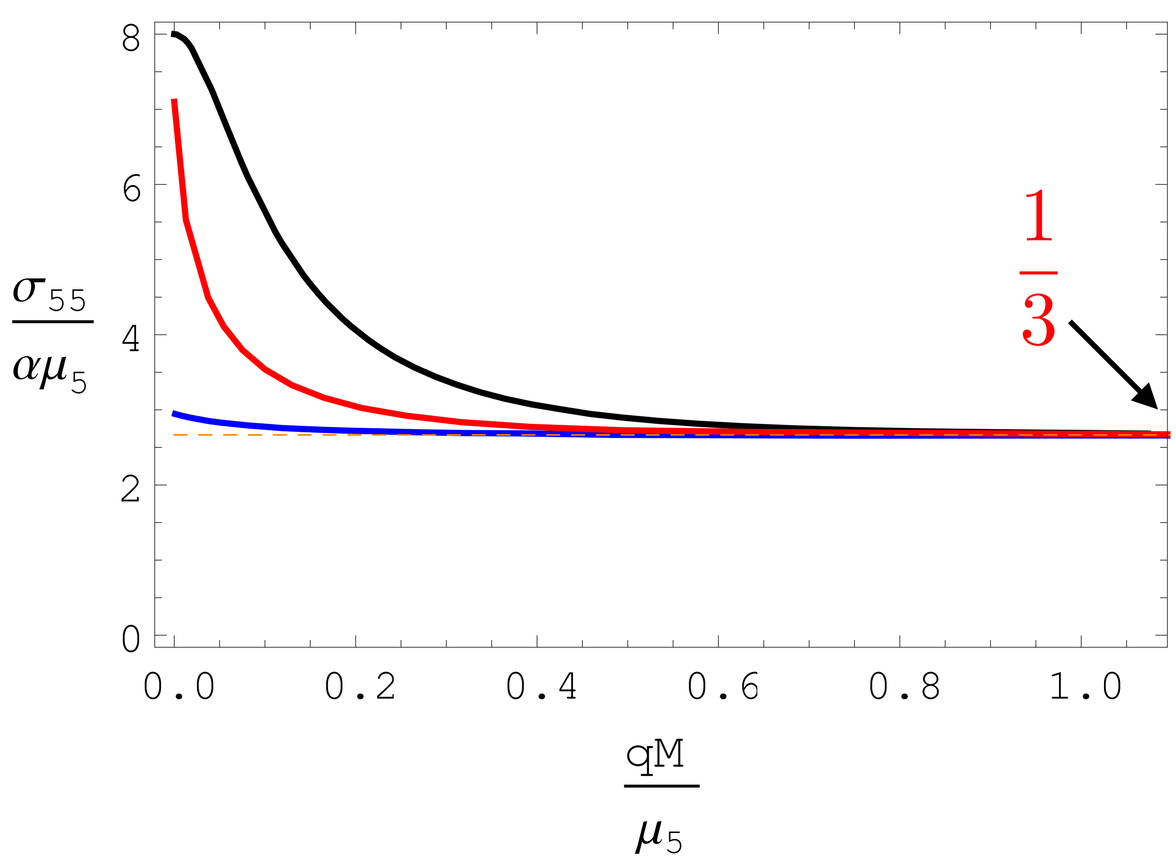
- Intuition: no chiral transport for large mass (strong breaking)
- BUT: *covariant* or *consistent* current ?

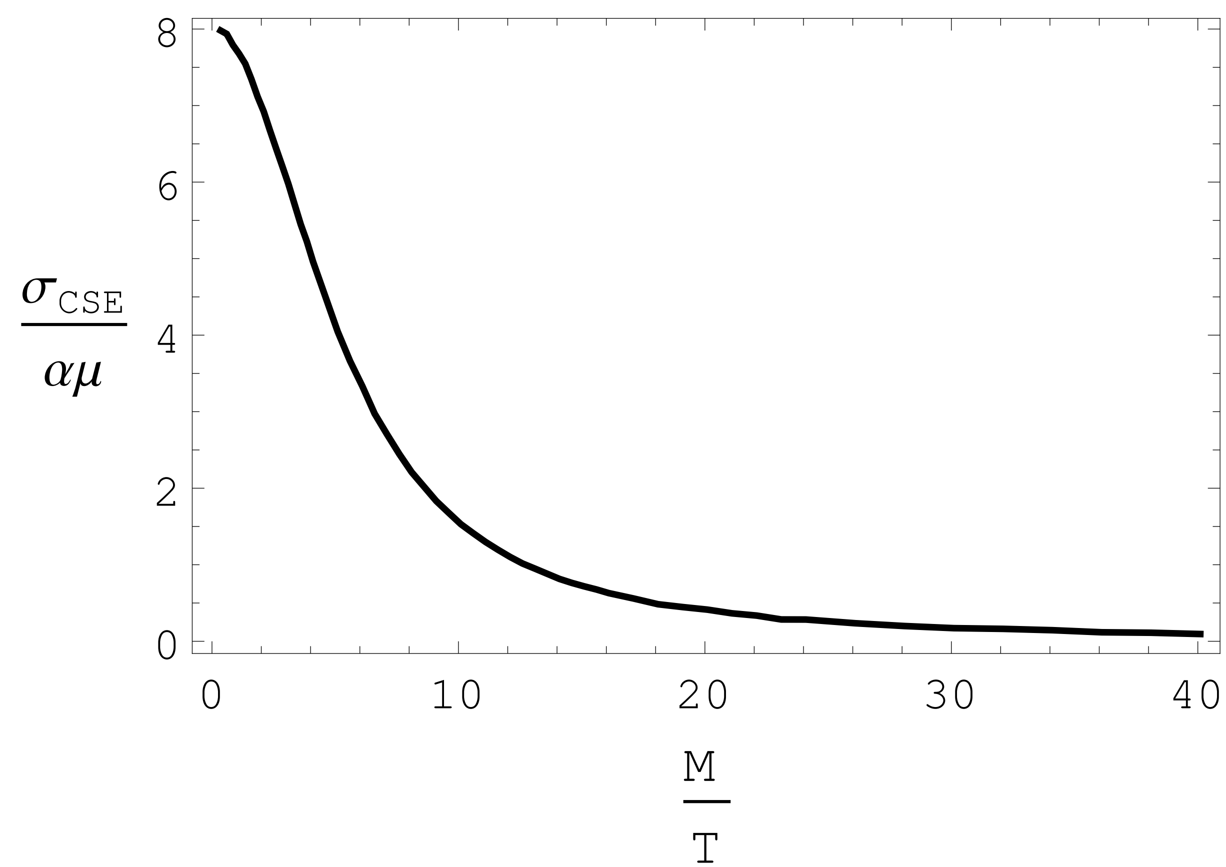
Symmetry breaking - I

CME and CSE

CME =0 all the way (Bloch!)

[Jimenez-Alba, K.L., Liu, Sun]





Consistent currents vanish !

Same for spontaneous breaking [Amado, Lisker, Yarom]

Dynamical gauge fields [Domingo, Gursoy]

Symmetry breaking - II

- In Holography "linear axion" background (massless scalar)

$$S = \int d^5x \sqrt{-g} \left(|\partial\phi|^2 \right) \quad , \quad \phi \approx kx + \cdots$$

- Background breaks translations eoms are homogeneous
- Graviton has mass

$$T^{0i} = T^{i0}$$

- Charge (Momentum) = Current (Energy-current)

1. Intuition: Momentum density, broken symmetry
2. Intuition: Energy current is dissipationless

Symmetry breaking - II

- Can extrinsic curvature term be seen in UV?

$$ds^2 = -fr^2dt^2 + \frac{dr^2}{r^2 f} + r^2 d\vec{x}^2 \qquad f = 1 + \frac{k^2}{r^2} - (1 - \frac{k^2}{4})\frac{r_H^4}{r^4}$$

Unusual power!

- Extrinsic curvature as additional variable

$$\delta S_{on-shell} = \int_\partial \sqrt{-g}(t^{\mu\nu}\delta g_{\mu\nu} + u^{\mu\nu}\delta K_{\mu\nu})$$

- Energy momentum tensor (Ward identity)

$$\Theta^{\mu\nu} = t^{\mu\nu} + u^{\mu\lambda}K_\lambda^\nu$$

- New term is due to gravitational Chern-Simons term

Symmetry breaking - II

- CME and CVE without new term

$$J^i = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) 2\Omega^i$$
$$T^{0i} = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2 - 4\lambda k^2\right) B^i$$

- Impossible in unitary theory
- CVE = 2 CME for energy current by Kubo formulas

- Including the new term

$$T^{0i} = \left(4\kappa\mu^2 + 32\pi^2\lambda T^2\right) B^i$$

- All is well!
- Energy current being dissipation wins!

