Nonlinear Propagation of Crossing Electromagnetic Waves in Vacuum due to Photon-Photon Scattering

Daniele Tommasini[*], Albert Ferrando[†], Humberto Michinel[*] and Marcos Seco[**]

[*]*Departamento de Física Aplicada. Universidade de Vigo. As Lagoas E-32004 Ourense, Spain.*
[†]*Interdisciplinary Modeling Group, InterTech. Departament d'Òptica, Universitat de València. Dr. Moliner, 50. E-46100 Burjassot (València), Spain.*
[**]*Departamento de Física de Partículas. Universidade de Santiago de Compostela. E-15706 Santiago de Compostela, Spain.*

Abstract. We review the theory for photon-photon scattering in vacuum, and some of the proposals for its experimental search, including the results of our recent works on the subject. We then describe a very simple and sensitive proposal of an experiment and discuss how it can be used at the present (HERCULES) and the future (ELI) ultrahigh power laser facilities either to find the first evidence of photon-photon scattering in vacuum, or to significantly improve the current experimental limits.

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Among other interesting phenomena [1, 2], Quantum Electrodynamics (QED) and non-standard theories of the electromagnetic field[3, 4] predict that the vacuum should behave like a kind of *virtual* electron-positron plasma, thus allowing for photon-photon ($\gamma\gamma$) scattering. However, the latter prediction has not yet been confirmed by experiment, not even indirectly. In fact, two possible strategies can be used for the quest of $\gamma\gamma$ scattering. On one hand, the cross section for the process will be maximum at a possible future photon-photon collider[5], based on an electron laser producing two beams of photons in the *MeV* range (i.e., having wavelengths in the range of a few *fm*). A second approach will be to perform experiments using ultrahigh power optical lasers, such as those that are already in use today[6, 7] or will be available in the near future[6, 8], in such a way that the high density of photons will compensate the smallness of the cross section. In this case, the photon energies are well below the electron rest energy, and the effect of photon-photon collisions due to the interchange of virtual electron-positron pairs can be expressed in terms of the effective Euler-Heisenberg nonlinear Lagrangian[9, 10]. This modifies Maxwell's equations for the average values of the electromagnetic quantum fields[11] and affects the properties of the QED vacuum[12].

Here, we will briefly review the theory for $\gamma\gamma$ scattering, and discuss some of the proposals for its experimental search. We will then consider the present and future ultrahigh power laser facilities, and show that, besides their important technological uses, they will provide a unique opportunity for the search of $\gamma\gamma$ scattering, taking into account the results of our recent works [13, 14]. Finally, we will describe a very simple and sensitive proposal of an experiment and discuss how it can be used at the present

(HERCULES) [7] and the future (ELI) [8] ultrahigh power laser facilities either to find $\gamma\gamma$ scattering, or to improve significantly the current experimental limits.

THE QED NONLINEAR WAVE EQUATION

Even in the absence of matter, photons can interact with each other by interchanging virtual electron-positron pairs in a loop. In this sense, the vacuum behaves like some kind of virtual electron-positron plasma. We will assume that the photon energy is well below the threshold for the production of electron-positron pairs, $2m_ec^2 \simeq 1MeV$. This means that we will only consider radiation of wavelengths $\lambda \gg 2 \times 10^{-13}m$, which is always the case in optical experiments. In this case, the QED effects can be described by the Euler-Heisenberg effective Lagrangian density[10, 1],

$$\mathscr{L} = \mathscr{L}_0 + \xi_l \mathscr{L}_0^2 + \xi_t \varepsilon_0^2 c^2 (\mathbf{E} \cdot \mathbf{B})^2, \tag{1}$$

being

$$\mathscr{L}_0 = \frac{\varepsilon_0}{2} \left(\mathbf{E}^2 - c^2 \mathbf{B}^2 \right) \tag{2}$$

the linear Lagrangian density and ε_0 and c the dielectric constant and the speed of light in vacuum, respectively. Here, $\mathbf{E}$ and $\mathbf{B}$ are the expectation values of the electromagnetic fields, that can be measured by the usual classical means e.g. using test charges.

