Fermions in a spontaneously generated holographic lattice

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Numerical Relativity and Holography

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Outline

1 Introduction
   - Inhomogeneity
   - Coupling
   - Instability

2 Backreaction
   - Expansion
   - Scalar

3 Phase Space
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   - Fermions

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Condensed Matter Theory

Semi-Classical explanation

- near the superconducting/normal transition line

Ginzburg-Landau functional

\[ F_G = a |\psi|^2 + \gamma |\nabla \psi|^2 + \frac{b}{2} |\psi|^4 \]

- \( a, b, \gamma \) are functions of \( B, T, \psi \) - “order parameter”

\( a = 0 \) at the critical temperature

at lower temperatures \( \psi \) turns on and \( F_G \) is extremized by

\[ |\psi|^2 = -\frac{a}{b} \]
**Condensed Matter Motivation**

**FFLO states** [Fulde, Ferrell, and Larkin, Ovchinnikov]

- near the superconducting/normal transition line

Ginzburg-Landau functional

\[ F_G = a|\psi|^2 + \gamma|\nabla \psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\eta}{2}|\nabla^2 \psi|^2 \]

- \(a, b, \gamma, \eta\) are functions of \(B, T\)

**Inhomogeneous Ground State**

- finite electron pair momentum
- translation/rotation symmetries broken

\[ \psi \sim e^{iqx} \]

- truncation of low-energy IIB string theory [Arean, Bertolini, Krishnan, Prochazka]
- holographic unbalanced superconductor [Bigazzi, Cotrone, Musso, Fokeeva and Seminara]
Lattice generation

Dynamical generation of a lattice found in
- Q-lattice, [Donos, Gauntlett]
- axion type, [Andrade, Withers]
- Einstein, Maxwell, Scalar ($F \wedge G$) (Pantelidou talk), [Donos, Gauntlett, Pantelidou]

Mechanism

gravity with $\Lambda_{AdS}$, scalar field $\phi$ of mass $m$ and charge $(q, 0)$ coupled to $U(1)$ vector potential $A_\mu$

$$S = \int d^4x \sqrt{-g} \left[ \frac{R + 6/L^2}{16\pi G} - \frac{1}{4} F_{AB} F^{AB} + S_\phi + S_{int} \right]$$
Gravitational duals related to inhomogeneity have been widely studied allows for calculation of many interesting properties:

- Drude peaks, \([Horowitz, Santos, Tong, \ldots]\)
- Transport Properties, \([Donos, Gauntlett, Blake, \ldots]\)
- Momentum relaxation (Kim talk), \([Hartnoll, Hoffman, \ldots]\)
- Quench (Withers talk) \([Withers]\)
- Metal-Insulator, \([Donos, Hartnoll, Goutraux, \ldots]\)

**Fermion spectral functions**

- explicit - \([Liu, Schalm, Sun, Zaanen]\)
- Q-lattice, \([Ling, Liu, Niu, Wu, Xian]\)
Hairy black hole

Asymptotics

near Boundary ($z \to 0$)

$$h \to 1, \quad A_t \sim \mu - \rho z, \quad \psi \sim \psi^\pm z^{\Delta^\pm}, \quad \Delta^\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2}$$

- chemical potential, $\mu$ and charge density, $\rho$
- $\langle O_{\Delta^\pm} \rangle = \sqrt{2}\psi^\pm$

Horizon

$z \to 1$

$$A_t \to 0, \quad h \to 0$$
Unbroken phase

Solution with \( \phi = 0 \)

\[
\begin{align*}
    ds^2 & = \frac{1}{z^2} \left[ -h(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)} \right], \\
    A_t & = \mu (1 - z) \\
    h(z) & = 1 - \left(1 + \frac{\mu^2}{4}\right) z^3 + \frac{\mu^2}{4} z^4
\end{align*}
\]

use scaling symmetries: \( z_H = 1 \), with \( z \in [0, 1] \)
\( \rightarrow \) AdS boundary at \( z \rightarrow 0 \)

Alternative:
self-dual under \( \vec{E} \leftrightarrow \vec{B} \)

\[
A_y = \beta x, \quad A_t = 0
\]
Introduction

Simple Scalar

Scalar Action

\[ S_\phi = -\int d^4 x \sqrt{-g} \left[ g^{AB} (D_A \phi)^* D_B \phi + m^2 |\phi|^2 \right], \quad D_\mu = \partial_\mu - i q A_\mu \]

\[ \partial^2_\phi + \left[ \frac{h'}{h} - \frac{2}{z} \right] \partial_z \phi + \frac{1}{h} \nabla^2_2 \phi - \frac{1}{h} \left[ \frac{m^2}{z^2} - \frac{q^2 A_t^2}{h} \right] \phi = 0 \]

- introduce $\vec{x}$-dependence

\[ \nabla^2_2 \phi = -k^2 \phi, \quad \phi \sim \psi(z) \cos kx \]

- fix $\mu$

- $\mu/r_+ \rightarrow$ eigenvalue in scalar equation

\[ \frac{T_0}{\mu_0} = \frac{3}{4\pi \mu_0} \left[ 1 - \frac{\mu_0^2}{12} \right] \]
Instability