[Copetti, Fernandez-Pendas, K.L., Megias]

Quenching the CME

- CME and CVE depend on equilibrium quantities T, μ

- Natural question: anomaly induced transport **far** from equilibrium physics

- Possible importance for Heavy Ion Collisions (magnetic field has already decayed in hydrodynamic regime)

- Holography allows both: study fast time evolution, quenches and anomalous transport

- Study CME via gravitational Chern-Simons term. (T)

What to look for

- "Minimal" setup: inject energy
- Equilibrium: energy — temperature $T_0 \longrightarrow T$
- CME in energy-momentum tensor
- First near equilibrium = hydro

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu + p\eta_{\mu\nu} + \hat{\sigma}_B(u_\mu B_\nu + u_\nu B_\mu)\,,$$

$$J_\mu = \rho u_\mu + \sigma_B B_\mu\,,$$

$$J_\mu^X = \rho_X u_\mu + \sigma_{B,X} B_\mu$$

- Landau frame

$$\hat{\sigma}_B = 0$$

$$\sigma_B = 24\alpha\mu - \frac{\rho}{\epsilon + p}\left(12\alpha\mu^2 + 32\lambda\pi^2 T^2\right)$$

$$\sigma_{B,X} = -\frac{\rho_X}{\epsilon + p}\left(12\alpha\mu^2 + 32\lambda\pi^2 T^2\right)$$

What to look for

- Energy current = Momentum density

$$T_{0i} = T_{i0}$$

- Momentum density = conserved charge

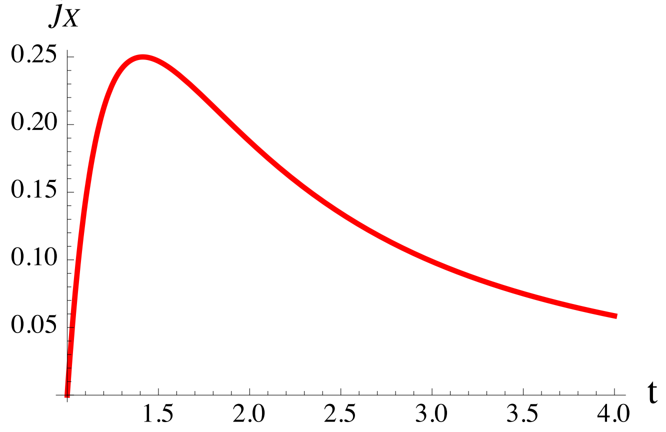
$$32\lambda\pi^2 T_0^2 \vec{B} = (\epsilon + p)\vec{v}$$

- Monitor response in tracer U(1) current

$$\vec{J}_X = 32\frac{\rho_X}{\epsilon + p}(T_0^2 - T^2)\pi^2\lambda\vec{B}$$

- Removing constants: benchmark near equilibrium curve

$$j_X = \frac{T^2/T_0^2 - 1}{T^4/T_0^4}$$



Holographic quench

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(\mathcal{R} - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{4q^2}F_X^2 - \frac{1}{2}(\partial\phi)^2 + \lambda\epsilon^{MNOPQ}A_M R^A_{BNO}R^B_{APQ}\right)$$

- Tracer U(1) in decoupling limit
- Holographic quench $\phi_0(t,\vec{x}) = \frac{1}{2}\eta\left(1+\tanh\frac{t}{\tau}\right)$

$$ds^2 = \frac{1}{z^2}\left(-f(t,z)e^{-2\delta(t,z)}dt^2 + \frac{dz^2}{f(t,z)} + d\vec{x}^2\right)$$

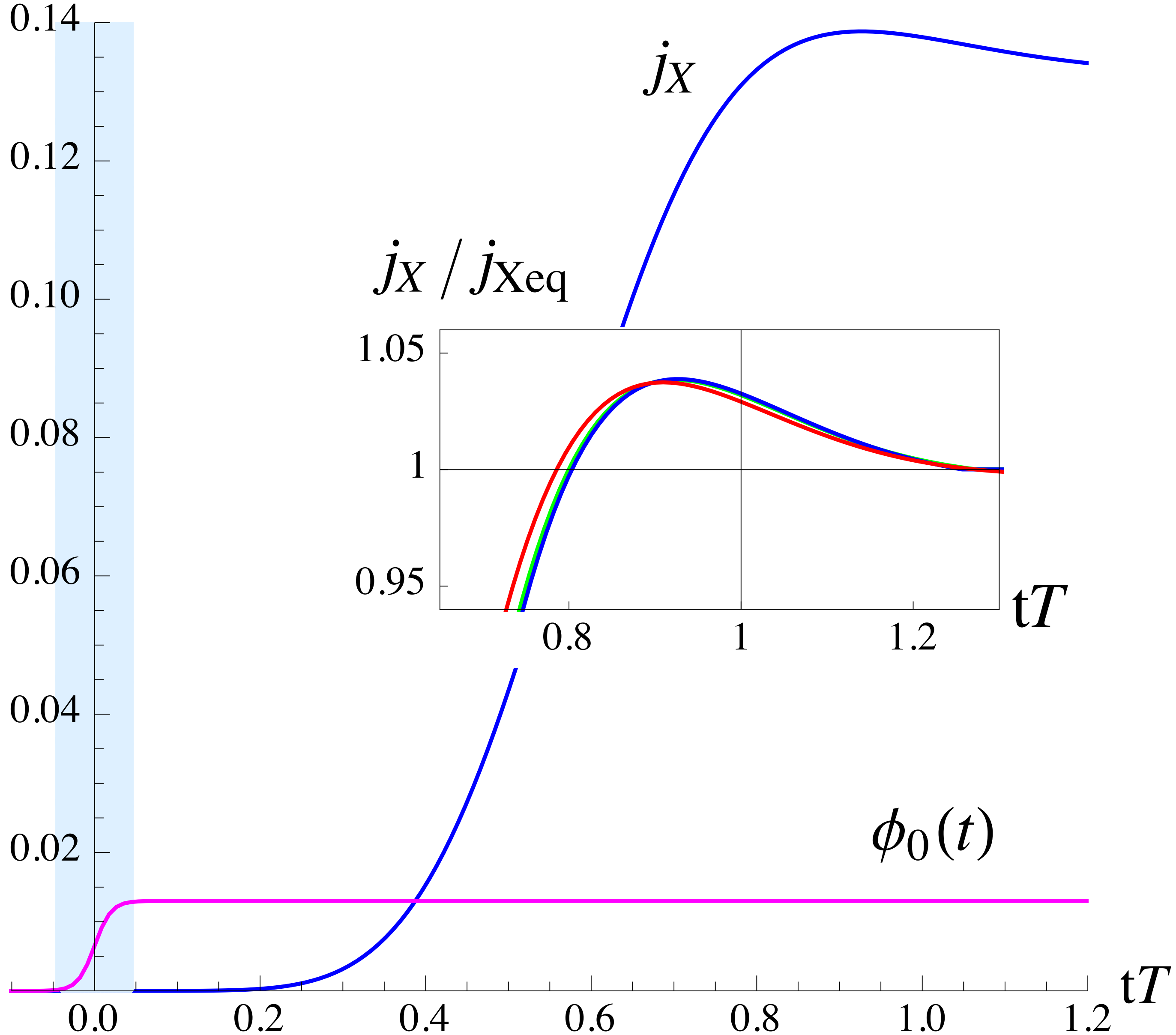
$$F_{xy} = B \qquad\qquad F_{X,0z} = \rho_X\, z\, e^{-\delta(t,z)}$$