The additional, nonlinear terms, that appear multiplying the parameters ξ_l and ξ_t in Eq. (1), are the only two Lorentz-covariant terms that can be formed with the electromagnetic fields at the lowest order above $\mathscr{L}_0$. Therefore, they appear as the first correction to the linear evolution in QED in all the alternative theories, such as the Born-Infeld theories [3, 4]. In QED, $\xi_t^{QED} = \frac{7}{4}\xi_l^{QED} \equiv \frac{7}{4}\xi$, being

$$\xi = \frac{8\alpha^2\hbar^3}{45m_e^4c^5} \simeq 6.7 \times 10^{-30}\frac{m^3}{J}. \tag{3}$$

This quantity has dimensions of the inverse of an energy density. This means that significant changes with respect to linear propagation can be expected when the electromagnetic energy density is such that $\xi\rho$ is not very small. While such intensities may have an astrophysical or cosmological importance, they are not achievable in the laboratory, as we shall discuss in the next section.

Once the electromagnetic fields are expressed in terms of the four-component gauge field $A^\mu = (A^0, \mathbf{A})$ as $\mathbf{B} = \nabla \wedge \mathbf{A}$ and $\mathbf{E} = -c\nabla A^0 - \frac{\partial \mathbf{A}}{\partial t}$, the equations of motion are given by the Variational Principle:

$$\frac{\delta\Gamma}{\delta A^\mu} = 0, \tag{4}$$

where $\Gamma \equiv \int \mathscr{L} d^4x$ is the QED effective action. Instead of studying the resulting equations for the fields $\mathbf{E}$ and $\mathbf{B}$, that can be found in the literature[11, 15], it is more convenient to consider the equations for the gauge field components A^μ. As we discussed in Ref. [13, 14], such equations admit solutions in the form of linearly polarized waves,

e.g. in the x direction, with $A^0 = 0$ and $\mathbf{A} = (A, 0, 0)$, provided that: i) the field A does not depend on the variable x (a *transversality* condition) and ii) $A(t, y, z)$ satisfies the single equation [13, 14]:

$$
\begin{aligned}
0 = \partial_y^2 A + \partial_z^2 A - \partial_t^2 A + \xi_l \varepsilon_0 c^2 \{ & \qquad (5) \\
\left[(\partial_t A)^2 - 3(\partial_y A)^2 - (\partial_z A)^2 \right] \partial_y^2 A + & \\
\left[(\partial_t A)^2 - (\partial_y A)^2 - 3(\partial_z A)^2 \right] \partial_z^2 A - & \\
\left[3(\partial_t A)^2 - (\partial_y A)^2 - (\partial_z A)^2 \right] \partial_t^2 A + & \\
4 \left(\partial_z A \partial_t A \partial_z \partial_t A - \partial_z A \partial_y A \partial_z \partial_y A + \partial_y A \partial_t A \partial_y \partial_t A \right) \}, &
\end{aligned}
$$

where $\partial_y \equiv \frac{\partial}{\partial y}$, $\partial_z \equiv \frac{\partial}{\partial z}$ and $\partial_t \equiv \frac{\partial}{c \partial t}$.

Hereafter, we will restrict our discussion to this case of linearly-polarized solution. Note that the plane-wave solutions of the linear Maxwell equations, such as $\mathscr{A} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$, where $\mathscr{A}$ is a constant, $\mathbf{k} = (0, k_y, k_z)$ and $\omega = c|\mathbf{k}|$, are still solutions of Eq. (5). However, we expect that the non-linear terms proportional to ξ, due to the QED correction, will spoil the superposition principle. As we shall see, this will imply a change in the propagation of crossing waves. On the other hand, no nonlinear effect appears for a parallel beam.

Of course, this nonlinearity is not a problem for the communication system: it can easily be seen that this vacuum effect is much smaller than that of the atmosphere. In fact, the non trivial question is the opposite: how can we detect such a nonlinearity? In order to have a significant effect, we need very high fields to compensate $\xi = 6.7 \times 10^{-30} m^3 / J$. Which is the maximum electromagnetic energy density that can be achieved?