Mechanism for Instability

Extremal black holes near horizon exhibit $AdS_2 \times \mathbb{R}^2$

- effective mass can be below 2D $m_{BF}^2$

$$m_{\text{eff}}^2 < 6m_{BF,2}^2 = -3/2$$
Instability

Imbalance + Inhomogeneity

\[ m_{\text{eff}}^2 = m^2 - 2q^2 + k^2 \]

Limits placed on maximum imbalance from zero-T instability

\[ q_{\text{min}}^2 = \frac{3 + 2\Delta(\Delta - 3) + 2k^2}{4} \]

and a limit on \( k^2 \)

\[ k_{\text{max}}^2 = 2q^2 - \frac{3}{2} - \Delta(\Delta - 3) \]
Interacting Scalar

Stringy considerations dictate $S_{\phi}$

\[
S_{int} = \int d^4x \sqrt{-g} \mathcal{L}_{int}
\]

\[
\mathcal{L}_{int} = \phi^* \left[ \eta G^{AB} D_A D_B + \eta' H^{ABCD} D_A D_B D_C D_D + \ldots \right] \phi + c.c.
\]

$G^{AB}$ and $H^{ABCD}$ may come from Einstein tensor, stress energy tensor, gauge or scalar fields, ... 

Interaction

Can include terms in the Lagrangian

\[
|F^{AB} \partial_B \phi|^2
\]

⇒ similar to Landau-Ginzburg gradient term
Earlier Coupling

- scalar coupled with Einstein tensor [J.A., E. Papantonopoulous, G. Siopsis]
  - used cosmology with vanishing $\Lambda$
  - entry/exit quasi-de Sitter
  - scalar-tensor theory with second order $\Psi$ eqn. [Sushkov]

[J.A., E. Papantonopoulous, G. Siopsis, K. Yeter]

$$S_{int} = \int d^4x \sqrt{-g} \left[ \eta G^{AB} (D_A \phi)^* D_B \phi - \eta' |D_A G^{AB} D_B \phi|^2 \right]$$

pick

$$G_{AB} = T^{EM}_{AB} + g_{AB} \mathcal{L}^{EM} = F_{AC} F_B^C - \frac{1}{2} g_{AB} F^2$$
Backreaction

Interaction term contribute linear terms in $\eta$, $\eta'$ to the full system's stress-energy tensor, electromagnetic current, and scalar equation

- Perturbatively solve the EMS equations

$$ds^2 = \frac{1}{z^2} \left[ -h(z, x)e^{-\alpha(z, x)} dt^2 + \frac{dz^2}{h(z, x)} + e^{\beta(z, x)} dx^2 + e^{-\beta(z, x)} \right]$$

$$A_t = A_t(z, x)$$

$$\phi = \phi(z, x)$$

Expansion in $\xi$

\begin{align*}
  h(z, x) &= h_0(z) + \xi^2 h_1(z, x) + \ldots, \quad \alpha = \xi^2 \alpha_1(z, x) + \ldots \\
  \phi &= \xi \phi_0(z, x) + \xi^3 \phi_1(z, x) + \ldots, \quad \beta = \xi^2 \beta_z(z, x) \\
  A_t &= A_{t0}(z) + \xi^2 A_{t1}(z, x) + \ldots \quad (1)
\end{align*}
The Hawking temperature is found as

$$\frac{T}{\mu} = -\frac{h'(1)e^{-\alpha(1)}}{4\pi \mu}$$

and chemical potential

$$\mu = A_t(0, x) = \mu_0 + \xi^2 \mu_1 + \ldots$$

- when \(r_H\) is scaled back in, \(\mu\) is constant

Scalar at \((\xi)\)

$$\partial_z^2 \phi + \left[ \frac{h'}{h} - \frac{2}{z} \right] \partial_z \phi + \frac{1}{h} \left( 1 - \eta \mu^2 z^4 - \eta' \mu^4 z^{10} \nabla_2^2 \right) \nabla_2^2 \phi$$

$$- \frac{1}{h} \left[ \frac{m^2}{z^2} - q^2 \frac{A_t^2}{h} \right] \phi = 0$$
Backreaction

In the limit $k \to \infty$, the $k$ term dominates the scalar equation at the horizon

$$\frac{T}{\mu} = \frac{3}{4\pi} \sqrt{\eta} \left( 1 - \frac{1}{12\eta} \right)$$

Large enough $\eta \Rightarrow$ produces a higher transition temperature than the homogeneous $k = 0$.