- To linear order in B

$$J^\mu_X = \lim_{z\to 0}\sqrt{-g}\,F_X^{\mu z}$$

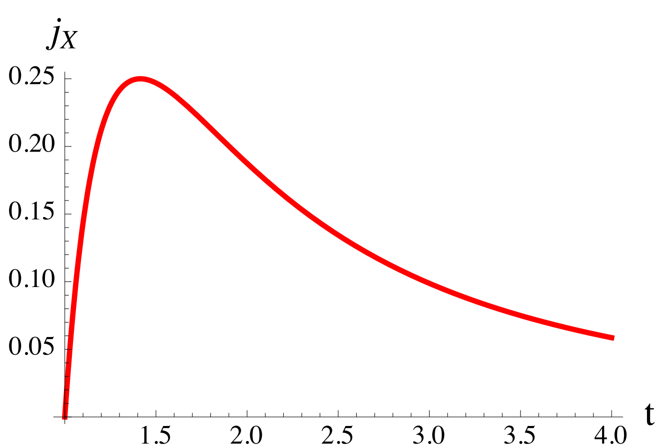
Holographic quench

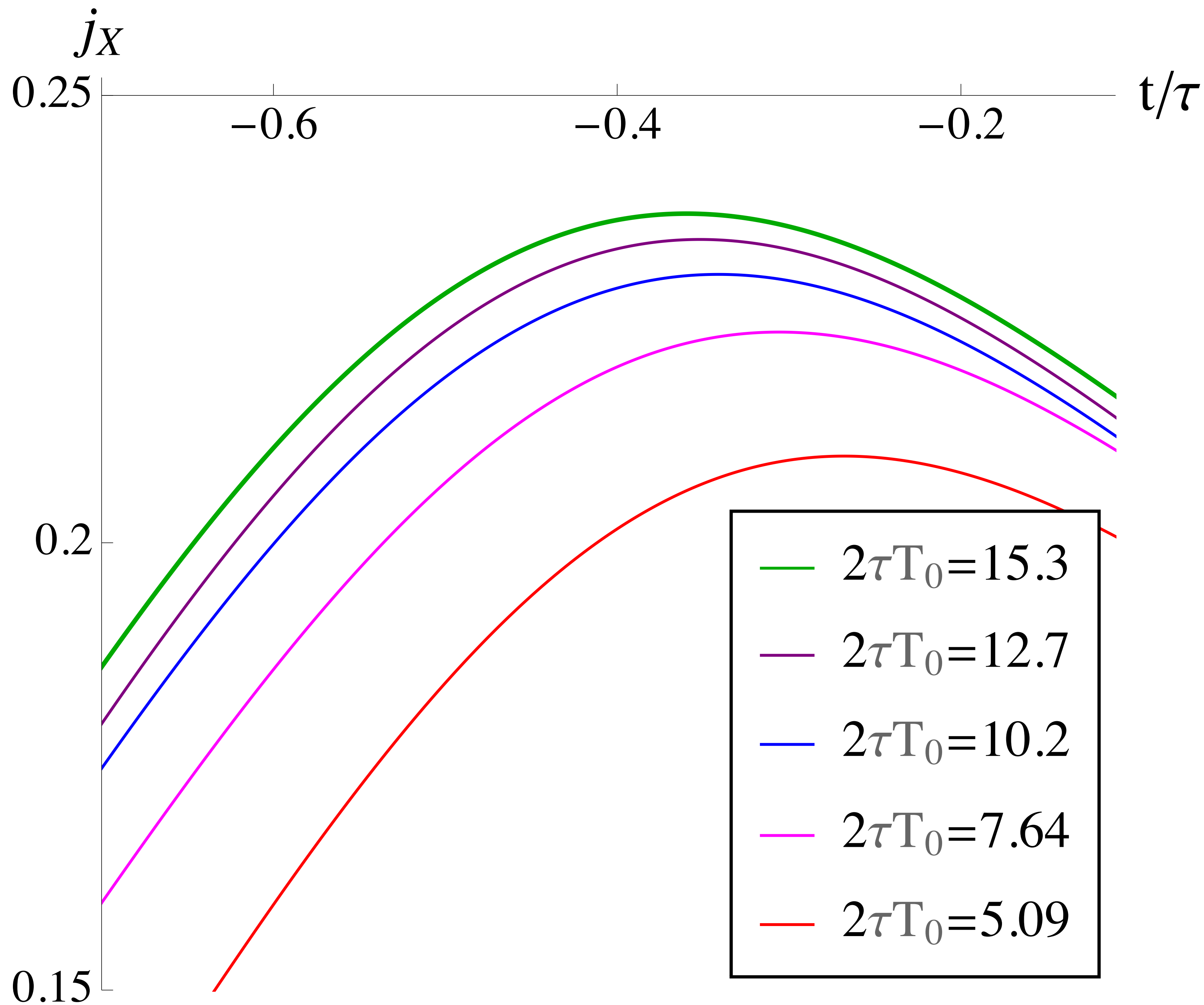
- Fast quenches $2\tau T < 1$



Holographic quench

- Very slow quenches

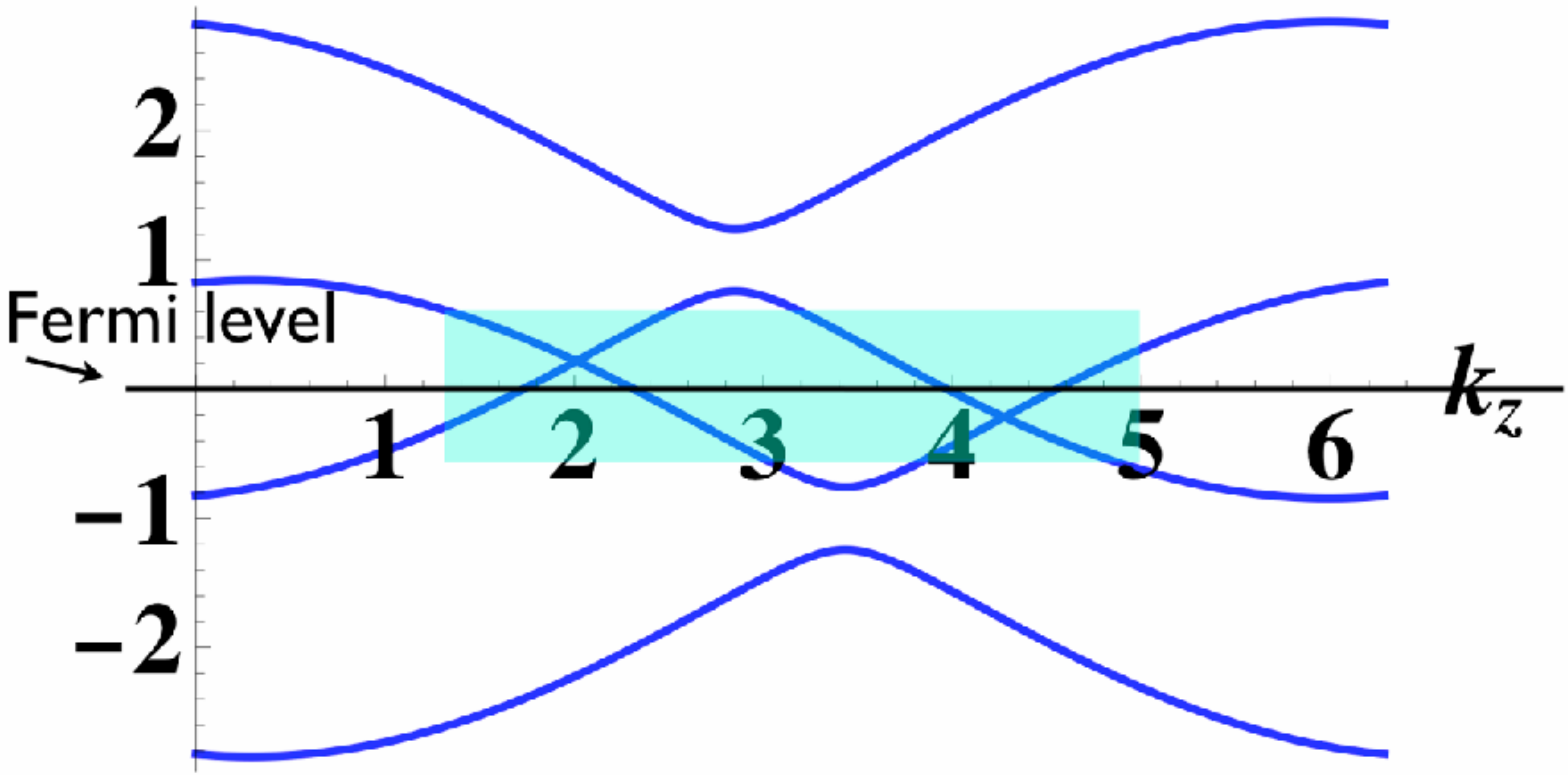




Cond-Mat applications of anomalous transport theory

WSM

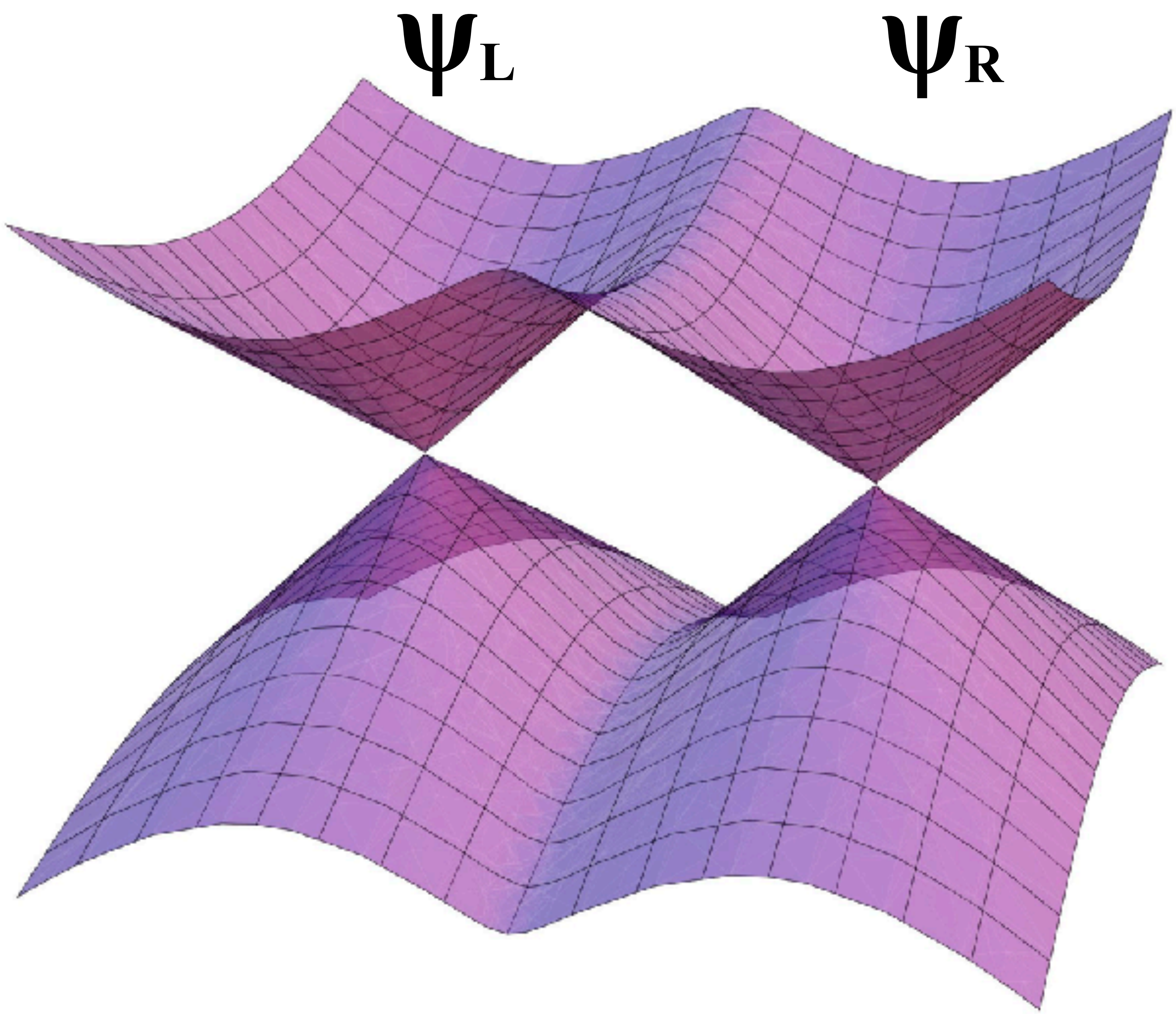
Band structure of WSM