ULTRAHIGH POWER LASER BEAMS

Ultra intense photon sources are available thanks to the discovery of chirped pulse amplification (CPA)[16] in the late 80's and optical parametric chirped pulse amplification (OPCPA) [17] in the 90's. Very recently, the HERCULES Laser [7] reached a peak intensity $I \sim 2 \times 10^{22} W\,cm^{-2}$, corresponding to a value $\xi \rho \sim 4 \times 10^{-12}$ for the product giving the importance of the nonlinear QED effects. In few years, the projected European Extreme Light Infrastructure (ELI) [8] will achieve $I \sim 10^{25} W\,cm^{-2}$, or $\xi \rho \sim 2 \times 10^{-9}$. Finally, for a more distant future there is the hope to achieve $I \sim 10^{28} W\,cm^{-2}$ [6], or $\xi \rho \sim 2 \times 10^{-6}$.

These are the highest values for the intensities of the electromagnetic field that are available (HERCULES) or will be available in the future. These values are enormous. The power per pulse that has been reached at HERCULES is $\simeq 3 \times 10^{14} W$, which is an order of magnitude larger than the total power used by Mankind at present! Of course, it is the ultrashort duration of the pulse ($\Delta t \sim 3 \times 10^{-14} s$) that make it possible to reach such intensities keeping the energy per pulse as small as $\sim 10 J$. On the other hand, at ELI [8] the power per pulse will be $10^{18} W$, an order of magnitude larger then the total power that the earth receives from the sun! (In that case, $\Delta t \sim 10^{-18} - 10^{-15} s$, so that the energy per pulse will still be reasonably small $\sim 1 - 10^3 J$.)

Of course, these facilities have very important technological and scientific applications. First, it has been proved that they can be used to produce beams of high energy charged particles after bombarding a plasma [6]. This effect is highly nonlinear, since it can produce kinetic energies of the order of the *MeV*s or even the *GeV*s (and possibly even higher) using the $\sim eV$ photons of the laser bumps [6]. This will provide cheap and reliable sources of radiation for cancer therapy and medical diagnostics, and may also help to ignite nuclear fusion. Moreover, it can be hoped that this surprising new form of accelerating particles can lead to a new era in particle physics. A second class of applications uses the small duration of the pulses, that will allow for an ultrashort time microscopy, which can be very important e.g. in nanotechnology.

Finally, as we shall see below, these ultrahigh power lasers will provide a very promising tool for the exploration of the nonlinearity of the vacuum, and in particular for the search of $\gamma\gamma$ scattering.

PREVIOUS PROPOSALS AND CURRENT LIMIT

We have seen that the QED radiative corrections imply a nonlinear wave equations for the vector potential A^μ. On the other hand, the modified Maxwell's equations for the average values of the electromagnetic quantum fields **E** and **B** were obtained long ago [11], and they were used to show that the QED vacuum exhibits the DC Kerr effect, implying birefringence [12]. More recently, several works have proposed different configurations that can actually be used for experimental tests of the nonlinear optical response of the vacuum, e.g. using harmonic generation in an inhomogeneous magnetic field[18], QED four-wave mixing[19], resonant interactions in microwave cavities[20], or laser-induced QED vacuum birefringence[21] which can be probed by x-ray pulses[22], among others[23]. All these proposals are very interesting and all of them deserve to be explored. They will provide different, independent tests for photon-photon scattering. However, all these cases will need some technological advance. For instance, x-ray probing of QED birefringence requires an extra free electron laser, which is not yet available. Other techniques like four-wave mixing processes[19] require the crossing of at least three beams, with the corresponding alignment problems. As we shall see in the next section, our proposal is considerably simpler than the previous ones, and it only requires a ultrahigh energy pulse.