$k_c^2$

$\eta'$ creates a limit new $k_{\text{max}}^2$

$\eta'$ a cutoff to compete with this effect and select a preferred finite
Inhomogeneous Solution, order $\xi$

Transition Temperature

- Inhomogeneous solutions possess higher transition temperature than homogeneous solution
- In CFT, dominant terms possess modulated order parameter $\langle O \rangle \sim \cos kx$

\[
\Delta = 1.7, 2, 2.5
\]
\[
\frac{\eta}{\mu^2} = 1.
\]
\[
\frac{\eta'}{\mu^4} = 0.005
\]
Inhomogeneous Solution

Critical Temperature, $\Delta = 1.7, 2., 2.5$, $\frac{\eta}{\mu^2} = 1.$, $\frac{\eta'}{\mu^4} = 0.005$

$\Rightarrow \eta'$ sets UV cutoff and selects the lattice size
Below $T_c$

**Expansion**

At each order in $\xi$ only a finite number of modes

- $O(1)$ - 0
- $O(\xi)$ - $k$
- $O(\xi^2)$ - 0, $2k$
- $O(\xi^3)$ - $k, 3k$

\[
\frac{T}{T_c} \approx 1 - \xi^2 \left( \alpha_{10}(1) + \frac{\mu_1}{\mu_0} - \frac{h'_{10}(1)}{3 - \mu_0^2/4} \right), \quad \frac{\langle O \rangle}{T_c} \sim \sqrt{1 - \frac{T}{T_c}}
\]

\[
\frac{\rho}{\mu^2} = - \frac{\partial_z A_t(0, x)}{[A_t(0, x)]^2} \approx \frac{\rho_0}{\mu_0^2} + \xi^2 \frac{\rho_1(x)}{\mu_0^2}, \quad \rho_1 \sim \cos 2kx
\]
Inhomogeneous Solution

The charge density is spatially inhomogeneous in presence of lattice and spatially homogeneous chemical potential

Charge and Scalar

\[ \Delta = 1, \ q = 0, \ \xi = .1, \ \eta \mu_0^2 = .41, \ \eta' / \mu_0^4 = 0.005 \]
Fermion phases

- (aspects of) pseudogap
- insulating antiferromagnet
- strange metal
- Fermi liquid
Fermionic phases

Spectral Function

Scattering light illuminates constituents

- extract Green’s function

\[ G = \frac{Z}{\omega - \nu_F(k - k_F) + \Sigma(\omega, k)} \]

- calculate the self-energy \( \Sigma \)

\[ S_{\text{fermion}} = i \int d^4x \sqrt{-g} \bar{\psi} \left[ \not{D} - m_f \right] \psi, \]
Fermion

Bloch Expansion

\[ \psi_{\alpha s} = \sum_{l=0,\pm 1,\pm 2,\ldots} \psi_{\alpha s}^l(z) e^{2i l k x} \]

Expand \( \psi \) in terms of parameter \( \xi \)

\[ \psi_{\alpha s}^l = \psi_{\alpha s}^0 + \xi^2 \psi_{\alpha s}^1 + \xi^4 \psi_{\alpha s}^2 + \ldots \]

\( G_R \) comes from the \( z \to 0 \) behavior

\[ \psi_{\alpha s}^l(z) \approx A_{\alpha s}^l z^{-m_f} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + B_{\alpha s}^l z^{m_f} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ G_R = BA^{-1} \]
Leading Order

Poles

at leading order, the Dirac equation

\[ \partial_z \psi_{\alpha s}^{0,l} = -i q f \mu_0 (1 - z) + \omega \sigma^2 \psi_{\alpha s}^{0,l} \]
\[ + \frac{k_x + 2 kl}{\sqrt{h(z)}} \sigma^3 \psi_{\alpha s}^{0,l} - \frac{k_y}{\sqrt{h(z)}} \sigma^1 \psi_{-\alpha}^{0,l} \]

and spectral function

\[ A^{0,l}(\omega, k_x, k_y) = \Im \left[ \frac{\psi_{+1}^{0,l}(\epsilon)}{\psi_{+2}^{0,l}(\epsilon)} + \frac{\psi_{-1}^{0,l}(\epsilon)}{\psi_{-2}^{0,l}(\epsilon)} \right] \]

► Same behavior at 1\textsuperscript{st} order
2\textsuperscript{nd} Order

- four nearest modes to \( l \) are excited
- \( l, l \pm 1 \) and \( l \pm 2 \)

Near the Fermi surface,

\[
G_R \sim \frac{A^0, l B^0, l}{(A^0, l + \xi^4 A^2, l)^2 - \xi^4 A^1, l-1 A^1, l+1}
\]

The type of (non)-Fermi fluid is selected by

\[
\nu_{k_l} = \frac{\sqrt{2}}{\mu_0} \sqrt{k_l^2 - \frac{q_f^2 \mu_0^2}{6}}
\]

Which also determines the expansion of \( G_R \) and size of pseudogap \( \Delta \)
Pseudogap

\( \nu_{kl} < 1/2 \rightarrow \) non-Fermi liquid

\[ \Delta \sim \xi^{1/2 \nu_{kl}} \]
small gap, broad peaks

\( \nu_{kl} > 1/2 \rightarrow \) Fermi liquid

\[ \Delta \sim \xi^2 \]
broad gap, tight peaks

\( \nu_{kl} = 1/2 \rightarrow \) marginal Fermi liquid
There is a summary slide that contains the following points:

- superconducting, strongly-coupled matter
- lattice mechanism
  - lattice structure
  - modulated charge density
- fermion pseudogap creation

Work to be done:

- effect on other types of states, insulators
- transport coefficients
- general features of other dynamical lattices?