$$\gamma^{\mu}\left(iD_{\mu}+\gamma_5 A^5_{\mu}\right)\Psi=0$$

Weyl semi-metal

Linear band touching



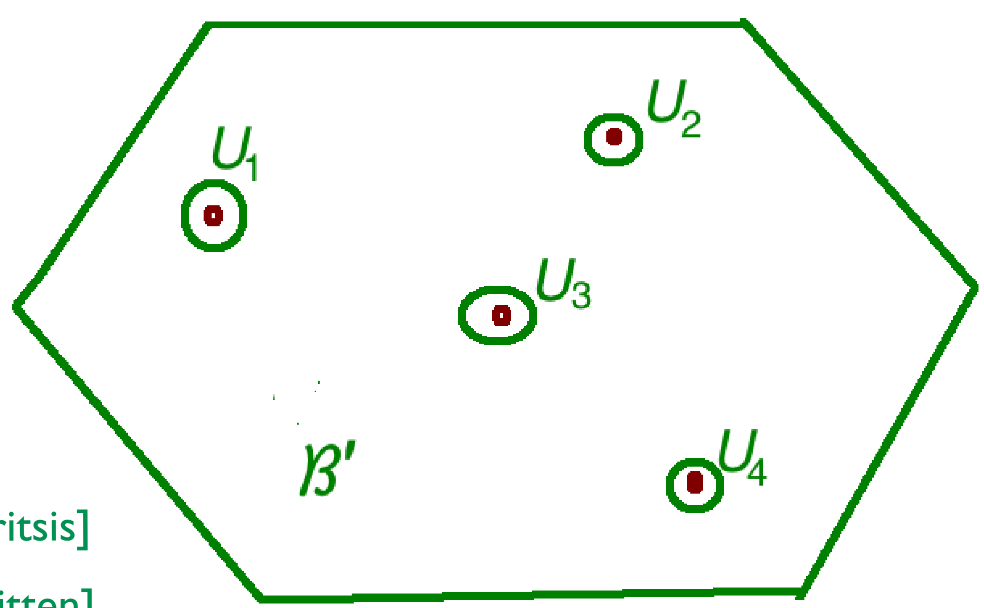
Weyl semi-metal

Topological constraint (Nielsen-Ninimiya)

Berry connection $$\mathcal{A} = \langle \psi(k) | \frac{\partial}{\partial k_i} | \psi(k) \rangle dk_i$$

$$\mathcal{F}_B = d\mathcal{A}$$

$$d\mathcal{F}_B = 0$$



[Kiritsis]

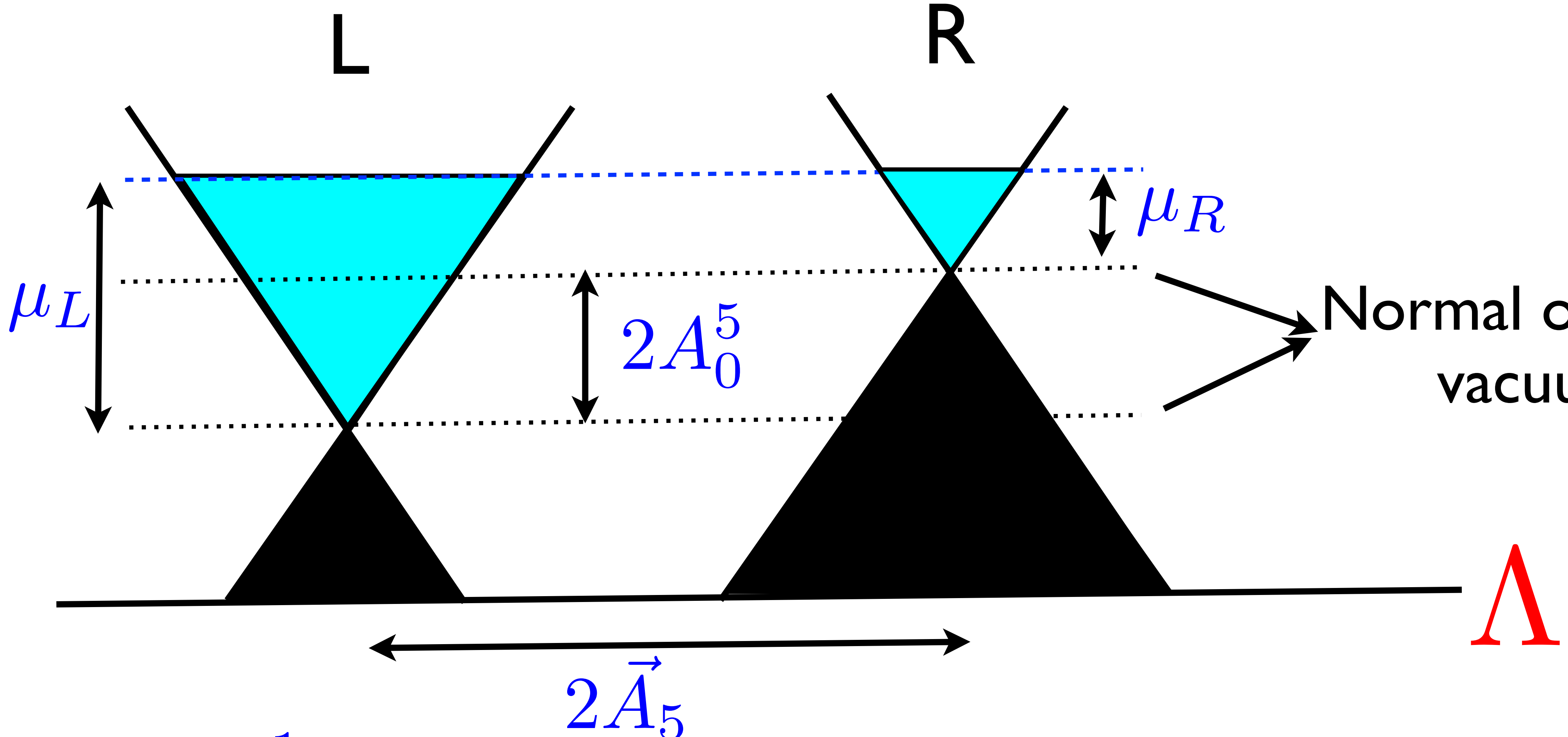
[Witten]