Here, we would like to mention a very recent result obtained by the PVLAS collaboration [24] that has searched for an evidence of birefringence of the vacuum in a *magnetic* field background [25]. Their *negative* result has been used to set the *current limit* on the cross section for $\gamma\gamma$ scattering at optical wavelengths. Assuming the QED effective Lagrangian, but treating ξ as a free phenomenological parameter, their results can be translated in the limit $\xi_{\exp} < 3.1 \times 10^{-26} m^3/J$. We see that this constraint is 4.6×10^3 times higher than the QED value. Note that in Ref. [24] this result was also stated as a limit on the photon-photon cross section, $\sigma_{\gamma\gamma} < 4.6 \times 10^{-62} m^2$, for $\lambda = 1064 nm$, and $\sigma_{\gamma\gamma} < 2.7 \times 10^{-60} m^2$, for $\lambda = 532 nm$.

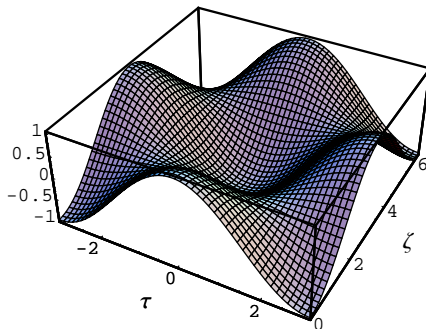# A NUMERICAL SOLUTION OF THE QED WAVE EQUATION

In Ref. [14] we have obtained the first numerical solution of the full nonlinear wave equation Eq. (5) in the case of two counter-propagating plane waves that travel along the z-axis, for simplicity having the same phase at the space-time origin. The corresponding analytical solution of the linear wave equation (that can be obtained by setting $\xi_l = 0$ in Eq. (5)) would be

$$
\begin{aligned}
A_{\text{lin}}(t,z) &= \frac{\mathscr{A}}{2}\left[\cos(kz - \omega t) + \cos(kz + \omega t)\right] \\
&= \mathscr{A}\cos(\omega t)\cos(kz),
\end{aligned}
\tag{6}
$$

where $\mathscr{A}$ is a constant amplitude, $\mathbf{k} = (0,0,k)$ is the wave vector, and $\omega = ck$ is the angular frequency. It is easy to see that Eq. (6) can also be considered as the analytical solution of the linear wave equation satisfying the boundary conditions

$$
\begin{aligned}
A(t,0) &= \mathscr{A}\cos(\omega t), \\
A\left(-\frac{\pi}{\omega},z\right) &= A\left(\frac{\pi}{\omega},z\right), \\
\partial_z A(t,0) &= 0,
\end{aligned}
\tag{7}
$$

where for convenience we chose as the integration interval a 'small' cuboid of time-dimension $2\pi/\omega$ and space-dimension $2\pi/k$.
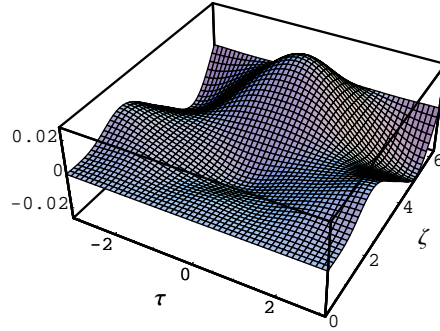

FIGURE 1. Numerical solution $A_{\text{num}}/\mathscr{A}$ of Eqs. (5) and (7) for $\xi\bar{\rho} = 0.00252$, as a function of the adimensional time and space coordinates $\tau \equiv \omega t$ and $\zeta \equiv kz$ (from Ref. [14]).

In Ref. [14], we have found the solution of the nonlinear Eq. (5) that satisfies the same boundary conditions of Eq. (7), using the same small cuboid as the integration interval. This solution is shown in Figs. 1 and 2 for a choice of parameters such that $\xi\varepsilon_0\mathscr{A}^2\omega^2 = 0.01$. The corresponding time-averaged value $\bar{\rho}$ of the energy density turns out to be approximately constant along the z-evolution, giving $\xi\bar{\rho} = \frac{\omega}{2\pi}\xi\int_{-\pi/\omega}^{\pi/\omega}\rho\,dt = 0.00252$. As we have discussed above, this value of the product $\xi\rho$ is several order of magnitude larger than what can be achieved in the laboratory in the next decades. Of course, we will use realistic values of ρ when we will present our proposals of experiments in
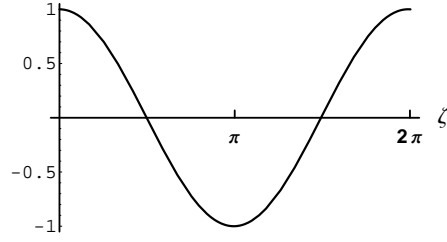
FIGURE 2. Zero-time plot of $A_{num}/\mathscr{A}$, as a function of the adimensional space coordinate $\zeta \equiv kz$ (from Ref. [14]).

the last sections. For the moment, it is interesting to note that even such an enormous energy density is still small enough so that the effect of the nonlinear terms gives a small correction to the linear evolution in the short distance. In fact, we see from Fig. 3 that the relative difference between the linear and the nonlinear evolution, in our integration interval which is of the order of the wavelength, is of the order of few percent, i.e. of the same order than the adimensional parameter $\xi \varepsilon_0 \mathscr{A}^2 \omega^2 = 0.01$. This result is not surprising, and will provide a justification for the perturbatively-motivated variational approach that we will use in the next sections.

FIGURE 3. Relative error $(A_{num} - A_{lin})/\mathscr{A}$ of the linear approximation, as a function of the adimensional time and space coordinates $\tau \equiv \omega t$ and $\zeta \equiv kz$ (from Ref. [14]).

However, from Fig. 3 itself, we can also appreciate that the difference between the linear and nonlinear behavior tends to increase along the z evolution, so that it can be expected that it will eventually become large after a distance much larger than the wavelength. In the next section, we will see an analytical argument that confirms this expectation.

Finally, in Fig. 4 we compare the z-evolution of the solutions A_{num} and A_{lin} in a greater detail for values of z around the second zero of the solutions. We see that A_{num} anticipates A_{lin}, and the corresponding phase shift can be evaluated numerically if we define an effective wave vector component k_z by computing the value z_0 corresponding to $A_{lin}(0, z_0) = 0$, and setting $k_z z_0 = 3\pi/2$. The numerical determination of the zero gives $\zeta_0 = kz_0 = 4.68885$, so that $k_z = 1.0050k$. We will provide a full explanation for this result in the following section.
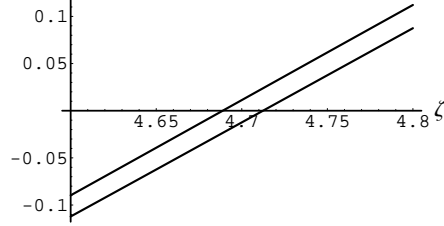
FIGURE 4. Detail of the zero-time functions $A_{\text{num}}/\mathscr{A}$ (upper curve) and $A_{\text{lin}}/\mathscr{A}$ (lower curve), for values of the adimensional space coordinate $\zeta \equiv kz$ close to the second zero (from Ref. [14]).

VARIATIONAL APPROXIMATION

Although it can be considered as an interesting achievement due to its simplicity and lack of previous approximations, in practice the numerical solution that has been discussed above can only be obtained in the special configuration of two counter-propagating, in-phase waves, and for short propagation (of the order of the wavelength). In Ref. [14], we have found a variational approximation that can be used to provide an *analytical* solution which is valid even for a long evolution. In particular, we have considered the same two counterpropagating waves as in the previous section, that would be described by Eq. (6) in the linear case. Note that any of the two crossing waves, $\frac{\mathscr{A}}{2}\cos(kz - \omega t)$ and $\frac{\mathscr{A}}{2}\cos(kz + \omega t)$, when taken alone, would be a solution of both the linear and nonlinear equations, provided that $\omega = ck$. However, their superposition would only solve the linear equations of motion. Taking into account the smallness of the product $\xi\rho$, and using the Variational Method, we have found the following analytical approximation

$$A = \mathscr{A}\cos(\omega t)\cos[(k+\chi)z], \tag{8}$$

where $\chi = \xi_l \varepsilon_0 \mathscr{A}^2 c^2 k^3 / 2 \simeq 2\xi_l \rho k$ is a parameter that describes the leading nonlinear effects.