BZ has no boundary !
(Torus)

$$\int \frac{d\mathcal{F}_B}{2\pi} = \sum_i \oint_{U_i} \frac{\mathcal{F}_B}{2\pi} = 0$$

(anti-)chiral fermion = (anti-)monopole

CME in WSMs

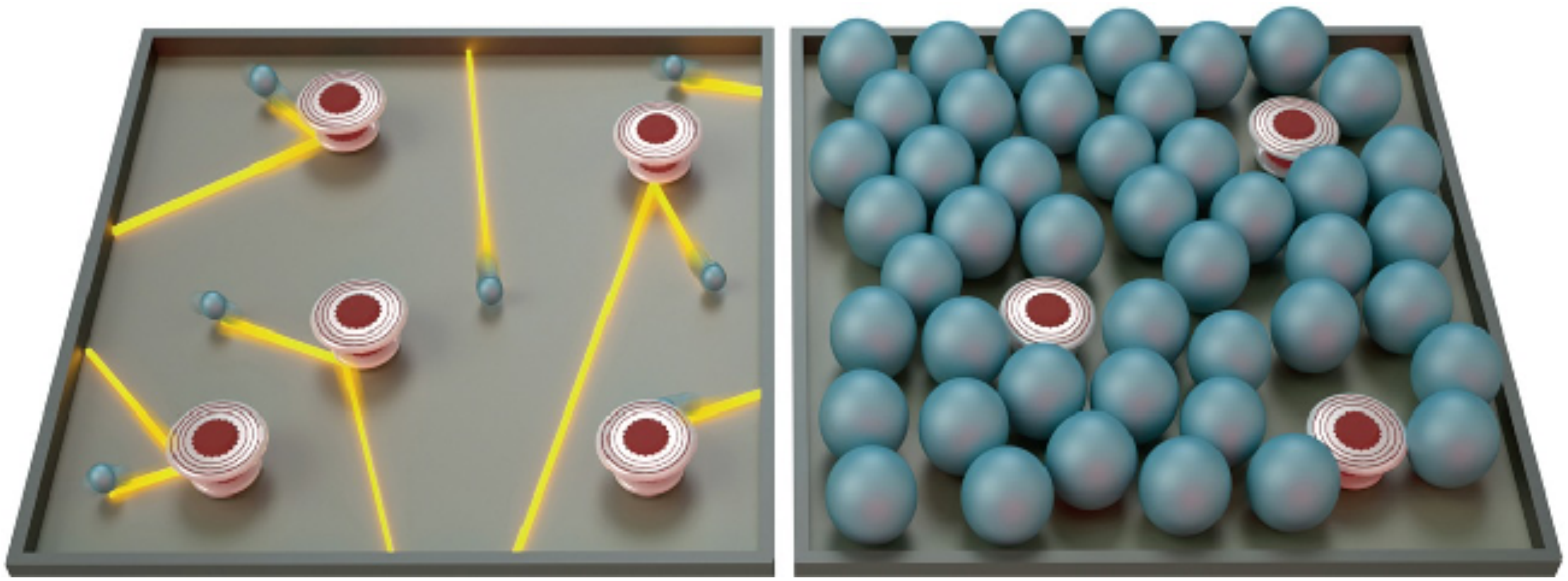


$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R) \qquad \mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $$\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0$$

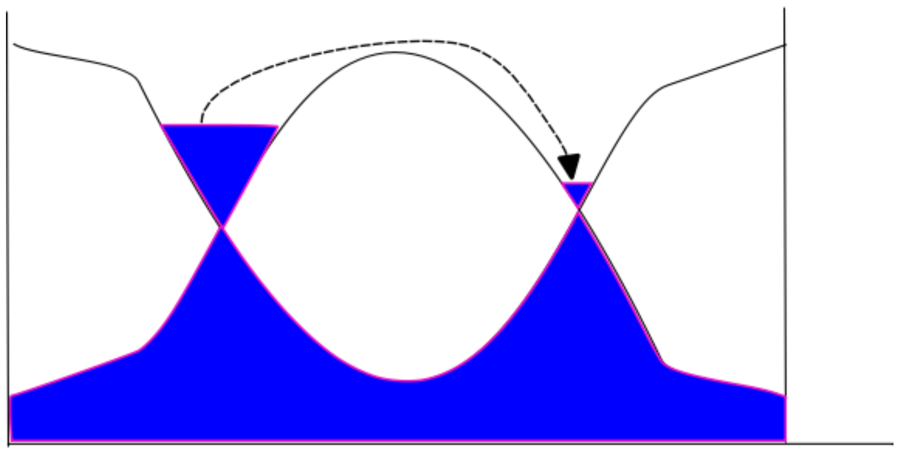
AHE: $$\vec{J}_{CS} = \frac{1}{2\pi^2}\vec{A}_5 \times \vec{E}$$

NMR and NTMR in WSM



[J. Zaanen, "Electrons go with the flow in exotic materials", *Science* Vol. 351, 6277]

If WSM is not strongly coupled, hierarchy of scattering times



$$\tau_{\text{inner}} < \tau_{\text{inter}} < \tau_{ee}$$

Kills $\vec{P}$ — Kills ρ_5, ϵ_5 — Is irrelevant

NMR and NTMR in WSM

NMR = Negative Magnetoresistivity

In equilibrium CME vanishes,
Induce non-equilibrium steady state

$$\dot{\rho}_5 = \frac{1}{2\pi^2}(\vec{E} - \vec{\nabla}\mu)\vec{B} - \frac{1}{\tau_5}\rho_5$$

$$\rho_5 = \chi_5 \mu_5 \qquad \vec{J} = \sigma \vec{E} + \frac{\mu_5}{2\pi^2}\vec{B}$$

$$\vec{J} = \left(\sigma + \frac{\tau_5 B^2}{(2\pi^2)^2 \chi_5}\right)(\vec{E} - \vec{\nabla}\mu)$$

NTMR via CME

Coupled charge and energy transport of chiral currents

$$\vec{J}_\epsilon = \left(\frac{a_\chi}{2}\mu^2 + a_g T^2\right)\vec{B}$$

$$\vec{J} = a_\chi \mu \vec{B}$$

$\Rightarrow$

$$\vec{J} = G_E(\vec{E} - \vec{\nabla}\mu) + G_T \vec{\nabla} T$$

$$G_E = \tau_5 \frac{a_\chi^2}{\det(\Xi)}\left(\frac{\partial \epsilon}{\partial T} - \mu \frac{\partial \rho}{\partial T}\right) B^2$$

$$G_T = \tau_5 \frac{2 a_g a_\chi}{\det(\Xi)} \frac{\partial \rho}{\partial T} B^2$$