As a result, we find that after a distance Δz the phase of the wave is shifted by a term

$$\Delta\phi = \chi\Delta z \simeq 2\xi_l \rho k \Delta z \tag{9}$$

Note also that the result of Eq. (8) can be stated equivalently by defining a wave vector as $k_z = k + \chi$, that satisfies a modified dispersion relation, $\omega = c(k_z - \chi)$.

In QED ($\xi_l = \xi$), for the same choice of parameters that was considered in the previous section, $\xi\rho \simeq \xi\varepsilon_0 \mathscr{A}^2 \omega^2 / 4 = 0.0025$, in agreement with the numerical simulation. In this case, we get $\chi = 0.005k$, which is two order of magnitude smaller than k. Our approximations can then be expected to be reasonably good even for the extremely large of ρ that we have chosen here. Note also that this corresponds to a value $k_z = k + \chi = 1.005k$, in agreement with the result that we obtained from the numerical simulation in the previous section.

In Fig. 5 we compare the corresponding analytical solution A_{var}, given in Eq. (8), with the numerical solution A_{num} of the QED wave equations that we have found in the previous section. Comparing with Fig. 3, we see that the variational solution is an
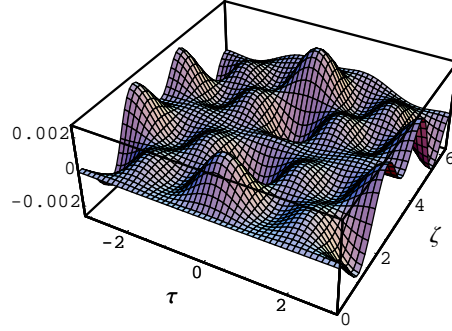
FIGURE 5. Relative error $(A_{\mathrm{num}} - A_{\mathrm{var}})/\mathscr{A}$ of the variational approximation, as a function of the adimensional time and space coordinates $\tau \equiv \omega t$ and $\zeta \equiv kz$.

order of magnitude closer to the numerical simulation than the linear evolution, Eq. (6), that was obtained by completely neglecting the nonlinear terms. This is a significant improvement in such a short distance. However, according to the previous discussion, in the perturbative regime corresponding to small values of the product $\xi \rho$, the variational solution is expected to be a good approximation even when a longer propagation distance is considered along the z-axis.

On the other hand, the agreement of the variational solution with the numerical simulation can be used as an additional, *a posteriori* justification for our analytical approach. Therefore, in Ref. [14] we have applied the Variational Method to different configurations, that do not allow for a direct integration of the full Eq. (5). The most promising one is the case of the scattering of a relatively low power wave with a counterpropagating high power wave, both traveling along the z axis. We have also allowed for an arbitrary initial phase difference between the two waves, assuming for simplicity that two waves have the same frequency. In this case, the main effect will be the modification of the low power beam due to the crossing with the high power one, which would be unaffected in a first approximation. Using our variational approach, we have found that the low power wave is modified, as compared to the linear behavior, as described by the following approximated analytical solution

$$A_l(t,z) = \alpha_0 \cos[(k + \eta)z - \omega t], \tag{10}$$

where $\eta = 2\xi_l \varepsilon_0 \mathscr{A}^2 c^2 k^3 \simeq 4\xi \rho k$, and ρ is the (average) energy density of the high power wave. Therefore, after a distance Δz the low power beam accumulates a phase shift

$$\Delta \Phi = \eta \Delta z \simeq 4\xi_l \rho k \Delta z. \tag{11}$$

We stress that this result only depends on the energy density of the high power wave, as far as it much larger than that of the low power wave. The initial phase difference is found to be irrelevant.

PROPOSAL OF EXPERIMENTS AT HERCULES AND AT ELI

In Ref. [14], we have proposed an experimental setup which can provide the simplest and most sensitive approach for the optical search of photon-photon scattering in vacuum. We have also showed that this porposal can be used at future exawatt facilities such as ELI [8] to detect the QED effect. Here, we will briefly review that idea, and show that it can also be used at *present* high power laser facilities, such as HERCULES [7], to search for non-QED effects, or to improve the current limit for photon-photon scattering.