Large B (ultraquantum limit): $\rho = \frac{|B|}{4\pi^2}\mu$

- G_E linear in B
- G_T vanishes

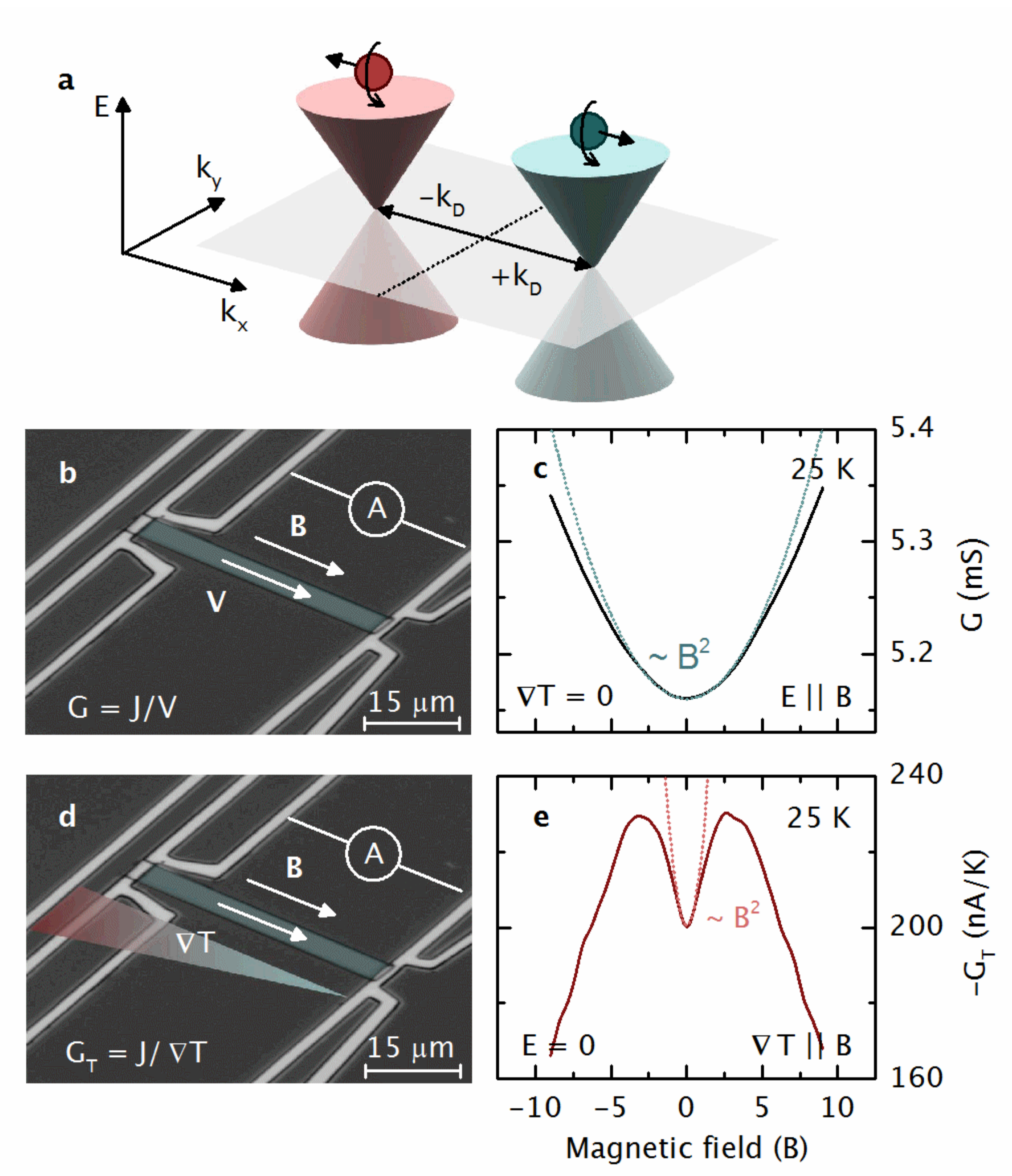
[Spivak, Andreev], [Lundgren, Laurell, Fiete] *kinetic theory*
[Lucas, Davison, Sachdev] *chiral fluids*

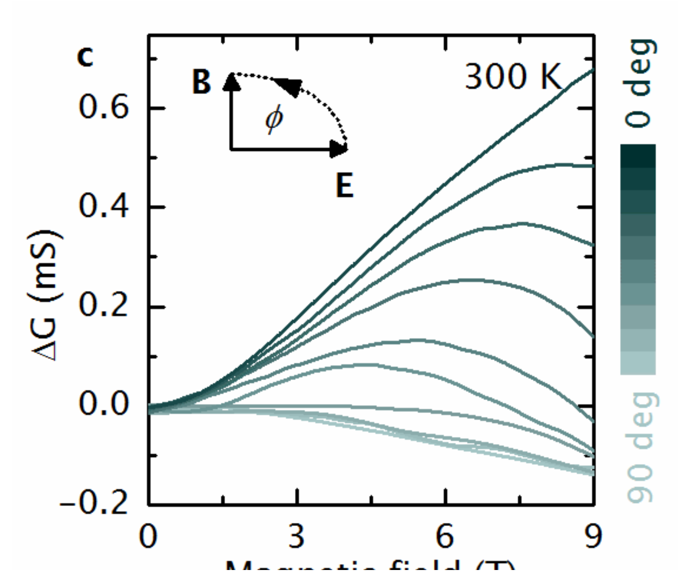
NMR and NTMR in NbP

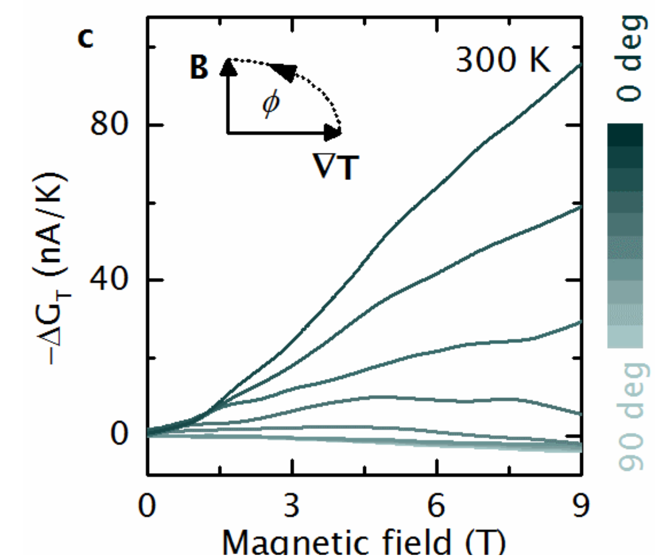
Experimental signatures of the mixed axial-gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth, Anna Corinna Niemann, Tobias Meng, Adolfo G. Grushin, Karl Landsteiner, Bernd Gotsmann, Fabian Menges, Marcus Schmidt, Chandra Shekhar, Vicky Sueß, Ruben Huehne, Bernd Rellinghaus, Claudia Felser, Binghai Yan, Kornelius Nielsch

arXiv:1703.10682 (Nature)



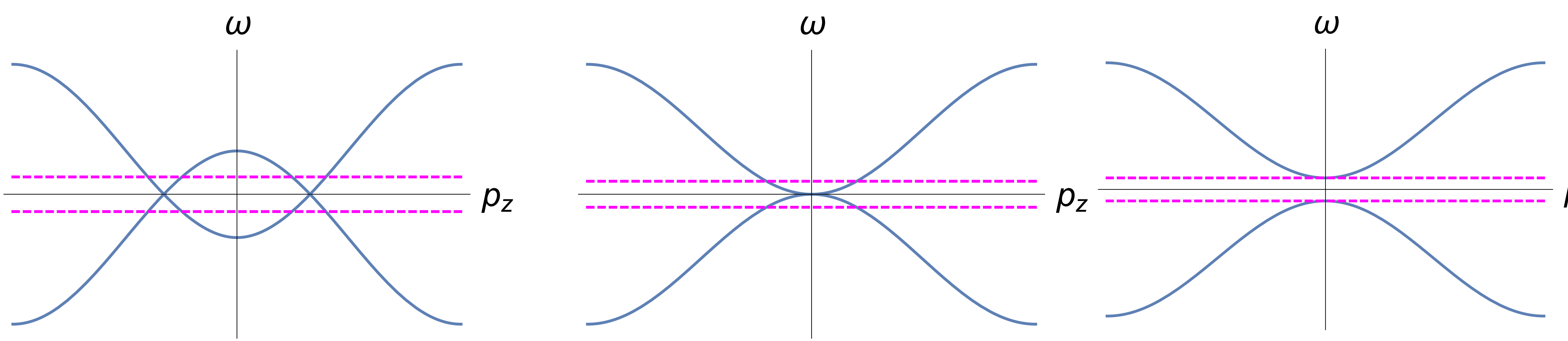




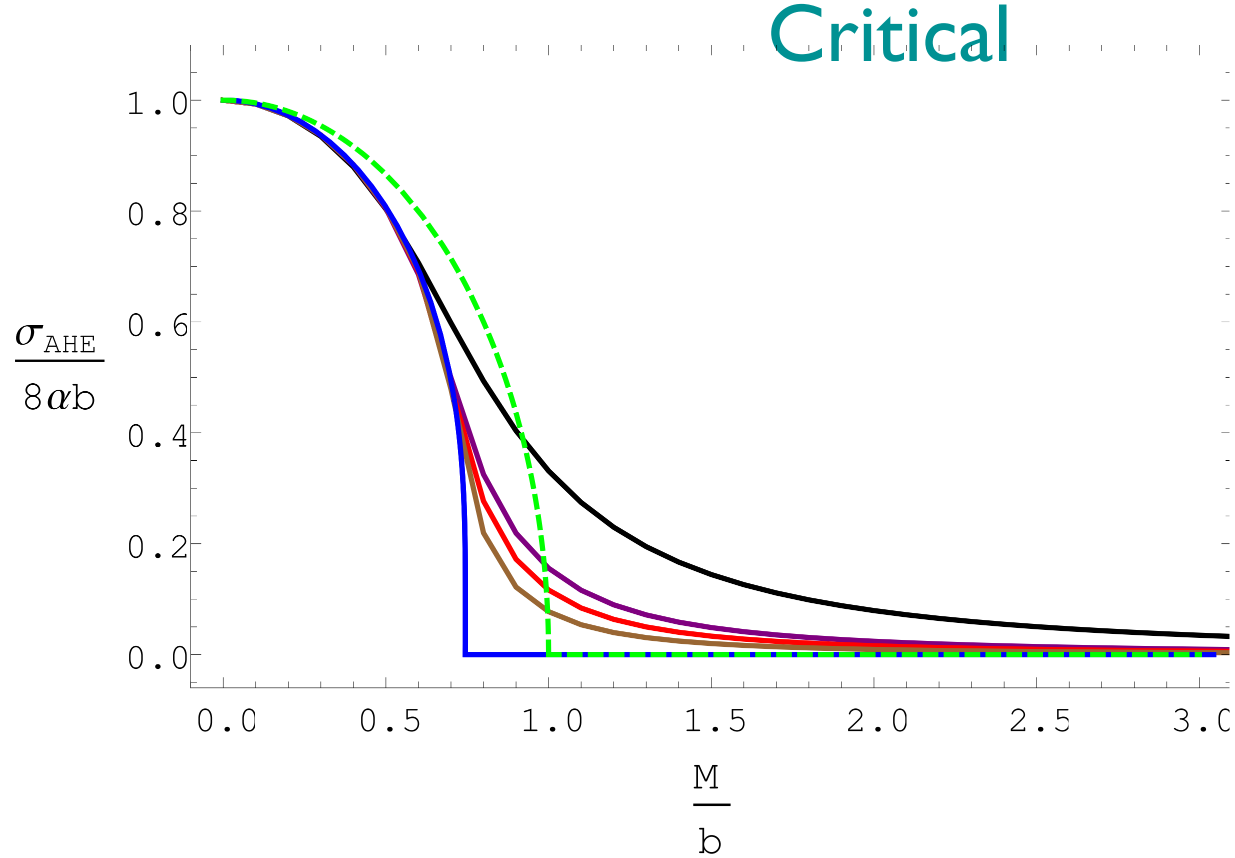
- Angle dependence
- NMR and NTMR show B^2 at small B
- NMR ~ linear for large B field
- NTMR vanishes for large B field

A prediction from Holography for WSMs

- Holographic model for quantum phase transition [K.L., Liu, Sun]



WSM Critical Trivial



$$\sigma_{xy} = 8\alpha A_z^5(0)$$

Odd viscosity

- Hall viscosity in 2D Quantum Hall states [Avron, Seiler, Zograf]
- Time reversal breaking necessary
- 2D : invariant ε tensor
- 3D: need some anisotropy

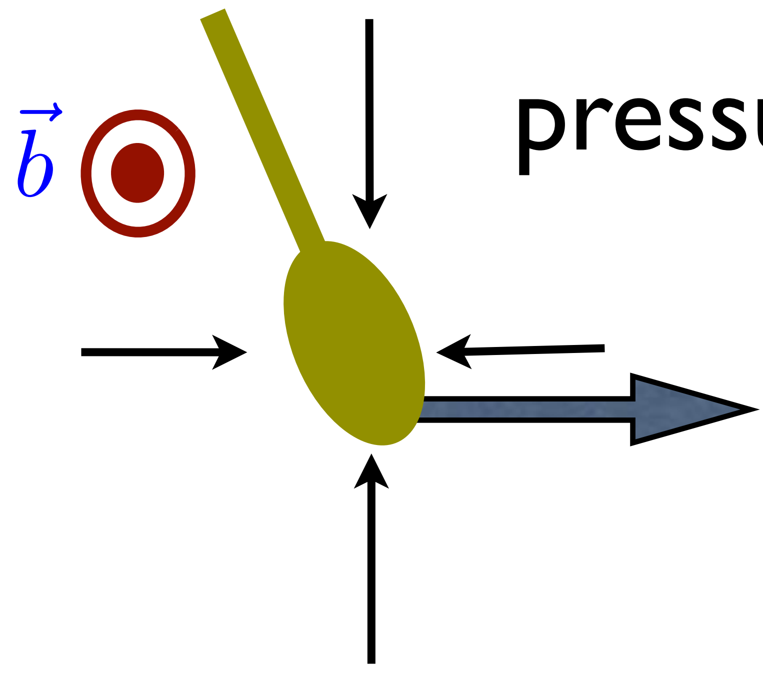
$$\tau_{xy} = \eta_{\perp} V_{xy} - \eta_{\perp}^{H} (V_{xx} - V_{yy})$$
$$\tau_{xz} = \eta_{||} V_{xz} + \eta_{||}^{H} V_{yz}$$
$$\tau_{yz} = \eta_{||} V_{yz} - \eta_{||}^{H} V_{xz}$$

[Landau, Lifshytz Vol. 10]

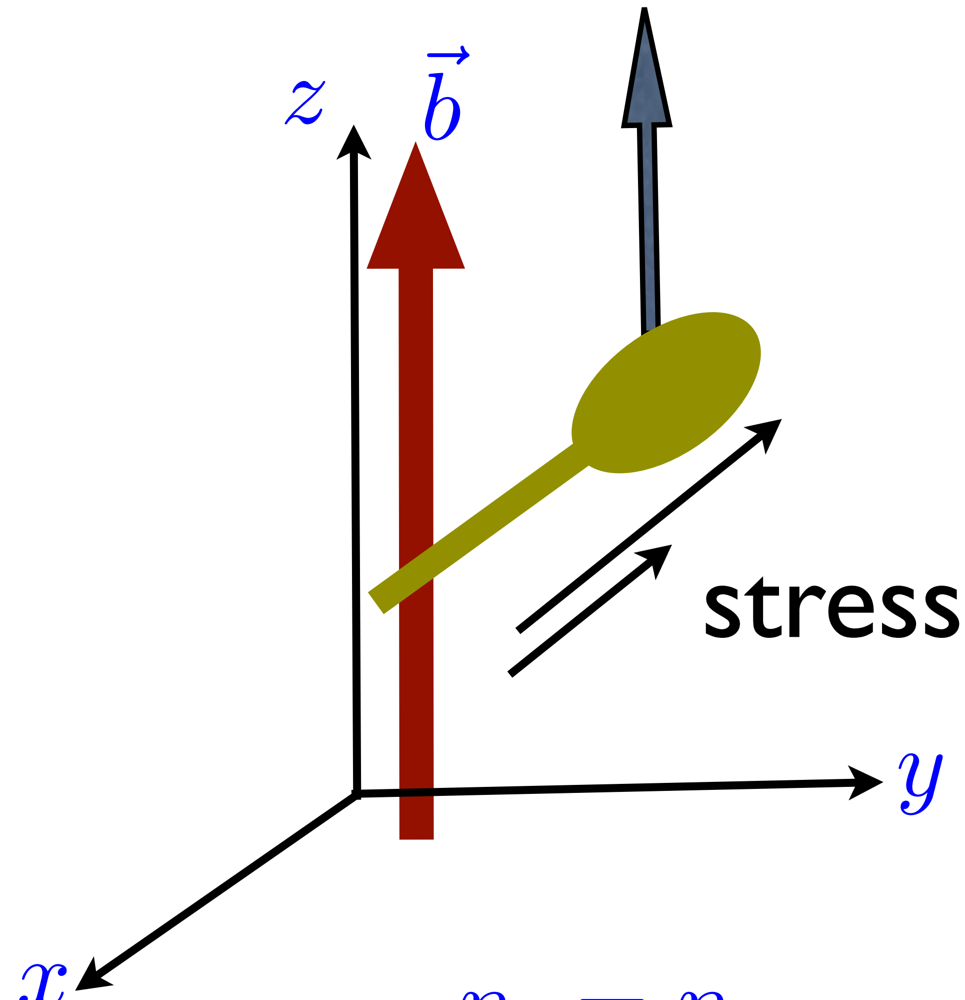
$$V_{ij} = \frac{1}{2}\left(\partial_i v_j + \partial_j v_i\right)$$