The proposed experiment can be described as follows. A common laser pulse is divided in two beams, A and B, one of which (say A) crosses at a 180^0 a very high power beam. As a result, the central part of the distribution of the beam A has acquired a phase shift $\Delta\Phi$ with respect to beam B, that has propagated freely.

In an experiment corresponding to the parameters of the ELI project in its first step we have pulses of wavelength $\lambda = 800nm$, intensity $I = 10^{25}Wcm^{-2}$ and duration $\Delta t = 10fs$ which are focused in a spot of diameter $d \approx 10\mu m$. From Eq. (11), using for ξ_l the QED value of ξ as given in Eq. (3), this results in a phase shift $\Delta\Phi \approx 2 \times 10^{-7}rad$ for beam A, which can be resolved comparing with the beam B which was not exposed to the effects of QED vacuum. Current techniques like spectrally resolved two-beam coupling, which can be applied for ultrashort pulses[26], can be used to this purpose, being sensible to phase shifts as small as $\Delta\Phi = 10^{-7}rad$. It is interesting to note that the sensitivity of this method for the detection of photon-photon scattering may be enhanced by a suitable choice of the combination of the intensity I, the wavelength λ and the time duration Δt that enter in Eq. (11). In fact, taking into account that $\Delta z \simeq c\Delta t$, Eq. (11) implies that the most sensitive experimental configuration will be that having the maximum value of the combination $I\Delta t/\lambda$.

Moreover, in an experiment corresponding to the parameters of the HERCULES laser [7], $\lambda = 810nm$, $I = 2 \times 10^{22}Wcm^{-2}$, $\tau = 30fs$, $d \approx 0.8\mu m$, the same configuration as above would give $\Delta\Phi \simeq 10^{-9}$ (from QED), which is possibly beyond the possibility of detection using current techniques. However, the experiment could find evidences of non-QED $\gamma\gamma$ scattering, or, if the result is negative, it can be used to substantially improve the current limit. Assuming as in Ref. [24] the same form of the effective Lagrangian than in QED, and taking $\Delta\Phi = 10^{-7}rad$ as the sensitivity in the phase shift measurement, we see that HERCULES can prove values of the ξ_l parameter down to the value $\sim 5 \times 10^{-28}$, which is almost two orders of magnitude lower than PVLAS current limit. Since the scattering cross section depends quadratically on ξ, HERCULES can improve the current limit on $\sigma_{\gamma\gamma}$ by a factor $(3 \times 10^{-26}/5 \times 10^{-28})^2 \simeq 4 \times 10^3$. This sensitivity may be increased by the upgrades of HERCULES that are programmed for the next months. Again, the most sensitive configuration will be that maximizing the product $I\Delta t/\lambda$.

Finally, it could be interesting to study the possibility of further enhancing the sensitivity of our proposed experiment by focalizing the high power pulse in the center of a cavity, keeping the intensity on the mirrors below the damage threshold. In this case, the high and low power pulse might be synchronized to cross each other several times. The number of times they meet would then multiply the value of the phase shift.

CONCLUSIONS

We have reviewed our recent proposal [13, 14] of a very simple experiment for the search of photon-photon scattering in vacuum. In this proposal, a low power laser pulse is made to cross with an ultrahigh power laser bump. $\gamma\gamma$ scattering will then be tested by measuring the phase shift of the low power laser beam, e.g. by comparing with a third low power laser beam. Even in the first step of ELI, we have found [14] that the resulting phase shift due to the QED-induced $\gamma\gamma$ scattering will be at least $\Delta\Phi \approx 2 \times 10^{-7} rad$, which can be measured with present technology. Moreover, in the present work we have also discussed the possibility of performing our proposed experiment at present ultrahigh power laser facilities, such as HERCULES. In particular, we have shown that HERCULES is already able either to detect $\gamma\gamma$ scattering, or to improve the current limit on the cross section $\sigma_{\gamma\gamma}$ at optical wavelengths by more than three orders of magnitude.

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