- In total: 3 shear, 2 "bulk" and 2 odd viscosities

Odd viscosity



$\eta_3 = \eta_\perp$

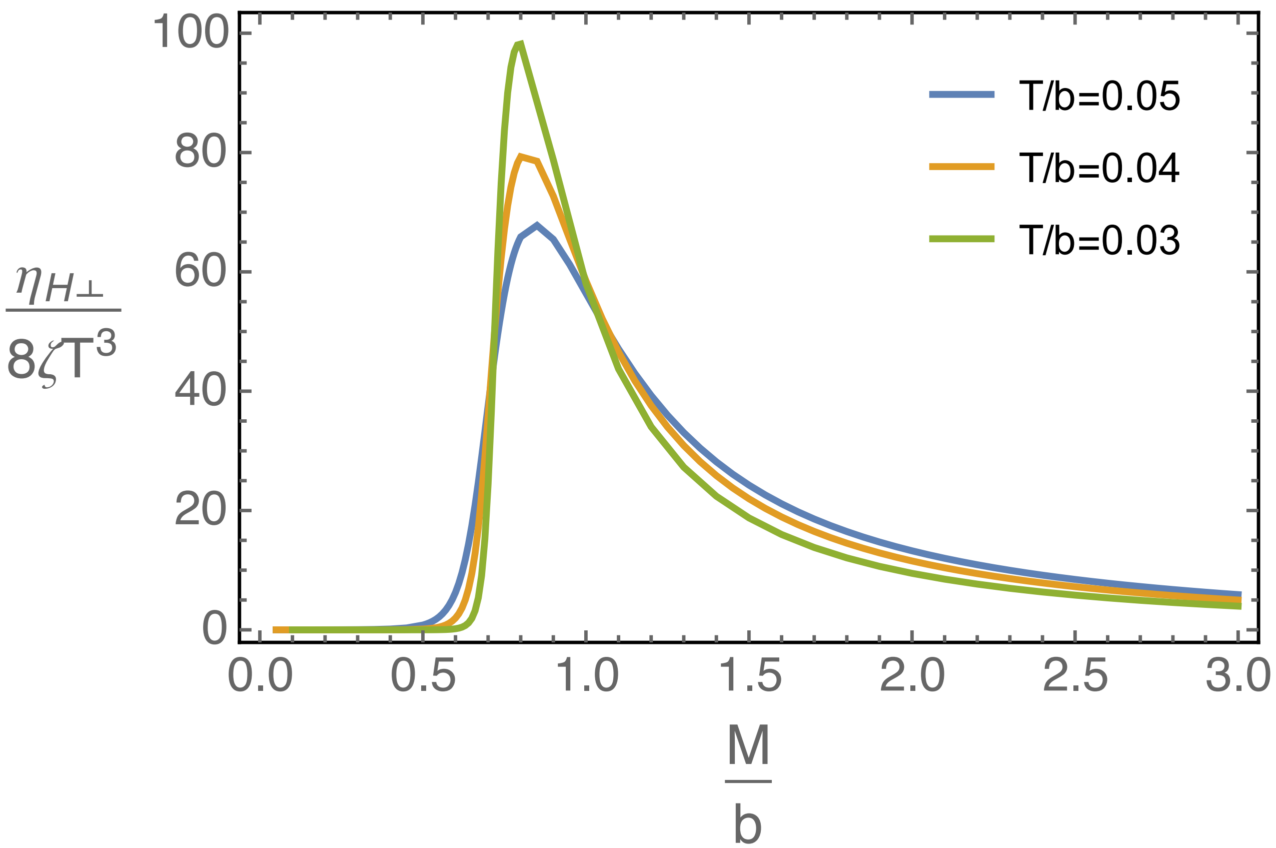


$\eta_4 = \eta_\parallel$

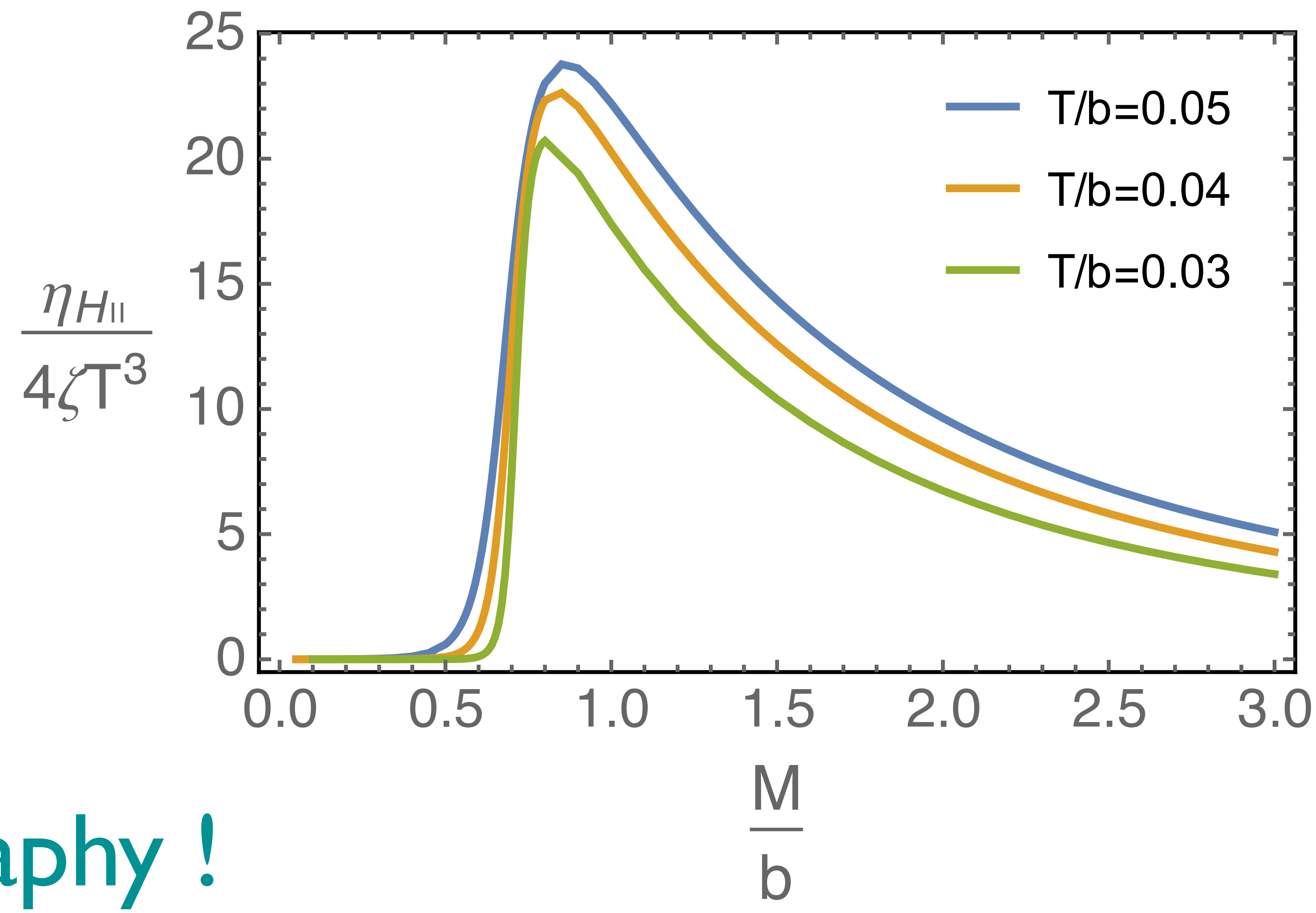
Odd viscosity

- Odd viscosities by adding gravitational anomaly term
- Probe IR region of geometry: Low T

transverse



parallel



- Prediction from Holography !
- Again: gravitational Anomaly at first order !

Summary

- Holography is efficient discovery tool for transport
- Fate of anomalous transport under symmetry breaking
- New questions:
  - Why do consistent currents vanish?
  - What is the extrinsic curvature in field theory?
- Anomalous transport far from equilibrium (QGP)
  - CME@RHIC vs. CME@LHC